Experiments with Michelson Interferometer $Lab\ Report\ \#4$

Landon Osborne (7159452) - *Physics BS*December 3rd, 2022

Abstract

Using the Michelson interferometer three experiments were performed, measuring the wavelength of a Helium Neon laser, determining the relationship between air pressure and the index of refraction of air, and measuring the index of refraction of a glass plate. The results include a measured wavelength of $0.6456 \pm 0.0187~\mu m$ for the He-Ne laser, a linear relation between air pressure and the index of refraction of air with a slope of $3.100 \times 10^{-6} \pm 7.9 \times 10^{-8} {\rm cmHg^{-1}}$ from which the index of refraction of air at atmospheric pressure was calculated to be $1.000236 \pm 5.604 \times 10^{-6}$. Finally the index of refraction of glass was found to be 1.507 ± 0.017 for the glass plate used in our experiment. Comparing these to the expected values of $0.6328~\mu m$ for the wavelength of He-Ne lasers in air, an index of refraction of 1.000293 for air at atomospheric pressure [1], and a index of refraction of 1.52 for window glass [1], it can be seen that the actual values are within the range of uncertainty for the measurements taken in the experiments.

Introduction

The Michelson interferometer is an optical interferometer setup that can be utilized to generate interference patterns. It functions byfocusing a light source through a lens and then using a beam splitter to separate the light into two beams. By reflecting these beams back at the beam splitter their amplitudes can be recombined because of the principle of superposition. A diagram of the Michelson interferometer used can be seen below in figure 1.

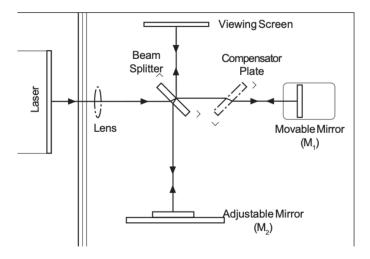


Figure 1: Diagram of the Michelson Interferometer used in all experiments from the PASCO scientific manual [2].

If the light source used has a highly correlated phase, such as a laser beam, then any difference in the lengths of the distance travelled by each split beam will cause them become out of phase with each other. This causes an interference pattern to become visible. The Michelson interferometer generates an interference pattern of alternating light and dark rings, where the light rings are referred to as fringes. A drawing of such an interference pattern is visible below in figure 2.



Figure 2: Visualization of the pattern the interference fringes create on the viewing screen from the PASCO scientific manual [2].

If the distance that one of the light beams travels changes, it will alter the interference pattern produced by causing the circular fringes to move radially. The number of fringes that pass a given point is directly related to how much the path length of the laser beam changes. The change in path length that causes the pattern move one fringe radially to the position of the next and return to its original pattern is exactly one half of the light's wavelength, because the light must travel the path twice. Using this principle the changes in the interference pattern can be used to determine the wavelength of the light source and the index of refraction of any medium the beam passes .

Experimental Methods

Wavelength of He-Ne Laser

For the first experiment the principles outlined above were used to measure the wavelength of the Helium-Neon laser that was the interferometer's light source. First setup the interferometer as shown above in figure 1 and align the laser beams to produce an interference pattern on the viewing screen. Next align an increment on the screen with the edge of one of the fringes. Then move Mirror #1 by rotating the micrometer knob and count the number of fringes that pass. By recording the distance the mirror moved, d, and the number of fringes that passed, N, the wavelength can be calculated with the formula

$$\lambda = \frac{2d}{N} \tag{1}$$

In order to reduce uncertainty in this measurement there are two items to consider. First is backlash, which is the slight mechanical slippage that occurs whenever the micrometer changes direction. To eliminate this source of uncertainty, make sure to turn the micrometer one full rotation before making any distance measurements and continue to turn it in the same direction while taking the measurement.

Second the micrometer must be properly calibrated. To do this turn the micrometer knob a set number of fringes and record the distance as d'. The actual distance moved, d, is equal to $N\lambda/2$, given the known wavelength of the He-Ne laser which is $0.6328\mu m$. Multiplying any distance measurements taken by d/d' will give the accurate calibrated distance.

Refraction index of air at varying pressures.

For this experiment the pressure of the air that one of the light beam travels through will be changed by using a vacuum cell. This change in pressure will alter the path length the light takes through the cell, altering the interference pattern on the viewing screen. A diagram of the setup for this experiment can be seen below in 3

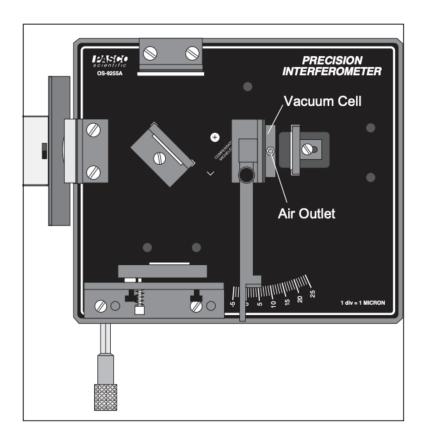


Figure 3: Diagram of the Michelson Interferometer set up to measure the index of refraction of air from the PASCO scientific manual [2].

By pumping air out of the vacuum cell and counting the number of fringes that pass an increment on the viewing screen, the relation between pressure and index of refraction can be calculated. Assuming the relation between the index of refraction and air pressure is linear, the slope of the line, m, can be found with the formula

$$m = \frac{n_i - n_f}{P_i - P_f} = \frac{N\lambda_0}{2d(P_i - P_f)} \tag{2}$$

The index of refraction for a given pressure, P is then given by

$$n = mP + 1$$

where the y-intercept is the index of refraction of a vacuum.

Refraction index of glass

Using a similar method to the one outlined above the index of refraction of glass can be calculated by varying the distance the light has to travel through a glass plate. This is accomplished by mounting the glass plate on a rotating stand in the path of the beam, as shown in the diagram below in figure 4

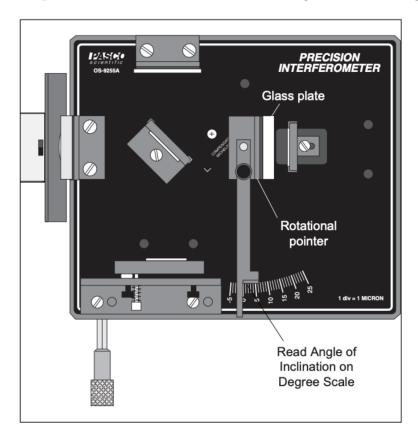


Figure 4: Diagram of the Michelson Interferometer set up to measure the from the PASCO scientific manual [2].

Rotating the glass plate will cause the beam to pass through more glass and alter the interference pattern produced on the viewing screen. By counting the number of fringes that pass after rotating the plate by an angle θ the refraction index of glass, n_g , can be calculated from the following formula

$$n_g = \frac{(2t - N\lambda_0)(1 - \cos\theta)}{2t(1 - \cos\theta) - N\lambda_0}$$
(3)

where t is the thickness of the glass plate, and λ_0 is the wavelength of the light in a vacuum, $0.632991\mu m$ for a He-Ne laser.

Raw Data

Wavelength of He-Ne Laser

Prior to starting this experiment we found a value for the calibration of the micrometer to be d/d' = 0.98875. The raw data taken from this experiment can be seen in table 1 below.

Trial	N	$d_{meas} (\mu m)$
1	25	8.0
2	25	8.0
3	30	10.0
4	30	10.2
5	25	8.0
6	20	6.5
7	20	6.5

Table 1: Table of data for the number of interference fringes, N, counted as Mirror 1 moved the distance d_{meas} . Note that these values for distance are uncalibrated.

For this experiment the uncertainty in the distance measurement is $\delta d = \pm 0.015 |d| \pm 0.5~\mu \mathrm{m}$ due to the distance displayed on the micrometer only being rated to within 1.5% of the actual distance traveled by the mirror and an additional 0.5 $\mu \mathrm{m}$ of uncertainty from the smallest increment on the micrometer.

Refraction index of air at varying pressures

The raw data taken from this experiment can be seen in table 2 below.

Trial	$P_i(\text{cmHg})$	$P_f(\text{cmHg})$	N
1	76	35.359	12
2	76	22.659	16
3	76	34.089	12

Table 2: Number of interference fringes, N, counted as the pressure in the vacuum cell decreased from P_i to P_f . 76 cmHg is the pressure of air at sea level and was used as the initial setting for the vacuum cell.

For this experiment the uncertainties were $\delta P = \pm 1.27 \text{cmHg}$ and a distance of the of the vacuum cell of $d = 3.0 \pm 0.1$ cm. Both of these uncertainties come from the smallest increments of measurement on the instruments.

Refraction index of glass

The raw data taken for this experiment can be seen in table 3 below.

Trial	$\Delta\theta$ (°)	N	n_g
1	10	98	1.508
2	10	95	1.485
3	10	101	1.532
4	10	96	1.492
5	10	99	1.516

Table 3: The index of refraction from each trial of the experiment calculated from the change in the angle of the glass plate, $\Delta\theta$, and the number of interference fringes that passed, N.

Results

Wavelength of He-Ne Laser

From the data displayed in table 1 the calibrated distance and wavelengths for each trial were calculated via equation 1 and are displayed in table 4 below.

Trial	$d (\mu m)$	$\lambda(\mu m)$
1	7.91	0.6328
2	7.91	0.6328
3	9.89	0.6593
4	10.13	0.6753
5	7.91	0.6328
6	6.43	0.6430
7	6.43	0.6430

Table 4: Table of data showing the calibrated mirror distance, d, for each trial. From this and the number of interference fringes counted for each trial in Table 1 the wavelength of the laser beam was calculated.

The uncertainty in the wavelength measurements was found with the formula

 $\delta\lambda = \frac{2}{N}\delta d$

the values for each trial can be seen below in table 5 below.

Trial	$\delta\lambda(\mu\mathrm{m})$
1	0.0495
2	0.0495
3	0.0519
4	0.0522
5	0.0495
6	0.0477
7	0.0477

Table 5: Calculated values of the wavelength uncertainty for each trial.

From this the average wavelength and associate uncertainty were calculated to be $\lambda=0.6546\pm0.0187~\mu\mathrm{m}$, where the average uncertainty is given by the formula

$$\delta \lambda_{avg} = \frac{\sqrt{(\delta \lambda_1)^2 + (\delta \lambda_2)^2 (\delta \lambda_3)^2 + \dots}}{7}$$

Refraction index of air at varying pressures

From the data seen in table 2 the slope of the the relation of index of refraction to pressure were calculated with equation 2. The value for each trial can be

seen below in table 6.

Trial	Slope $(cmHg^{-1})$
1	3.115×10^{-6}
2	3.165×10^{-6}
3	3.021×10^{-6}

Table 6: Calculated slopes of the linear relationship between air pressure and the index of refraction of air for each trial.

From this the uncertainty in the slope was calculated with the formula

$$\delta m = |m| \sqrt{\left(\frac{\delta d}{d}\right)^2 + \left(\frac{\delta \Delta P}{\Delta P}\right)^2}$$

for which the values for each trial can be seen below in table 7.

Trial	$\Delta P \text{ (cmHg)}$	$\delta m (cmHg^{-1})$
1	40.641	1.42×10^{-7}
2	53.341	1.29×10^{-7}
3	41.911	1.36×10^{-7}

Table 7: Calculated values for the uncertainty in the slope of air pressure vs the index of refraction of air for each trial.

These values gave an average slope of $m=3.100\times 10^{-6}\pm 7.9\times 10^{-8}$. Using this slope the index of refraction at atmospheric pressure, 76 cmHg, is $n=1.000236\pm 5.604\times 10^{-6}$. This linear relation is visualized below in figure 5.

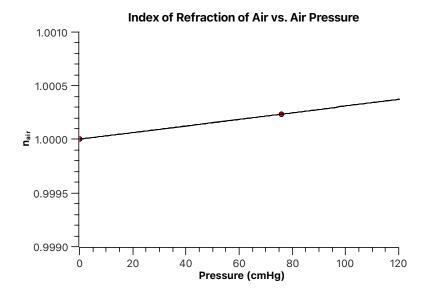


Figure 5: Graph showing the linear relation between air pressure and the index of refraction of air. At 76 cmHg the calculated index of refraction is n = 1.000236.

Refraction index of glass

From the data above the average index of refraction of glass was calculated to be $n_g=1.507$. The uncertaintiy in the index of refraction of glass was found by the standard deviation of the values for n_g given in table 3. The formula for the standard deviation is

$$\delta n_g = \sqrt{\frac{\Sigma (n_i - 1.507)^2}{5}}$$

This was found to be $\delta n_g = \pm 0.017$, so the final measured value for the refraction index of glass is $n_g = 1.507 \pm 0.017$.

Discussion

The accepted values for each of these experiments are 0.6328 for the wavelength of He-Ne lasers, 1.000293 for the refraction index of light at atmospheric pressure, and 1.52 for the refraction index of standard window glass,

which was the most likely material for the glass plate used in our exper-Comparing these to the measured values of $0.6546 \pm 0.0187 \mu m$, $1.000236 \pm 5.604 \times 10^{-6}$, and 1.507 ± 0.017 for each experiment respectively it can be seen that the actual expected values are within the range of uncertainty for the values of the measurements. Most of the uncertainty in the measurements came from the instruments used in the experiment, so this indicates that the experiments were accurate in measuring these various attributes. If I were to repeat this experiment, it would be interesting to attempt to use some sort of photo multiplier circuit to count the number of fringes. This could help in a number of ways, perhaps most importantly by allowing more fringes to be counted allowing for more accurate measurements. Because for these experiments all of the fringes were counted by eye, it meant that there was a limit to how many fringes could be counted accurately before losing track of the fringes due to eyestrain. By using a PHT circuit or camera to count the number of fringes it would make it easier to do more trials counting higher number of fringes, greatly reducing the uncertainty in the experiments.

References

- [1] "Refractive index," Nov 2022. [Online]. Available: https://en.wikipedia.org/wiki/Refractive_index
- [2] "Precision interferometer manual pasco scientific." [Online]. Available: https://cdn.pasco.com/product_document/ Precision-Interferometer-Manual-OS-9255A.pdf