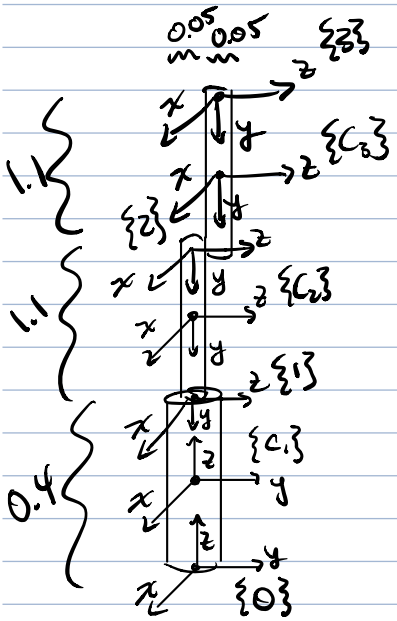
ME 699 HW2 - Derivations



$$T^0_{C_1}(q_1) = \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_1(q_1, q_2) = \begin{bmatrix} C_{q_1}C_{q_2} & -C_{q_1}S_{q_2} & -S_{q_1} & 0 \\ S_{q_1}C_{q_2} & -S_{q_1}S_{q_2} & C_{q_1} & 0 \\ -S_{q_2} & -C_{q_2} & 0 & 0.35 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_{C_2}(q_1, q_2) = \begin{bmatrix} C_{q_1}C_{q_2} & -C_{q_1}S_{q_2} & -S_{q_1} & 0.55\,C_{q_1}S_{q_2} \\ S_{q_1}C_{q_2} & -S_{q_1}S_{q_2} & C_{q_1} & 0.55\,S_{q_1}S_{q_2} \\ -S_{q_2} & -C_{q_2} & 0 & 0.35+0.55\,C_{q_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_2(q_1, q_2, q_3) = \begin{bmatrix} C_{q_1}C_{q_2}C_{q_3} - C_{q_1}S_{q_2}S_{q_3} & -C_{q_1}C_{q_2}S_{q_3} - C_{q_1}S_{q_2}C_{q_3} & -S_{q_1} & 1.05\,C_{q_1}S_{q_2} \\ S_{q_1}C_{q_2}C_{q_3} - S_{q_1}S_{q_2}S_{q_3} & -S_{q_1}C_{q_2}S_{q_3} - S_{q_1}S_{q_2}C_{q_3} & C_{q_1} & 1.05\,S_{q_1}S_{q_2} \\ -S_{q_2}C_{q_3} - C_{q_2}S_{q_3} & S_{q_2}S_{q_3} - C_{q_2}C_{q_3} & 0 & 0.35+1.05\,C_{q_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_{C_3}(q_1, q_2, q_3) = \begin{bmatrix} C_{q_1}C_{q_2}C_{q_3} - C_{q_1}S_{q_2}S_{q_3} & -C_{q_1}C_{q_2}S_{q_3} - C_{q_1}S_{q_2}C_{q_3} & -S_{q_1} & 0.55(C_{q_1}C_{q_2}S_{q_3} + C_{q_1}S_{q_2}C_{q_3}) - 0.1S_{q_1} + 1.05C_{q_1}S_{q_2} \\ S_{q_1}C_{q_2}C_{q_3} - S_{q_1}S_{q_2}S_{q_3} & -S_{q_1}C_{q_2}S_{q_3} - S_{q_1}S_{q_2}C_{q_3} & C_{q_1} & 0.55(S_{q_1}C_{q_2}S_{q_3} + S_{q_1}S_{q_2}C_{q_3}) + 0.1C_{q_1} + 1.05\,S_{q_1}S_{q_2} \\ -S_{q_2}C_{q_3} - C_{q_2}S_{q_3} & S_{q_2}S_{q_3} - C_{q_2}C_{q_3} & 0 & -0.55(S_{q_2}S_{q_3} - C_{q_2}C_{q_3}) + 0.35 + 1.05\,C_{q_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_3(q_1, q_2, q_3) = \begin{bmatrix} C_{q_1}C_{q_2}C_{q_3} - C_{q_1}S_{q_2}S_{q_3} & -C_{q_1}C_{q_2}S_{q_3} - C_{q_1}S_{q_2}C_{q_3} & -S_{q_1} & 1.15(C_{q_1}C_{q_2}S_{q_3} + C_{q_1}S_{q_2}C_{q_3}) - 0.1S_{q_1} + 1.05C_{q_1}S_{q_2} \\ S_{q_1}C_{q_2}C_{q_3} - S_{q_1}S_{q_2}S_{q_3} & -S_{q_1}C_{q_2}S_{q_3} - S_{q_1}S_{q_2}C_{q_3} & C_{q_1} & 1.15(S_{q_1}C_{q_2}S_{q_3} + S_{q_1}S_{q_2}C_{q_3}) + 0.1C_{q_1} + 1.05\,S_{q_1}S_{q_2} \\ -S_{q_2}C_{q_3} - C_{q_2}S_{q_3} & S_{q_2}S_{q_3} - C_{q_2}C_{q_3} & 0 & -1.15(S_{q_2}S_{q_3} - C_{q_2}C_{q_3}) + 0.35 + 1.05\,C_{q_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* useless

$$J_{\omega_{C_1}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad J_{\omega_{C_2}} = \begin{bmatrix} 0 & -S_{q_1} & 0 \\ 0 & C_{q_1} & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad J_{\omega_{C_3}} = \begin{bmatrix} 0 & -S_{q_1} & -S_{q_1} \\ 0 & C_{q_1} & C_{q_1} \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_{v_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad J_{v_{c_2}} \begin{bmatrix} z_0 \times (O_{c_2} - O_0), & z_1 \times (O_{c_2} - O_1), & 0 \\ & & 0 \end{bmatrix}$$

$$J_{v_{c_3}} = \begin{bmatrix} z_0 \times (O_{c_3} - O_0), & z_1 \times (O_{c_3} - O_1), & z_2 \times (O_{c_3} - O_2) \end{bmatrix}$$

$$D(q) = \sum_{i=1}^{3} \left(m_i J_{v_{ci}}^T J_{v_{ci}} + J_{\omega_{ci}}^T R_{ci} I_{c_i} R_{c_i}^T J_{\omega_{ci}} \right)$$

$$I_{c_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.083 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_{c_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.083 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_{c_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.33 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$m_{c_1} = m_{c_2} = m_{c_3} = 1$$

$$P = \sum_{i=1}^{3} m_i g \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} c_i^0(q)$$

$$P = \begin{bmatrix} 0 & 0 & g \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0.2 \end{pmatrix} + \begin{bmatrix} 0 & 0 & g \end{bmatrix} \begin{pmatrix} 0.55 C_{q_1} S_{q_2} \\ 0.55 S_{q_1} S_{q_2} \\ 0.35 + 0.55 C_{q_2} \end{pmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & g \end{bmatrix} \begin{pmatrix} 0.5\left(C_{q_1} C_{q_2} S_{q_3} + C_{q_1} S_{q_2} C_{q_3}\right) - 0.1 S_{q_1} + 1.05 C_{q_1} S_{q_2} \\ 0.5\left(S_{q_1} C_{q_2} S_{q_3} + S_{q_1} S_{q_2} C_{q_3}\right) + 0.1 C_{q_1} + 1.05 S_{q_1} S_{q_2} \\ -0.5\left(S_{q_2} S_{q_3} - C_{q_2} C_{q_3}\right) + 0.35 + 1.05 C_{q_2} \end{pmatrix}$$

$$P = 0.2g + (0.35 + 0.55 C_{q_2}) g + (0.55 C_{q_2 + q_3} + 0.35 + 1.05 C_{q_2}) g$$

$$g_k = \frac{\partial P}{\partial q_k}$$

$$G(q) = \begin{bmatrix} g_1, g_2, g_3 \end{bmatrix}^T$$

$$g_1 = 0$$

$$g_2 = -\left(1.6 s_{q_2} + 0.55 s_{q_2+q_3}\right) g$$

$$g_3 = -0.55 s_{q_2+q_3} g$$

$$C(q, \dot{q}) \cdot \dot{q} = \sum_{i=1}^{3} c_{ijk}(q) \dot{q}_i, \quad c_{ijk} = \frac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right)$$

Used Symbolic Toolbox in Matlab to solve for C, it's pretty ugly though, so the actual values were copy-pasted directly into our Julia file.