Notes from Lab Manual

I. Properties of Laser Light

- Laser is monochromatic with wavelength $\lambda = 632.8$ nm
- Using a He-Ne laser 90% He gas & 10% Ne gas in experiment

II. Quantum theory

- Molecules absorb energy in discrete amounts from the frequency of photons because of different energy levels in the atoms
- $h\nu = e_m - e_n$

III. Non Ideal Behavior

A) Doppler Broadening

Exercise III.1 We know average KE of atom is $\frac{1}{2} k_B T$

Taking $T \cong 400°K$ & $M_{NE} \cong 20 \times 1.67 \times 10^{-27}$ kg

$$\frac{1}{2} k_B T = \frac{1}{2} m_{NE} v^2$$

$$v = \sqrt{\frac{k_B T}{m_{NE}}} = \sqrt{\frac{1.380649 \cdot 10^{-23} \frac{m^2 kg}{s^2 K} \cdot 400 K}{3.34 \cdot 10^{-26} kg}}$$

$$v = 406 \text{ m/s} \qquad v/c = \cancel{1.366}\ 1.36 \times 10^{-6}$$

Exercise III.2 ~~Band~~ ~~frequenc~~ Wave length of He-Ne laser is $\lambda = 6.328 \times 10^{-7}$ m so by the wavelength-frequency relation of light

$$c = \lambda f$$

the frequency is $f = 4.738 \times 10^{14}$ Hz

Exercise III.3 From the formula

$$\cancel{f = \nu_0 (1 + v/c)} \quad \nu_f = \nu_0 (1 + v/c)$$

we find that the Doppler broadened spectral line, ~~ν_0 to be~~ which is centered around $\nu_0 = 4.738 \times 10^{14}$ Hz

→ cont.

Ex. III.3 cont.

From this we find the width of the Doppler broadened spectrum by taking the calculated $\nu_{f_{max}}$ and subtracting ν_0

$$(\nu_{f_{max}} - \nu_0) = 2 \cdot 6.443 \times 10^8 \text{ Hz} = 1.289 \times 10^9 \text{ Hz}$$

B) Modes of Oscillation

To resonate the cavity length in the laser must be equal to an integral number of half wavelengths of the radiation. So the cavity is resonant for frequencies

$$\nu_n = \frac{nc}{2L}$$

Exercise III.4 For cavity length of $L = 0.2$ m and frequency of $\nu = 4.738 \cdot 10^{14}$ Hz

~~$n = 6.32 \times 10^5$~~

$n = 632183$

Exercise III.5 For each new observable line there is an increase in n which also increases the frequency so the ~~maximum frequency in the Doppler broadened band corresponds to the highest n. from this~~

$$\Delta\nu = \nu_{n+1} - \nu_n$$

~~$\nu_{max} = (\nu_0 + 6.443 \times 10^8 \text{ Hz}) = \frac{n_{max} c}{2L}$~~

$\nu_n = 4.738 \cdot 10^{14}$ $\quad \nu_{n+1} = 4.7389 \cdot 10^{14}$

~~so $n_{max} =$~~ $\quad \boxed{\nu_{diff} = (\nu_{n+1} - \nu_n) = 5.609 \times 10^8 \text{ Hz}}$

~~$n+1$~~

~~$n_{n+1} = 632001$ so $\nu_{n+1} = 4.74 \cdot 10^{14}$ Hz~~

Range of band = 4.7389×10^{14} Hz $\pm 0.6455 \cdot 10^9$ Hz $= 4.736846 \cdot 10^{14}$ thru $4.736833 \cdot 10^{14}$

so there are only three ~~visi~~ observable lines within the Doppler Broadened Gain Curve

Exercise III.6 Cohenence length is maximum difference in pathlength between interfering beams such that an interference pattern is still visible

$$L_c = \frac{c}{\Delta\nu} \qquad \Delta\nu = 1.289 \times 10^9 \text{ Hz}$$

$$\boxed{L_c = 0.2326 \text{ m}}$$

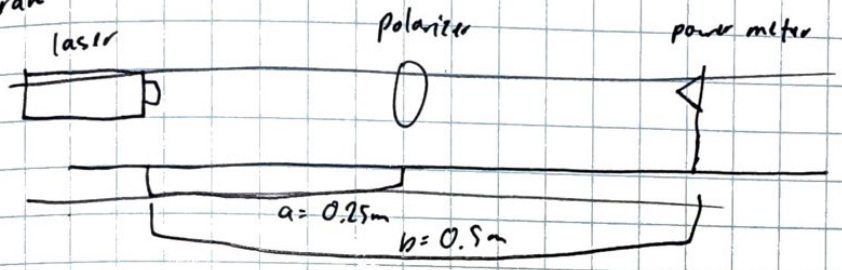
IV. Experimental Analysis of Laser Light

A). Polarization

- Degree of polarization defined as $P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

Exercise IV.7 Measure Intensity $P(\theta)$ as a fcn of θ the angle of the Polarizer

~~Diagram~~



Data 3:45pm

$\theta \pm 1°$ (°)	(μW) $P(\theta) \pm 1\%$	$\sigma P(\theta)$ (μW)
0°	0.86	
10°	0.86	
20°	0.82	
30°	0.84	
40°	0.84	
50°	0.84	
60°	0.84	
70°	0.84	
80°	0.84	
90°	0.84	
100°	0.84	

Ex. IV.7 cont.

Previous data was wrong. The laser was too far and not properly aligned so the readings were just background light.

Trial #2 : 3:58 pm 10-3-22

New set up Polarized distance from laser is 6 cm
laser meter is 11 cm

$\theta \pm 1°$	$P(\theta)$ (μW)	$\sigma P(\theta)$ (μW)
0°	0.38	±0.0038
10°	0.33	±0.0033
20°	0.29	±0.0029
30°	0.28	±0.0028
40°	0.28	±0.0028
50°	0.30	±0.0030
60°	0.33	±0.0033
70°	0.37	±0.0037
80°	0.43	±0.0043
90°	0.48	±0.0048
100°	0.52	±0.0052
110°	0.56	±0.0056
120°	0.58	±0.0058
130°	0.58	±0.0058
140°	0.57	±0.0057

Ex IV.7 cont.

$\theta \pm 1°$	$P(\theta)$ (μW)	σP (1%) (μW)
150°	0.53	~~0.0053~~
160°	0.48	~~0.0048~~
170°	0.43	~~0.0043~~
180°	0.38	~~0.0038~~
~~190°~~		
~~200°~~		
~~210°~~		
~~220°~~		
~~230°~~		

Too further find the maximum and minimum Intensity ~~we~~ will take further measurements ~~around~~ from 20°–30° (min) and from 120°–130° max

4:26 pm

Minimum

$\theta \pm 10$	$P(\theta)$ (μW)	σP (1%) (μW)
30°	0.28	~~0.0028~~
32°	0.27	~~0.0027~~
34°	0.27	~~0.0027~~
~~38°~~ 36°	0.28	~~0.0028~~
38°	0.28	~~0.0028~~
40°	0.28	~~0.0028~~

Minimum of $P(\theta)$ is $0.27 \pm$ ~~0.0027~~ μW

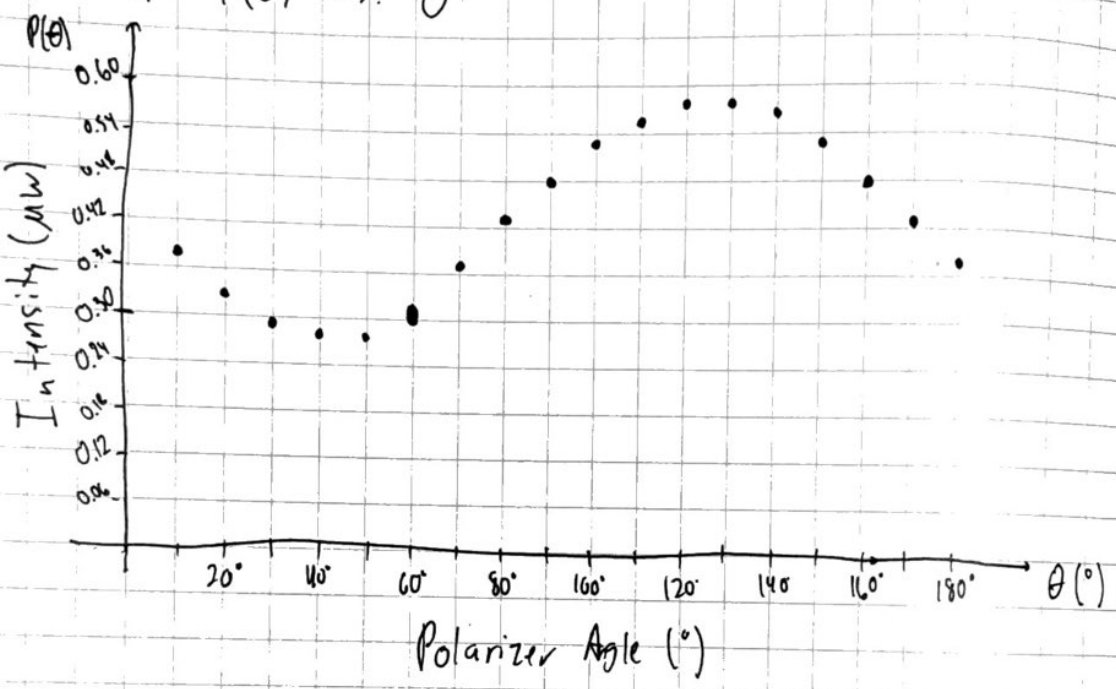
Ex. IV.7 cont. →

Maximum

$\theta \pm 1°$	$P(\theta)$ (μW)	σP (1%) (μW)
120	0.58	~~0.~~0.0058
122	0.58	~~0.~~0.0058
124	0.58	~~0.~~0.0058
126	0.59	~~0.~~0.0059
128	0.58	~~0.~~0.0058
130	0.58	~~0.~~0.0058

Maximum of $P(\theta) = 0.59 \pm 0.00059\ \mu W$

Graph of $P(\theta)$ vs. θ



~~Error Discussion~~:

Discussion and notes:

For the experiment there were two main sources of error. The first was in the polarizer and was due to the precision of the marks for the angles. The second, larger, error source was background interference & noise measured in the power meter. The steps we took to reduce this error included shutting the door to attempt to block some sunlight and standing behind the device to hopefully not reflect any light into it.

Ex. IV.7 cont. →

Analysis:

Degree of Polarization of the laser light

$$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad I_{max} = 0.59 \pm 0.0059\,\mu W$$

$$I_{min} = 0.27 \pm 0.0027\,\mu W$$

$$P = \frac{0.32 \pm 0.0006\,\mu W}{0.86 \pm 0.0006\,\mu W} = \cancel{0.37 \pm 0.37 \pm 0.006\,\mu W}$$

$$0.37 \pm 0.006\,\mu W$$

Error Propogation

$$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad \frac{\sigma P}{|P(\theta)|} = 1\%$$

~~Make I_{max}~~

$$\cancel{\frac{P_1(\theta)\left(1 \pm \frac{\sigma P_1}{|P_1(\theta)|}\right) - P_2(\theta)\left(1 \pm \frac{\sigma P_2}{|P_2(\theta)|}\right)}{P_1(\theta)\left(1 \pm \frac{\sigma P}{|P_1(\theta)|}\right) + P_2(\theta)\left(1 \pm \frac{\sigma P_2}{|P_2(\theta)|}\right)}}$$

$$\delta P = \sqrt{\left(\frac{\partial P}{\partial I_{max}} \sigma I_{max}\right)^2 + \left(\frac{\partial P}{\partial I_{min}} \sigma I_{min}\right)^2}$$

$$\frac{\partial P}{\partial I_{max}} = \frac{2 I_{min}}{(I_{max} + I_{min})^2} \qquad \frac{\partial P}{\partial I_{min}} = \frac{-2 I_{max}}{(I_{max} + I_{min})^2}$$

$$I_{max} = 0.59\,\mu W \quad \sigma I_{max} = 0.0059\,\mu W$$

$$I_{min} = 0.27\,\mu W \quad \sigma I_{min} = 0.0027\,\mu W$$

$$\delta P = \sqrt{\left(\frac{2(0.27\,\mu W)}{(0.59\,\mu W + 0.27\,\mu W)^2} \cdot (0.0059\,\mu W)\right)^2 + \left(\frac{2(0.59\,\mu W)}{(0.59\,\mu W + 0.27\,\mu W)^2} (0.0027\,\mu W)\right)^2}$$

$$\delta P = \pm 0.006$$

$$\boxed{P + \delta P = 0.37 \pm 0.006\,\mu W}$$

B). Beam Diameter

Exercise IV.8 for beam with radius R and $r = \sqrt{x^2+y^2}$

(1) $I(x,y) = I(r) = I_0 \exp\left(\frac{-2r^2}{R^2}\right)$

$P_{tot} = \int_0^{2\pi}\int_0^{\infty} I(r)\, r\, dr\, d\theta = 2\pi \int_0^{\infty} I_0 \exp\left(\frac{-2r^2}{R^2}\right) dr = I_0 \sqrt{\frac{\pi R}{8}} \quad \frac{I_0 \pi R^2}{2}$

5:32pm

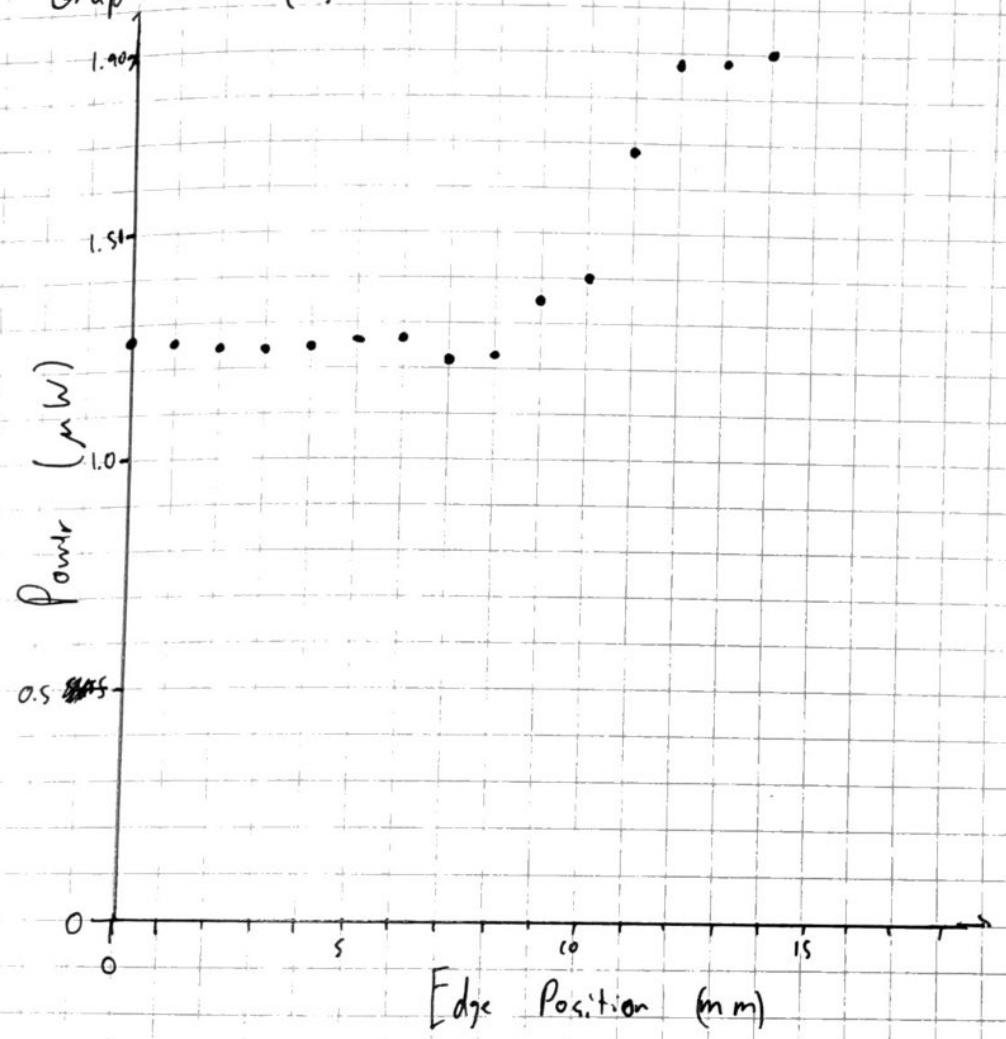
$I_{tot} = \frac{P_{tot}}{A} = \frac{I_0}{2}$

Exercise IV 9: Move blade out

$\sigma x = 10\mu m$ x (mm)	P μ(W)	σ P (μW)
0	1.27	0.0127
1	1.26	0.0126
2	1.24	0.0124
3	1.24	0.0124
4	1.25	0.0124
5	1.27	0.0127
6	1.27	0.0127
7	1.21	0.0121
8	1.23	0.0123
9	1.35	0.0135
10	1.40	0.0140
11	1.69	0.0169
12	1.87	0.0187
13	1.87	0.0187
14	1.90	0.0190

Ex. IV.9 cont.→ Notes; ~~the~~ when we stood ~~to~~ close the impressions ~~de~~ caused the table to sink slightly and the measurement of the Intensity to change. I think this was because the laser was ~~too high on~~ positioned near the top of the power meter which caused any motion or pressure on the table to change the ~~intensity readings.~~ Power readings.

Graph of P(x) vs. x



~~Exercise IV.10 Deriving an equation for I(x,y) based on the data collected in IV.9~~

Analysis: from the above graph, it can be seen that the power sharply increases from 8-9 mm and plateaus at 13 mm. From this we can make a rough estimate of the beam's diameter, when the power increases is the first edge and once in plateaus then the entire beam has been crossed. So the beam diameter is ~ 4.5 mm.

Exercise IV.10 To create a theoretical model for the beam Intensity as a fcn of distance, $I(x,y)$, we can use the measured Beam diameter of 4.5 mm. so $R = 2.25$ mm

$$I(x,y) = I_0 \exp\left(\frac{-2(x^2+y^2)}{(2.25\times10^{-3}\,m)^2}\right)$$

By converting to polar coordinates and integrating we can find the total Power ~~which is related to~~

$$I(r) = I_0 \exp\left(\frac{-2r^2}{5.0625\times10^{-6}\,m^2}\right)$$

$$P_{tot} = \int_0^a\int_0^{2\pi} I(r)\,dr\cdot r\,d\theta = 2\pi I_0 \int_0^a r\exp\left(\frac{-2r^2}{5.0625\times10^{-6}\,m^2}\right)dr$$

$$P_{tot} = \cancel{I_0\ 3.181\times10^{-5}\,m^2}\ I_0\cdot 7.95216\cdot10^{-6}\,m^2$$

~~From our data graph we can approximate P_{tot}~~

Based on the data collected, $P_{tot} = P_{plateau} - P_{background}$

$$P_{tot} \approx 1.90\,\mu W - 1.25\,\mu W$$
$$P_{tot} \approx 0.65\,\mu W$$

so from this $I_0 = 8.17\times10^4\ \mu W/m^2$

~~$I_0 = 2.04\times10^4\ \mu W/m^2$~~ ~~8.17~~

making ~~our~~ the equation for beam intensity

$$I(x,y) = \left(\cancel{2.04}\ 8.17\times10^4\,\frac{\mu W}{m^2}\right)\exp\left[\frac{-2(x^2+y^2)}{(2.25\times10^{-3}\,m)^2}\right]$$

Error Analysis:

Because ~~the~~ the data for the beam diameter was taken from ~~our~~ the graph which had 1mm intervals the uncertainty is very high with $\sigma x = \pm 1$mm for the beam diameter and ± 0.5mm for the beam Radius ~~for~~ in the equation. The uncertainty for the power is the same at $\frac{\sigma P}{|P(x)|} = 1\%$.

cont. →

Exercise IV.10 cont. →

Error Propogations

$$\delta I_0 = \sqrt{\left(\frac{\partial R}{\partial I_0}\sigma R\right)^2 + \left(\left(\frac{\partial P}{\partial I_0}\right)\sigma P\right)^2}$$

$$I = \int_0^R I_0$$

$$\frac{\partial R}{\partial I_0} = \cancel{\frac{4\pi^3 e^{*(-2r^2/R^2)}}{R^3}}$$

$$= \pi R$$

$$\sigma R = \pm 0.5\,mm$$

$$R = 2.25\,mm$$

$$\frac{\partial P}{\partial I_0} = 1 \qquad \sigma P = 0.0315$$

$$\delta I_0 = \pm 0.0315\,\frac{\mu W}{m^2}$$

Exercise IV.11

$$\frac{1}{e}\cdot P_{tot} = \frac{1}{e}\cdot 0.65\,\mu W = 0.239\,\mu W$$

$$1.90\,\mu W - 0.239\,\mu W = 1.66\,\mu W$$

this corresponds to ~ 10-11 mm on the graph so ~~10.5 mm is~~ from the maximum at 13 mm the beam ~~diameter~~ radius is

~~13 mm - 11.5 mm = 2.5 mm~~

13 mm - 11 mm = 2 mm

To find power inside beam we must integrate the Intensity fcn from 0 to the beam ~~radius~~ diameter of 2 mm

$$P_{ins} = 2\pi\int_0^{1.0\times10^{-3}m}\left(8.17\times10^{4}\,\frac{\mu W}{m^2}\right) r e^{\left(\frac{-2r^2}{(2\cdot10^{-3}m)^2}\right)}\,dr$$

$$\boxed{P_{ins} = 0.443\,\mu W}$$

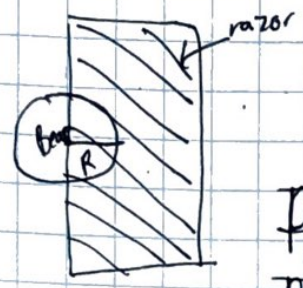
And as a fraction of P_{tot}

$$\frac{P_{ins}}{P_{tot}} = 68.15\%$$

Exercise IV.12 1) Calculate how much total Intensity is reduced when you move the knife edge within one beam radius of the beam axis.

2) Calculate how much Intensity is reduced when you move the knife edge one beam radius beyond the beam axis.

1)



How much is intensity reduced when half blocked

$$P_1 = \int_0^{\pi}\int_0^{R} I_0 e^{-\frac{2r^2}{R^2}} r\,dr\,d\theta \qquad P_1/A = I_1$$

$$P_{tot} = \int_0^{2\pi}\int_0^{\infty} I_0 e^{-\frac{2r^2}{R^2}} r\,dr\,d\theta = \cancel{P_0} \qquad \frac{P_{tot}}{A} = I_{tot}$$

$$\frac{I_1}{I_{tot}} \qquad I_1 = 0.2219/\tfrac{1}{2}\pi(0.002^2) = \cancel{3.766\times10^4\ \mu W/m^2}\ 3.532\times10^4\ \mu W/m^2$$

$$I_{tot} = 0.5133/\pi(0.002^2) = 4.0850\times10^4\ \mu W/m^2$$

$$\frac{I_1}{I_{tot}} = 86.4\%$$

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2) Note: At 3:50 pm on October 6th 2022 after asking the TA's for help because our laser was too dim to take measurements it was pointed out that we never switched the laser cover off making all our previous data and measurements useless. Going to start over now from Exercise IV.7. While costly and frustrating, many of the calculations and analysis we used were correct but just with bad data so hopefully moving forward we will be able to go at a faster pace through the things we already did.

A) Polarization

Exercise IV.7 Polarizer

Measure intensity $P(\theta)$ as a fcn of Polarizer angle

first start with 0°, increase 10° at a time and record reading from power meter

3:55pm Oct. 6th

θ (°)	$P(\theta)$ (μW)	σP (μW) (1%)
0°	159	1.59
10°	87	0.87
20°	35	0.35
30°	6	0.06
40°	3	0.03
50°	28	0.28
60°	76	0.76
70°	142	1.42
80	219	2.19
90	296	2.96
100	366	3.66
110	~~444~~ 417	4.17
120	~~446~~ 444	4.44
130	~~420~~ 446	4.46
140	~~375~~ 420	4.20
150	~~307~~ 375	3.75
160	~~236~~ 307	3.07
170	~~152~~ 236	2.36
180	159	1.59

Maxima is between 120° & 130°
Minima is between 30° & 40°

Max

θ	$P(\theta)$ (μW)	σP
120°	444	4.44
122°	447	4.47
124°	448	4.48
126°	448	4.48
128°	447	4.47
130°	446	4.46

max = 448 μW

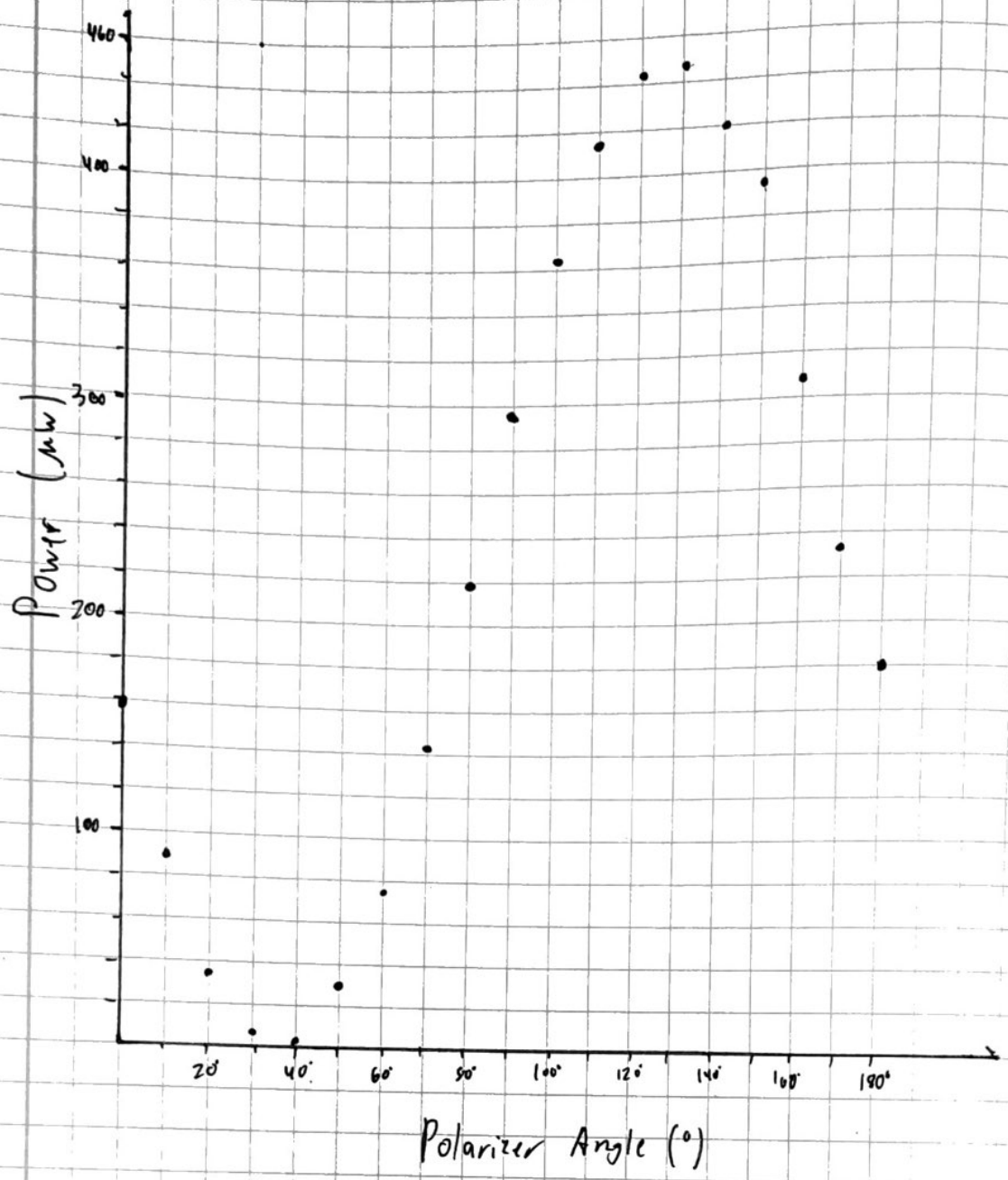
Min

θ(°)	$P(\theta)$ (μW)	σP
30	6	0.06
32	3	0.03
34	2	0.02
36	1	0.01
38	2	0.02
40	3	0.03

min = 1 μW

Errors: Error could come from two sources the power meter which is rated to ±1% so $\frac{\sigma P}{|P(\theta)|} = \frac{1}{100}$ and the polarizer which has 2° of precision so the ~~error~~ uncertainty is ±1°

Graph of $P(\theta)$ vs. θ



$I_{max} = 448 \pm 4.48\,\mu W$ $I_{min} = 1 \pm 0.01\,\mu W$

for $P = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

$$\sigma P = \sqrt{\left(\frac{\partial P}{\partial I_{max}}\sigma I_{max}\right)^2 + \left(\frac{\partial P}{\partial I_{min}}\sigma I_{min}\right)^2}$$

$\sigma I_{max} = \pm 4.48\,\mu W$

$\sigma I_{min} = \pm 0.01\,\mu W$

$$\frac{\partial P}{\partial I_{min}} = \frac{2 I_{min}}{(I_{max} + I_{min})^2} \qquad \frac{\partial P}{\partial I_{max}} = \frac{-2 I_{max}}{(I_{max} + I_{min})^2}$$

$\sigma P = 6.28 \times 10^{-5}$

$$\boxed{P = 0.9955 \pm 6.28 \times 10^{-5}}$$

Conclusion: This laser has a very high degree of Polarization which makes sense because the majority of the laser beam is polarized.

B) Beam Diameter

Exercise IV.8

$$I(x,y) = I_0 \cancel{e^{-x}} e^{\left(-\frac{2x^2}{R^2}\right)} e^{\left(-\frac{2y^2}{R^2}\right)}$$

$$P_{tot} = \int_0^\infty \int_0^\infty I_0 e^{-\frac{2x^2}{R^2}} e^{-\frac{2y^2}{R^2}} dx\, dy \qquad r^2 = x^2 + y^2$$

$$= \int_0^{2\pi} \int_0^\infty I_0 r e^{-\frac{2r^2}{R^2}} dr\, r d\theta$$

$$= 2\pi \int_0^\infty I_0 r e^{-\frac{2r^2}{R^2}} dr$$

$$P_{tot} = \frac{I_0 \pi R^2}{2}$$

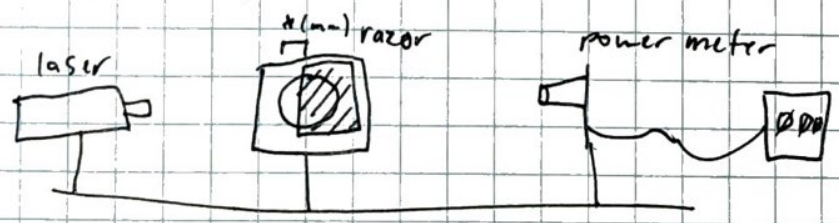
$$I_{tot} = \frac{P_{tot}}{\pi R^2} = \frac{I_0}{2}$$

Exercise IV.9

Measure total Power as a fcn of razor edge Position

Start with razor fully covering beam. Then in 1mm increments move razor across beam profile until Power reading plateaus. Take reading from power meter at every 1mm increment

Diagram



$\sigma x = \pm 5\mu m$

x (mm)	P(x)	σ P (%)
0	0.54	0.0054
1 ~~1~~	0.54	0.0054
2 ~~2~~	0.54	0.0054
3 ~~3~~	0.54	0.0054
4 ~~4~~	0.54	0.0054
5 ~~5~~	0.54	0.0054
6 ~~6~~	0.54	0.0054
7 ~~7~~	0.54	0.0054
8 ~~8~~	0.54	0.0054
9 ~~9~~	~~0.58~~ 0.58	0.0058
9.25 ~~10~~	~~0.54~~ 1.56	0.0156
9.5 ~~11~~	~~0.58~~ 13.1	0.131
9.75 ~~12~~	~~1.58~~ 206	2.06
10 ~~13~~	571	5.71
10.25 ~~14~~	726	7.26
10.5	~~7~~738	7.38
10.75	740	7.40
11	741	7.41
12	741	7.41
13	741	7.41
14	741	7.41

Decided to take finer increments in the actual beam diameter

finer measurments

first beam edge

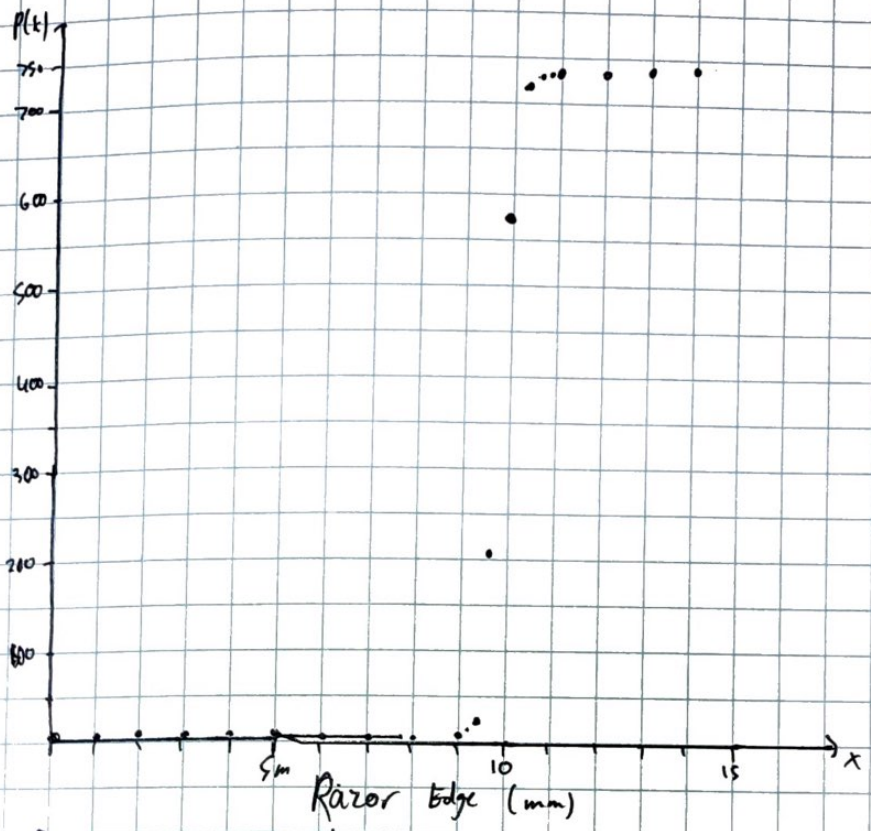
x	P(x)	σ P
8.5	0.54	0.0054
8.55	0.53	.0053
8.6	0.53	.0053
8.65	0.53	.0053
8.7	0.53	.0053
8.75	0.54	.0054
8.80	0.54	.0054
8.85	0.54	.0054
8.90	0.54	.0054
8.95	~~0.58~~ 0.55	.0055
9.00	0.57	.0057

Plateau

x	P(x)
10.5	726
10.55	725
10.6	725
10.65	725

From out measurments, we saw the first edge at $x = 9.00$ mm and the plateau around 10.50 mm with uncertainty of ± 0.25 mm for the x location of both.

Graph of $P(x)$ vs. x



From the graph and data it can be seen how the general ~~Finer graph from~~ background Power is ~0.54 μW and the plateau has a value of ~741 μW which is the total power of the laser plus the background.

Exercise IV.10 To create the theoretical expression that matches the data ~~the~~ the measured beam diameter is

$$2R = (10.50 \pm 0.25 \text{ mm}) - (9.00 \pm 0.25 \text{ mm})$$

$$2R = (1.5 \pm 0.35) \text{ mm}$$

with error $\sigma R = \sqrt{(0.25)^2 + (0.25)^2} = \pm 0.35$ mm

so

$$\boxed{R = 0.75 \pm 0.18 \text{ mm}}$$

from this we can plug into our expression

cont. →

IV.10 cont. →

$$I(r) = I_0 \exp \frac{-2r^2}{(0.75)^2}$$

and then Integrate to find the total Power

$$P_{tot} = \int_0^{2\pi}\int_0^{\infty} I_0 r e^{-2r^2/(0.75\times10^{-3}m^2)^2}\, dr\, d\theta$$

$$P_{tot} = I_0 \cdot 8.84 \times 10^{-7}\ m^2$$

So b/c $P_{tot} = P_{plat} - P_{back} = 741\,\mu W - 0.58\,\mu W = 740.42\,\mu W$

$$I_0 = 8.38 \times 10^8 \frac{\mu W}{m^2}$$

so the full equation for beam intensity is

$$I(r) = \left(8.38\times10^8 \frac{\mu W}{m^2}\right) \exp \frac{-2r^2}{(0.75\times10^{-3}m)^2}$$

Then Propogating the error

$$\sigma P_{tot} = \sqrt{(7.41)^2 + (0.0058)^2} = \pm 7.41\,\mu W$$

$$\sigma I_0 = \pm 8.31 \times 10^6 \frac{\mu W}{m^2}$$

Exercise IV.11

$$\frac{1}{e} \cdot P_{tot} = \frac{1}{e} \cdot 740.42\,\mu W \pm 7.41\,\mu W$$

$$= 272.38\,\mu W \pm 2.72\,\mu W$$

distance from edge of beam to $272.38\,\mu W$

$$d_{beam} = 10.5 \pm 0.25\,mm - 9.75 \pm 0.25\,mm$$

$$d_{beam} = 0.75 \pm 0.35\,mm \qquad \sigma d = \sqrt{(0.25)^2 + (0.25)^2} = \pm 0.35\,mm$$

$$r_{beam} = 0.38 \pm 0.18\,mm$$

for fraction of total power inside beam

$$P = \int_0^{2\pi}\int_0^{0.38\times10^{-3}m} \left(8.38\cdot10^8 \frac{\mu W}{m^2}\right) r e^{-2r^2/(0.75\cdot10^{-3}m)^2}$$

cont. →

IV.11 cont.

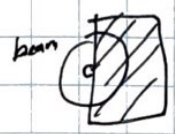
$P_{ins} = 297.326 \frac{mW}{m^2}$ $\sigma P_{ins} = \pm 2.72 \mu W$

Fraction of power inside Beam Diameter = $297.33 \pm 2.72 \, mW/m^2 / 740.42 \pm 7.41 \, mW/m^2$

$\boxed{\text{Fraction} = 40.15 \pm 1.35\%}$

$\sigma\% = \sqrt{\left(\frac{2.72}{297.33}\right)^2 + \left(\frac{7.41}{740.42}\right)^2}$

Exercise IV.12 1. Calculate how much is total intensity reduced when razor moved within one radius of beam axis



2. Calculate how much Intensity is reduced when razor is moved one R beyond beam axis

1.) $P_1 = \int_0^{\pi} \int_0^{3.8 \cdot 10^{-4} m} \left(8.38 \cdot 10^{8} \frac{mW}{m^2}\right) r e^{-2r^2/(0.75 \cdot 10^{-3})^2} dr$

$P_1 = 148.663 \, mW$ $\sigma P_1 = \pm 1.36 \mu W$

$I_1 = P_1 / A = 148.663 \pm 1.36 \, mW/m^2 / \left(\frac{1}{2}\pi (0.38 \cdot 10^{-3} \pm 0.18 \cdot 10^{-3} m)^2\right)$

$I_1 = 6.55 \times 10^6 \frac{mW}{m^2}$ $\sigma I_1 = \sqrt{\left(\frac{1.36}{148.663}\right)^2 + \left(\frac{\frac{1}{2}\pi \sqrt{2\left(\frac{0.18 \cdot 10^{-3}}{0.38 \cdot 10^{-3}}\right)^2}}{\frac{1}{2}\pi (0.38 \cdot 10^{-3})^2}\right)^2}$

$\frac{I_1}{I_{tot}} = \frac{6.55 \cdot 10^6 \pm 4.64 \cdot 10^4 \, mW/m^2}{4.38 \cdot 10^8 \pm 8.38 \cdot 10^6 \, mW/m^2} = 4.694 \pm 4.64 \cdot 10^{-4} \, mW/m^2$

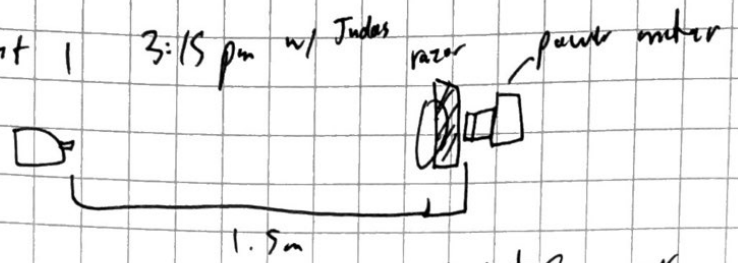
$\frac{I_1}{I_{tot}} =$

C.) Beam Divergence

Ex IV.13 Measure Beam Diameter at large distance and close to out put port to find Beam divergence

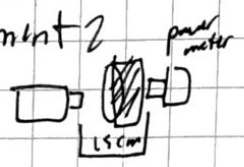
Divergence = $2\Delta\theta$

Experiment 1 3:15 pm w/ Judas



~~Data~~ ~~P_{tot} = 711 mW~~

Experiment 2



① ~~Start~~ by measuring total beam power, then closing the razor until it reaches a percentage of the original power. The distance from the first edge where it ~~starts~~ decreasing to the fraction, is the beam diameter. By measuring this from far away and up close we can measure the beam ~~div~~ divergence

x_{edge} = 97.7% P_{tot} x_{frac} = 22.8%

1. Data P_{tot} = 711 μW

d_1 = 1.5m x_{Edge} = 10.45 mm P_{edge} = 694 μW D = 1.95 mm

$x_{fraction}$ = 8.50 mm P_{frac} = 16.2 μW σ_x = 0.05 mm

2. Data P_{tot} = 756 μW

d_2 = 0.0015m x_{edge} = 10.94 mm P_{edge} = 738.612 μW

x_{frac} = 10.14 mm P_{frac} = 17.28 μW D = 0.8 mm

Analysis

1.) distance of 1.5m

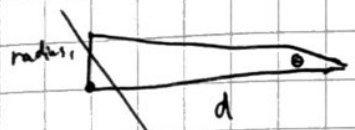
measured beam diameter = 1.95 mm = 10.45mm - 8.50 mm

$\sigma_{diameter} = \sqrt{(0.05)^2 + (0.05)^2} = \pm 0.07$ mm

cont.

Beam diameter$_1$ = 1.95 ± 0.07 mm

radius$_1$ = 0.98 ± 0.035 mm



$\tan\theta = \frac{rad_1}{d}$ $\quad \theta_1 = \tan^{-1}\left(\frac{rad_1}{d}\right)$ $\quad \theta_1 = 0.037°$

$\sigma\,\theta_1 = \tan^{-1}\left(\sigma\,\frac{rad_1}{d}\right)$ ~~$\sigma\left(\frac{rad_1}{d}\right) = \pm 0.0506$~~

~~$\sigma\left(\frac{rad_1}{d}\right) = \sqrt{\left(\frac{.035\,mm}{0.98\,m}\right)^2 + \left(\frac{.07\,mm}{1.95\,mm}\right)^2} = \pm 0.0506 \cdot \left(\frac{rad_1}{d}\right)$~~

~~$= 0.00254$~~

~~$\sigma\,\theta = \tan^{-1}(0.0503 \pm 0.00$~~

~~$\sigma\,\theta = \tan^{-1}(0.0503 + 0.00254) - \tan^{-1}(0.0503 - 0.00254)$~~

$$\sigma\left(\frac{rad_1}{d}\right) = \frac{0.98 \cdot 10^{-3}\,m}{1.5\,m} \cdot \sqrt{\left(\frac{0.035 \cdot 10^{-3}\,m}{0.98\,m}\right)^2 + \left(\frac{(0.5 \cdot 10^{-3})}{1.5\,m}\right)^2}$$

$$\sigma\,\frac{rad_1}{d} = \pm\, 2.33 \cdot 10^{-5}$$

$$\sigma\,\theta_1 = \tan^{-1}(6.533 \cdot 10^{-4} + 2.33 \cdot 10^{-5}) - \tan^{-1}(6.533 \cdot 10^{-4})$$

~~$= \pm 0.037$~~

$\sigma\,\theta_1 = \pm\, 0.00134°$ $\qquad \theta_1 = 0.037° \pm 0.00134°$

$= 0.645 \pm 0.0233$ mili rad

2.) Beam Diameter 2 = 10.94 mm - 10.14 mm = 0.8 mm

σ diam$_2$ = ~~$\pm$ 0.0 mm~~ $\pm$ 0.07 mm

Radius 2 = 0.4 mm $\pm$ 0.035 mm $\qquad d = 0.015$ m

$\tan\theta_2 = \frac{rad_2}{d}$ $\qquad \theta_2 = \tan^{-1}\frac{rad_2}{d} = 1.53°$

$\sigma\,\theta_2 = \tan^{-1}(2.66 \cdot 10^{-2} + 2.496 \cdot 10^{-3}) - \tan^{-1}(2.66 \cdot 10^{-2}) = \pm\, 0.1429°$

$\theta_2 = 1.53 \pm 0.143°$ = ~~$26.7 \cdot 10^{-3}$~~ 26.7 ± 2.50 mili rad

cont →

~~Beam divergence = $2\Delta\theta$~~

$$\cancel{= 2\cdot\left((26.7 - 0.645) \pm \sqrt{(0.0237)^2 + (2.5)^2}\right)}$$

$$\cancel{= 2\cdot(26.01 \pm 2.5)}$$

~~Divergence = 52.1 ± 5.0 milliRad~~

Note: Calculated beam divergence incorrectly, was thinking about it in terms of linear measurements not the beam diverging over distance

from this the new formula for beam divergence is

$$\theta = 2\tan^{-1}\left(\frac{r_{beam_1} - r_{beam_2}}{x_1 - x_2}\right) \approx 2\cdot\left(\frac{r_{beam_1} - r_{beam_2}}{x_1 - x_2}\right)$$

r_{beam_1} = ~~1.95 ± 0.07 mm~~ 0.98 ± 0.04 mm

$r_{beam_2} = 0.4 \pm 0.04$ mm

$x_1 = 1.5\,m \pm 5\cdot10^{-4}\,m$

$x_2 = 0.015\,m \pm 5\cdot10^{-4}\,m$

Divergence = 2·0.78 milli rads = 1.56 mrad

Error Analysis

$$\sigma_{num} = \sqrt{2\cdot(0.04)^2} = \pm 5.65\cdot10^{-5}\,m$$

$$\sigma_{den} = \pm 7.07\cdot10^{-5}\,m$$

$$\sigma_{quo} = 3.91\cdot10^{-4}\left(\sqrt{\left(\frac{5.65\cdot10^{-5}}{5.5\cdot10^{-4}}\right)^2 + \left(\frac{7.07\cdot10^{-4}}{1.4985}\right)}\right) = \pm 3.81\cdot10^{-5}$$

$$\sigma_{divergence} = 2\left(\tan^{-1}(3.91\cdot10^{-4} + 3.81\cdot10^{-5}) - \tan^{-1}(3.91\cdot10^{-4})\right)$$

$$= \pm 7.619\cdot10^{-5}\text{ rad} = (\pm 0.09\text{ milliRad})\,2 = \pm 0.16\text{ mrad}$$

~~Beam div = 0.78 mm ± 0.08 milliradians~~

Beam divergence = 1.56 mrad ± 0.16 mrad

Ex. IV.14

From Fraunhofer diffraction

$$\Delta\theta = 1.22\left(\frac{\lambda}{D_0}\right)$$

compare with mesured divergence

$\lambda = 632.8$ nm $\quad D_0 =$ ~~0.80 mm ± 0.07 mm~~

$= 0.0008 \pm 0.00007$ m

~~$\Delta\theta = 1.22 \cdot \frac{632.8}{(0.8 \pm 0.07)}$~~

$$\Delta\theta = 1.22 \cdot \frac{632.8 \text{ nm}}{8 \cdot 10^{-4} \text{ m}}$$

~~= 965.02~~

$$\Delta\theta = 9.65 \cdot 10^{-4} \pm 7.76 \cdot 10^{-5} \text{ rad}$$

$$= 0.965 \pm 0.077 \text{ milliRad}$$

the theoretical value is larger than our measured value

Also we could not find any record for the manufacturer's value for the divergence of the Laser

D) Beam Waist

for TEM_{00} mode beam waist d and collimated distance L are given by

$$d = \left(\frac{4\lambda}{\pi}\right)\left(\frac{f}{D_0}\right)$$

$$L = 2\left(\frac{4\lambda}{\pi}\right)\left(\frac{f}{D_0}\right)^2$$

w/ D_0 initial beam diameter & f = focal length of lens

IV.15 $\quad D_0 = 0.8 \pm 0.07$ mm

Note: We couldn't find the focal length f for our lens anywhere. Asked the TA and she didn't know either so we'll come back to this.

cont. We decided to measure the focal length ourselves and found it was approximately 10cm, with an uncertainty of around ±0.5 cm

$d = \left(\frac{4\lambda}{\pi}\right)\left(\frac{f}{D_0}\right) = 1.07 \times 10^{-4}$ m $\qquad L = 2\left(\frac{4\lambda}{\pi}\right)\left(\frac{f}{D_0}\right)^2 = 0.0252$ m

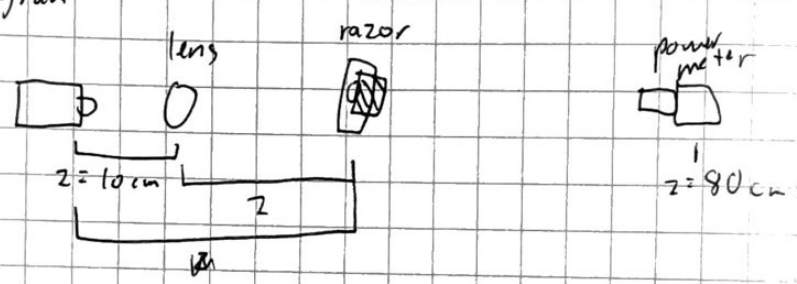
Ex IV.16 Pass laser through lens w/ focal length f = and measure beam Diameter D(z) at a number of axial positions

Plot diameter as a fcn of z & compare w/ prediction

Note: We're starting this before we know our focal length of lens this shouldn't affect our data or measurements but we cannot compare our results w/o it.

~~Procedure:~~

Diagram



Procedure

① lens 10cm away Power meter 80cm away
Vary position of razor measuring diameter of beam each time with distance from lens.

② Measure the beam diameter by taking the difference of the point where power is at 97.7% and 2.28% using the method from earlier.

③ By varying the position of the razor we can collect data of how the beam changes with the axial distance

Notes $\sigma z = \pm 0.005$ m (metersticK)

$\sigma x = \pm 0.00005$ m $= \pm 5 \cdot 10^{-5}$ (micrometer)

$\sigma D = \pm 0.0001$ m $= \pm 10^{-4}$ m (difference of x_1 & x_2)

start 5:20pm w/ Judas $P_{tot} = 739\ \mu W$ $P_1 = 0.977\ P_{tot} = 722\ \mu W$
$P_2 = 0.0[illegible]\ P_{tot} = 16\ \mu W$

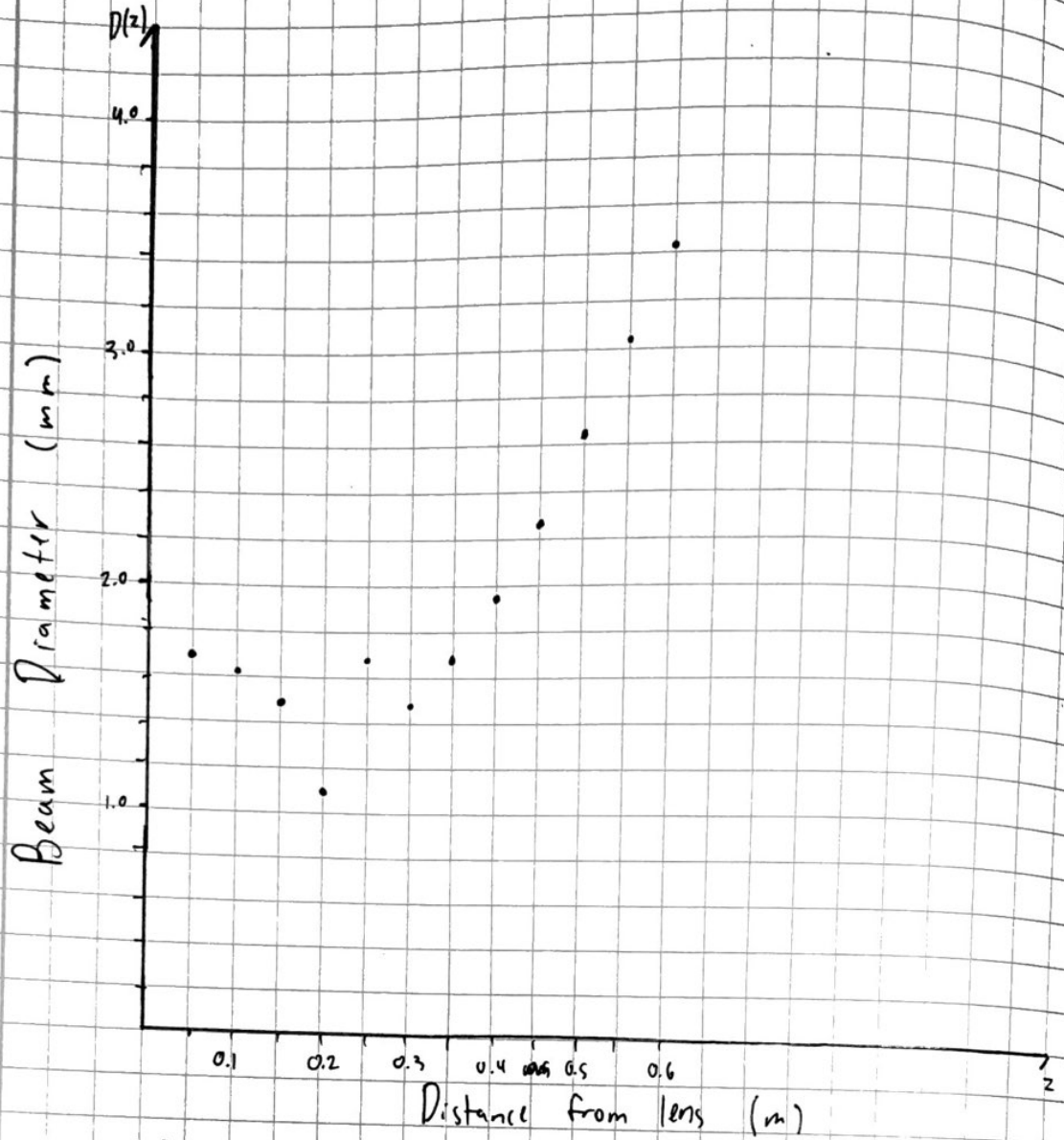
Z (m)	x_1 (mm)	x_2 (mm)	D(Z) (mm)
0.05 m	12.50	10.82	1.68
0.10	12.69	11.07	1.62
0.15	12.60	11.11	1.49
~~0.20~~	~~12.01~~	~~10.60~~	~~1.41~~
0.20	12.14	11.03	1.11
0.25	12.41	10.74	1.67
0.30	11.92	10.45	1.47
0.35	11.97	10.30	1.67
0.40	12.05	10.10	1.95
0.45	12.16	9.90	2.26
0.50	12.50	9.87	2.63
0.55	12.60	9.56	3.04
0.60	12.75	~~10.30~~ 9.32	~~3.43~~ 3.43

From our data the beam waist is between 0.10 m and 0.25 m so we're going to take more resolution in these pts.

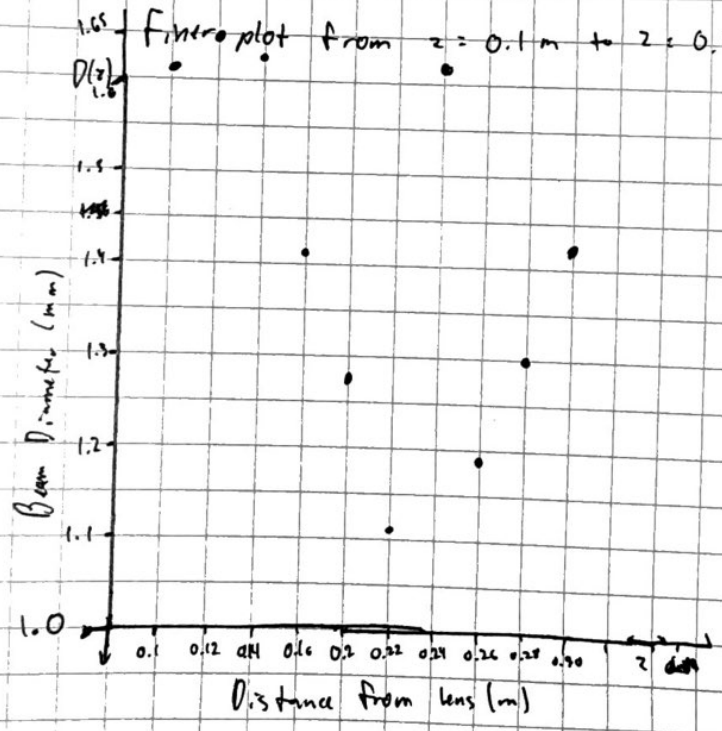
Z (m)	x_1 (mm)	x_2 (mm)	D(Z) (mm)
0.12	12.78	11.13	1.65
0.14	12.79	11.15	1.64
0.16	12.55	11.14	1.41
0.18	12.41	11.14	1.27
0.20	12.23	11.12	1.11
0.22	12.02	10.39	1.63 - ☆
0.24	11.97	10.78	1.19
0.26	11.90	10.6	1.30
0.28	11.98	10.55	1.43

IV. 16 cont'd

Plot D(z) vs. z



Finer plot from z = 0.1 m to z = 0.3 m



IV.16 cont→

Comparison with Prediction from IV.15

IV.15 1st Error propogation for d, minimum beam waist, and L, the depth of field

$$\delta d = |d| \cdot \sqrt{\left(\frac{\delta f}{f}\right)^2 + \left(\frac{\delta D_0}{D_0}\right)^2}$$

$$\delta d = 1.01 \times 10^{-4}\,m \cdot \sqrt{\left(\frac{.005\,m}{0.1\,m}\right)^2 + \left(\frac{7 \times 10^{-5}\,m}{8 \times 10^{-4}\,m}\right)^2}$$

$$\delta d = 1.02 \times 10^{-5}\,m$$

$$\boxed{d = 1.01 \times 10^{-4} \pm 1.02 \times 10^{-5}\,m}$$

$$\delta L = |L| \cdot \sqrt{2\left(\frac{\delta f}{f}\right)^2 + 2\left(\frac{\delta D_0}{D_0}\right)^2}$$

$$\delta L = (0.0252\,m) \cdot \text{"} \quad \text{"}$$

$$\delta L = 3.59 \times 10^{-3}\,m$$

$$\boxed{L = 0.0252 \pm 0.0036\,m}$$

From our measurements, we ~~[illegible]~~ measured a minimum beam waist of 1.1 mm at ≈ 0.2 m away from the lens.

The ~~field of depth~~ depth of field length was from 0.2 m to 0.22 m or about 2 cm which is fairly close to our calculated prediction.

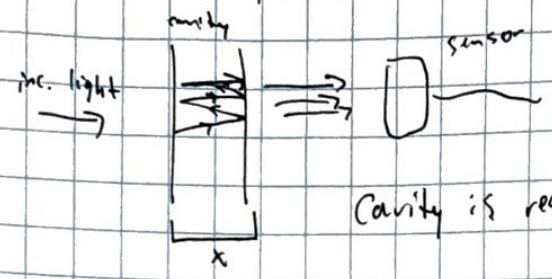
However our minimum beam waist was much higher than the prediction at almost a full order of magnitude larger

It may be due to the divergence of the laser beam as we had our lens around 10 cm away from the exit aperture and not right in front of it, where we made our measurement for Initial beam diameter.

Another possible source of error could be from how we kept the power meter at 0.8 m away from the lens instead of moving it along the beam axis w/ the razor edge. This may have led to some beam divergence past the razor edge and effected the accuracy of our Diameter measurements by altering the power meter readings

E) FABRY-PEROT Interferometer

- Actually a spectrometer that shows intensity distribution of laser light as a fcn of frequency on oscilloscope



$\lambda = 632.8$

Cavity is resonant for $x = \frac{n\lambda}{2}$ n = any Integer

Think #1 The intensity graph will be periodic because as the distance x increases it will eventually reach another half wave length and resonate again leading to a spike in intensity.

As x goes from $\frac{n\lambda}{2}$ to $\frac{(n+1)\lambda}{2}$ ~~at~~

$I\left(\frac{n\lambda}{2}\right)$ will equal $I\left(\frac{(n+1)\lambda}{2}\right)$ and in between it will be decreased because of interference of the waves.

#2 Better for monochromatic wavelengths because the wavelengths are bunched closer together, meaning the distance between the mirrors, x, will be closer for each wavelengths resonant distance. ~~the of the~~.
If the wavelengths are more spread out than the resonant peaks of each individual frequency will be more spread out and have different patterns of interference as the x distance varies.

Next Set up FP Interferometer

- Turn on ~~the~~ Laser and Hook up Analyzer & controller
- Align Laser
- Configure waveform on oscilloscope to show resonant peaks and ~~transverse~~ longitudinal modes
- Print out trace of scope showing modes and sweep voltage.

Measurements & Analysis

We can use the Voltage of the sweep fcn to convert to the distance x between the mirrors

from the Data sheet

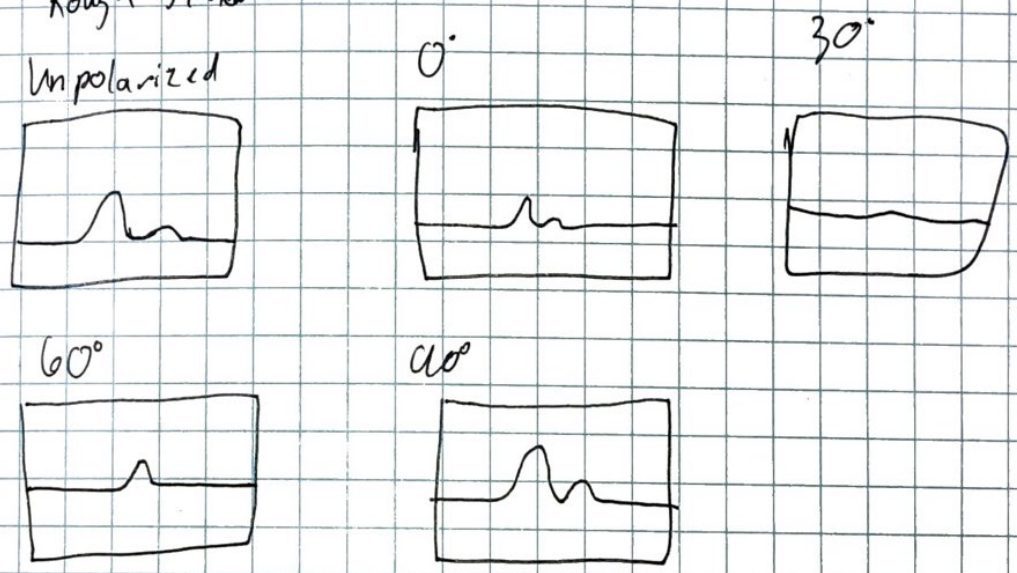
$\sim 4 nm / Volt$

Sweeps through 0 - 250 V

5. Mode Polarization

~~Find~~ Determine Polarization of Individual modes
Place Polarizer in front of spectrum analyzer
Save waveforms of modes at each different angle of the Polarizer to USB. Take every 30°.

Rough Sketches



6. Mode Spacing

- free Spectral range of 7.5 GHz while ramp input is swept from 0 to 40 V

To estimate maximum change in length of FB cavity
Amplitude of ramp is 250 V so our free spectral range is $6.25 \cdot 7.5\ GHz = 47\ GHz$
So the maximum change in length is $\frac{c}{f} = \boxed{6.4\ mm}$ cont.

from the scope cursors we can see the first resonant mode is at -50V on our ramp sweep and the next resonant mode is at 25V

so the approximate mode spacing in Volts is 75V ±2V

from the Spectral Analyzer Data sheet the conversion is a 4 nm/Volt

so mode spacing is 75 V · 4 $\frac{nm}{V}$ = 300 nm which makes sense because this is approximately $\lambda/2$

mode spacing = 300 ± 8 nm

for the FWHM of the Doppler Broadened gain curve we use the scope cursors to find the difference in Voltage between each side of the resonant peak

$$\Delta V = 8.00\,V \pm 2\,V$$

to convert to frequency and wavelength

$$\Delta f = \frac{7.5\,GHz}{40\,V} \cdot \Delta V = 1.50\,GHz \pm 0.38\,GHz$$

$$\boxed{\Delta f = 1.50 \pm 0.38\,GHz}$$

which is close to our previously calculation in section III.5

to find $\Delta\lambda$ we use

$$\Delta f = -(c/\lambda^2)\,\Delta\lambda$$

$$\Delta\lambda = -\left(\frac{\lambda^2}{c}\right)\Delta f$$

$$\boxed{\Delta\lambda = 2.0\times10^{-12} \pm 0.5\times10^{-13}\,m}$$

Coherence length $L_c = \frac{c}{\Delta f} = 0.199 \pm 0.040\,m$

which is fairly close to our first calculation of $L_c = 0.2326\,m$ in III.5

from our scope caps we saw clusters @ 1 div apart w/ div. = 2.5 ms
When zoomed into one cluster we saw ~~that~~ from the .csv data ~~that~~ the mode maximums were 1.036 divisions apart w/ div. = 2.5 ms

So the mode spacing is approximately $1.036 \text{ div} \cdot \frac{0.75 \text{ GHz}}{\text{div.}}$

= 777 MHz mode spacing

E) Zeeman Effect

Magnetic field changes ~~frequency distribution~~ energy levels of gases in laser changing the distribution of frequencies & the polarization of the modes.

Note: Couldn't find 2nd magnet to set up magnetic field across Laser Beam. ~~Could~~ did not get to take any measurements.

Exercise IV.11 & IV.12

I've decided to rework these problems here because I need the space to fully work this out the right way

~~[illegible]~~

IV.11 When E field decayed to 1/e of maximum that is the beam diameter

$$P \propto |E|^2 \qquad E\text{ field} \cdot \frac{1}{e} \rightarrow \frac{1}{e^2} \cdot \text{Power} \Rightarrow \vec{r} = R$$

$$P(r) = 2\pi \int_0^R I_0 r e^{-2r^2/R^2} dr$$

$$= \frac{P(R)}{P_0} = \frac{2\pi I_0}{\frac{\pi}{2} R^2 I_0} \int_0^R r e^{-2r^2/R^2} dr$$

$$\frac{P(R)}{P_0} = \frac{4}{R^2} \cdot \frac{R^2(e^2-1)}{4e^2}$$

$$\text{Fraction of Power Inside Beam} = \frac{e^2-1}{e^2} = \boxed{86.47\ \%}$$

Exercise IV.12 How much is Intensity reduced when Razor edge is one radius away from the beam axis?

How much when Razor edge is one Radius beyond the beam axis?

1. Razor edge at one radius before the beam axis

$$P_1 = I_0 2\pi \int_A r e^{-2r^2/R^2} dr$$

$$P_1 = I_0 \int_{-\infty}^{\infty} e^{-2y^2/R^2} dy \cdot \int_{-R}^{\infty} e^{-2x^2/R^2} dx$$

$$P_1 = I_0 \cdot \sqrt{\frac{\pi}{2}} R \cdot \frac{1}{2}\sqrt{\frac{\pi}{2}}\left(R + \mathrm{erf}(\sqrt{2})R\right)$$

$$P_1 = I_0 \frac{\pi}{4} R^2 \left(1 + \mathrm{erf}(\sqrt{2})\right)$$

as a fraction of total Power, P_{tot}, where you can find

$$P_{tot} = I_0 \int_{-\infty}^{\infty} e^{-2y^2/R^2} dy \cdot \int_{-\infty}^{\infty} e^{-2x^2/R^2} dx = \frac{1}{2} I_0 \pi R^2$$

$$\frac{P_1}{P_{tot}} = \frac{\cancel{I_0} \frac{\pi}{4} \cancel{R^2}\left(1 + \mathrm{erf}(\sqrt{2})\right)}{\frac{1}{2}\cancel{I_0}\cancel{\pi}\cancel{R^2}} = \frac{1}{2}\left(1 + \mathrm{erf}(\sqrt{2})\right)$$

$$= 0.97725$$

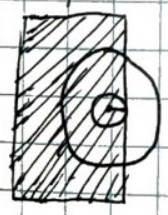
$$= \boxed{97.73\%}$$

When applied to our measured P_{tot} of

$P_{tot} = 741 \pm 7.41\ \mu W$

$$\boxed{P_1 = 724 \pm 7.24\ \mu W}$$

2.) Razor Edge at one Radius beyond the beam axis



$$P_2 = I_0 \int_{-\infty}^{\infty} e^{-2y^2/R^2} dy \cdot \int_{R}^{\infty} e^{-2x^2/R^2} dx$$

$$P_2 = I_0 \sqrt{\frac{\pi}{2}} R \cdot \frac{1}{2}\sqrt{\frac{\pi}{2}} R\left(1 - \mathrm{erf}(\sqrt{2})\right) = \frac{\pi}{4} R^2 I_0 \cdot \left(1 - \mathrm{erf}(\sqrt{2})\right)$$

$$\frac{P_2}{P_{tot}} = \frac{\frac{\cancel{\pi}}{4} \cancel{R^2} \cancel{I_0}\left(1 - \mathrm{erf}(\sqrt{2})\right)}{\frac{\cancel{\pi}}{2} \cancel{R^2} \cancel{I_0}} = \frac{1}{2}\left(1 - \mathrm{erf}\sqrt{2}\right) = 0.02275$$

$$= 2.28\%$$

when applied to the total Power measured

$$P_{tot} = 741 \pm 7.41\ \mu W$$

$$\boxed{P_2 = 168 \pm 0.168\ \mu W}$$

So to measure the beam diameter, first find the total beam power by measuring the beam unobstructed. Then move the razor edge until the measured power is 97.7% of the total. This position is the first edge of the beam. Next move the razor edge until the measured power is 2.3% of the total. This is the far edge of the beam. The difference of these two positions is the beam diameter.

Discussion and Conclusion

This lab was difficult for us and we had several setbacks and had to re-do several experiments due to initial mistakes and errors. Eventually we got onto the right track.

Our ultimate results from our experiments are

Degree of Polarization $\rho = 0.9955 \pm 6.28 \times 10^{1}$

Beam Profile $R = 0.75 \pm 0.18$ mm

$I_0 = 8.38 \times 10^{2} \pm 8.38 \times 10^{6}\ \frac{\mu W}{m^2}$

Initial Beam Diameter

$D_0 = 0.80 \pm 0.07$ mm

Beam Divergence 1.56 ± 0.16 mrad

Beam waist minimum

$d = 411 \pm 0.1$ m

Mode Spacing 777 MHz

Compare these to expected values from manual & specs

Ideal

$\rho = 0.998$

$D_0 = 0.48$ mm

Divergence $= 1.7 \pm 0.051$ mrads

Beam waist $d = 0.101$ mm

Mode spacing 1090 MHz

Ultimately our results were pretty far from ideal and we only came close on the polarization and divergence. There could have possibly been some error with our method of measuring beam diameter or maybe the beam divergence skewed our power meter readings.

In conclusion this lab taught me some very important ideas & techniques for experimental procedure and error analysis & propagation.

If I could re-do this lab I would slow down our pace and take more detailed measurements and notes on the experiment setup and procedure and do more of the difficult analysis and error propagation outside of lab time if possible.