Learning with Multigraph Convolutional Filters

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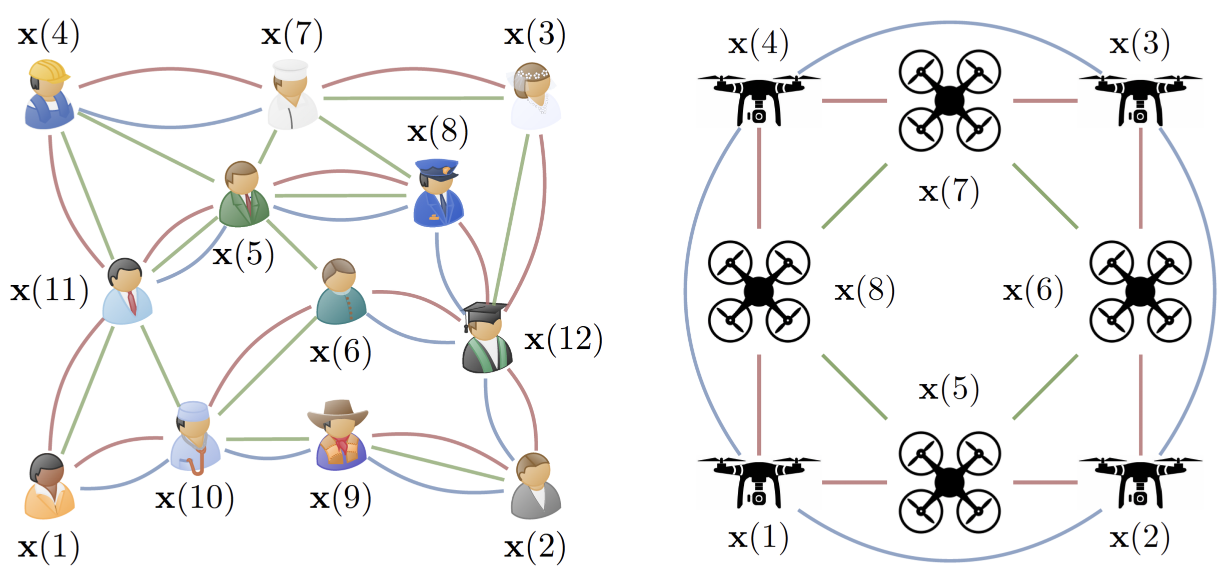
Introduction

A multigraph M is a pair $(\mathcal{V}, \{\mathcal{E}_i\}_{i=1}^m)$, where $\mathcal{V}$ is a set of nodes and $\{\mathcal{E}_i\}_{i=1}^m$ is a collection of edge sets, such that each pair $(\mathcal{V}, \mathcal{E}_i)$ constitutes a graph with nodes in $\mathcal{V}$ and edges in $\mathcal{E}_i$.



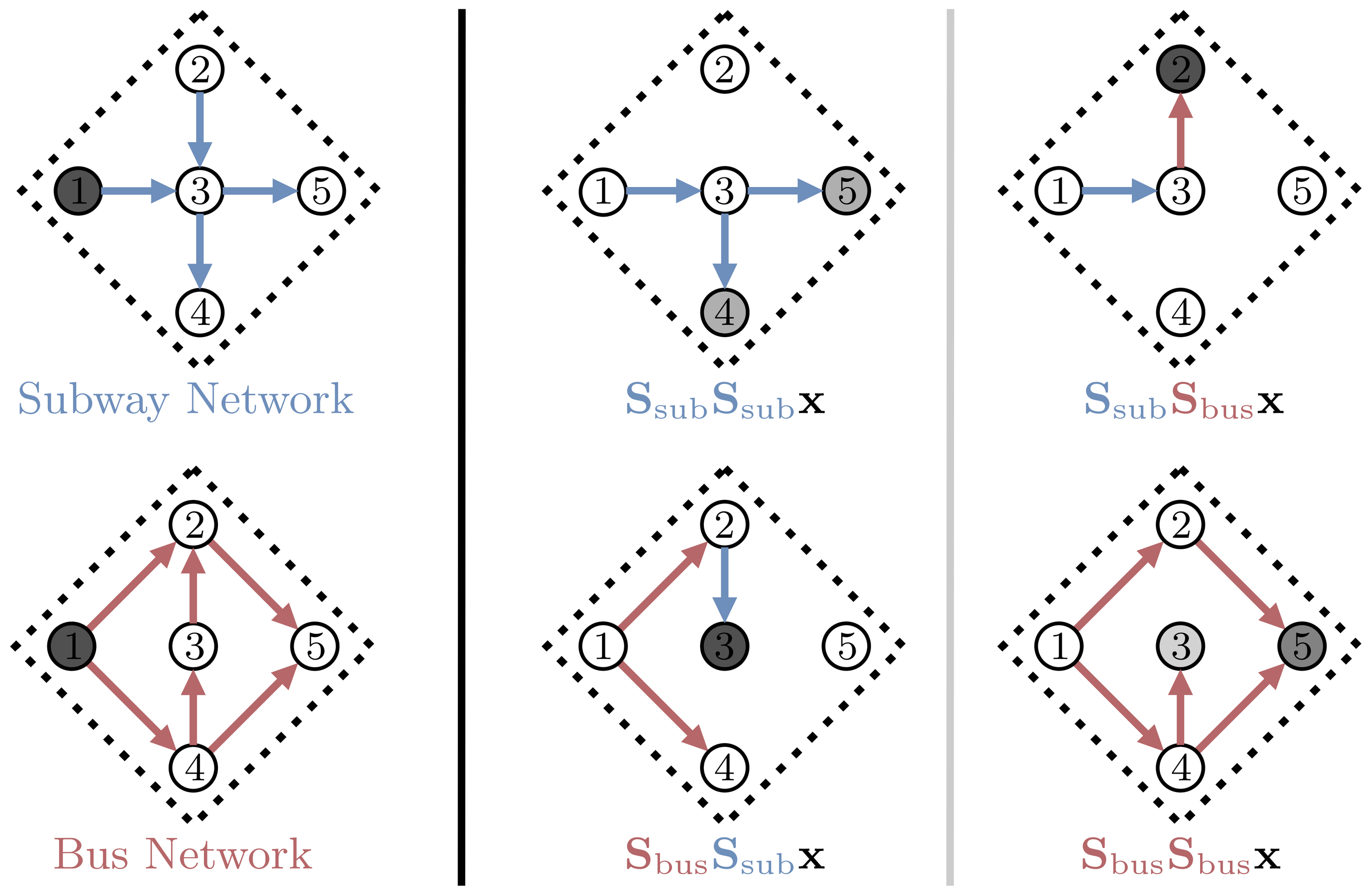
Multigraphs appear in many settings, such as social networks, wireless communication systems and autonomous systems.

Multigraph Signal Processing (MSP)

The convolutional multigraph signal processing framework is obtained as a particular instantiation of a generic algebraic signal model $(\mathcal{A}, \mathcal{M}, \rho)$, where $\mathcal{A}$ is a multinomial algebra with generators t_i, $\mathcal{M} = \mathbb{R}^n$ and ρ is the homomorphism given by $\rho(t_i) = \mathbf{S}_i$, where $\mathbf{S}_i$ is the matrix representation of $(\mathcal{V}, \mathcal{E}_i)$.

Convolutional Signal Processing on Multigraphs

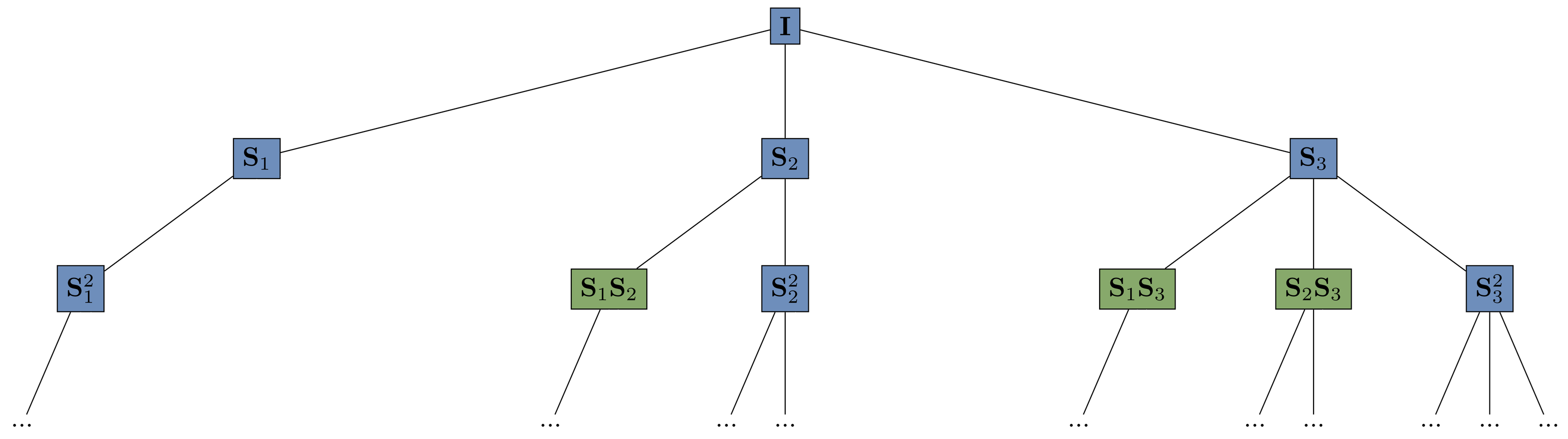
- **Signals** are vectors in $\mathbb{R}^m$. The i-th component of the signal $\mathbf{x} \in \mathbb{R}^n$ lives on the i-th node in $\mathcal{V}$.
- **Convolutional filters** are multivariable polynomials $\mathbf{H}(\mathbf{S}_1, \dots, \mathbf{S}_m)$, where $\mathbf{S}_i$ are the independent variables.
- The convolution results in the signal $\mathbf{y} = \mathbf{H}(\mathbf{S}_1, \dots, \mathbf{S}_m)\,\mathbf{x}$. We say $\mathbf{x}$ is diffused by $\mathbf{H}(\mathbf{S}_1, \dots, \mathbf{S}_m)$ on the multigraph $(\mathcal{V}, \{\mathcal{E}_i\}_{i=1}^m)$.



Convolutional diffusions on multigraphs capture unique dynamics of information that cannot be captured with the traditional GSP model on the individual graphs that constitute the multigraph.

Operator Pruning

By scaling either the number of operators or diffusion order, the number of monomials in the multigraph filter increases exponentially. To reduce computation expense through removing redundant operators, we present an operator pruning algorithm which identifies and removes operators that are nearly commutative.

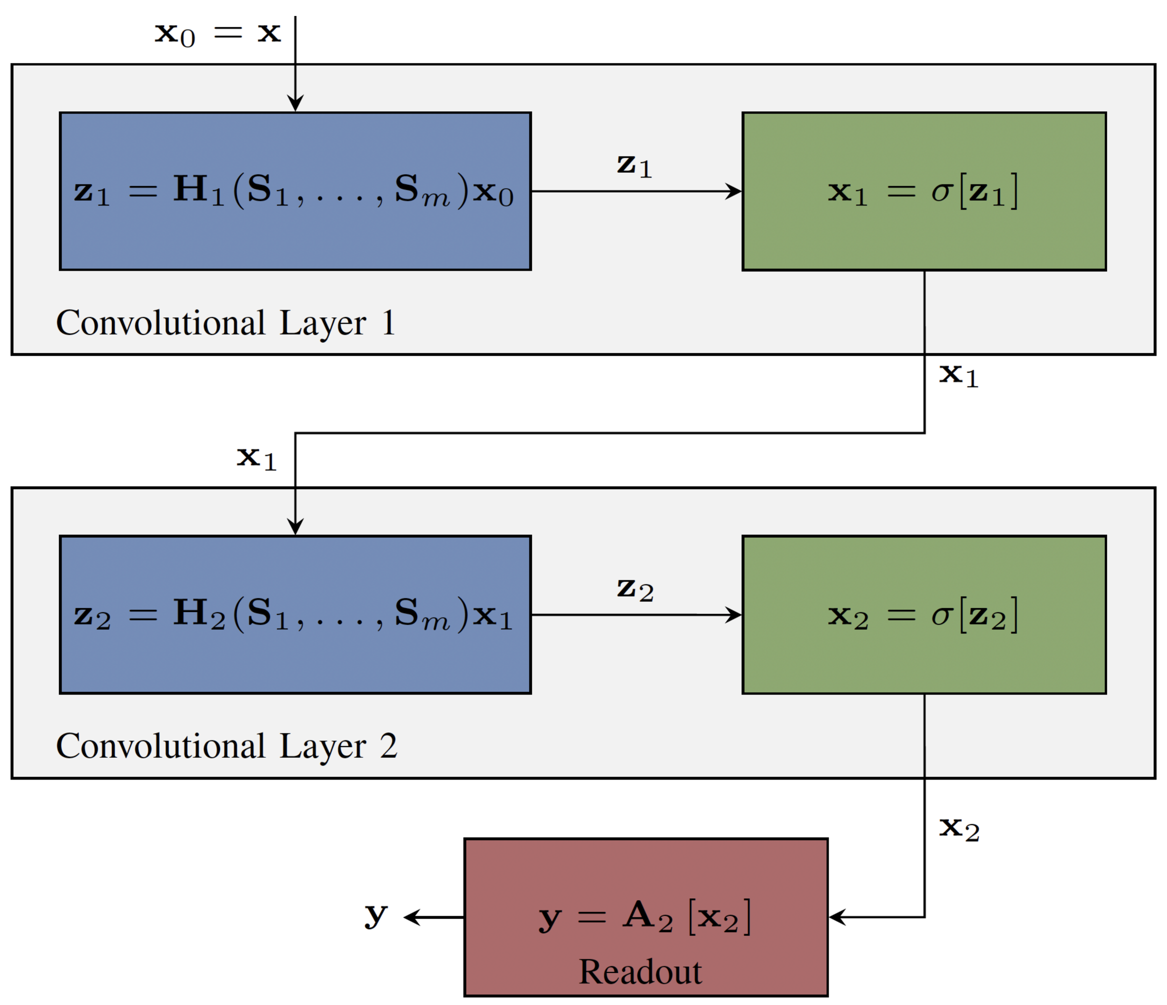


Lemma (*The pruning error does not propagate*)

Let $\|\mathbf{S}_i\|_2 \le 1$ for all i. Let $[\mathbf{S}_i, \mathbf{S}_j] = \mathbf{S}_i\mathbf{S}_j - \mathbf{S}_j\mathbf{S}_i$ with $\|[\mathbf{S}_i, \mathbf{S}_j]\|_2 \le \epsilon \ll 1$ for some i, j. Then, it follows that $\left\|\mathbf{S}_{i_1}^{k_1} \cdots \mathbf{S}_{i_m}^{k_m}[\mathbf{S}_i, \mathbf{S}_j]\mathbf{S}_{j_1}^{\ell_1} \cdots \mathbf{S}_{j_m}^{\ell_m}\right\|_2 \le \epsilon$ for all i_r, j_r, k_r, ℓ_r.

Multigraph Convolutional Neural Networks (MGNN)

Through the recursive application of multigraph filters and pointwise nonlinearities, we create an expressive learning architecture for multigraph signals. Using training samples, we tune the multigraph filters to learn a mapping from the input signal to a target representation, parameterized by the underlying multigraph. Once trained, the mapping can be applied to new multigraph signals.



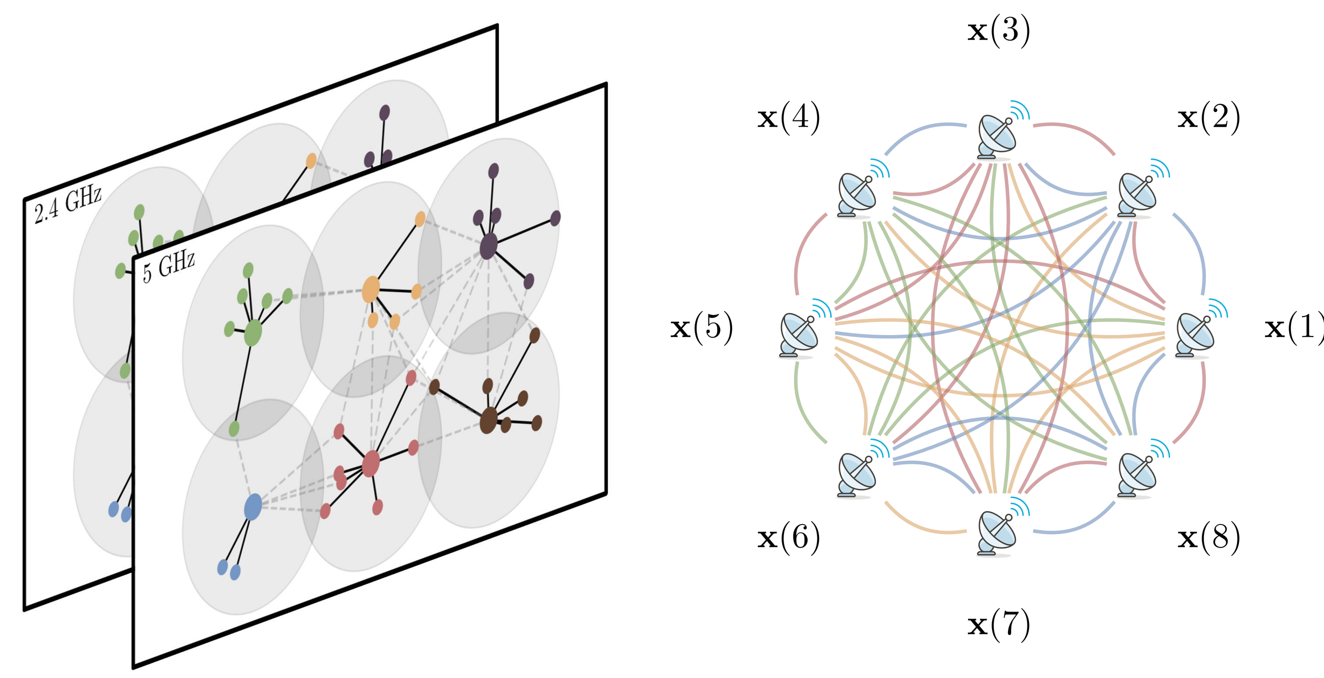
- A *multigraph perceptron* $\sigma_\ell(\mathbf{H}(\mathbf{S}_1, \dots, \mathbf{S}_m))$ is a composition of a pointwise nonlinearity σ_ℓ and a convolutional filter $\mathbf{H}(\mathbf{S}_1, \dots, \mathbf{S}_m)$.
- The **learnable parameters** are the coefficients of the polynomial $\mathbf{H}(\mathbf{S}_1, \dots, \mathbf{S}_m)$.

Selection Sampling and Pooling: To reduce dimensionality at each layer of the architecture, we aggregate community feature information onto a pre-selected set of nodes. With careful selection of nodes, we retain the structural and signal information necessary to learn informative filters, while reducing computational expense.

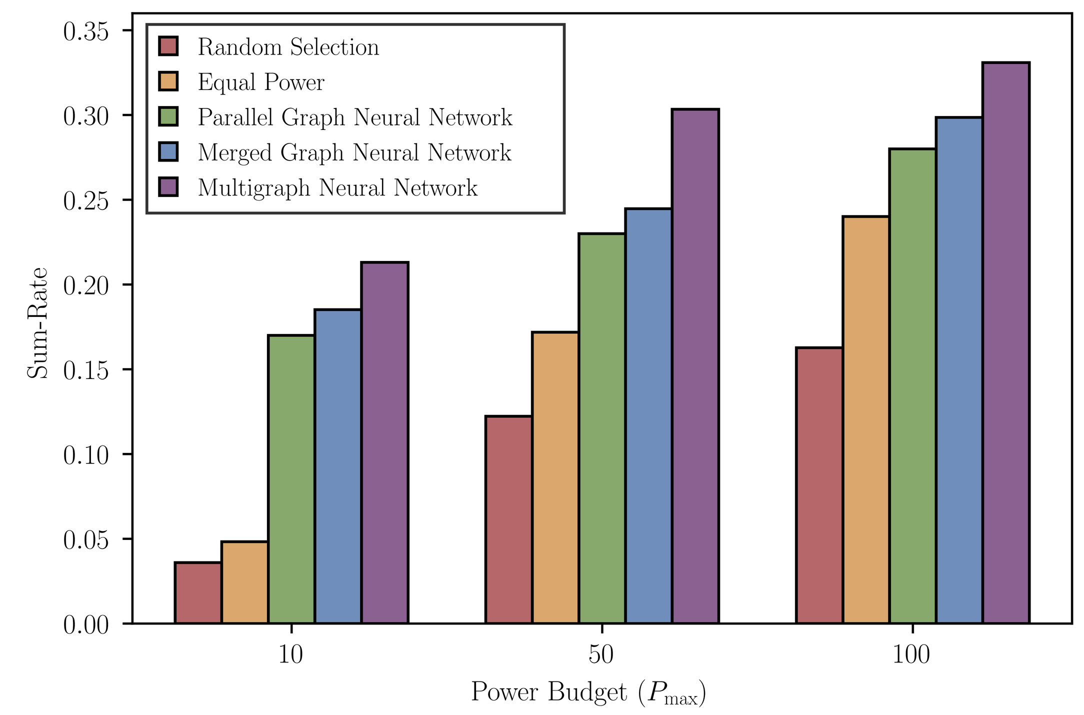
- Pooling preserves *spectral structural* properties of data across layers.

Experiments

Goal: Maximize the sum-rate of a multi-user interference channel through controlling the allocation of transmit power, where transmitters can choose to transmit over the 2.4GHz or 5GHz frequency band.



- Each transmitter-receiver pair is modeled as a node, with channels over the 2.4GHz and 5GHz frequency bands represented as multi-edges. Self-loops track the channel between a transmitter-receiver pair, with all other edges representing sources of interference.
- All transmitters are limited by a shared power budget. The constrained optimized problem is solved using a primal-dual learning method.



Results: Across different power budgets and noise regimes, we find that the Multigraph Convolutional Neural Network achieves the highest sum-rate as compared to heuristics and different GNN architectures. The expressiveness of the multigraph filters yields power allocations that significantly improve the throughput of the system.

Future Work

- Multigraph filters can be applied to other multigraphs with equivalent spectral representations, but it is unclear whether or not MGNNs possess the same transferability property as GNNs.
- If filters are restricted to be Lipschitz or Integral Lipschitz, do MGNNs show the same stability properties as GNNs?
- There is much work remaining to improve selection sampling for multigraphs, which would improve the pooling strategy for MGNNs.