

Problem

How can we **efficiently identify** the **influential interactions** in large models?

Medical Record:
A 52-year-old woman has a 10-kg weight loss. Her hemoglobin concentration is 7.5 g/dL and leukocyte count is 41,800/mm³. Bone marrow biopsy shows cellular hyperplasia with many immature granulocytic cells.

Model's Decision:
Diagnose with Chronic Myeloid Leukemia

Model Explanation:
10-kg weight loss + cellular hyperplasia
52-year-old + immature granulocytic cells



The Big Ben **clock** tower peering over the city of London.

- (Faithfulness Problem) Marginal attribution approaches like SHAP/LIME scale, but don't capture important interactions.
- (Efficiency Problem) Prior SOTA still requires tens of thousands of model inferences, which can be prohibitive for complex models such as LLMs.

Formulation: For input \mathbf{x} = "Her acting fails to impress", let $f(\mathbf{x}_S)$ be the output of the LLM under *masking pattern* S . If $S = \{1, 2, 4, 5, 6\}$, then \mathbf{x}_S is "Her acting [MASK] fails to impress".

Faithfulness at Scale

We aim to learn an interpretable approximation of f denoted \hat{f} . We define **faithfulness** as the predictive power of \hat{f} :

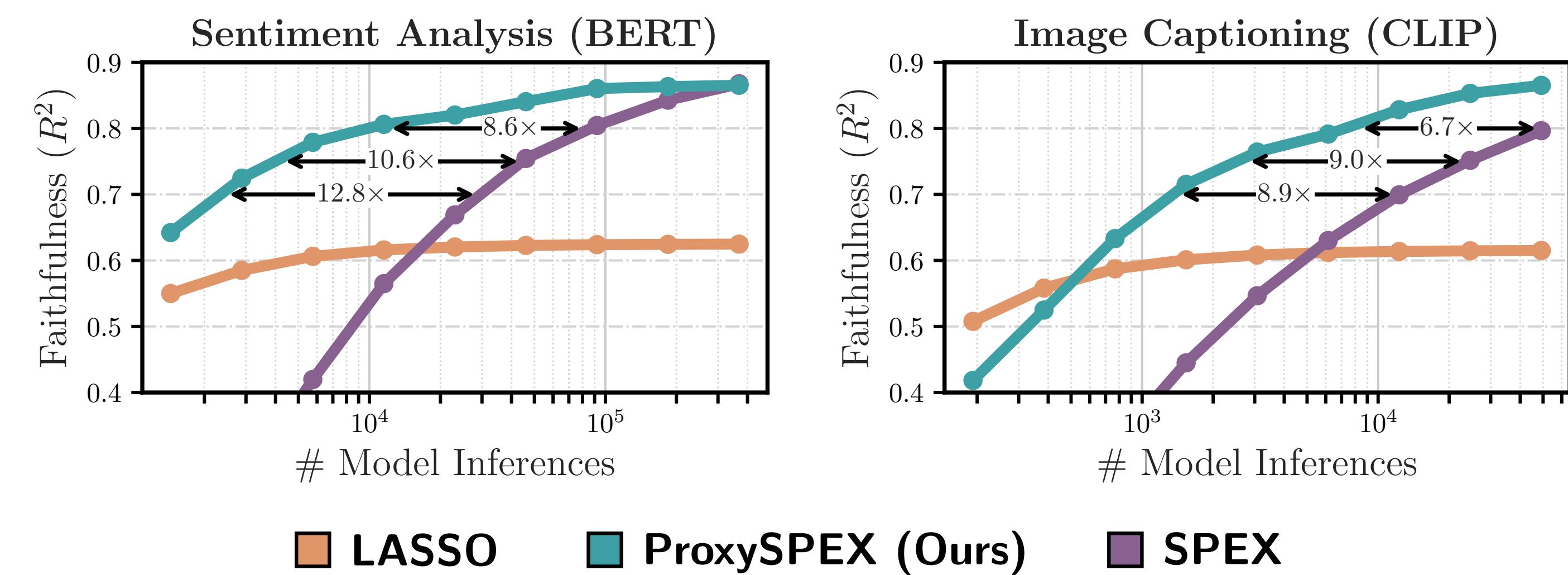
$$R^2 = 1 - \frac{\|\hat{f} - f\|^2}{\|f - \bar{f}\|^2}, \text{ where } \|f\|^2 = \sum_{S \subseteq [n]} f(\mathbf{x}_S)^2, \bar{f} = \frac{1}{2^n} \sum_{S \subseteq [n]} f(\mathbf{x}_S).$$

$f(\mathbf{x}_S)$ has a unique decomposition under the Fourier transform, expressed as:

$$F(\mathbf{x}_T) = \frac{1}{2^n} \sum_{S \subseteq [n]} (-1)^{|S \cap T|} f(\mathbf{x}_S), \quad f(\mathbf{x}_S) = \sum_{T \subseteq [n]} (-1)^{|S \cap T|} F(\mathbf{x}_T).$$

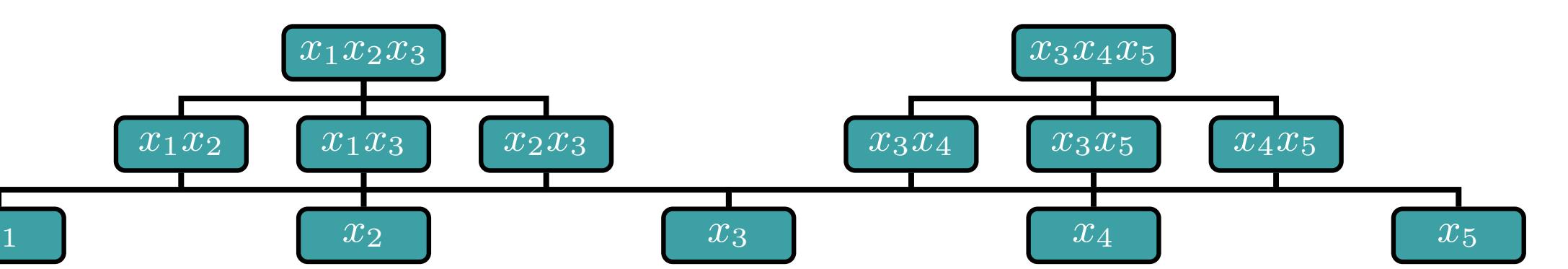
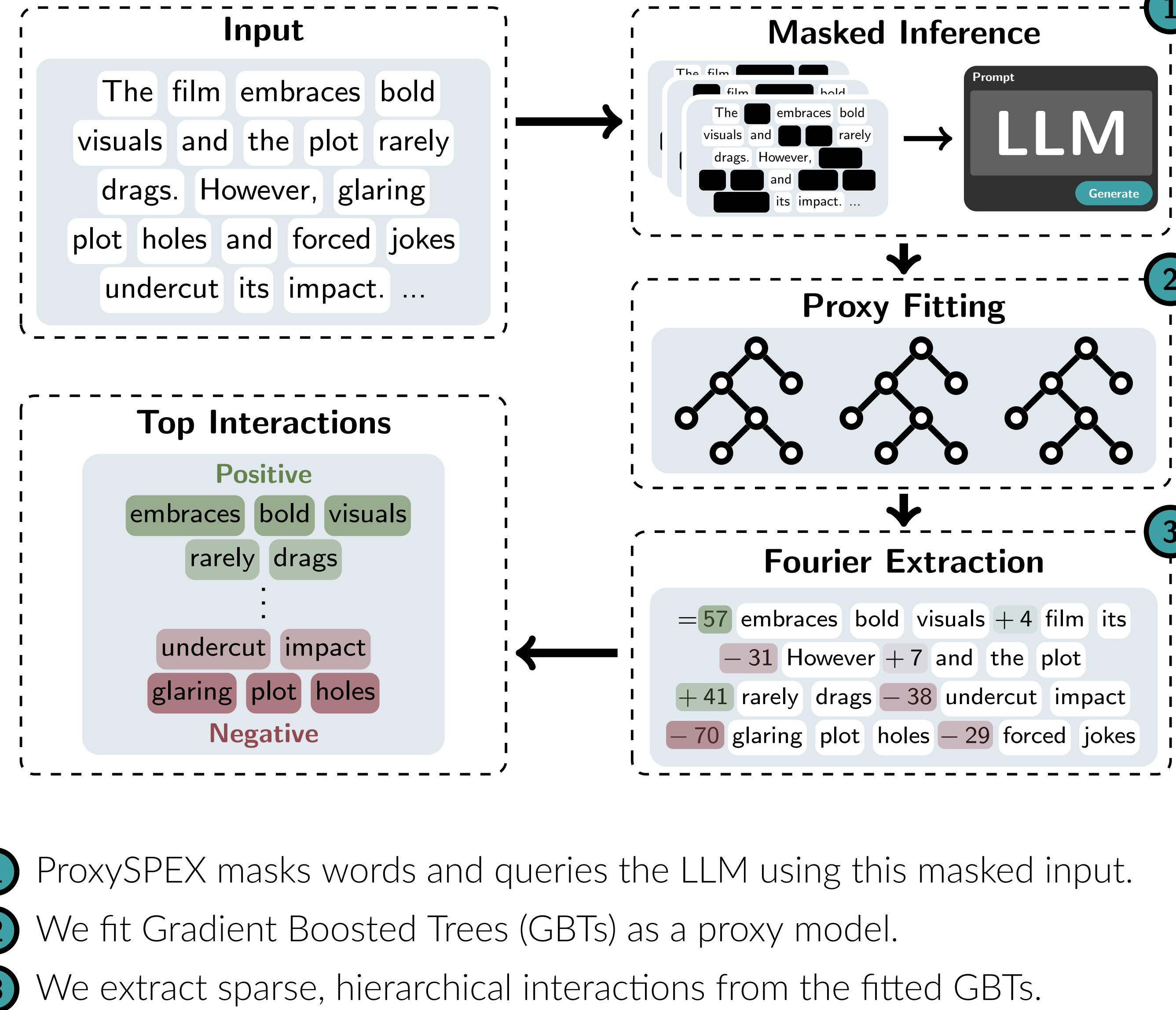
Empirically, we observe that $F(\mathbf{x}_T) \approx 0$ for most T (**sparsity**), large $F(\mathbf{x}_T)$ are **low degree** such that $|T| \leq d$ for some small d , and are correlated (**hierarchy**).

Key Insight: influential interactions are **sparse**, **low-degree**, and **hierarchical**.



Result: ProxySPEX reduces inference cost by **~10x** compared to prior SOTA.

Algorithm



Example: Hierarchical interaction structure extracted from GBTs.

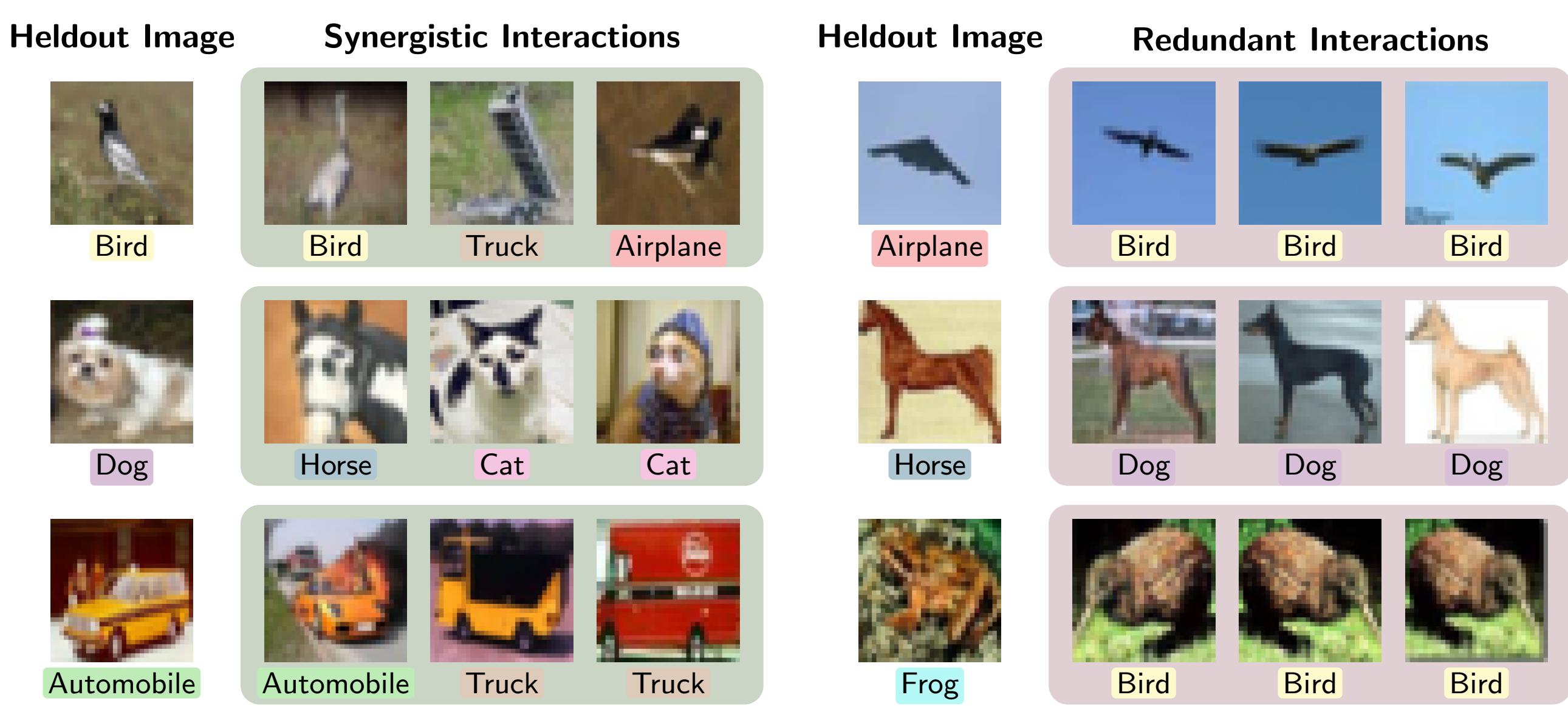
Can we apply ProxySPEX to attribution for model training data, inputs, and internal components?

Experiments: Data Interaction Attribution

Data attribution measures how each training sample influences the prediction of a particular test point \mathbf{z} on class c . We generalize this framework to capture interactions between training samples. For a model trained on S :

$$f(S) \triangleq (\text{logit for } c \text{ on } \mathbf{z}) - (\text{highest incorrect logit on } \mathbf{z}).$$

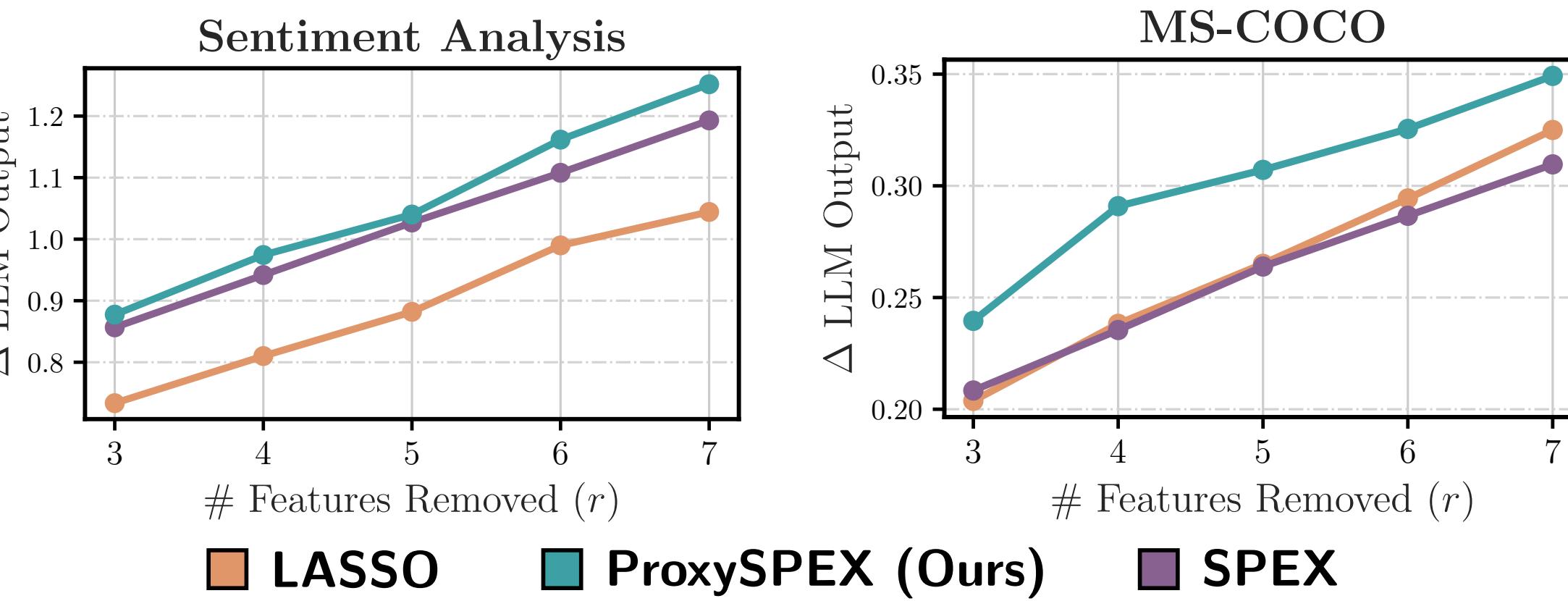
ProxySPEX is the first method efficient enough to learn interactions between training data.



Synergistic Interactions: Combined influence is more than the sum of parts.
Redundant Interactions: Combined influence is less than the sum of parts.

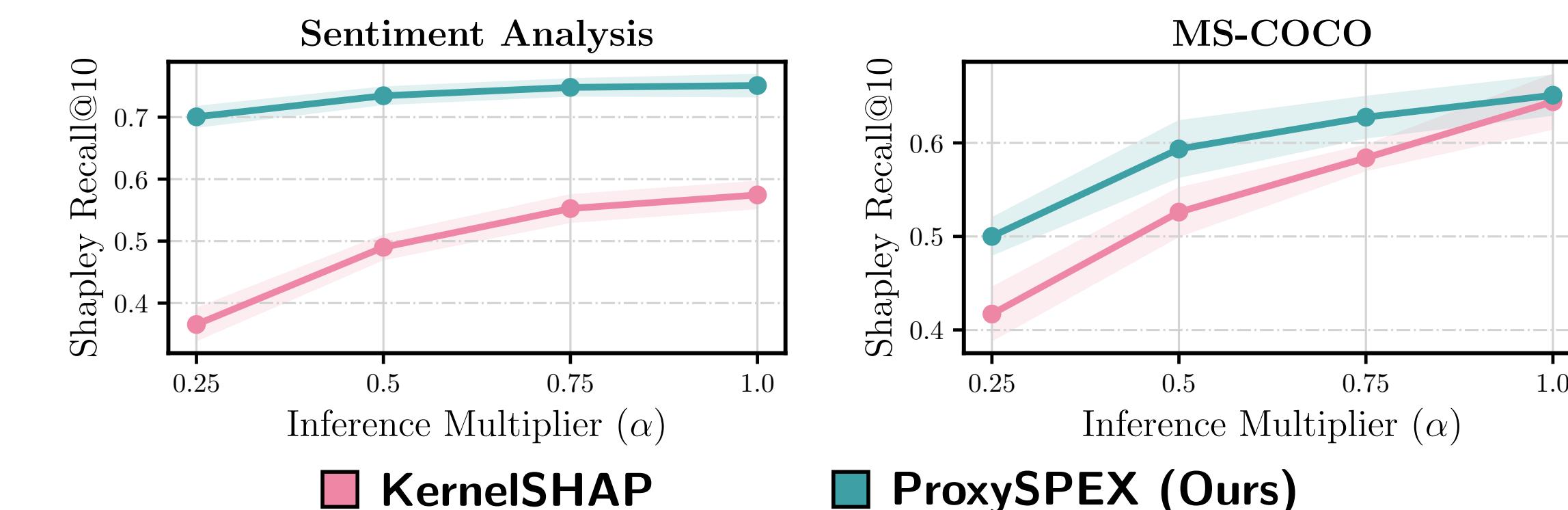
Experiments: Input Feature Interaction Attribution

Feature Removal: By accounting for interactions, ProxySPEX identifies a small set of features to remove that changes the model output.



We use DistilBERT for Sentiment Analysis and CLIP-ViT-B/32 for MS-COCO. Both plots measure a normalized change in logits.

Efficient Shapley Value Estimation: ProxySPEX is SOTA at estimating Shapley values of input features when we don't have much data.

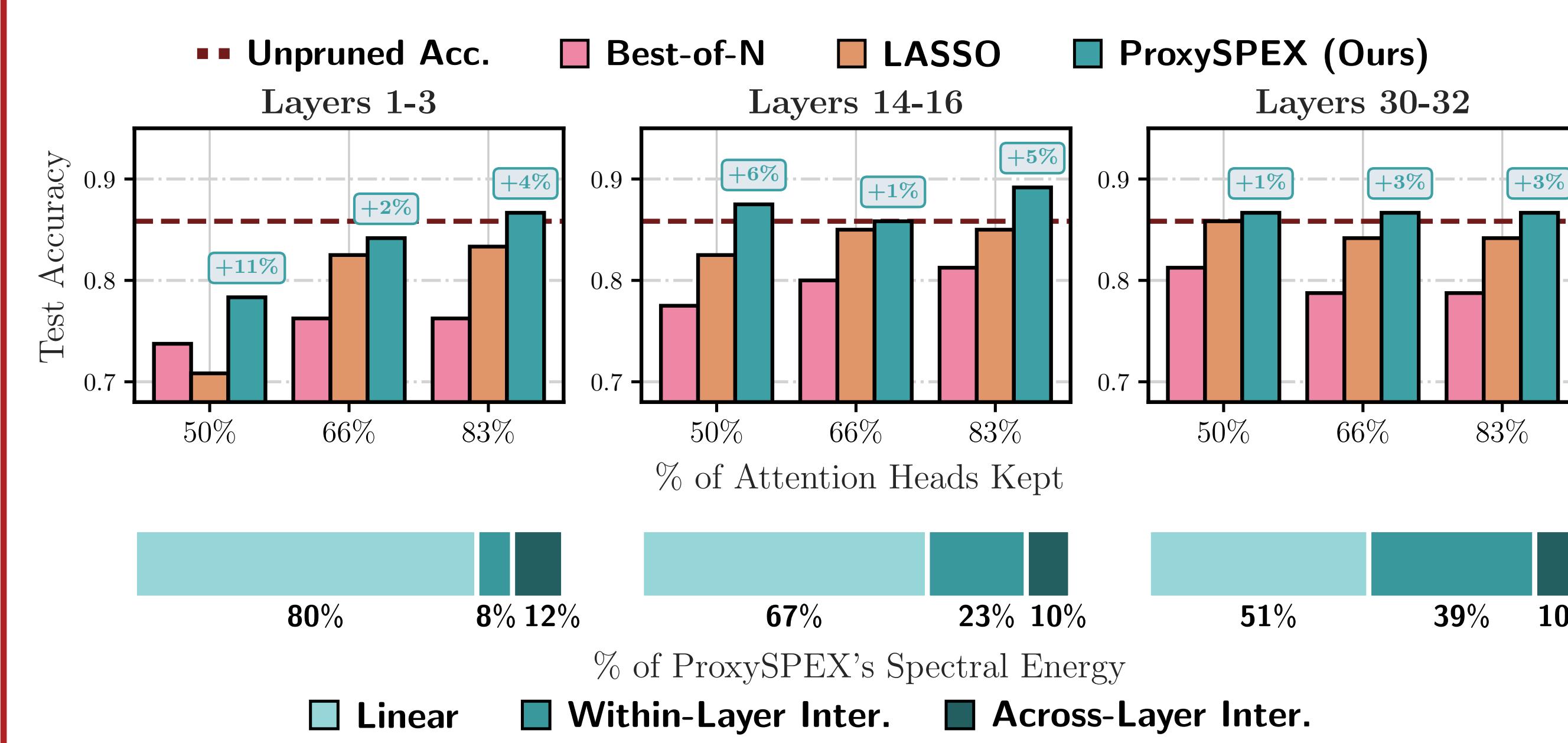


For multipliers $\alpha \in \{0.25, 0.5, 0.75, 1.0\}$, recall of the top ten Shapley values after $\alpha \cdot n \log_2(n)$ inferences.

Experiments: Attention Head Interaction Attribution

To determine which attention heads \mathcal{H} are important for a task, define $\text{LLM}_S(\cdot) \triangleq$ model with only heads $S \in \mathcal{H}$. Then we define the function:

$$f(S) \triangleq \text{Accuracy of } \text{LLM}_S \text{ on MMLU training set.}$$



Attention head pruning for Llama-3.1-8B-Instruct for MMLU (high-school-us-history) across different layers. Unpruned accuracy shown by dashed line. The faithfulness of ProxySPEX leads to superior performance.

Uniquely, ProxySPEX can be used to measure the amount of **interaction energy** between heads, both within and across layers, offering exiting new tools for understanding computation in LLMs.