Sound Synthesis by Connecting a Chaotic Map to a Bank of Resonators

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ABSTRACT

A chaotic map can be connected to a bank of resonators in order to realize a sound synthesizer. If the right parameters are chosen, this kind of sound synthesizer can explore the edge of chaos, producing tones that sound neither too random nor too simplistic.

In this work, an example synthesizer is created by connecting the De Jong chaotic map to banks of resonators. A multichannel contact microphone called "The Hexapad" is used to excite the sound synthesizer.

1. INTRODUCTION

1.1 Linear Network Synthesis

For decades, engineers and scientists have studied the synthesis of linear electrical circuit networks. These networks have served a variety of purposes. Many of the networks were simple (e.g. low order), but some higher-order networks have traditionally been designed for carrying out steep filtering applications [1]. By analogy, similar results have been derived for linear acoustical and linear mechanical networks [2].

Indeed, quite a bit is known about linear network synthesis. For example, if and only if a finite driving-point impedance at a point in a circuit is positive real, then a finite passive linear network exists that realizes such a driving-point impedance [3]. This knowledge has helped guide the study of linear network synthesis in a very systematic way [1].

1.2 Nonlinear Network Synthesis

In contrast, the study of nonlinear network synthesis is greatly complicated by the fact that so many nonlinear phenomena are possible, even with low-order systems. Some nonlinear *analysis* methods have been established [4], but nonlinear network *synthesis* is quite challenging and dependent on the specific application. Often, when studying nonlinear systems, one resorts to "exhaustive simulation" of the system to try to study its properties.

That is the strategy employed by Stephen Wolfram regarding the study of simple cellular automata [5]. He enumerates all of the simple cellular automata by representing each one as a number. He then systematically simulates all of them, looking for ones with interesting behaviors. For example, program 1599 demonstrates complex behavior – its state only starting to repeat itself after 8282 time steps [5].

1.3 Nonlinear Sound Synthesis

Over the past few decades, researchers of sound synthesis have simulated a variety of nonlinear networks. Over time, more and more nonlinear networks have been studied, and various kinds of behavior could be found. It should be kept in mind that, compared with linear networks, so many more different kinds of nonlinear networks are possible, so that it becomes a challenge to try them all out. However, some nonlinear networks have already been studied for sound synthesis.

1.3.1 Low-Order Nonlinear Sound Synthesis

Low-order nonlinear networks can be simulated at the audio sampling rate in order to realize sound synthesizers. For example, Chua's circuit [6] or a Duffing or Van der Pol oscillator [7] can be simulated at the audio sampling rate. These are low-order systems. Their dynamics are limited due to their relatively simple nature, although it can be interesting to traverse the edge of the bifurcations through careful parameter adjustment in real time.

In contrast, chaotic maps have been used by some researchers to synthesize audio [8, 9, 10]. However, signals synthesized directly using chaotic maps tend to sound like random noise. Although such chaotic signals may formally be deterministic, the human auditory system may not necessarily able to hear the structure, and so these signals can potentially sound like noise.

1.3.2 Higher-Order Nonlinear Sound Synthesis

Higher-order methods may have the potential to have richer dynamics, potentially sounding more musical. **This is in fact the scientific challenge of the work – finding higher-order nonlinear synthesizers that work well for creating experimental music.** Accordingly, it has been suggested to try connecting chaotic maps with delay lines (e.g. digital waveguides—see Figure 1) in order to bring some more order to the chaos. For example, in 1993, Rodet suggested connecting a violin nonlinearity to a delay line [11]. More recently, Berdahl et al. explored this approach with a wider variety of chaotic maps [12]. This enabled the exploration of the "edge of chaos" of the very ordered and harmonic sound of a waveguide crossed with the potentially noise-like sound of a chaotic map.

1.4 Present Work

The present work aims to explore a new series of nonlinear networks for sound synthesis. This work proposes and ex-

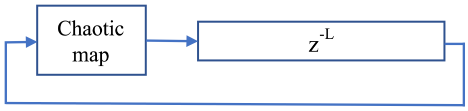

Figure 1. It was previously suggested to try the architecture of a delay line z^{-L} connected in feedback with a chaotic map [12].

plores the connection of a chaotic map in feedback with a bank of resonators. This architecture is depicted in Figure 2. Such resonators can be tuned to a wide variety of frequency combinations. For example, modal synthesis applications would choose the resonance frequencies to match those of certain physical objects [13, 14, 15, 16]. However, many other frequency combinations can be explored, which is one of the advantages of this technique.

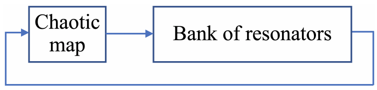

Figure 2. The present work explores the sonic possibilities of connecting a chaotic map in feedback with a bank of resonators.

If the frequencies of the resonator bank line up in a harmonic series, then qualitatively, the synthesizer behaves somewhat like the architecture shown in Figure 1. However, if the frequencies of the resonator do not line up in a harmonic series, as is the general case, then the system has the potential to oscillate at a series of different resonator-supported frequencies. That can make the behavior less predictable and potentially more interesting.

2. DESIGNING AN EXAMPLE INSTRUMENT

2.1 Acoustic Sensors

It's intriguing to think about playing these kinds of sound synthesizers using inputs from acoustic sensors. This design is inspired by other instruments with acoustic piezoelectric inputs such as the Korg Wavedrum, the Kalichord [17], and the Tangible Virtual Vibrating String [18].

In this case, the authors have realized *The Hexapad*, which consists of ten contact microphones, which can be tapped, scraped, struck, brushed, and otherwise excited acoustically and/or gesturally in order to play a synthesizer. Figure 3 shows the second author tapping on *The Hexapad*. The audio received from these sensors was passed through a first-order highpass filter at 500Hz in order to improve the sound quality.

2.2 A Random Resonator

Next, a specific synthesizer is designed to exemplify the technique outlined in Figure 2. A synthesizer with eight channels of output is designed in order to realize a surround-sound instrument as excited by *The Hexapad*.

The character of the sound synthesizer is affected by banks of randomly-tuned resonators, whose pitches are chosen from some prespecified distribution. For example, one way to tune a resonator is for the pitch to be chosen as a fundamental frequency from a whole-tone scale.

This can be realized in Max using the `reson~` object as illustrated in Figure 4. The Q is also randomly selected to provide some more variety in the sounds. As can be

Figure 3. The second author playing *The Hexapad* instrument. Although *The Hexapad* has ten sensors, this project is using only the eight sensors located around the perimeter.

seen in Figure 4, a bang sent into the object will cause the resonance frequency and Q to again be randomly selected. This enables a handy automation of the object's parameters over time and a method for doing something akin to changing chords.

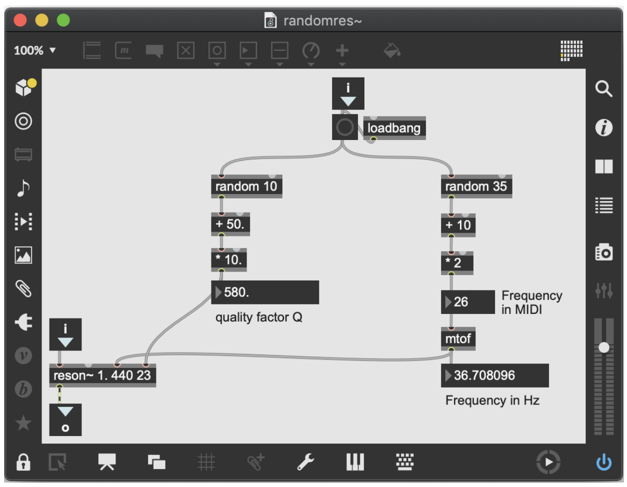

Figure 4. An example version of `randomres~` that implements a single random resonance frequency selected from a whole-tone scale with a random quality factor Q.

2.3 Eight Random Resonators in Parallel

A single resonance frequency may tend to have rather a dull sound. Therefore, it can be useful to combine multiple resonances in parallel to realize a multi-frequency resonator [13, 19, 16, 7]. In Max, this can be achieved by placing eight of the `randomres~` abstractions in parallel.

2.4 Connection to the De Jong Chaotic Map

A wide variety of chaotic maps can be used with this general sound synthesis technique. [1] The authors decided to use the Peter De Jong chaotic map [2] for this instrument, as the map has some useful properties.

The De Jong chaotic map by itself is described by the following difference equations, which are controlled by way of four parameters a, b, c, and d:

$$x_n = \sin(2\pi a y_{n-1}) - \cos(2\pi b x_{n-1}) \tag{1}$$

$$y_n = \sin(2\pi c x_{n-1}) - \cos(2\pi d y_{n-1}), \tag{2}$$

where x_n and y_n are the internal state variables. Due to the usage of the trigonometric functions, the state variables are bounded by the range $[-2 \quad 2]$, so the system will never explode.

Finally, the instrument DSP is completed by putting the De Jong map in feedback with the random banks of resonators. In the following, input signals l_n and m_n are implemented as well for external excitation of the model:

$$u_n = \sin(2\pi a y_{n-1}) - \cos(2\pi b x_{n-1}), \tag{3}$$

$$v_n = \sin(2\pi c x_{n-1}) - \cos(2\pi d y_{n-1}), \tag{4}$$

$$x_n = \texttt{randomres8\~{}}((l_n+u_n),(l_{n-1}+u_{n-1}),...), \tag{5}$$

$$y_n = \texttt{randomres8\~{}}((m_n+v_n),(m_{n-1}+v_{n-1}),...). \tag{6}$$

If the signal vector size is set to 1 in Max, then the model can be implemented as shown in Figure 5.

To utilize all eight inputs from *The Hexapad*, the signal flow diagram was duplicated four times, resulting in the complete signal flow diagram shown in Figure 6.

After some testing, it was decided to choose the resonance frequencies as the fundamental frequencies of a two-octave natural minor scale. Each time a bang is sent to the resonators, their pitches are changed, realizing a kind of chord progression.

2.5 Demo of Properties

A demonstration video shows how the a, b, c, and d parameters affect the sound. For this, the parameters are controlled with potentiometers on a MIDI controller. The MIDI values from the potentiometers were mapped from the range of $[0 \quad 127]$ (standard MIDI control range) to the range of $[-2 \quad 2]$, which were found to be a good range for experimenting with the De Jong maps.

The demonstration video also shows how the acoustic sensing enables expressive control of the synthesizer. [3]

[1] `https://en.wikipedia.org/wiki/List_of_chaotic_maps`

[2] `http://paulbourke.net/fractals/peterdejong/`

[3] `https://drive.google.com/file/d/1D0rDJnZjEp7Wjy-rMk9C3azgydMiSqhw`

3. COMPOSED MUSIC

A short étude titled *Resonator Soirée* [4] was created to showcase the instrument. The patches receive eight inputs from eight of the Hexapad's piezoelectric contact microphones and pass the signals through the De Jong maps, which output chaotically processed signals (see Figure 6). By striking *The Hexapad* and changing parameter values, the second author was able to explore sounds from the De Jong maps and create an interesting, eight-channel musical étude which focused on the development of timbre and rhythm from the De Jong maps. Some of the sounds created by changing these parameters were stable harmonic pitches, unstable inharmonic pitches, chaotic noise, and rhythmic beating frequencies. The beating frequencies were the most interesting to the second author, so he based the étude around exploiting those rhythmic beating frequencies in musical ways. The sounds from the instrument are accompanied by percussion in order to give the music a driving pulse.

4. CONCLUSIONS

This work investigated sound synthesizers made by connecting chaotic maps in feedback with banks of resonators. As with other fields of study involving nonlinear network synthesis, a large number of diverse networks remain to be investigated by simulation. This is the way forward in nonlinear science as proposed by Stephen Wolfram [5]. In experimental music, the emphasis may tend to be on nonlinear networks that can resonate at several frequencies simultaneously.

As exemplified by the composition *Resonator Soirée*, this architecture realizes an intriguing sound synthesis technique for exploring the edge of chaos [12], and we continue to look forward to expanding these results using a variety of chaotic maps and resonance frequency tunings.

Moreover, it is important to think about how to excite these kinds of sound synthesizers using high-fidelity signals. In the present work, this was realized using a multichannel contact microphone called *The Hexapad*.

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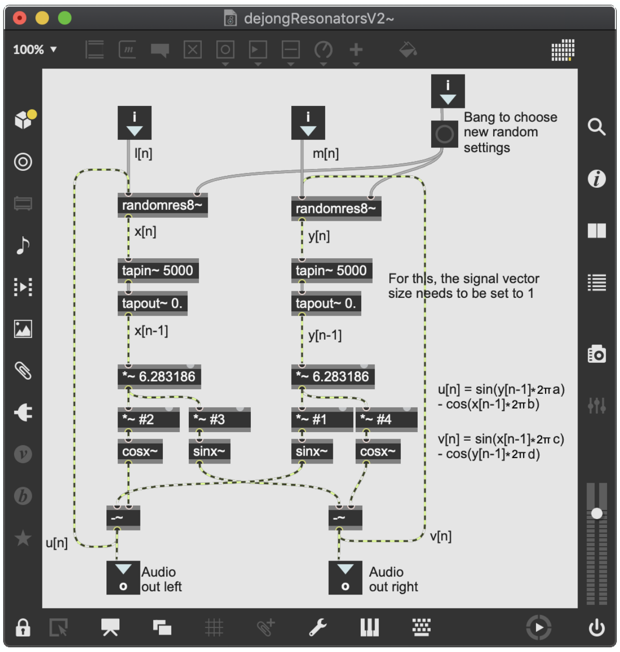

Figure 5. Here the model represented in Figure 2 is implemented in detail "in stereo" for the De Jong chaotic map. (Note: The signal vector size must be set to 1 in Max in order for the `tapin~/tapout~` connections to delay the audio by only a single sample.)

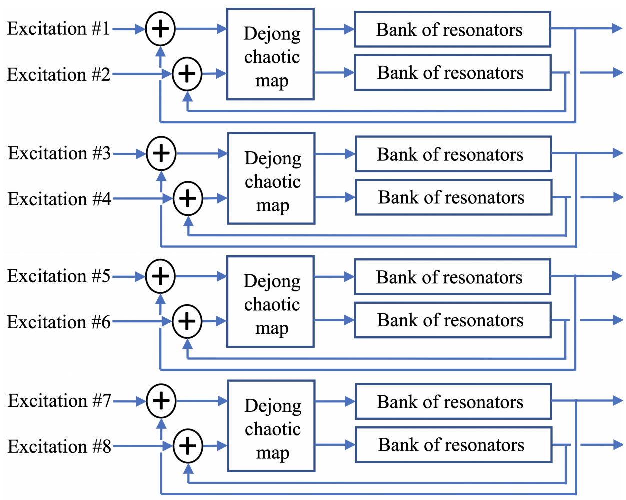

Figure 6. Complete signal flow diagram for the final version of the synthesizer for *The Hexapad*. The feedback signals are summed with excitation audio signals that come from the eight piezoelectric sensors.