Conversion from $M\omega$ to f (kHz)

We often encounter the f-mode frequency expressed in terms of the dimensionless product

$$M\omega,$$

where M is the gravitational mass in geometric units ($G = c = 1$). To convert to a physical frequency in kilohertz, we proceed as follows.

Step 1. Restore units

In geometric units, the solar mass corresponds to a time:

$$M_\odot = \frac{GM_\odot}{c^3} \approx 4.9254909 \times 10^{-6}\,\mathrm{s}.$$

For a neutron star of mass M, expressed in solar masses,

$$M\ [\mathrm{s}] = \left(4.9254909 \times 10^{-6}\,\mathrm{s}\right)\left(\frac{M}{M_\odot}\right).$$

Step 2. Angular to cyclic frequency

The physical frequency is

$$f = \frac{\omega}{2\pi}.$$

Substituting $\omega = (M\omega)/M$ gives

$$f\ [\mathrm{Hz}] = \frac{M\omega}{2\pi\, M[\mathrm{s}]}.$$

Step 3. Convert to kHz

Finally, converting to kilohertz,

$$f\ [\mathrm{kHz}] = \frac{M\omega}{2\pi\, M[\mathrm{s}] \times 10^3}.$$

Step 4. Numerical form

Inserting $M[\mathrm{s}] = 4.9254909 \times 10^{-6}(M/M_\odot)$, we obtain

$$f\ [\mathrm{kHz}]\ \approx\ \frac{M\omega}{3.093 \times 10^{-2}\,(M/M_\odot)}.$$

Equivalently,

$$\boxed{f\ [\mathrm{kHz}]\ \approx\ 32.35\,\frac{M\omega}{M/M_\odot}}.$$