Attitude Dynamics and Control of a Nano-Satellite Orbiting Mars

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Abstract

This document outlines the work completed for Tasks 1 through 6 of the ASEN 5010 Capstone Project. These tasks focus on simulating and analyzing the orbit and attitude dynamics of a nano-satellite in Mars orbit. Results are derived using a Python-based simulation framework and validated through analytical and numerical approaches.

Introduction

The project involves a nano-satellite in a circular Low Mars Orbit (LMO), performing various attitude maneuvers depending on whether it is in sunlit, communication, or science mode. The satellite must align its body-fixed reference frames accordingly using different pointing strategies such as sun-pointing, nadir-pointing, or GMO (Geostationary Mars Orbit) mothership communication alignment. Each of the following tasks contributes to a component of this overall attitude control simulation.

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1 Task 1: Orbit Simulation

Using circular orbital dynamics, the inertial position and velocity vectors of the satellite are computed. The Python implementation utilizes a function `orbit_sim()` that takes orbital elements and uses a 3-1-3 Euler rotation sequence to convert to inertial coordinates. The directional cosine matrix (DCM) is calculated via the function `Euler313toDCM()`.

LMO at $t = 450$ s

$$\vec{r}_{LMO} = \begin{bmatrix} -669.29 \\ 3227.50 \\ 1883.18 \end{bmatrix} \quad \text{(km)}, \quad \vec{v}_{LMO} = \begin{bmatrix} -3.256 \\ -0.798 \\ 0.210 \end{bmatrix} \quad \text{(km/s)}$$

GMO at $t = 1150$ s

$$\vec{r}_{GMO} = \begin{bmatrix} -5399.15 \\ -19697.64 \\ 0.0 \end{bmatrix} \quad \text{(km)}, \quad \vec{v}_{GMO} = \begin{bmatrix} 1.397 \\ -0.383 \\ 0.0 \end{bmatrix} \quad \text{(km/s)}$$

2 Task 2: Orbit Frame Orientation

This task calculates the orientation of the orbit frame O with respect to the inertial frame N. The transformation is done through a 3-1-3 Euler angle sequence using symbolic algebra in SymPy to represent $\theta(t)$, the true anomaly as a function of time. The DCM $[HN]$ is computed and evaluated at $t = 300$ s.

$$HN(t = 300s) = \begin{bmatrix} -0.0465 & 0.8741 & 0.4834 \\ -0.9842 & -0.1229 & 0.1277 \\ 0.1710 & -0.4698 & 0.8660 \end{bmatrix}$$

3 Task 3: Sun-Pointing Reference Frame Orientation

When the spacecraft is on the sunlit side (positive $\hat{n}_2$ coordinate), it must align its $+\hat{b}_3$ axis (solar panel normal) with the sun, assumed to be in the $+\hat{n}_2$ direction. To build an orthonormal frame, the -$\hat{n}_1$ axis is used to define $+\hat{b}_1$.

$$R_sN(t = 0s) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \vec{\omega}_{Rs/N} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4 Task 4: Nadir-Pointing Reference Frame Orientation

In science mode on the shadowed side of Mars, the satellite must point its sensor $(+\hat{b_1})$ directly toward the nadir direction (toward Mars center). The reference frame is constructed from the relative position vector and the local orbit track direction.

$$R_nN(t = 330s) = \begin{bmatrix} 0.0726 & -0.8706 & -0.4866 \\ -0.9826 & -0.1461 & 0.1148 \\ -0.1710 & 0.4698 & -0.8660 \end{bmatrix}, \quad \vec{\omega}_{R_n/N} = \begin{bmatrix} 0.000151 \\ -0.000416 \\ 0.000766 \end{bmatrix}$$

5 Task 5: GMO-Pointing Reference Frame Orientation

In communication mode, the satellite aligns its -b_1 (antenna direction) with the direction of the GMO mothership. The relative vector between the spacecraft and GMO is used to compute the DCM R_cN, and angular velocity $\omega_{R_c/N}$ is evaluated both analytically and numerically for verification.

$$R_cN = \begin{bmatrix} 0.2655 & 0.9609 & 0.0784 \\ -0.9639 & 0.2663 & 0.0 \\ -0.0209 & -0.0755 & 0.9969 \end{bmatrix}$$

$$\omega_{R_c/N}(\text{Numerical}) = \begin{bmatrix} 1.978 \times 10^{-5} \\ -5.465 \times 10^{-6} \\ 1.913 \times 10^{-4} \end{bmatrix}, \quad \omega_{R_c/N}(\text{Analytical}) = \begin{bmatrix} 1.978 \times 10^{-5} \\ -5.465 \times 10^{-6} \\ 1.913 \times 10^{-4} \end{bmatrix}$$

6 Task 6: Attitude Error Evaluation

Attitude error is evaluated using Modified Rodrigues Parameters (MRPs) and relative angular velocity between the body frame and reference frames. These are computed using functions like `DCM2MRP` and vector subtraction logic in `values.py`.

Sun-Pointing

$$\sigma_{B/R} = \begin{bmatrix} -0.7754 \\ -0.4739 \\ 0.0431 \end{bmatrix}, \quad \omega_{B/R} = \begin{bmatrix} 0.01745 \\ 0.03054 \\ -0.03840 \end{bmatrix}$$

Nadir-Pointing

$$\sigma_{B/R} = \begin{bmatrix} 0.2623 \\ 0.5547 \\ 0.0394 \end{bmatrix}, \quad \omega_{B/R} = \begin{bmatrix} 0.01685 \\ 0.03093 \\ -0.03892 \end{bmatrix}$$

GMO-Pointing

$$\sigma_{B/R} = \begin{bmatrix} 0.0170 \\ -0.3828 \\ 0.2076 \end{bmatrix}, \quad \omega_{B/R} = \begin{bmatrix} 0.01730 \\ 0.03066 \\ -0.03844 \end{bmatrix}$$

7 Task 7: Numerical Attitude Simulator

[Placeholder for Task 7 results.]

8 Task 8: Sun Pointing Control

[Placeholder for Task 8 results.]

9 Task 9: Nadir Poniting Control

[Placeholder for Task 9 results.]

10 Task 10: GMO Pointing Control

[Placeholder for Task 10 results.]

11 Task 11: Mission Scenario Simulation

[Placeholder for Task 11 results.]

12 Conclusion

This report presents a comprehensive simulation of a nano-satellite's orbit and attitude dynamics around Mars. Tasks 1-11 have been implemented and validated using a custom Python script.