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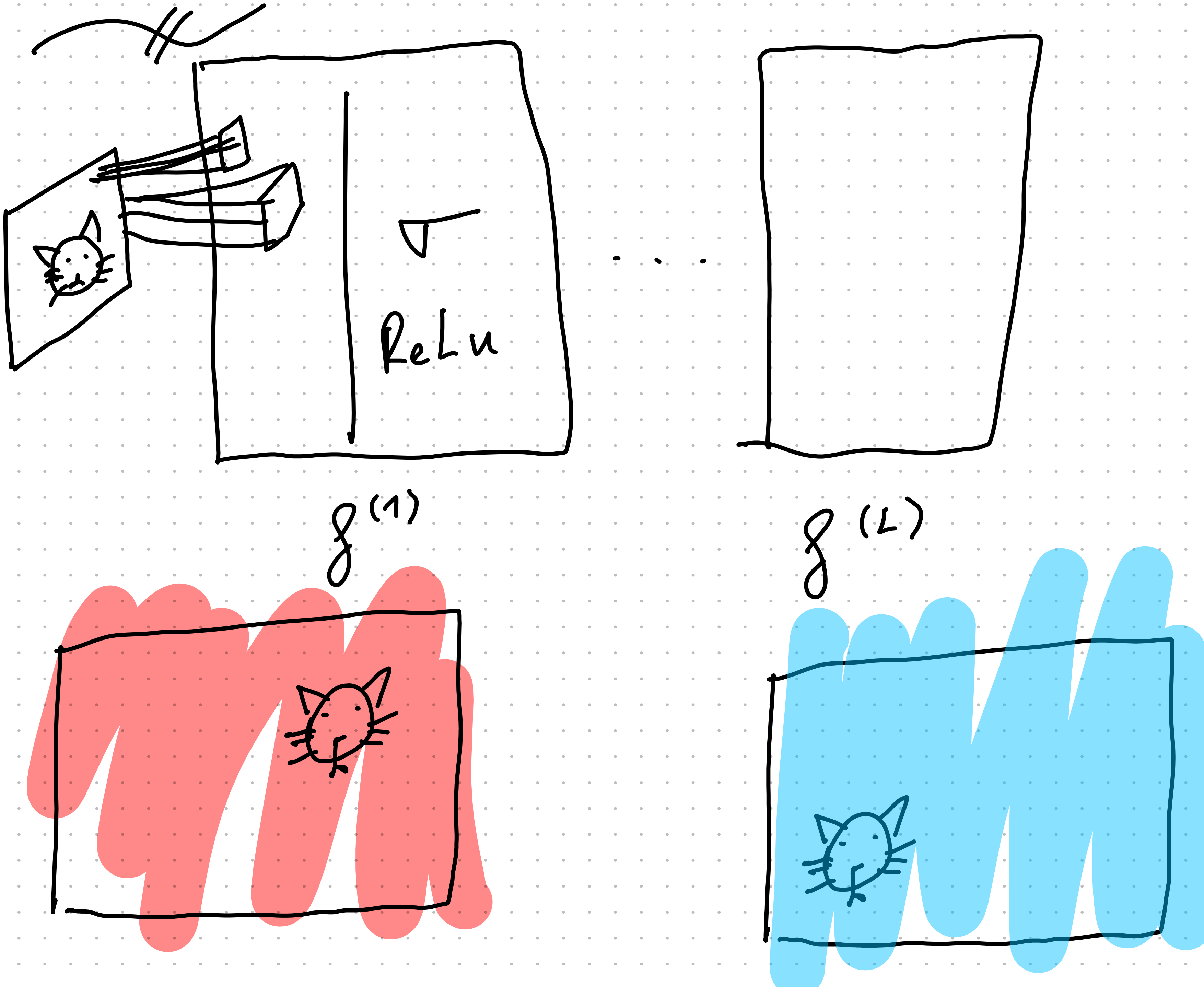
Glaze, Segarra

icml 21

CNNs

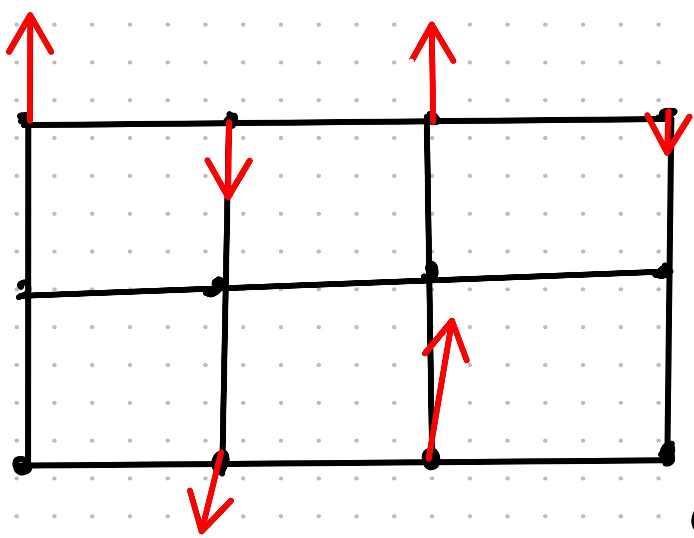


GNNs

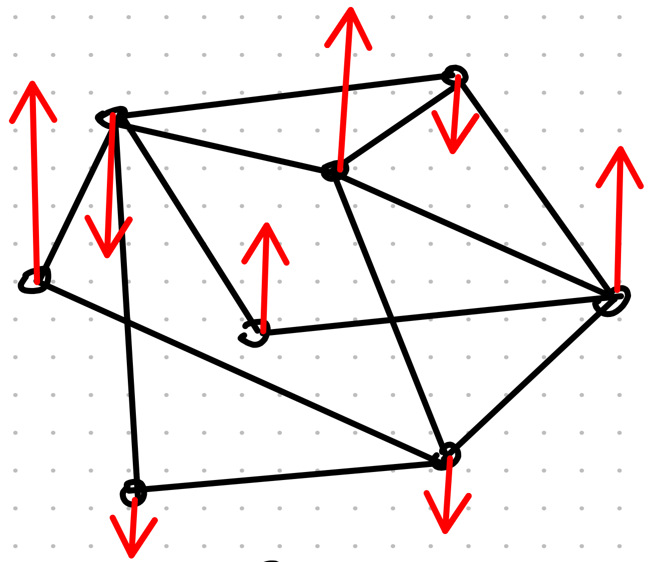
$$G = (V, E)$$

vertices

edges



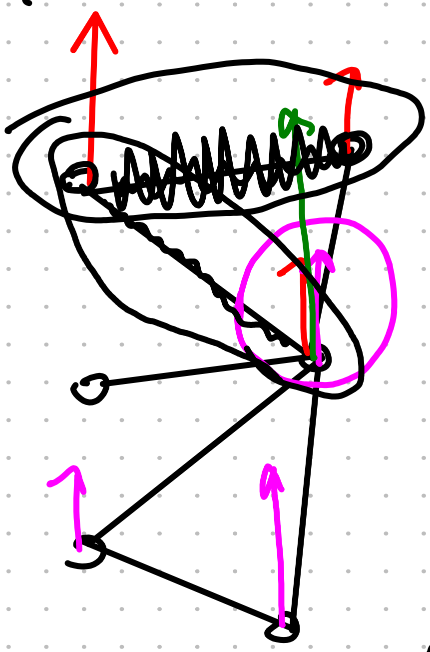
grid

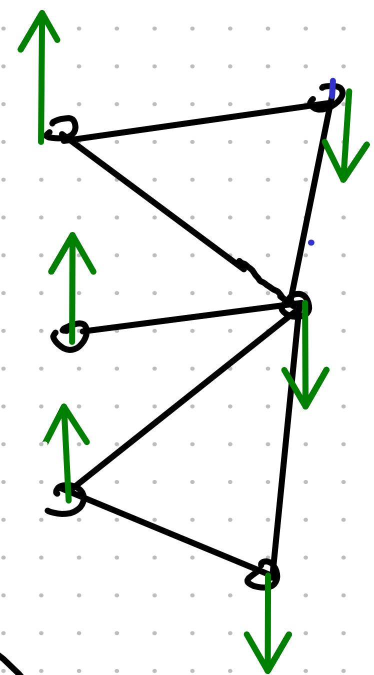


graph

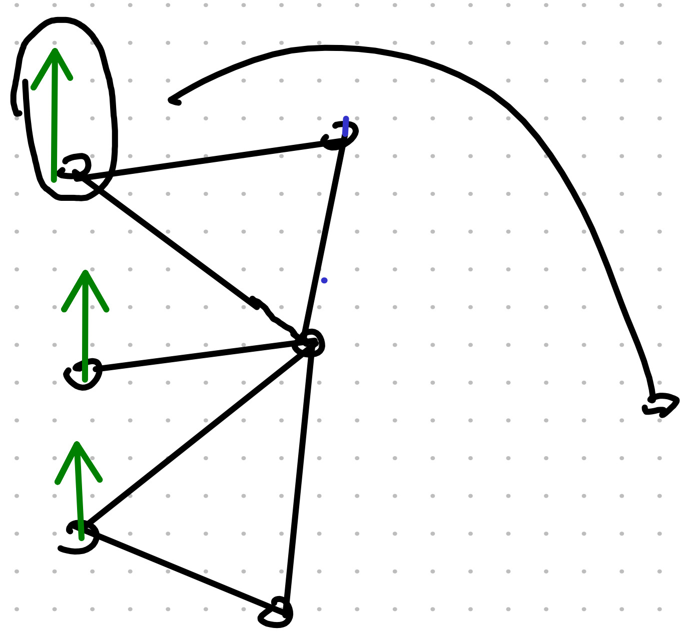
$$f : V \to \mathbb{R}^d$$

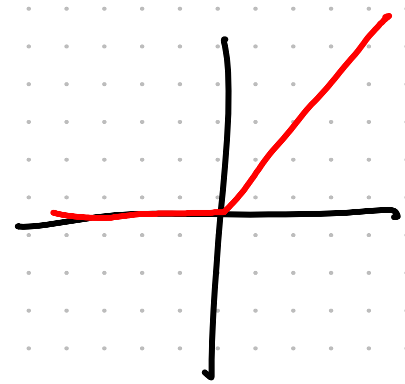
CNNs → GNNs



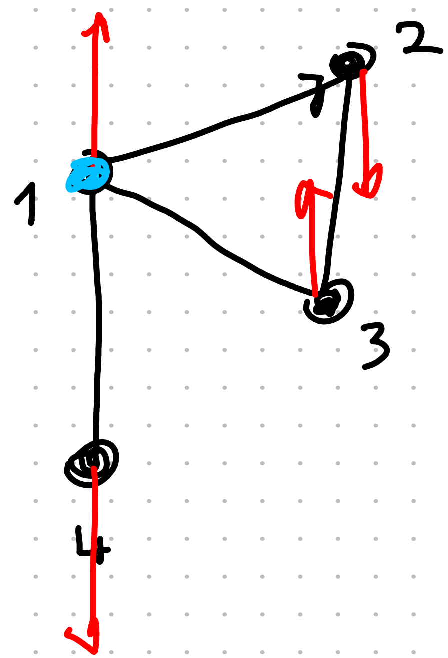


ReLu



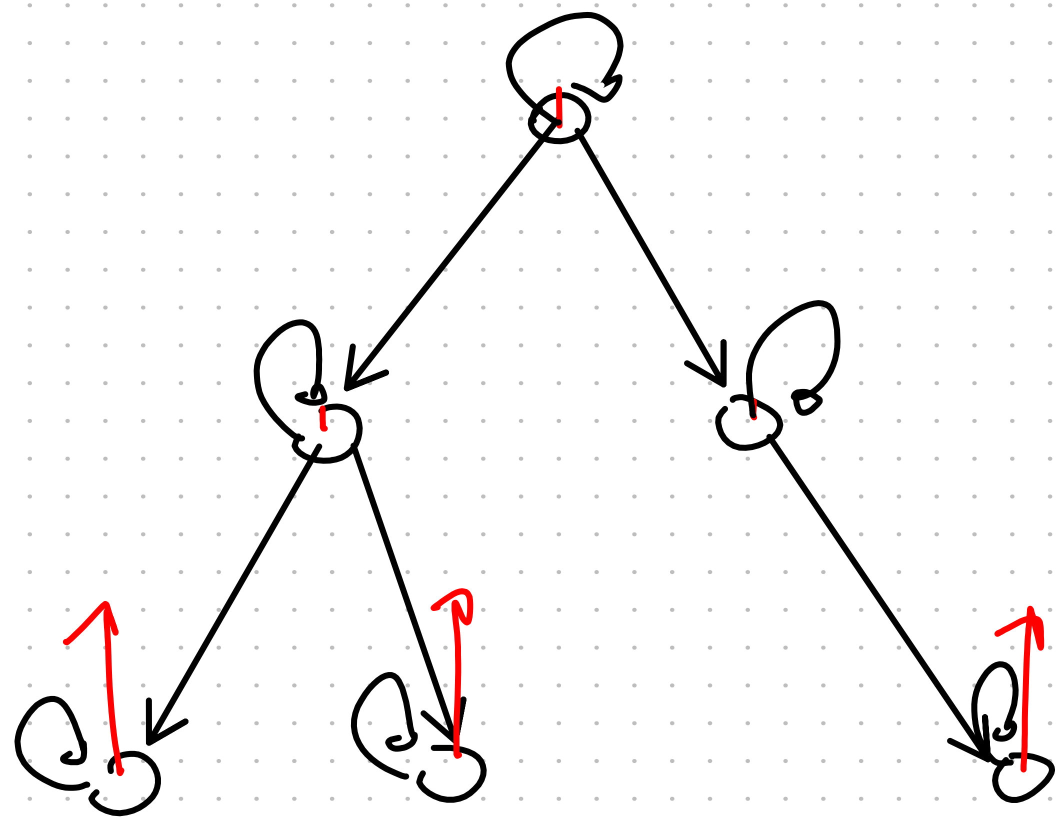


$$A = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad L = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$



$$f = \begin{array}{c c c} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0.7 \\ -0.1 \\ 0.5 \\ -0.9 \end{bmatrix} & \begin{array}{c} o \\ + \\ + \\ + \end{array} \end{array}$$

$$D(f) = [Lf]_1 = 3(0.7) - (-0.1) - 0.5 - (-0.9)$$



GNNs

$\mathrm{GNN} : f \mapsto g$

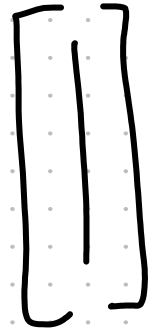
$$\mathrm{GNN}(f) = \left(\cdots\cdots \sigma \circ D \circ \sigma \circ D \right)(g)$$

$G = (V, E)$

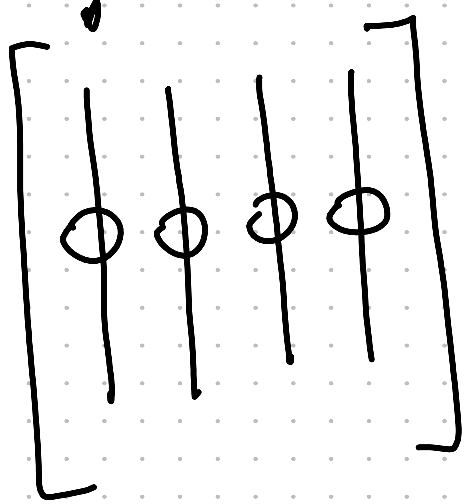
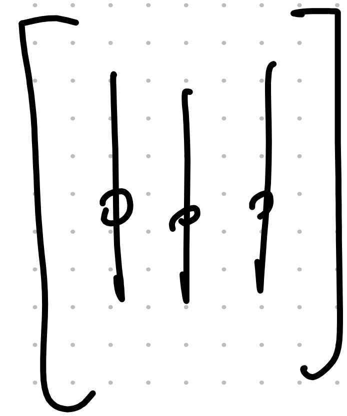
(v, f)

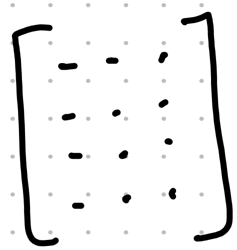
$V_1 = V \setminus V_0 \to (v, f)$

$V_0 \subseteq V \to (v, f, \ell)$

$f =$ 

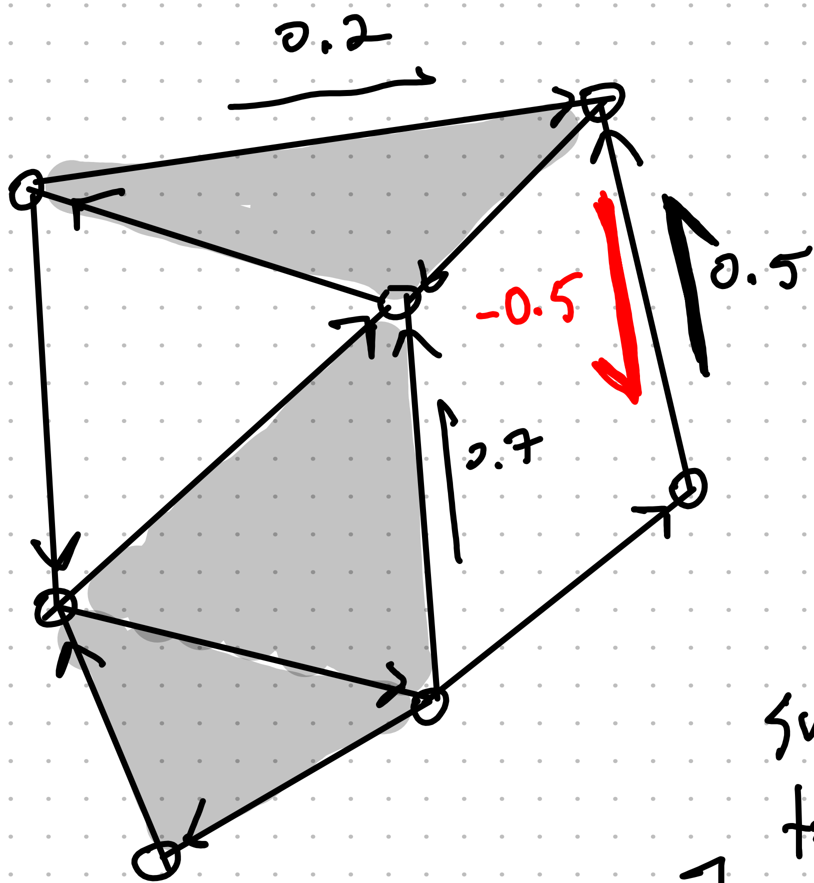
channel

$F =$  $\rightarrow$ 

$W =$ 

filter coeff.

$$D \underbrace{F^e}_{e^{th}\text{ input data}} W^e$$

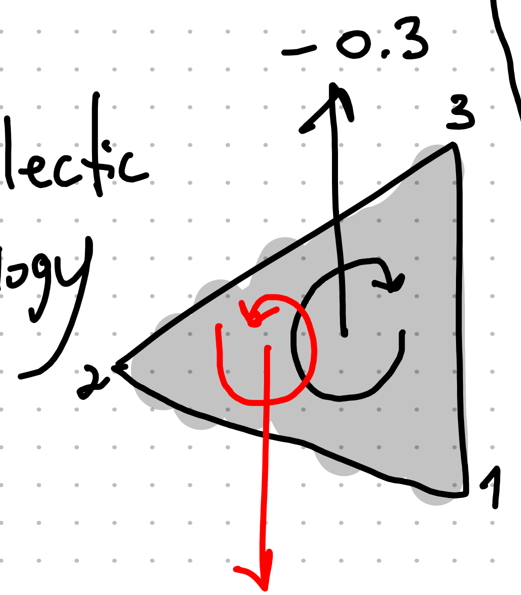


nodes - X_0

edges - X_1

triangles - X_2

symplectic topology



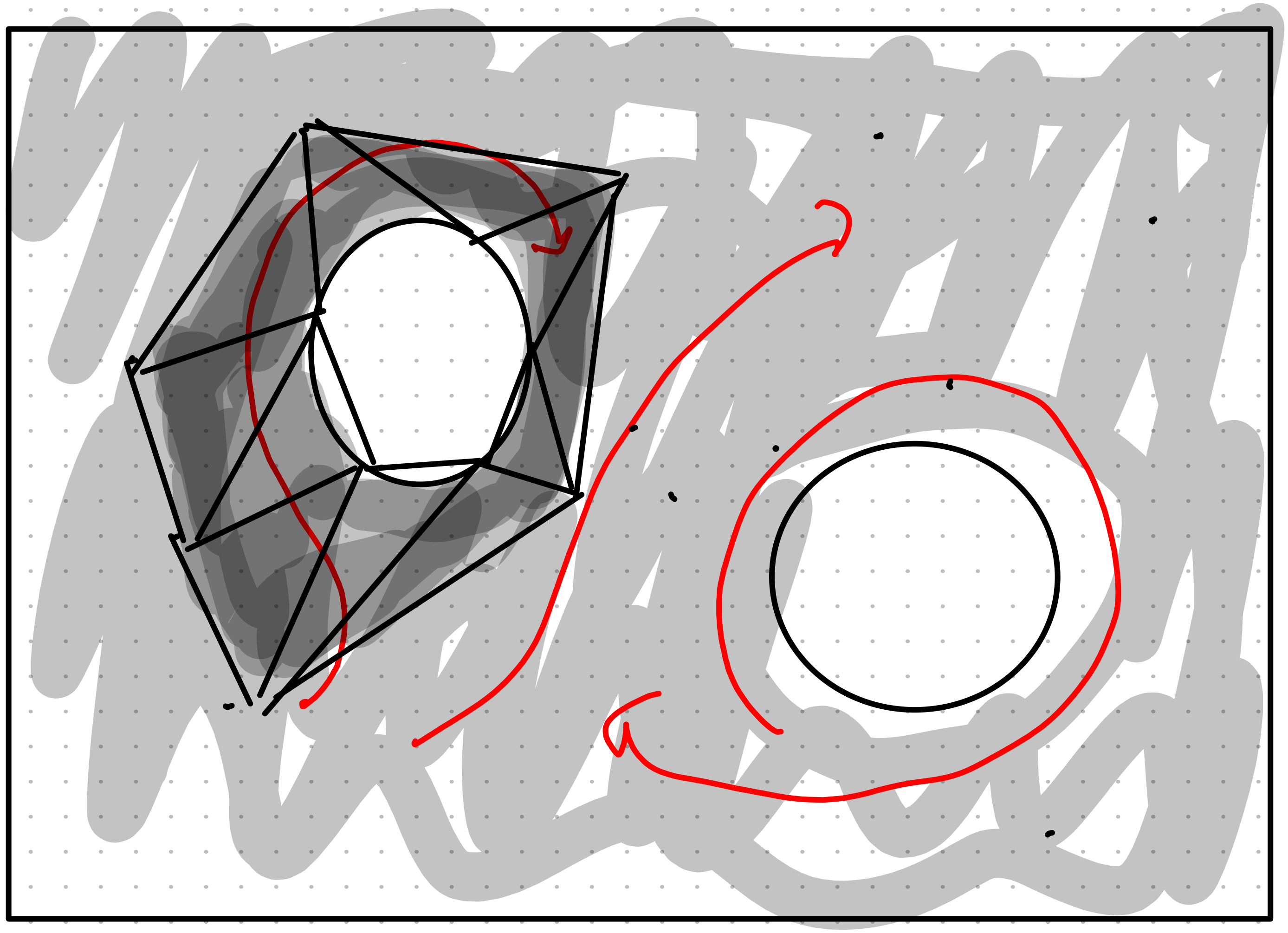
[1, 2, 3]
[3, 1, 2]
[2, 3, 1]

[2, 1, 3]
[1, 3, 2]
[3, 2, 1]

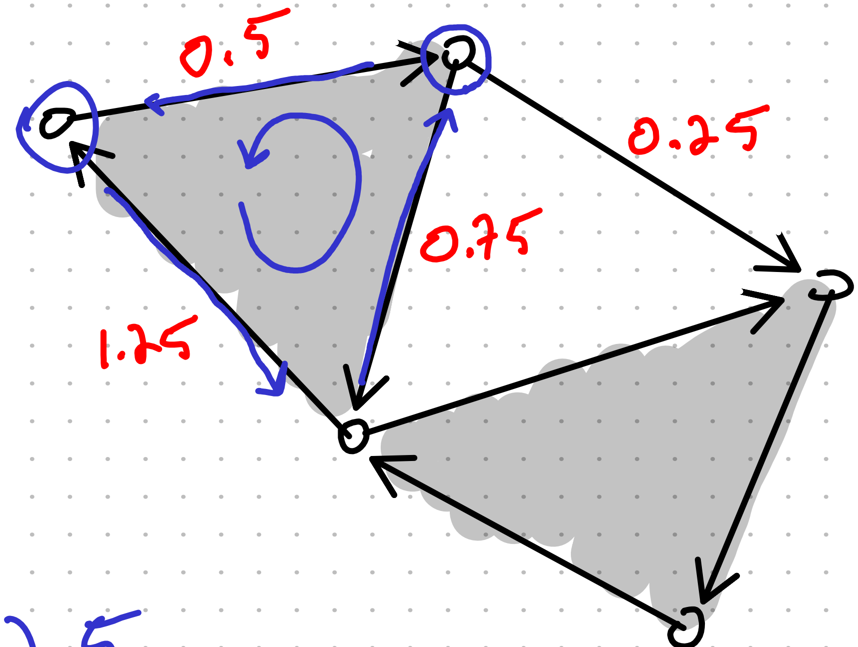
0-chains : $f: X_0 \to \mathbb{R}$

1-chains : $\omega: X_1 \to \mathbb{R}$

2-chains : $\Omega: X_2 \to \mathbb{R}$



1-chains | $\omega : X_1 \to \mathbb{R}$



- divergence
- curl
- harmonic

$\circlearrowright \to 2.5$

$\circlearrowleft \to -2.5$

B_1 | $\nabla \cdot$: vec. fields $\to$ scalar fields
| 1-chains $\to$ 0-chains

B_2^T | $\nabla \times$: 1-chains $\to$ 2-chains

B_1 : div B_1^T : derivative

B_2^T : curl B_2 : boundary

$$\boxed{B_1 B_2 = 0}$$

↳ homology

$\{B_1, B_2, B_3, \ldots\}$

$B_K B_{K+1} = 0$

$B_{K+1}^T B_K^T = 0$

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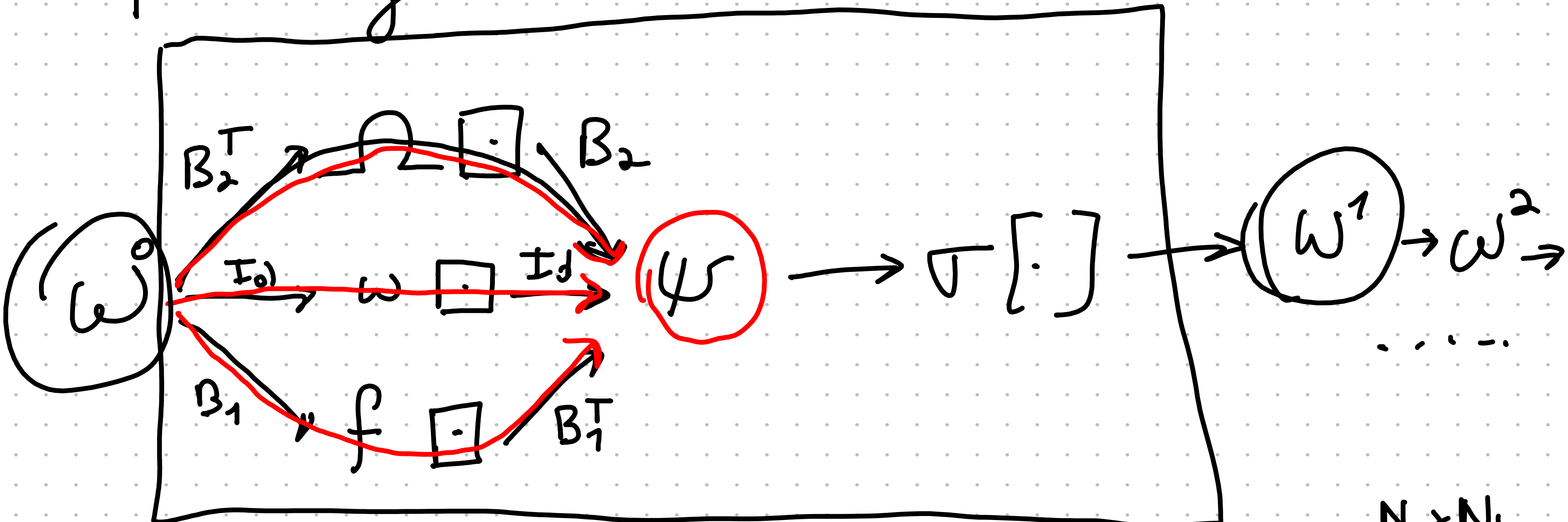
$$f \xrightarrow{D} g \xrightarrow{\nabla} h$$



LFW

$N_0 = |X_0|$

$N_1 = |X_1|$

$N_2 = |X_2|$

$B_1 \in \mathbb{R}^{N_0 \times N_1}$

$B_2 \in \mathbb{R}^{N_1 \times N_2}$

$I_d \in \mathbb{R}^{N_1 \times N_1}$

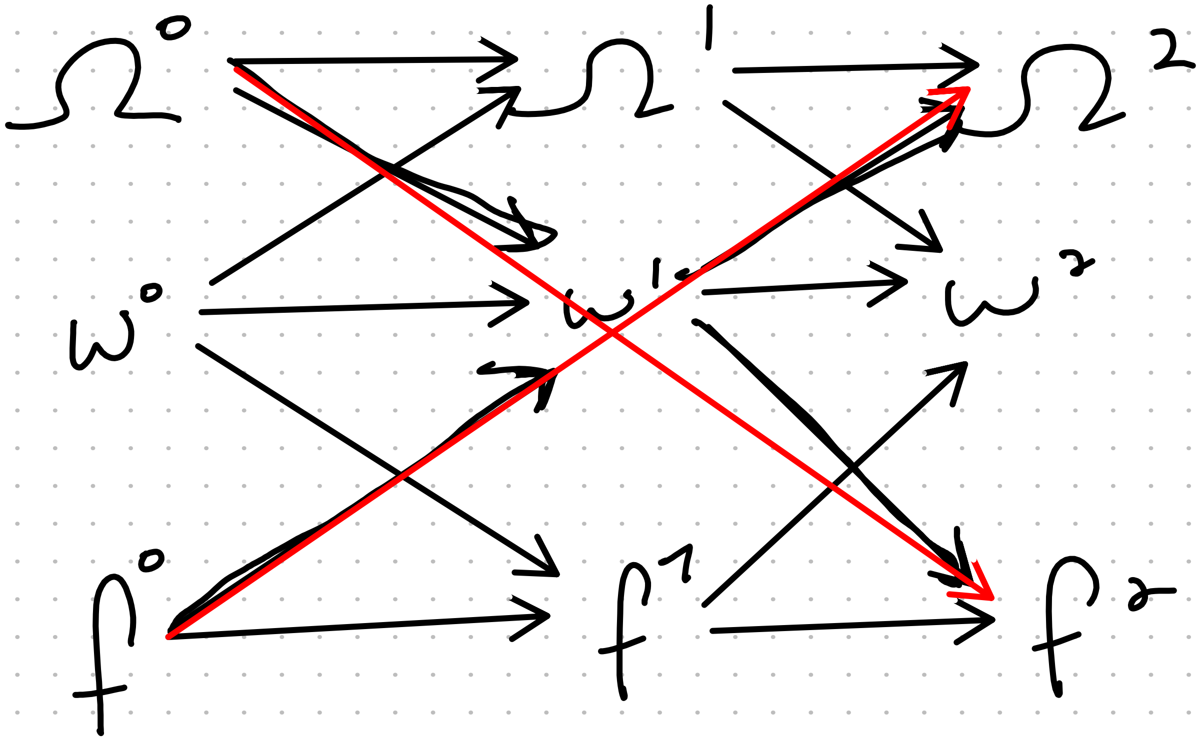
Ω : 2-chain (Δ-signal)

ω : 1-chain (/-signal)

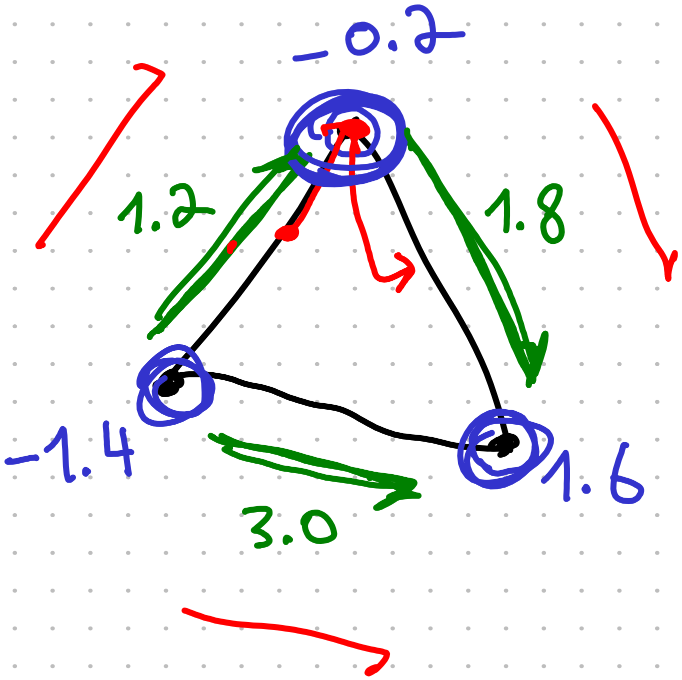
f : 0-chain (•-signal)

Eric Bunch et al

$$\boxed{B_k B_{k+1} = 0}$$



Claim nonlinearities are provably necessary for SNNs



ω

$$\boxed{B_1 \omega = f}$$

$$\boxed{B_1^T f = \beta}$$

ω

$$\Omega = B_2^T \omega$$

$$\beta = B_2 \Omega$$