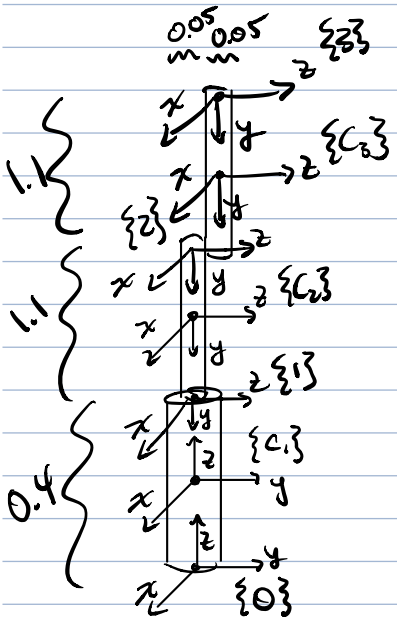
ME 699 HW2 - Derivations



$$T^0_{C_1}(q_1) = \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_1(q_1, q_2) = \begin{bmatrix} c_{q_1}c_{q_2} & -c_{q_1}s_{q_2} & -s_{q_1} & 0 \\ s_{q_1}c_{q_2} & -s_{q_1}s_{q_2} & c_{q_1} & 0 \\ -s_{q_2} & -c_{q_2} & 0 & 0.35 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_{C_2}(q_1, q_2) = \begin{bmatrix} c_{q_1}c_{q_2} & -c_{q_1}s_{q_2} & -s_{q_1} & 0.55\,c_{q_1}s_{q_2} \\ s_{q_1}c_{q_2} & -s_{q_1}s_{q_2} & c_{q_1} & 0.55\,s_{q_1}s_{q_2} \\ -s_{q_2} & -c_{q_2} & 0 & 0.35 + 0.55\,c_{q_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_2(q_1, q_2, q_3) = \begin{bmatrix} c_{q_1}c_{q_2}c_{q_3} - c_{q_1}s_{q_2}s_{q_3} & -c_{q_1}c_{q_2}s_{q_3} - c_{q_1}s_{q_2}c_{q_3} & -s_{q_1} & 1.05\,c_{q_1}s_{q_2} \\ s_{q_1}c_{q_2}c_{q_3} - s_{q_1}s_{q_2}s_{q_3} & -s_{q_1}c_{q_2}s_{q_3} - s_{q_1}s_{q_2}c_{q_3} & c_{q_1} & 1.05\,s_{q_1}s_{q_2} \\ -s_{q_2}c_{q_3} - c_{q_2}s_{q_3} & s_{q_2}s_{q_3} - c_{q_2}c_{q_3} & 0 & 0.35 + 1.05\,c_{q_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_{C_3}(q_1, q_2, q_3) = \begin{bmatrix} c_{q_1}c_{q_2}c_{q_3} - c_{q_1}s_{q_2}s_{q_3} & -c_{q_1}c_{q_2}s_{q_3} - c_{q_1}s_{q_2}c_{q_3} & -s_{q_1} & 0.6(c_{q_1}c_{q_2}s_{q_3} + c_{q_1}s_{q_2}c_{q_3}) - 0.1s_{q_1} + 1.05c_{q_1}s_{q_2} \\ s_{q_1}c_{q_2}c_{q_3} - s_{q_1}s_{q_2}s_{q_3} & -s_{q_1}c_{q_2}s_{q_3} - s_{q_1}s_{q_2}c_{q_3} & c_{q_1} & 0.6(s_{q_1}c_{q_2}s_{q_3} + s_{q_1}s_{q_2}c_{q_3}) + 0.1c_{q_1} + 1.05\,s_{q_1}s_{q_2} \\ -s_{q_2}c_{q_3} - c_{q_2}s_{q_3} & s_{q_2}s_{q_3} - c_{q_2}c_{q_3} & 0 & -0.6(s_{q_2}s_{q_3} - c_{q_2}c_{q_3}) + 0.35 + 1.05c_{q_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_3(q_1, q_2, q_3) = \begin{bmatrix} c_{q_1}c_{q_2}c_{q_3} - c_{q_1}s_{q_2}s_{q_3} & -c_{q_1}c_{q_2}s_{q_3} - c_{q_1}s_{q_2}c_{q_3} & -s_{q_1} & 1.15(c_{q_1}c_{q_2}s_{q_3} + c_{q_1}s_{q_2}c_{q_3}) - 0.1s_{q_1} + 1.05c_{q_1}s_{q_2} \\ s_{q_1}c_{q_2}c_{q_3} - s_{q_1}s_{q_2}s_{q_3} & -s_{q_1}c_{q_2}s_{q_3} - s_{q_1}s_{q_2}c_{q_3} & c_{q_1} & 1.15(s_{q_1}c_{q_2}s_{q_3} + s_{q_1}s_{q_2}c_{q_3}) + 0.1c_{q_1} + 1.05\,s_{q_1}s_{q_2} \\ -s_{q_2}c_{q_3} - c_{q_2}s_{q_3} & s_{q_2}s_{q_3} - c_{q_2}c_{q_3} & 0 & -1.15(s_{q_2}s_{q_3} - c_{q_2}c_{q_3}) + 0.35 + 1.05c_{q_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J_{\omega_{C_1}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad J_{\omega_{C_2}} = \begin{bmatrix} 0 & -s_{q_1} & 0 \\ 0 & c_{q_1} & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad J_{\omega_{C_3}} = \begin{bmatrix} 0 & -s_{q_1} & -s_{q_1} \\ 0 & c_{q_1} & c_{q_1} \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_{v_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad J_{v_{c_2}} \begin{bmatrix} z_0 \times (O_{c_2} - O_0), & z_1 \times (O_{c_2} - O_1), & 0 \\ & & 0 \end{bmatrix}$$

$$J_{v_{c_3}} = \begin{bmatrix} z_0 \times (O_{c_3} - O_0), & z_1 \times (O_{c_3} - O_1), & z_2 \times (O_{c_3} - O_2) \end{bmatrix}$$

$$D(q) = \sum_{i=1}^{3} \left(m_i J_{v_{ci}}^T J_{v_{ci}} + J_{\omega_{ci}}^T R_{ci} I_{c_i} R_{c_i}^T J_{\omega_{ci}} \right)$$

$$I_{c_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.083 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_{c_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.083 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_{c_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.33 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$m_{c_1} = m_{c_2} = m_{c_3} = 1$$

$$P = \sum_{i=1}^{3} m_i g \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} c_i^0(q)$$

$$P = \begin{bmatrix} 0 & 0 & g \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0.2 \end{pmatrix} + \begin{bmatrix} 0 & 0 & g \end{bmatrix} \begin{pmatrix} 0.55 C_{q_1} S_{q_2} \\ 0.55 S_{q_1} S_{q_2} \\ 0.35 + 0.55 C_{q_2} \end{pmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & g \end{bmatrix} \begin{pmatrix} 0.6\left(C_{q_1} C_{q_2} S_{q_3} + C_{q_1} S_{q_2} C_{q_3}\right) - 0.1 S_{q_1} + 1.05 C_{q_1} S_{q_2} \\ 0.6\left(S_{q_1} C_{q_2} S_{q_3} + S_{q_1} S_{q_2} C_{q_3}\right) + 0.1 C_{q_1} + 1.05 S_{q_1} S_{q_2} \\ -0.6\left(S_{q_2} S_{q_3} - C_{q_2} C_{q_3}\right) + 0.35 + 1.05 C_{q_2} \end{pmatrix}$$

$$P = 0.2g + (0.35 + 0.55 C_{q_2}) g + (0.6 C_{q_2 + q_3} + 0.35 + 1.05 C_{q_2}) g$$

$$g_k = \frac{\partial P}{\partial q_k}$$

$$G(q) = \begin{bmatrix} g_1, g_2, g_3 \end{bmatrix}^T$$

$$g_1 = 0$$

$$g_2 = -\left(0.555 s_{q_2} + 0.65 s_{q_2+q_3} + 1.055 s_{q_2}\right) g$$

$$g_3 = -0.65 s_{q_2+q_3}\, g$$

$$C(q, \dot{q}) \cdot \{ c_{kj} = \sum_{i=1}^{3} c_{ijk}(q)\, \dot{q}, \quad c_{ijk} = \tfrac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right)$$

Tried using Symbolic toolbox to solve for D(q), did not help.