

# **7. LATENT SEMANTIC INDEXING**

# Latent Semantic Indexing

Vector space retrieval is vague and noisy

- Based on index terms
- Unrelated documents might be included in the answer set
  - apple (company) vs. apple (fruit)
- Relevant documents that do not contain at least one index term are not retrieved
  - car vs. automobile

## Observation

- The user information need is more related to concepts and ideas than to index terms

Despite its success and widespread use the vector space retrieval model suffers from some problems. The important insight is that terms may indicate a concept a user is interested in, but there does not necessarily exist a one to one correspondence between terms and concepts. As a consequence retrieval results may contain irrelevant documents, and relevant documents may be missed.

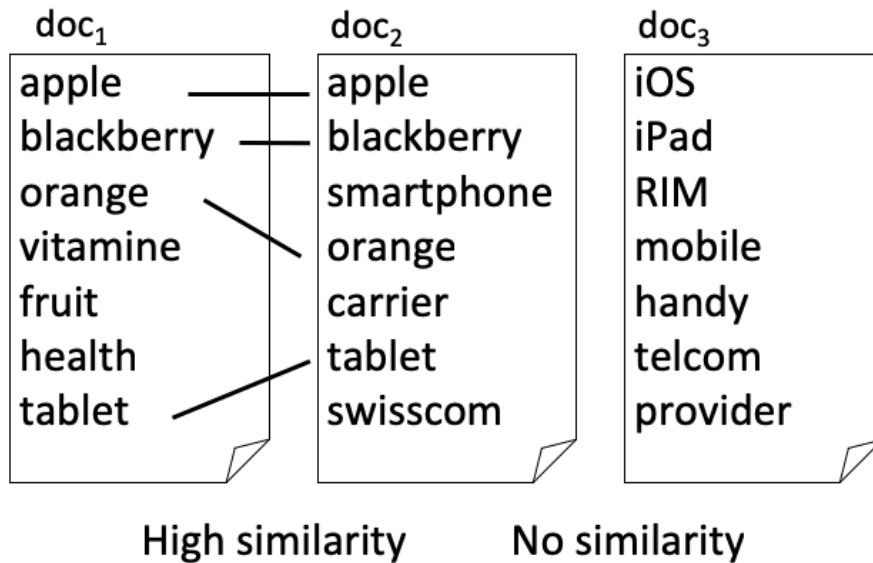
## The Problem

Vector Space Retrieval handles poorly the following two situations

1. *Synonymy*: different terms refer to the same concept, e.g. car and automobile
  - Result: poor recall
2. *Homonymy*: the same term may have different meanings, e.g. apple, model, bank
  - Result: poor precision

These problems are related to the fact that the same concepts can be expressed through many different terms (synonyms) and that the same term may have multiple meanings (homonyms). Studies show that different users use the same keywords for expressing the same concepts only 20% of the time.

### Example: 3 documents



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Let's illustrate the problem resulting from synonyms and homonyms by an example. Among these three documents at the level of terms  $doc_1$  and  $doc_2$  are (seem) highly related, whereas  $doc_3$  has no similarity with the other two documents. With human background knowledge it is however easy to see that in reality  $doc_2$  and  $doc_3$  are closely related, as they talk about mobile communications, whereas  $doc_1$  is completely unrelated to the others as it is about health and nutrition.

## Key Idea

Map documents and queries into a lower-dimensional space composed of higher-level concepts

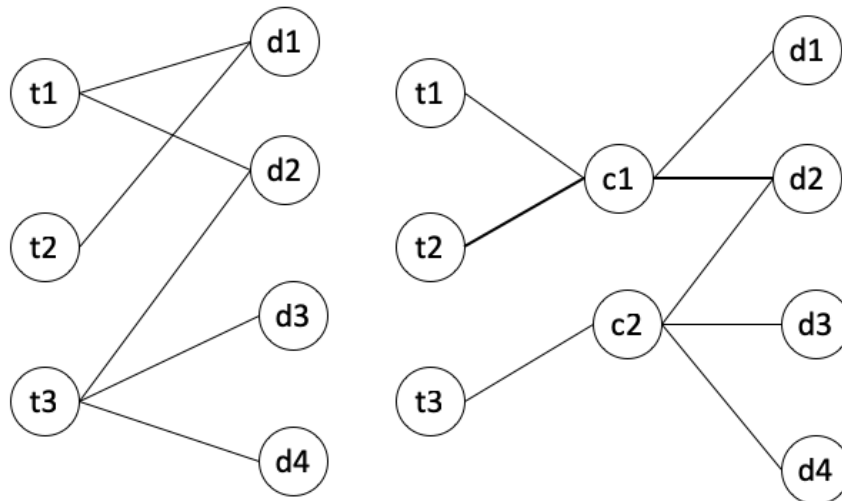
- Each concept represented by a combination of terms
- Fewer concepts than terms
- Vehicle = {car, automobile, wheels, auto car, motor car}

### Dimensionality reduction

- Retrieval (and clustering) in a reduced concept space might be superior to retrieval in the high-dimensional space of index terms

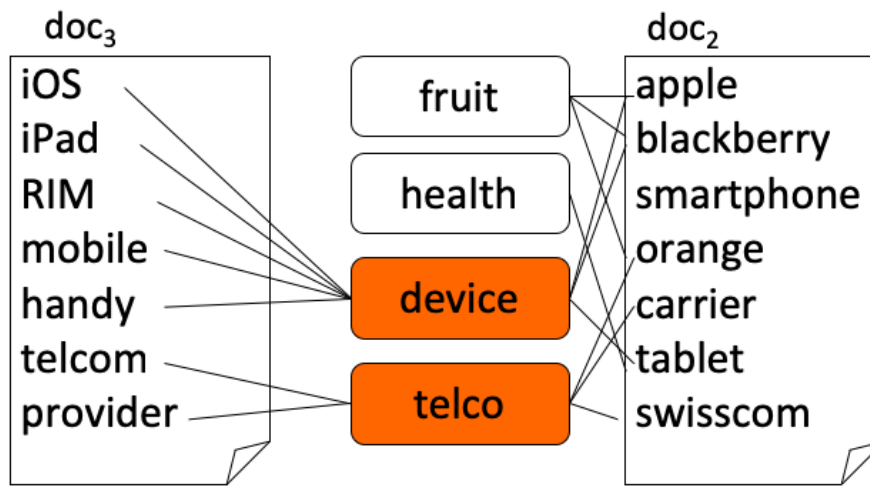
Thus it would be an interesting to base information retrieval methods on the use of concepts, instead of terms. To that end it is first necessary to define a "concept space" to which documents and queries are mapped, and compute similarity within that concept space. This idea is developed in the following. The concept space should ideally have much lower dimension than the term space, whose dimensionality is determined by the size of the vocabulary.

## Using Concepts for Retrieval



This figure illustrates the approach: rather than directly relating documents  $d$  and terms  $t$ , as in vector space retrieval, there exists an intermediate layer of concepts  $c$  to which both queries and documents are mapped. The concept space can be of a smaller dimension than the term space. In this small example we can imagine that the terms  $t_1$  and  $t_2$  are synonyms and thus related to the same concept  $c_1$ . If now a query  $t_2$  is posed, in the standard vector space retrieval model only the document  $d_1$  would be returned, as it contains the term  $t_2$ . By using the intermediate concept layer the query  $t_2$  would also return the document  $d_2$ .

## Example: Concept Space



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Applying this idea to our example before, we can imagine to have a concept space consisting of four concepts, two related to health, two related to mobile communication. When we consider now doc 2 and doc3 we can already recognize much better the close conceptual relationship the two documents have.

## Similarity Computation in Concept Space

Concept represented by terms, e.g.

device = {iOS, iPad, RIM, mobile, handy,  
tablet, apple, blackberry}

Document represented by concept vector, counting  
number of concept terms, e.g.

$\text{doc}_1 = (4, 3, 3, 1)$

$\text{doc}_3 = (0, 0, 5, 2)$

Similarity computed by scalar product of normalized  
concept vectors

We may consider concepts as being represented by sets of terms, and documents by concept vectors that count how many concept terms occur in the document. Using this approach we would obtain the non-normalized concept vectors  $\text{doc}_1 = (4, 3, 3, 1)$ ,  $\text{doc}_2 = (3, 1, 3, 3)$  and  $\text{doc}_3 = (0, 0, 5, 2)$ .



## Result

$\text{doc}_1 = (4,3,3,1)$

apple  
blackberry  
orange  
vitamine  
fruit  
health  
tablet

$\text{doc}_2 = (3,1,3,3)$

apple  
blackberry  
smartphone  
orange  
carrier  
tablet  
swisscom

$\text{doc}_3 = (0,5,5,2)$

iOS  
iPad  
RIM  
mobile  
handy  
telcom  
provider

$\text{Similarity}(\text{doc}_1, \text{doc}_2) = 0.245$

$\text{Similarity}(\text{doc}_2, \text{doc}_3) = 0.3$

$\text{Similarity}(\text{doc}_1, \text{doc}_3) = 0.22$

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After normalizing these vectors we compute the cosine similarities among the resulting concept vectors and obtain:

$\text{Sim}(2,3) = 0.3$

$\text{Sim}(1,2) = 0.245$

$\text{Sim}(1,3) = 0.22$

This result shows that indeed documents 2 and 3 are the more related ones, though still some confusion remains due to the high number of synonyms occurring in these documents.

## Basic Definitions

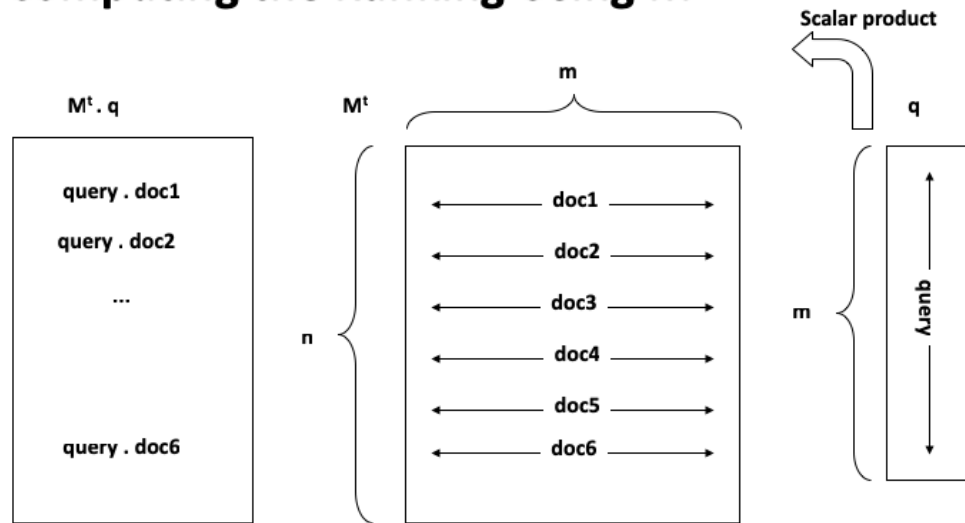
Problem: how to identify and compute “concepts” ?

Consider the term-document matrix

- Let  $M_{ij}$  be a term-document matrix with  $m$  rows (terms) and  $n$  columns (documents)
- To each element of this matrix is assigned a weight  $w_{ij}$  associated with  $t_i$  and  $d_j$
- The weight  $w_{ij}$  can be based on a tf-idf weighting scheme

The problem is to identify a method that identifies and characterizes important concepts in document collections. One approach would be to perform this task manually, e.g. by using a predefined ontology and let users annotate documents using terms of the ontology. This is an approach that has been used in libraries, but is labor intensive. Thus we will now present a method that performs the task of concept identification and document classification by concepts automatically. Starting point for the method is the term-document matrix that we have introduced for vector space retrieval with weights based on a tf-idf weighting scheme.

## Computing the Ranking Using M



We can understand the process of producing a retrieval results in vector space retrieval as a matrix operation. This is illustrated in this figure. The ranking is the result of computing the product of a query vector  $q$  with the term-document matrix  $M$ . We assume that all columns in  $M$  and  $1$  are normalized to 1.

In vector space retrieval each row of the matrix  $M^t$  corresponds to

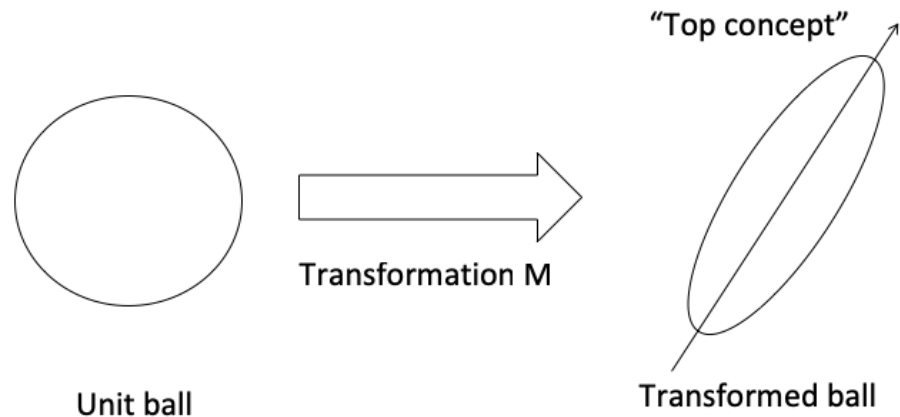
- A. A document
- B. A concept
- C. A query
- D. A query result



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## Identifying Top Concepts

Key Idea: extract the essential features of  $M^t$  and approximate it by the most important ones



One way to understand of how concepts can be extracted from a document collection is to consider the effect of the term-document matrix on data. If we apply this matrix to a (high-dimensional) unit ball it will distort this ball into an ellipsoid. This ellipsoid will have one direction with the strongest distortion. We may think of this direction as corresponding to a particularly important concept of the document collection.

## Singular Value Decomposition (SVD)

Represent Matrix  $M$  as  $M = K.S.D^t$

- $K$  and  $D$  are matrices with orthonormal columns

$$K.K^t = I = D.D^t$$

- $S$  is an  $r \times r$  diagonal matrix of the singular values sorted in decreasing order where  $r = \min(m, n)$ , i.e. the rank of  $M$
- Such a decomposition always exists and is unique (up to sign)

To extract this particularly important directions of a matrix mapping, a standard mathematical construction from linear algebra is used, the singular value decomposition (SVD). SVD decomposes a matrix into the product of three matrices. The middle matrix  $S$  is a diagonal matrix, where the elements of this matrix are the singular values of the matrix  $M$ .

## Construction of SVD

$K$  is the matrix of eigenvectors derived from  $M.M^t$

$D$  is the matrix of eigenvectors derived from  $M^t.M$

Algorithms for constructing the SVD of a  $m \times n$  matrix have complexity  $O(n^3)$  if  $m \leq n$

Formally the SVD can be computed by constructing eigenvectors of matrices derived from the original matrix  $M$ . This computation can be performed in  $O(n^3)$ . Note that the complexity is considerable, which makes the approach computationally expensive. There exist however also approximation techniques to perform this decomposition more efficiently.

## Interpretation of SVD

We can write M as sum of outer vector products

$$M = \sum_{i=1}^r s_i k_i \otimes d_i^t$$

The  $s_i$  are ordered in decreasing size

By taking only the largest ones we obtain a good "approximation" of M (least square approximation)

The singular values  $s_i$  are the lengths of the semi-axes of the hyperellipsoid E defined by

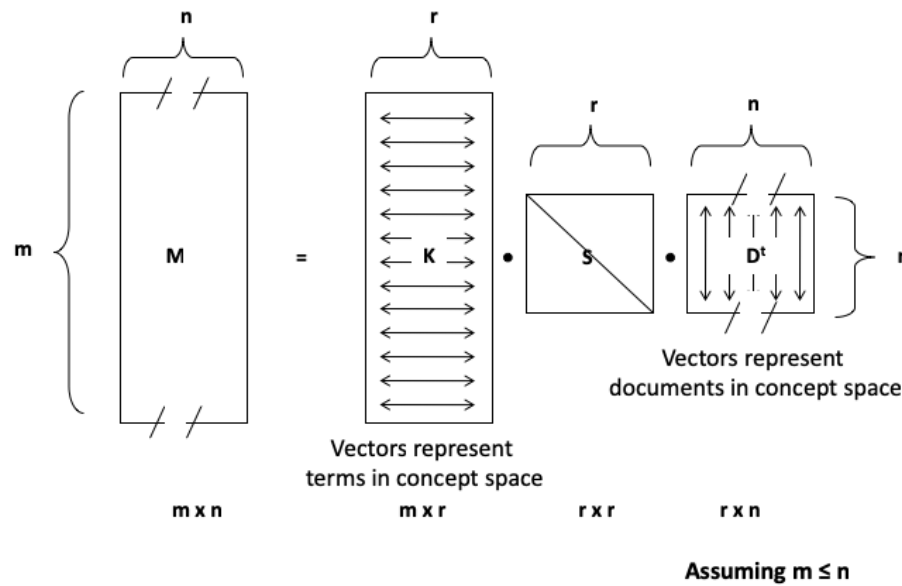
$$E = \{Mx \mid \|x\|_2 = 1\}$$

One way to understand of how the SVD extracts the important concepts from the term-document matrix is the following: the decomposition can be used to rewrite the original matrix as the sum of components that are weighted by the singular values. Thus we can obtain approximations of the matrix by only considering the larger singular values. The SVD after eliminating less important dimensions (smaller singular values) can be interpreted as a least square approximation to the original matrix. The symbol  $\otimes$  denotes the **outer product** of two vectors,  $d_i$  is the  $i$ -th row of D.

The singular values have also a geometrical interpretation, as they tell us how a unit ball ( $\|x\|=1$ ) is distorted when the linear transformation defined by the matrix M is applied to it. We can interpret the axes of the hyperellipsoid E as the dimensions of the concept space.

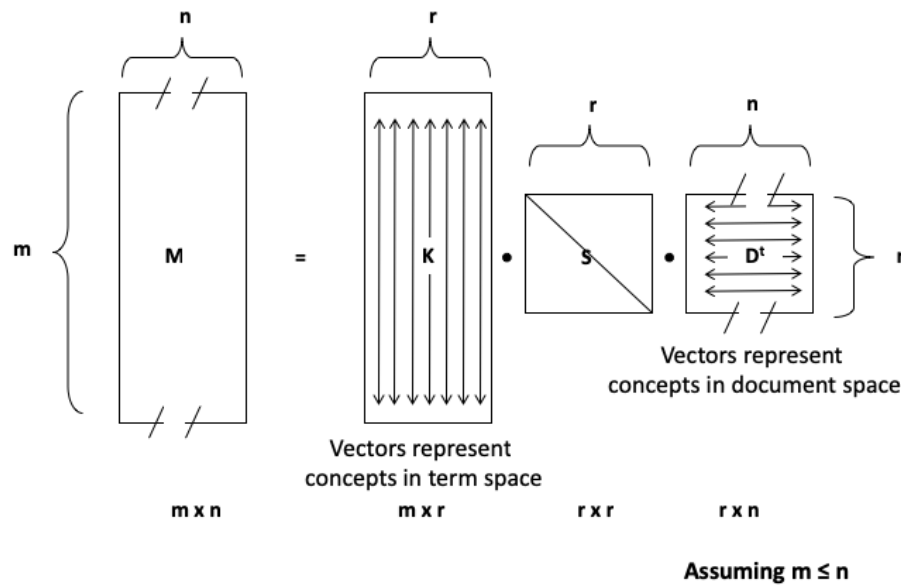


## Illustration of SVD



This figure illustrates the structure of the matrices generated by the SVD. In general,  $m \leq n$ , i.e., the number of documents may be larger than the size of the vocabulary. The rows in  $K$  can then be interpreted as the representation of terms in the concept space, and the rows in  $D$  as the representation of documents in the concept space.

## Illustration of SVD – Another Perspective



We can also see of how the concepts are represented by terms respectively documents. In particular, each concepts is now described as a weighted vector of terms.

## Latent Semantic Indexing

In the matrix  $S$ , select only the  $s$  largest singular values

- Keep the corresponding columns in  $K$  and  $D$

The resultant matrix is called  $M_s$  and is given by

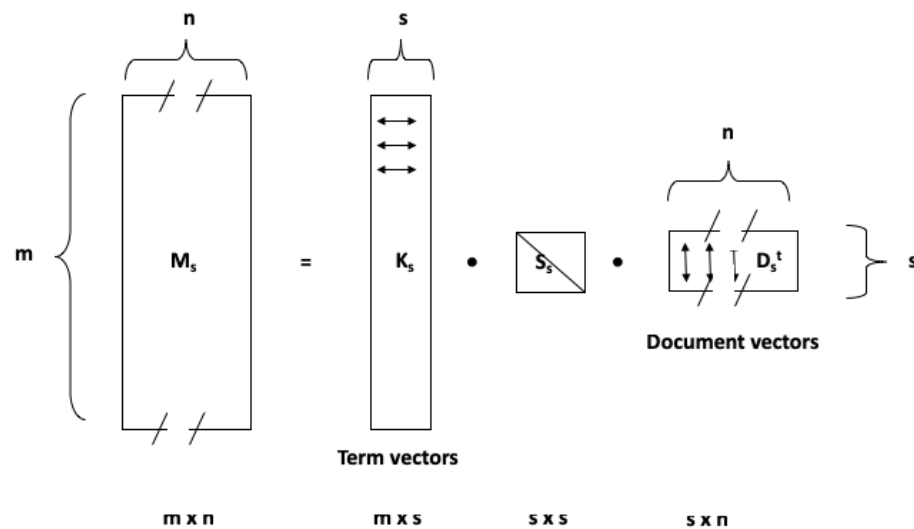
- $M_s = K_s \cdot S_s \cdot D_s^t$  where  $s, s < r$ , is the dimensionality of the concept space

The parameter  $s$  should be

- large enough to allow fitting the characteristics of the data
- small enough to filter out the non-relevant representational details

Using the singular value decomposition, we can now derive an "approximation" of  $M$  by taking only the  $s$  largest singular values in matrix  $S$ . The choice of  $s$  determines on how many of the "important concepts" the ranking will be based on. The assumption is that concepts with small singular value in  $S$  are rather to be considered as "noise" and thus can be neglected. The resulting method is called Latent Semantic Indexing.

## Illustration of Latent Semantic Indexing



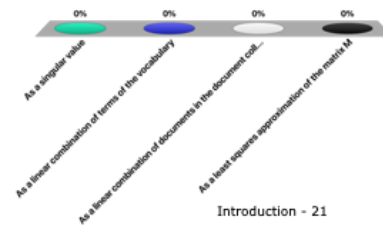
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This figure illustrates the structure of the matrices after reducing the dimensionality of the concept space to  $s$ , when only the first  $s$  singular values are kept for the computation of the ranking. The rows in matrix  $K_s$  correspond to term vectors, whereas the columns in matrix  $D_s^t$  correspond to document vectors. By using the cosine similarity measure between columns of matrix  $D_s^t$  the similarity of documents can be computed.

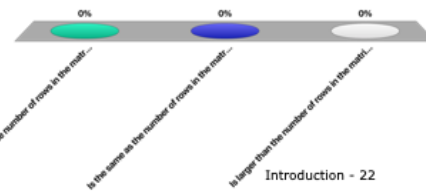
Applying SVD to a term-document matrix  $M$ . Each concept is represented

- A. As a singular value
- B. As a linear combination of terms of the vocabulary
- C. As a linear combination of documents in the document collection
- D. As a least squares approximation of the matrix  $M$



## The number of term vectors in the SVD for LSI

- A. Is smaller than the number of rows in the matrix  $M$
- B. Is the same as the number of rows in the matrix  $M$
- C. Is larger than the number of rows in the matrix  $M$



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## Answering Queries

Documents can be compared by computing cosine similarity in the concept space, i.e., comparing their columns  $(D_s^t)_i$  and  $(D_s^t)_j$  in matrix  $D_s^t$

A query  $q$  is treated like one further document

- it is added as an additional column to matrix  $M$
- the same transformation is applied as for mapping  $M$  to  $D$

After performing the SVD the similarity of different documents can be determined by computing the cosine similarity measure among their representation in the concept space (the columns of matrix  $D_s^t$ ). Queries are considered like documents that are added to the document collection. Answering queries then corresponds then to computing the similarity between the query considered as a document and the documents in the collection.

## Mapping Queries

Mapping of M to D

$$M = K.S.D^t$$

$$S^{-1}.K^t.M = D^t \quad (\text{since } K.K^t = 1)$$

$$D = M^t.K.S^{-1}$$

Apply same transformation to q:

$$q^* = q^t.K_s.S_s^{-1}$$

Then compare transformed vector by using the standard cosine measure

$$\text{sim}(q^*, d_i) = \frac{q^* \bullet (D_s^t)_i}{|q^*| |(D_s^t)_i|}$$

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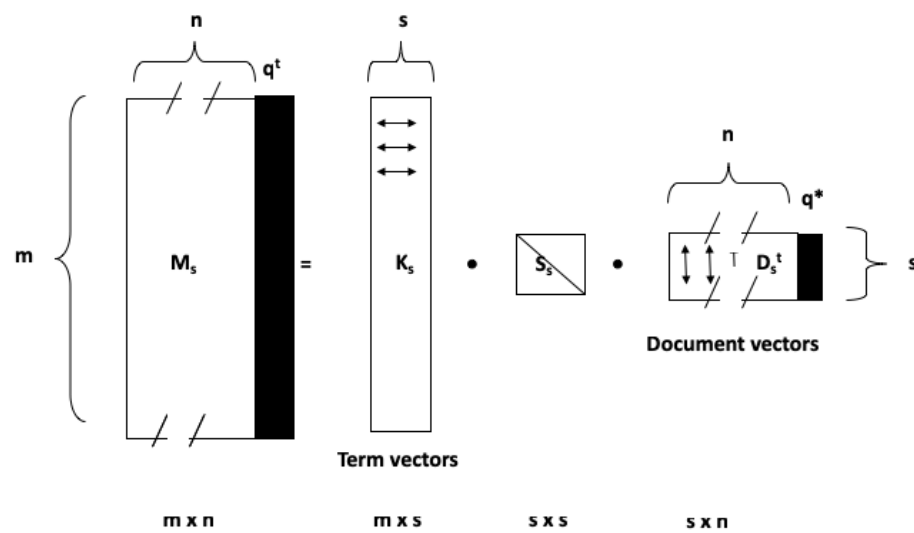
This construction works as follows: when a new column (the query) is added to M, we have to apply the same transformation to this new column, as to the other columns of M, in order to produce the corresponding column in the matrix  $D_s^t$ , representing documents in the concept space. We exploit the fact that  $K_s^t.K_s = 1$ .

Since  $M_s = K_s.S_s.D_s^t$  we obtain  $S_s^{-1}.K_s^t.M_s = D_s^t$  or  $D_s = M_s^t.K_s.S_s^{-1}$ .

This is the transformation that is applied to the query vector q to obtain a query vector  $q^*$  in the concept space. After that step, the similarity of the query to the documents in the concept space can be computed.  $((D_s^t)_i)$  denotes the i-th column of matrix  $D_s^t$



## Illustration of LSI Querying



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This figure illustrates of how a query vector is treated like an additional document vector.

## Example: Documents

B1 A Course on Integral Equations  
B2 Attractors for Semigroups and Evolution Equations  
B3 Automatic Differentiation of Algorithms: Theory, Implementation, and Application  
B4 Geometrical Aspects of Partial Differential Equations  
B5 Ideals, Varieties, and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra  
B6 Introduction to Hamiltonian Dynamical Systems and the N-Body Problem  
B7 Knapsack Problems: Algorithms and Computer Implementations  
B8 Methods of Solving Singular Systems of Ordinary Differential Equations  
B9 Nonlinear Systems  
B10 Ordinary Differential Equations  
B11 Oscillation Theory for Neutral Differential Equations with Delay  
B12 Oscillation Theory of Delay Differential Equations  
B13 Pseudodifferential Operators and Nonlinear Partial Differential Equations  
B14 Sinc Methods for Quadrature and Differential Equations  
B15 Stability of Stochastic Differential Equations with Respect to Semi-Martingales  
B16 The Boundary Integral Approach to Static and Dynamic Contact Problems  
B17 The Double Mellin-Barnes Type Integrals and Their Applications to Convolution Theory

This is an example of a (simple) document collection that we will use in the following as running example.

## Example (SVD, s=2)

$$\begin{array}{ccc}
 \mathbf{K}_s & \mathbf{S}_s & \mathbf{D}_s \\
 \begin{pmatrix} -0.0154227 & -0.422647 \\ -0.0242622 & -0.382996 \\ -0.178994 & -0.196573 \\ -0.603612 & 0.0992276 \\ -0.668904 & 0.135237 \\ -0.0143585 & -0.354329 \\ -0.0123052 & -0.174656 \\ -0.0063823 & -0.0905044 \\ -0.150975 & 0.119194 \\ -0.0816699 & 0.0728611 \\ -0.150975 & 0.119194 \\ -0.178994 & -0.196573 \\ -0.142064 & 0.0983069 \\ -0.00711902 & -0.144923 \\ -0.0954645 & 0.0687813 \\ -0.203256 & -0.579569 \end{pmatrix} & \begin{pmatrix} 4.52655 & 0 \\ 0 & 2.74066 \end{pmatrix} & \begin{pmatrix} -0.147774 & 0.0493447 \\ -0.147774 & 0.0493447 \\ -0.0568424 & -0.634717 \\ -0.312508 & 0.12142 \\ -0.00481714 & -0.187237 \\ -0.0240726 & -0.0608051 \\ -0.00815196 & -0.336379 \\ -0.36892 & 0.197629 \\ -0.0391324 & 0.0516819 \\ -0.314476 & 0.129042 \\ -0.405113 & -0.26937 \\ -0.405113 & -0.26937 \\ -0.33055 & 0.148005 \\ -0.314476 & 0.129042 \\ -0.281123 & 0.0855504 \\ -0.00271846 & -0.0637277 \\ -0.0529817 & -0.414944 \end{pmatrix}
 \end{array}$$

In the following we give a complete illustration of computing LSI for our running example. This is SVD for Term-Document Matrix from our running example.

$$\begin{pmatrix} -0.076442 & -0.605047 \end{pmatrix} = \begin{pmatrix} 0 \\ 2.14007 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1.44692 \end{pmatrix} K_s^{-1} \begin{pmatrix} -0.0154227 & -0.422647 \\ -0.0242622 & -0.382996 \\ -0.178994 & -0.196573 \\ -0.603612 & 0.0992276 \\ -0.668904 & 0.135237 \\ -0.0143585 & -0.354329 \\ -0.0123052 & -0.174656 \\ -0.0063823 & -0.0905044 \\ -0.150975 & 0.119194 \\ -0.0816699 & 0.0728611 \\ -0.150975 & 0.119194 \\ -0.178994 & -0.196573 \\ -0.142064 & 0.0983069 \\ -0.00711902 & -0.144923 \\ -0.0954645 & 0.0687813 \\ -0.203256 & -0.579569 \end{pmatrix} S_3^{-1} \begin{pmatrix} 0.220919 & 0. \\ 0. & 0.364876 \end{pmatrix}$$

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## Ranked Result

s=2

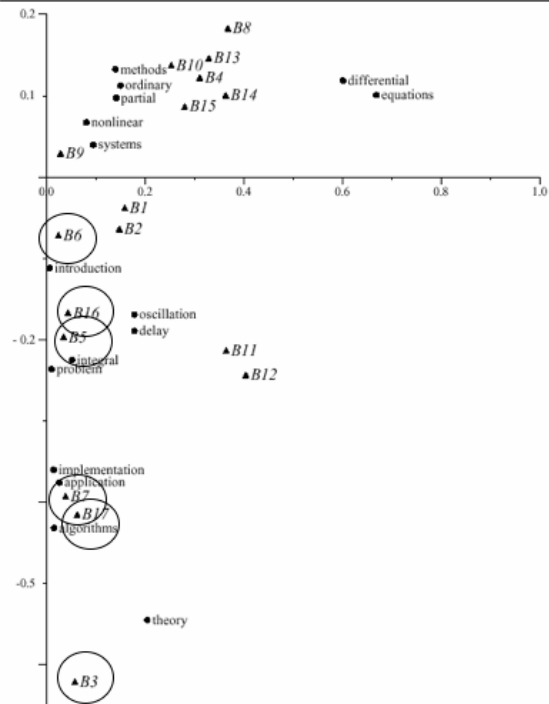
$$\begin{pmatrix} 0.999999 & \{17\} \\ 0.999339 & \{3\} \\ 0.996554 & \{16\} \\ 0.995009 & \{5\} \\ 0.994859 & \{7\} \\ 0.968592 & \{6\} \\ 0.653706 & \{12\} \\ 0.653706 & \{11\} \\ -0.168923 & \{15\} \\ -0.19534 & \{1\} \end{pmatrix}$$

s=4

$$\begin{pmatrix} 0.992173 & \{17\} \\ 0.970698 & \{16\} \\ 0.837632 & \{3\} \\ 0.537269 & \{12\} \\ 0.537269 & \{11\} \\ 0.434723 & \{7\} \\ 0.348928 & \{5\} \\ -0.101838 & \{6\} \\ -0.125203 & \{15\} \\ -0.131291 & \{4\} \end{pmatrix}$$

This is the ranking produced for the query for different values of s.

## Plot of Terms and Documents in 2-d Concept Space



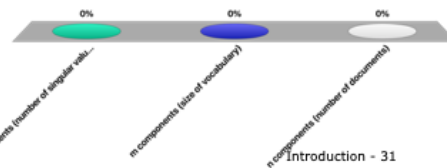
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Since in our example the concept space has two dimensions, we can plot both the documents and the terms in this 2-dimensional space. It is interesting to observe of how semantically "close" terms and documents cluster in the same regions. This illustrates very well the power of latent semantic indexing in revealing the « essential concepts » in document collections.

A query transformed into the concept space for LSI has ...

- A.  $s$  components (number of singular values)
- B.  $m$  components (size of vocabulary)
- C.  $n$  components (number of documents)



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## Discussion of Latent Semantic Indexing

Latent semantic indexing provides an interesting conceptualization of the IR problem

### Advantages

- It allows reducing the complexity of the underlying concept representation
- Facilitates interfacing with the user

### Disadvantages

- Computationally expensive
- Poor statistical explanation

From a modelling perspective LSI suffers from poor explanatory power. For example, the least squares approximation concept is related to the assumption that term frequencies are normally distributed, which is in contradiction with the observation that term frequencies are power law distributed.



## **Alternative Techniques**

### **Probabilistic Latent Semantic Analysis**

- Based on Bayesian Networks

### **Latent Dirichlet Allocation**

- Based on Dirichlet Distribution
- State-of-the-art method for concept extraction

### **Same objective of creating a lower-dimensional concept space based on the term-document matrix**

- Better explained mathematical foundation
- Better experimental results

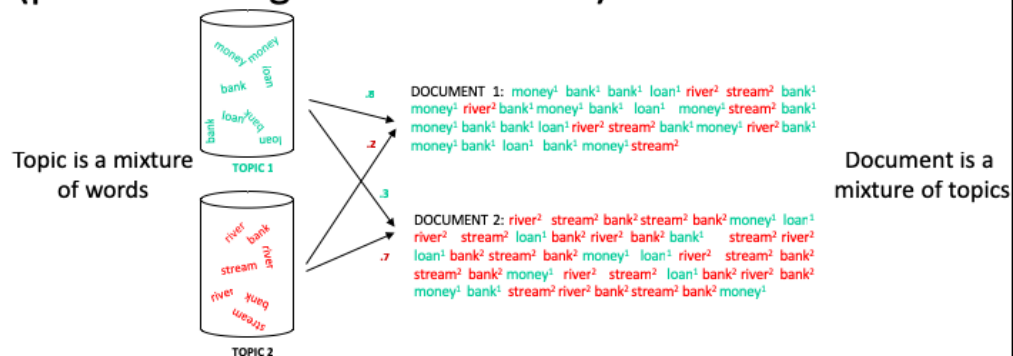
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The conceptual problems of LSI have triggered significant efforts in developing better suited models for concept extraction. The most recent and successful result of this has been the development of a method based on the use of Dirichlet distributions. Like LSI it produces a concept space, where concepts are represented as term vectors. The method has better theoretical foundations and empirically it produces better results, and is nowadays considered as the golden standard. The approach is mathematically more involved than LSI, and therefore we will not be able to develop this method in this course.

# Latent Dirichlet Allocation (LDA)

**Idea:** assume a document collection is (randomly) generated from a known set of topics (probabilistic generative model)



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This illustration shows the basic idea underlying LDA: it is assumed that there exist topics that are represented as mixtures of words. In the example we see words that are related to two meanings of “bank”, the financial institution and the river bank, and are represented by different related words. Then, documents are supposed to be generated from a mixture of topics, where the mixture is defined by some weights, as indicated in the figure. As a result each document contains terms from its associated topics in the right proportion.

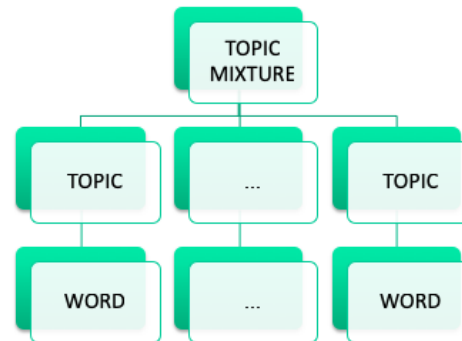
Similar to the probabilistic language model used in information retrieval, we have here another example of a probabilistic generative model.

## Document Generation using a Probabilistic Process

For each document, choose a mixture of topics

For every word position, sample a topic from the topic mixture

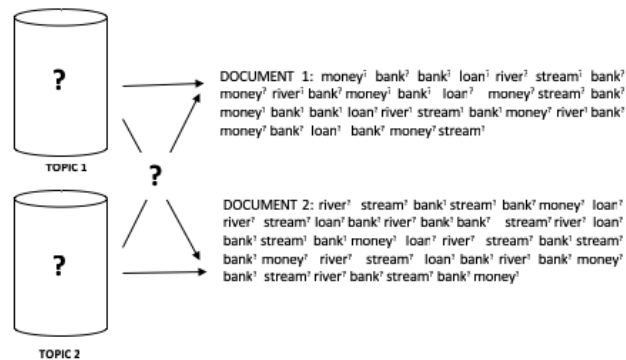
For every word position, sample a word from the chosen topic



Given the underlying model we can define a probabilistic process for generating documents.

# LDA: Topic Identification

**Approach:** Inverting the process: given a document collection, reconstruct the topic model



The challenge is to determine the probabilistic model. The situation we consider in practice is that a set of documents is given, and we intend to reconstruct the (most likely) probabilistic process that has generated this document collection. This is a difficult problem to solve.

## Latent Dirichlet Allocation

Topics are **interpretable** unlike the arbitrary dimensions of LSI

Distributions follow a Dirichlet distribution

Construction of topic model is mathematically involved, but computationally feasible

Considered as the state-of-the art method for topic identification

We do not present the details of LDA here, as the method is mathematically fairly involved. However, it is important to note that it is considered today as the state-of-the-art method of topic detection.

## Use of Topic Models

### Unsupervised Learning of topics

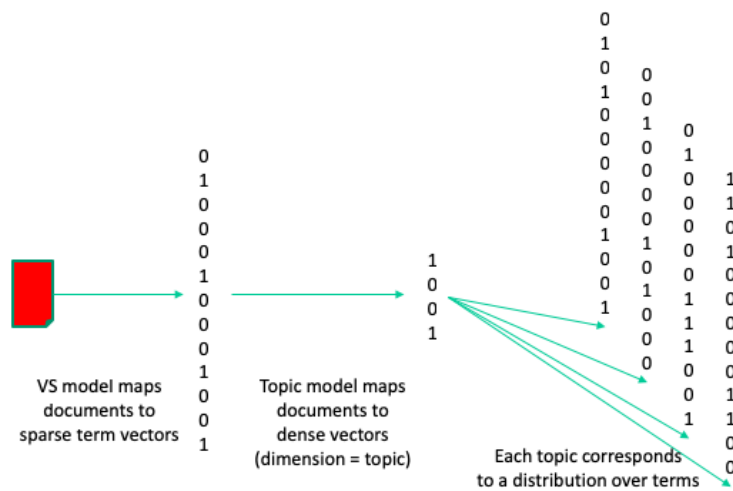
- Understanding main topics of a topic collection
- Organizing the document collection

Use for document retrieval: use topic vectors instead of term vectors to represent documents and queries

Document classification (Supervised Learning): use topics as features

Topic models provide a representation of documents at a conceptual level. Therefore they are applied in many different contexts, including document retrieval and classification.

## Summary



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This illustration summarizes the connection among the vector space model, which was introduced for information retrieval and topic models that produce low-dimensional (dense) representation of documents.

## 8. WORD EMBEDDINGS



## Word Context

The neighborhood of a word expresses a lot about its meaning

- “You shall know a word by the company it keeps”  
(J. R. Firth 1957)

government debt problems turning into banking crises as has happened in  
saying that Europe needs unified banking regulation to replace the hodgepodge

↩ These words will represent *banking* ↗

Word:  $w = \text{banking}$

Context:  $C(w) = \{\text{government, debt, problems, turning, into, crises, as, has, happened, in}\}$

Word-Context occurrence:  $(w, c), c \in C(w)$

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With latent semantic indexing we have exploited the idea that words that occur together in documents have a likelihood to have also some shared meaning. This idea is actually quite old, and has been already expressed, for example, by Firth in 1957. We will now exploit this idea in a slightly different way, by focusing on a much smaller context of a word than its document. We will just consider the local neighborhood of the word within a phrase. Typically we will consider all the neighborhoods that we can find in all documents of a large document collection.

For each word  $w$  we can identify its neighboring words, which will call its context and denote by  $C(w)$ . The context can be determined by choosing a certain number of preceding and succeeding words, e.g. a typical number used is 5.

## Similarity-Based Representation

One of the most successful ideas in natural language processing

- Two words are considered as similar, when they have similar contexts
- Context captures both syntactic and semantic similarity
  - Syntactic: e.g. king – kings
  - Semantic: e.g. king – queen
- Implicitly used with the term-document matrix  $M$ 
  - Less localized context
  - No distinction between context and word

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This idea is one of the main ideas used (successfully) in natural language processing. The neighborhood captures not only semantic relationships among words (as would also co-occurrence in the same document). It can at the same time also capture syntactic relationships. This is a main difference to methods based on document co-occurrence like LSI.

A second important difference is that methods based on this idea distinguish between a word occurring as the center of a context and as a member of context. For example, it is very unlikely the word “dog” would have in its context a second occurrence of the word “dog”. Thus dog as a “context word” needs to be treated differently from dog as the word of interest. This distinction is not made in purely co-occurrence based methods.

## Idea: Word Embeddings

Try to model how likely a word and a context occur together

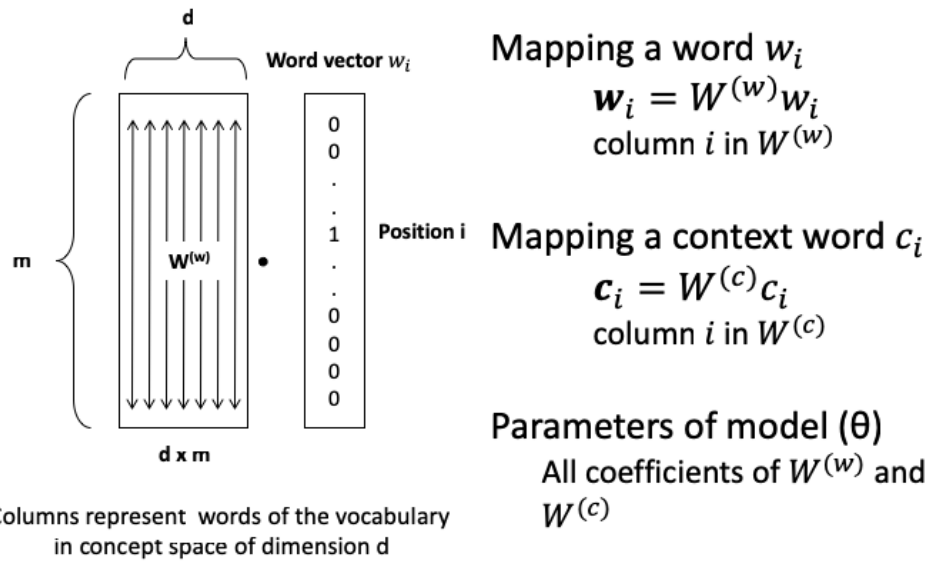
Approach:

- Map words into a low-dimensional space (e.g.,  $d=200$ )
- Map context words into the same low-dimensional space
  - A different mapping as before for the same words
- Interpret the vector distance (product) as a measure for how likely the word and its context occur together
- Due to projection in low-dimensional space, similar words and contexts should be close

The basic idea for word embeddings is simple. Both words and context words are mapped into the same low-dimensional space. Their representations in that space should be similar, if the word and the context word occur often together. Thus we capture the information on word context in a low-dimensional representation. Through the dimensionality reduction the expectation is that words and contexts that play similar roles would also move together, and thus we could capture syntactic and semantic similarity.

Dimensionality reduction is helpful, since vocabularies can become very large, in particular if not only words but also short phrases are included (e.g. tens of millions of entries). Thus data would be very sparse in these spaces and it would be difficult to model similarity. The low-dimensional spaces have on the other hand typically a few hundred dimensions.

## Overview of the Model



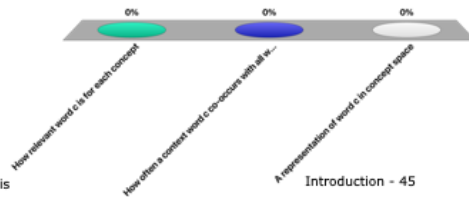
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We introduce the basic notations we will use in the following. Note that the size of the vocabulary  $m$  is typically very large (e.g. millions), whereas the size of the low-dimensional space  $d$  is typically in the hundreds. The model will be fully specified by the two matrices  $W^{(w)}$  and  $W^{(c)}$  used to map words and context words. For convenience we will denote in the following the coefficients of these two matrices by  $\theta$ .

## A column of matrix $W^{(c)}$ represents

1. How relevant word  $c$  is for each concept
2. How often a context word  $c$  co-occurs with all words
3. A representation of word  $c$  in concept space



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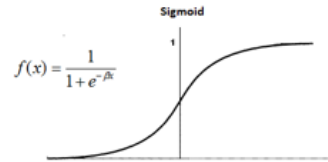
## Learning the Model from the Data

Consider a pair  $(w, c)$

Does it origin from the data?

Model this as a probability (sigmoid)

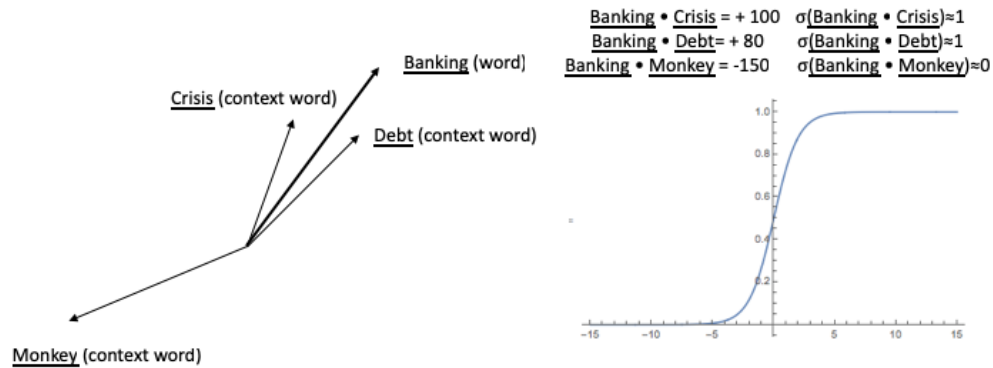
$$P(D = 1|w, c, \theta) = \frac{1}{1 + e^{-c \cdot w}} = \sigma(c \cdot w)$$



Intuition: convert similarity value in vector space into a probability

Differently to LSI, where the (parameters of the) model have been derived by a mathematical operation (SVD), the idea in word embeddings is to learn the parameters directly from the data. To start with, we consider a word-context pair  $(w, c)$  and ask the question whether it comes from the data. We would like that (ideally) the model gives us a probability of 1 if this is the case. In order to derive a probability from the model (the embedding into a low-dimensional space), we proceed as follows: we first take the scalar product of the embedded word and the context word vectors (this is where the parameters are hidden). This product will be large positive when the vectors are similar and large negative when they are opposite. In order to turn those values into a quantity that can be interpreted as probability (i.e. a value in  $[0, 1]$ ) we apply the sigmoid function. It produces values close to 0 for large negative arguments, and values close to 1 for large positive arguments. The use of the sigmoid function in this model is for exactly the same reasons as it is for its use in binary logistic regression, converting a similarity value into a probability. Apart from mapping the values to  $[0, 1]$  it also is insensitive to large outliers.

## Illustration of Word Embedding



**Problem: finding  $\theta$**

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Here we illustrate the embedding of words and context words in the same low-dimensional space. The idea is that the context words that typically co-occur together with a word, have similar embedding vectors, whereas others (like monkey, which is rarely occurring together with bank) have very different ones. The sigmoid function then converts the similarity values obtained from the scalar products of the embedding vectors into the interval  $[0, 1]$ .

## Finding Parameters

Formulate an optimization problem:

- Assume we have positive examples  $D$  for  $(w, c)$ , as well as negative examples  $\tilde{D}$ , not occurring in the document collection
- Find  $\theta$  such that the overall probability is maximized

$$\theta = \operatorname{argmax}_{\theta} \prod_{(w,c) \in D} P(D = 1 | w, c, \theta) \prod_{(w,c) \in \tilde{D}} P(D = 0 | w, c, \theta) =$$

$$\operatorname{argmax}_{\theta} \sum_{(w,c) \in D} \log \sigma(\mathbf{c} \cdot \mathbf{w}) + \sum_{(w,c) \in \tilde{D}} \log \sigma(-\mathbf{c} \cdot \mathbf{w})$$

Now we assume that we do not only know positive examples of word-context occurrences, but also negative ones, i.e. pairs that do not occur together. We can similarly model this case as a probability  $P(D = 0 | w, c, \theta)$ . If we have now a (big) collection of positive and negative samples  $D$  and  $\tilde{D}$ , the overall probability of such a collection to occur can be expressed as the product of all individual probabilities. Given this overall probability, the problem we would like to solve is to find parameters  $\theta$  that maximize this probability. This is then the best model we can obtain under the assumptions that have been made.



## Loss Function

Define a loss function (to be minimized)

$$J(\theta) = \frac{1}{s} \sum_{t=1}^s J_t(\theta)$$

$$J_t(\theta) = -\log(\sigma(\mathbf{c} \cdot \mathbf{w}_t)) - \sum_{k=1}^K \log(\sigma(-\mathbf{c}_k \cdot \mathbf{w}_t))$$

$\mathbf{c}_k$  are negative examples of context words taken from a set  $P_n(w_t)$

Following the previous derivation we can define a loss function  $J$  that optimizes the probabilities if minimized. (We transform the maximization problem into a minimization problem by defining a loss function  $J(\theta)$ ) The loss function is defined over all terms in the vocabulary. For each term  $w$  it consists of a first component corresponding to a positive example, a  $(w, c)$  pair that occurs in the document collection, and a couple of negative examples for the same word. The question is how to obtain the negative examples.

## Obtaining Negative Samples

Negative samples are taken from

$$P_n(w) = V \setminus C(w)$$

### Empirical approach

- If  $p_w$  is the probability of word  $w$  in collection, choose the word with probability  $p_w^{3/4}$
- Less frequent words are sampled more often
- Practically: approximate the probability by sampling a few non-context words

The method for learning the model relies on having negative samples. How to obtain them? We can choose any word from the vocabulary  $V$  that is not contained itself in the context. In order to model the occurrences we do this by considering the frequency at which the words occur in the document collection. Doing so, experience shows that high frequency words (e.g. stopwords) would however be favored too much, reducing the quality of the model. Thus the probability of word occurrence used for sampling is moderated by an exponent (in practice  $3/4$ ). Also for obtaining the statistics on word probabilities for practical purposes not the whole data collection is used, but just a small sample of words not occurring in the context.

## Stochastic Gradient Descent

For every  $(\mathbf{w}_t, \mathbf{c})$  in  $D$ , update  $\theta$

$$\theta^{new} = \theta^{old} - \alpha \nabla_{\theta} J_t(\theta^{old})$$

For  $J_t$  updates only affect rows in  $W^{(w)}$  and  $W^{(c)}$  that correspond to  $\mathbf{w}_t, \mathbf{c}$  and  $\mathbf{c}_k$ , e.g.,

$$\mathbf{w}_t^{new} = \mathbf{w}_t^{old} - \alpha \frac{\partial}{\partial \mathbf{w}_t} J_t(\mathbf{w}_t^{old}, \mathbf{c}^{old}, \mathbf{c}_t^{old}, \dots, \mathbf{c}_t^{old})$$

Finally, given the loss function, finding the optimal parameters  $\theta$  can then be done using standard methods from machine learning. Then  $\theta$  can be determined using gradient descent, i.e. standard search for minima using derivatives. More precisely, we will apply stochastic gradient descent. We update the parameters for each sample, separately. Standard gradient descent would update the values of data only after having seen all samples, which would take extremely long given the very large number of samples. When stochastic gradient descent is used, only rows that contain the word or some context word in the loss function need to be updated. This implies that the data has to be organized that these rows can be efficiently accessed (e.g. using hashing).

## Computing the Derivative

Some notation

$$x = \mathbf{c} \cdot \mathbf{w}, p = \sigma(x), z_k = -\mathbf{c}_k \cdot \mathbf{w}, q_k = \sigma(z_k)$$

Then  $J_t(\mathbf{w}, \mathbf{c}, \mathbf{c}_1, \dots, \mathbf{c}_K) = -\log(p) - \sum_{k=1}^K \log q_k$

$$\frac{\partial J_t}{\partial \mathbf{w}} = \frac{\partial J_t}{\partial p} \frac{\partial p}{\partial x} \frac{\partial x}{\partial \mathbf{w}} + \sum_{k=1}^K \frac{\partial J_t}{\partial q_k} \frac{\partial q_k}{\partial z_k} \frac{\partial z_k}{\partial \mathbf{w}}$$

$$\frac{\partial J_t}{\partial p} = -\frac{1}{p}, \frac{\partial p}{\partial x} = p(1-p), \frac{\partial x}{\partial \mathbf{w}} = \mathbf{c}$$

Finally  $\frac{\partial J_t}{\partial \mathbf{w}} = -(1-p)\mathbf{c} + \sum_{k=1}^K (1-q_k) \mathbf{c}_k$

Computing the derivative reveals that the updates to the parameters of the word embedding model are very simple computations. They involve the computation of the probability values for the words and context words and multiplying them with the existing embedding vectors. This (incomplete) computation shows how the weights of the embedding vector  $\mathbf{w}$  of the word are updated.

Exercise: complete the missing parts of the computation.

## Result

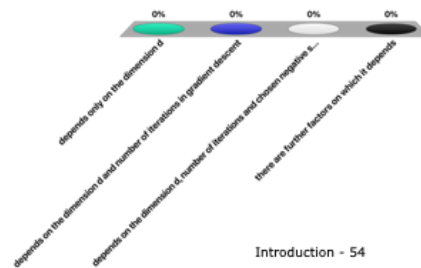
Matrices  $W^{(w)}$  and  $W^{(c)}$  that capture information on word similarity

- Words appearing in similar contexts generate similar contexts and vice versa
- Hence, mapped to similar representations in lower dimensional space
- Use  $W = W^{(w)} + W^{(c)}$  as the low-dimensional representation

As a result we obtain two models mapping words into a low dimensional space for different reasons. In practice, the final model is constructed as the sum of the two matrices. Though, there exists no theoretical insight, why exactly this models work well, practice shows that they are organizing words in interesting ways, capturing many semantic and syntactic relationships.

## A word embedding ...

1. depends only on the dimension  $d$
2. depends on the dimension  $d$  and number of iterations in gradient descent
3. depends on the dimension  $d$ , number of iterations and chosen negative samples
4. there are further factors on which it depends



## Alternative Approaches

### CBOW (Continuous Bag of Words Model)

- Predict word from context

GLOVE: exploits the following observation

	x = solid	x = gas	x = water	x = random
$P(x \text{ice})$	large	small	large	small
$P(x \text{steam})$	small	large	large	small
$\frac{P(x \text{ice})}{P(x \text{steam})}$	large	small	$\sim 1$	$\sim 1$

Technically similar

The model we have discussed is one variant of a number of similar models that have been proposed recently. One interesting model is GLOVE, which is based on the observation that ratios of probabilities can be used to capture semantic relationships among terms. For example, the term solid is much more likely to co-occur with ice, than with water, and similarly steam co-occurs much more likely with gas than with ice. On the other hand, water co-occurs frequently with both. So the ratio of probabilities captures semantics on the meaning of solid and gas, beyond co-occurrence.

Technically most of these models follow similar approaches, first modeling some observation on how co-occurrence can capture semantic relationship, and then using gradient descent methods to learn the parameters.

## Properties of Word Embeddings

1. Similar terms are clustered
2. Syntactic and semantic relationships encoded as linear mappings
3. Dimensions can capture meaning

Word Embeddings have received recently a lot of attention as they exhibit fascinating capacity to capture different types of relationships among words. We will illustrate some of those in the following.



## Glove: Nearest words to Frog

Nearest words to  
frog:

1. frogs
2. toad
3. litoria
4. leptodactylidae
5. rana
6. lizard
7. eleutherodactylus



litoria



leptodactylidae



rana



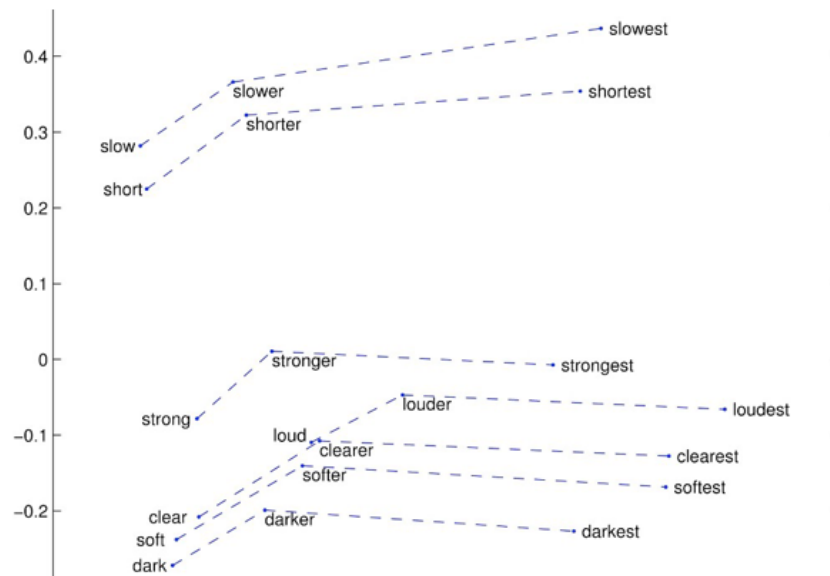
eleutherodactylus

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A first property is that similar words are grouped together. Here is a nice example, produced with Glove, that illustrates the point. (The underlying data is from Wikipedia).

## Syntactic Relationships



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Word embeddings also capture syntactic relationships, like singular-plural or comparative and superlative, as shown here. This type of visualizations is obtained by projecting from the d-dimensional word embedding space (appropriately) into a 2-dimensional space.

## Word Analogies: semantic relationships

man:woman :: king:?

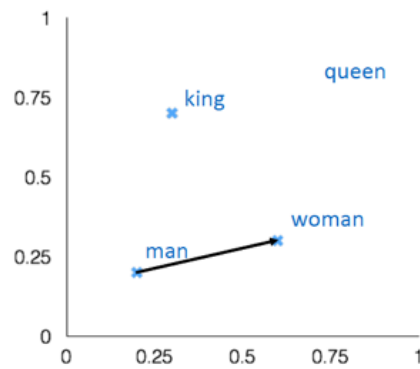
+ king [ 0.30 0.70 ]

- man [ 0.20 0.20 ]

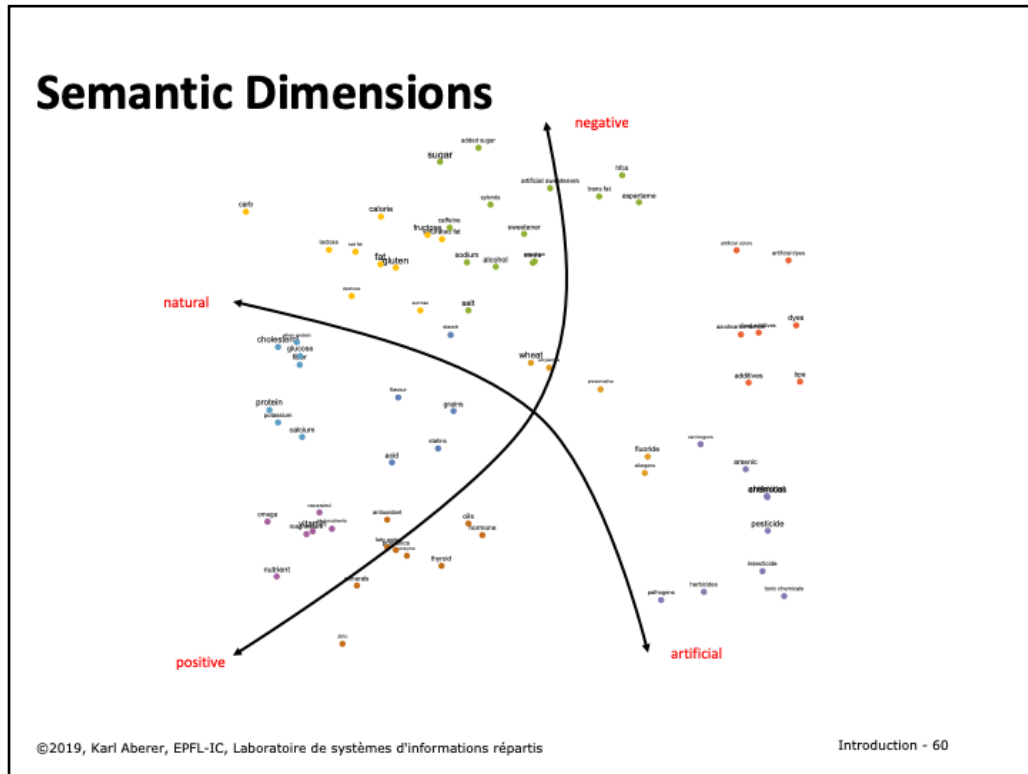
+ woman [ 0.60 0.30 ]

---

queen [ 0.70 0.80 ]



Even more interestingly, word embeddings enable “computing” with relationships (this is called the word analogy task). Analogies translate into linear mappings!



Furthermore, word embeddings can capture semantic meaning in dimensions. In this example, data on nutrition has been analyzed and terms indicating qualities of foods are extracted. The word embedding clearly organizes it according to properties that are human understandable, e.g. natural vs. artificial ingredients.

## **Use of Word Embedding Models**

### **Document Search**

- Use word embedding vectors as document representation

### **Thesaurus construction and taxonomy induction**

- Search engine for semantically analogous / related terms

### **Document classification**

- Use of word embedding vectors of document terms as features

# Impact

- Latent semantic indexing

**Indexing by latent semantic analysis**

S Deenwester, ST Dumais, GW Fumas... - Journal of the ..., 1990 - search.proquest.com

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**Indexing by latent semantic analysis**

S Deenwester, ST Dumais, GW Fumas... - Journal of the ..., 1990 - Wiley Online Library

A new method for automatic indexing and retrieval is described. The approach is to take  
218\_Grammatical (2).pdf is higher-order structure in the association of terms with documents  
in order to improve the detection of relevant documents on the basis of ...  
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- Latent Dirichlet allocation

**Latent dirichlet allocation**

DM Blei, AY Ng, MJ Jordan - Journal of machine Learning research, 2003 - jmlr.org

Cited by 17791 Related articles All 123 versions Web of Science: 5278 Cite Save

**Latent dirichlet allocation**

DM Blei, AY Ng, MJ Jordan - Journal of machine Learning research, 2003 - jmlr.org

We describe latent Dirichlet allocation (LDA), a generative probabilistic model for collections  
of discrete data such as text corpora. LDA is a three-level hierarchical Bayesian model, in  
which each item of a collection is modeled as a finite mixture over an underlying set of ...  
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+8000

- Word embeddings

**Distributed representations of words and phrases and their compositionality**

T Mikolov, I Sutskever, K Chen, GS Corrado... - Advances in neural ..., 2013 - papers.nips.cc

Cited by 3370 Related articles All 23 versions Cite Save

**Distributed representations of words and phrases and their compositionality**

T Mikolov, I Sutskever, K Chen, GS Corrado... - Advances in neural ..., 2013 - papers.nips.cc

The recently introduced continuous Skip-gram model is an efficient method for learning high-  
quality distributed vector representations that capture a large number of precise syntactic  
and semantic word relationships. In this paper we present several improvements that make  
the Skip-gram model more expressive and enable it to learn higher quality vectors more  
rapidly. We show that by subsampling frequent words we obtain significant speedup, and  
also learn higher quality representations as measured by our tasks. We also introduce ...  
Cited by 11619 Related articles All 37 versions 10

+8000

# References

## Course material based on

- Christopher D. Manning, Prabhakar Raghavan and Hinrich Schütze, Introduction to Information Retrieval, Cambridge University Press. 2008 (<http://www-nlp.stanford.edu/IR-book/>)
- Course Information Retrieval by TU Munich (<http://www.cis.lmu.de/~hs/teach/14s/ir/>)
- [http://videlectures.net/deeplearning2015\\_manning\\_language\\_vectors/](http://videlectures.net/deeplearning2015_manning_language_vectors/)

## Relevant articles

- Deerwester, Scott, et al. "Indexing by latent semantic analysis." *Journal of the American society for information science* 41.6 (1990): 391.
- Mikolov, Tomas, et al. "Distributed representations of words and phrases and their compositionality." *Advances in neural information processing systems*. 2013.
- Goldberg, Yoav, and Omer Levy. "word2vec explained: Deriving mikolov et al.'s negative-sampling word-embedding method." *arXiv preprint arXiv:1402.3722* (2014).