Johnson Noise Lab Phys 128AL 10/18/22

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Experiment 1: Gain

Johnson noise is on scale of nV

Gain $g(f) = \frac{V_o(f)}{V_i(f)}$

V_o: RMS Voltage of output (amplifier filter chain

V_i: RMS Voltage of input (White noise generator)

Question: What is the Refresh rate of the the monitor?

$\frac{50000 \text{ Hz}}{640 \text{ px}} = 78.13 \text{ Hz}$

2:45 pm Begin Experiment 1

The WNG set to Voltage of 10.9 mV AC

Pre-amp ~~was~~ set to 1×10^4 gain

To check the signals and that the pre-amp gain is working we use dB scale to compare the difference to background

Just white noise : $dB = 20 \log_{10}(10^{-3} V) = -60$

W/ Pre amp $dB = 20 \log(10^4) = 80$

~~Dr~~ 10/20 Data ended up being corrupted so we are going to re-do this experiment.

~~2.3 Analysis~~ WNG V = 15 mV AC

Pre amp gain setting : $2 \cdot 10^3$

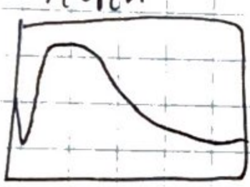
bandpass ~~[illegible]~~ 3 kHz - 10 kHz

Output Spec. Analyzer avg background of spectral Analyzer ≈ 30 μV rms

Max. V_{rms} = ~~[illegible]~~ 110 mV rms @ 5.000 kHz

shows clear band from 3kHz to 10kHz

Sketch



Output of generator should be ~ 10 μV

Analysis

From our fitted data for gaussian ~~$g(f) = (2382.2)e^{-(x-6761.75)^2/}$~~

$$g(f) = Ae^{-(f-\mu)^2/2\sigma^2}$$

A: 2382.21 μ: 6761.75 σ: 3411.83

δA: ± 27.474 $\delta\mu$: ± 48.836 $\delta\sigma$: ± 50.754

~~$$\delta g = \sqrt{\left(\frac{\partial g}{\partial A}\delta A\right)^2 + \left(\frac{\partial g}{\partial \mu}\delta\mu\right)^2 + \left(\frac{\partial g}{\partial \sigma}\delta\sigma\right)^2}$$~~

10/20

3. Johnson Noise

3.1 Theory

RMS Voltage V_i across any Resistor R at temp. T

$$dV_i^2 = 4RK_BT\,df$$

This measurement creates a modified frequency dependent resistance R_f from capacitance in system, C_i, forming a low-pass filter

$$R_f = \frac{R}{1+(2\pi fRC)^2}$$

Inclusion of amplifier creates amplified voltage dependent on V_i

$$dV^2 = [g(f)]^2\,dV_i^2$$

Leads to final eqn. for expected RMS Voltage at spectral analyzer

$$V^2 = 4RK_BTG$$

w/ $$G = \int_0^\infty \frac{[g(f)]^2}{1+(2\pi fRC)^2}\,df$$

But total

$$V_{meas}^2 = V^2 + V_{system}^2$$

3.2 Methodology

Measure all Resistors — R resolution : 100 mΩ = 0.1 Ω

Tool: Amprobe 37XR-A — R Accuracy = ±(0.5% reading + 8 dgts)

~~C resolution : 0.01 nF~~

~~C accuracy : ±(3.0% reading + 10 dgts)~~

didn't end up using this feature

3:55pm

Resistor	Actual Resistance (Measured) (Ω)	Uncertainty
Short	0 Ω	± 0.8 Ω
1kΩ, 1%	0.996 kΩ	± 0.012 kΩ
10kΩ, 1%	9.99 kΩ	± 0.13 kΩ
20kΩ, 1%	20.10 kΩ	± 0.18 kΩ
35.2kΩ, 1%	35.22 kΩ	± ~~0.18~~ 0.26 kΩ
48.7 kΩ, 1%	48.65 kΩ	± 0.32 kΩ
100 kΩ 1%	100.5 kΩ	± 1.3 kΩ

Resistor	Measured Capacitance	Uncertainty
Short	0.08 nF	
1kΩ, 1%	0.08 nF	
10kΩ, 1%	0.08 nF	
20kΩ	0.07 nF	
35.2 kΩ	0.07 nF	
48.7 kΩ	0.07 nF	
100 kΩ 1%	0.08 nF	

How much of systems capacitance is seen by the resistor

From pre-Amp specs Impedance - 100 MΩ +25pF

High input impedance / low output impedance

The Resistor is "looking into" the base of the amplifier which has very high input impedance of 100 MΩ.

Therefore the capacitance on the emitter side of the pre-amp is more important

Fr

C

E

From Google - capacitance of BNC cable is ~ 80 pF / meter

Wires	($\delta L = \pm 0.001$ m) Length	Capacitance $\delta C = \pm 0.1$ pF	
Resistors 1kΩ - 100 kΩ	0.938 m	75 pF	Used these
BNC #1	0.950 m	76	
BNC #2	0.950 m	76	
BNC #3	1.85 m	148	
Resistor (Short)	0.80 m	64	

Characteristic Capacitance of Electronics

Name	Rated ~~Impedance~~ Capacitance	
Pre-Amp	25 pF	used these
Filter	50 pF	
Spectrum Analyzer	15 pF	

Sum of Capacitance: In Series Capacitors add inversely

$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots \qquad \delta C_{tot} = |C| \cdot \left(\left(\frac{\delta C_1}{C_1}\right)^2 + \cdots \right)^{1/2}$$

$C_{tot} = \cancel{5.3 \text{ pF} \pm 0.05 \text{ pF}}$ $= \pm 0.045$ pF

$= 6.0$ pF ± 0.05 pF

Experiment Start 5:10 pm

Gain: 2×10^3

Room Temp 22°C

Band analysis - 3 kHz - 10 kHz

Avg: on Num: Avgs. - 1500

(δV) = from manual ± 0.3 dB

$\delta V_{rms} = \pm 7.02\%$ $\pm 0.02\%$

Resistor	Trial	Band Voltage (V_{rms})	
1kΩ	1	5.897×10^{-4} V	
	2	5.907×10^{-4} V	V_{rms} Avg: $5.9744 \cdot 10^{-4}$ V
	3	5.928×10^{-4} V	
	4	5.935×10^{-4} V	
	5	6.205×10^{-4} V	
10kΩ	1	1.455×10^{-3} V	
	2	1.458×10^{-3} V	V_{RMS} Avg: 1.46×10^{-3}
	3	1.455×10^{-3} V	
	4	1.458×10^{-3} V	
	5	1.456×10^{-3} V	

Resistor	Trial	Band Voltage (V_{rms})
20k	1	2.015×10^{-3} V
	2	2.025×10^{-3} V
	3	2.016×10^{-3} V
	4	2.015×10^{-3} V
	5	2.014×10^{-3} V
35.2k	1	2.625×10^{-3} V
	2	2.633×10^{-3} V
	3	2.635×10^{-3} V
	4	2.631×10^{-3} V
	5	2.627×10^{-3} V
48.7k	1	3.819×10^{-3} V
	2	3.82×10^{-3} V
	3	3.82×10^{-3} V
	4	3.819×10^{-3} V
	5	3.822×10^{-3} V
100k	1	4.253×10^{-3} V
	2	4.233×10^{-3} V
	3	4.23×10^{-3} V
	4	4.239×10^{-3} V
	5	4.251×10^{-3} V
Short	1	~~3.131×10^{-3} V~~
	2	~~3.149×10^{-3} V~~
	3	~~3.168×10^{-3} V~~
	4	~~3.186×10^{-3} V~~ ~~3.136×10^{-3} V~~
	5	~~3.13×10^{-3} V~~
Ground coupling	1	3.861×10^{-4} V
	2	3.858×10^{-4} V
	3	3.875×10^{-4} V
	4	3.853×10^{-4} V
	5	3.872×10^{-4} V

~~Room~~

V_{rms} Avg: 2.017×10^{-3}

end of lab 10/2?

restart lab from this pt. same settings

Room temp: 27°C

V_{rms} avg = 2.6302×10^{-3}

V_{rms} avg = 3.820×10^{-3}

V_{rms} 4.241×10^{-3}

mistake w/ plug-in

~~avg = 3.1478×10^{-3}~~

3.908×10^{-4}
3.912×10^{-4}
3.922×10^{-4}
3.913×10^{-4}
3.914×10^{-4}

V_{rms} avg = 3.8638×10^{-4}

There is difference between short & ground coupling and we expect that because instead of going to ground the current is going back and forth on the wire, causing capacitive pickup and interference with itself.

Analysis

$$V_{RMS} = \frac{1}{\sqrt{2}} \cdot V_p$$

$$G = \int_0^\infty \frac{|g(f)|^2}{(1+(2\pi f CR)^2)} df$$

$$2V_{RMS}^2 = V_p^2$$

Integral converges once $g(f)$ begins to fall off outside of band gain. ~~[illegible]~~ It is possible to integrate from zero to infinity but not necessary because infinite frequency is not possible

$$G(1k\Omega) = \int_{3kHz}^{10kHz} \left(\frac{|g(f)|^2}{1+(2\pi f (6\,pF)(996\,\Omega))^2} df \right) = 2.91970 \cdot 10^{10}$$

$$\frac{V_p^2}{4k_B G} = R = \frac{2V_{RMS}^2}{4k_B G} \qquad k_B = 1.380649 \cdot 10^{-23}\ J/K$$

$G(10k\Omega) = 2.91974 \times 10^{10}$

$G(20k\Omega) = 2.91965 \times 10^{10}$

$G(35.2\,k\Omega) = 2.91953 \times 10^{10}$

$G(48.7\,k\Omega) = 2.91939 \times 10^{10}$

$G(100\,k\Omega) = 2.91833 \times 10^{10}$

$\Rightarrow$

$R\ (\Omega)$	$\frac{2V_{RMS}^2}{4k_B G}$
996	2.587×10^5
9990	2.440×10^6
20100	4.865×10^6
35,220	8.391×10^6
48,500	1.791×10^7
100,500	2.214×10^7

Fit data for temperature later

Error Propogation

$C = 6.0\,pF \qquad \delta C = \pm 0.05\,pF$

$V_{RMS} \qquad \frac{\delta V_{RMS}}{V_{rms}} = \pm 7.02\%$

$V_{RMS}^2 \qquad \delta V_{RMS}^2 = |V_{RMS}^2| \sqrt{2\left(\frac{\delta V_{RMS}}{V_{RMS}}\right)^2} = 9.93\%$

~~δG from python quad integrate covariance~~

$$\delta G = \sqrt{\cancel{\left(\frac{\partial G}{\partial g}\delta g\right)^2} + \left(\frac{\partial G}{\partial C}\delta C\right)^2 + \left(\frac{\partial G}{\partial R}\delta R\right)^2 + \left(\frac{\partial G}{\partial A}\delta A\right)^2 + \left(\frac{\partial G}{\partial \mu}\delta\mu\right)^2 + \left(\frac{\partial G}{\partial \sigma}\delta\sigma\right)^2}$$

Used this formula in Python and got this list for each value of resistor

$\delta G = [7.585 \times 10^8,\ 7.585 \times 10^8,\ 7.586 \times 10^8,\ 7.586 \times 10^8,\ 7.589 \times 10^8,\ 7.597]$

Full error propogation for R vs $2V_{rms}^2/4k_B G$ later $\rightarrow$

Experiment 3 Liquid Nitrogen

Proc: Put liquid N in dewar

Put resistor tail in let come to temp.

Resistor	Trial	Band V_{rms} (V)	
1k	1	4.587×10^{-4}	
	2	4.572×10^{-4}	
	3	4.561×10^{-4}	
	4	4.582×10^{-4}	
	5	4.579×10^{-4}	
10k	1	8.469×10^{-4}	
	2	8.404×10^{-4}	
	3	8.333×10^{-4}	
	4	8.362×10^{-4}	
	5	8.348×10^{-4}	
20k	1	1.114×10^{-3}	
	2	1.11×10^{-3}	
	3	1.109×10^{-3}	
	4	1.107×10^{-3}	
	5	1.109×10^{-3}	
35.2k	1	1.413×10^{-3}	
	2	1.415×10^{-3}	
	3	1.419×10^{-3}	
	4	1.42×10^{-3}	
	5	1.418×10^{-3}	
48.7k	1	1.623×10^{-3}	
	2	1.618×10^{-3}	
	3	1.61×10^{-3}	
	4	1.605×10^{-3}	
	5	1.611×10^{-3}	
100k	1	~~2.776 × 10^-3~~	2.104×10^{-3}
	2	~~2.775 × 10^-3~~	2.105×10^{-3}
	3	~~2.77[illegible] × 10^-3~~	2.099×10^{-3}
	4	~~2.762 × 10^-3~~	2.106×10^{-3}
	5	~~2.765 × 10^-3~~	2.108×10^{-3}

Resistor	Trial	Band V_{rms} (V)
Short	1	3.927×10^{-4}
	2	3.921×10^{-4}
	3	3.919×10^{-4}
	4	3.92×10^{-4}
	5	3.923×10^{-4}

It wasn't necessary to chill the short tail because the resistance was 0 so there could be no difference in resistance from temp.

R (Ω)	$\frac{2 V_{rms}^2}{4 k_B G}$
996	6.895×10^4
9900	6.809×10^5
20,100	1.337×10^6
35,220	2.299×10^6
48,650	3.034×10^6
100500	5.306×10^6

Analysis and Data Fitting

Room temprature

From our data for R vs. $\frac{2V_{rms}^2}{4k_B G}$. we fit ~~an~~ to a linear function which is

$$y = mx$$

where y is $\frac{2V_{rms}^2}{4k_B G}$, x is R, and m is the temperature T.

~~[illegible]~~ Also including our uncertainty in our linear fit required taking the uncertainty of Y.

$$\delta Y = |Y| \sqrt{\left(\frac{\partial V_{rms}^2}{V_{rms}^2}\right)^2 + \left(\frac{\delta G}{G}\right)^2}$$

Including this uncertainty in our regression we found a value of $T = 249.74 \pm 10.54$ K for room temperature compared to actual room temp of 22°C or 298 K so ~~Abs~~ 0 = -226.78 ± 10.54 °C

For ~~liquidd~~ nitrogen we found a value of

$$T = 62.98 \pm 2.654\ K$$

where the actual value is 77 K

so Abs zero = -258.98 ± 2.654 °C

Comparing these values to absolute zero ~~the~~ they are both far from the accepted values

For room temperature the value was ~ 50°C away with ± 10°C of uncertainty, so we were off by ~ 16.8%

For Liquid Nitrogen the value was ~ 15°C away with ± 2.6°C of uncertainty, so we were off by ~ 18.2%

Given our results ~~Johson~~ Johnson Noise is partially useful for measuring temperature. It gave results that were far from the expected values but appeared to be more accurate and less uncertain at lower temperatures

Boltzmann constant k_B

For this we used the formula

$$T = \frac{2V_{RMS}^2}{4RG}$$

and fit to a linear function $y = mb$

where $Y = \frac{V_{RMS}^2}{2RG}$, $x = T$, & $m = k_B$

The error propagation for this formula, which we included in our curve fitting program is

$$\delta Y = M_Y \sqrt{\left(\frac{\delta V_{RMS}^2}{V_{rms}^2}\right)^2 + \left(\frac{\delta G}{G}\right)^2 + \left(\frac{\delta R}{R}\right)^2}$$

for each value of V_{RMS}^2, G, & R

Room temp.

From our curvefit data we calculated

$$k_B = 1.153 \times 10^{-23} \pm 4.913 \times 10^{-25} \text{ J/K}$$

where the accepted value is $k_B = 1.381 \times 10^{-23}$ J/K

Liquid N_2

From our curvefit data we calculated

$$k_B = 1.129 \times 10^{-23} \pm 4.784 \times 10^{-25} \text{ J/K}$$

These values are fairly close to k_B and are at the same order of magnitude varying by only $\sim 2.3 - 2.6 \times 10^{-24}$

Discussion & conclusion

I was surprised by this lab at how close we could come to the values of physical constants like the boltzmann constant and absolute zero, simply be measuring the amount of noise through resistors at different temperatures. It seemed difficult because of the random nature of noise and the uncertainty that comes with it, but it worked quite well, allowing us to calculate these constants to values on the same order of magnitude as the real values.

I think that to improve our findings, it would be beneficial to characterize our Gain over the pre-amp more accurately and hopefully get a better fitting Gaussian for the gain fcn. Ours was okay, but more focus and data on this part could be crucial because all of our calculations involve the Band Gain and the error associated with it.

It could also be better to work in colder temperatures b/c there would be less background noise, allowing our results to be more accurate.