

### IE 3013

*Sigmoid activation in feed-forward neural networks.* Let  $z$  be the weighted input to a node and let  $a$  be activation out of the same node. The sigmoid activation function is

$$a = f(z) = \frac{1}{1 + e^{-z}}$$

Show that  $f'(z) = a(1 - a)$ . To be clear,  $f'(z)$  is the derivative of  $f$  with respect to  $z$ . You will need to use this result when implementing backpropagation.

$$f'(z) = -(1 + e^{-z})^{-2} (-e^{-z}) = \frac{e^{-z}}{(1 + e^{-z})^2}$$

$$\begin{aligned} a(1 - a) &= \frac{1}{1 + e^{-z}} \left( 1 - \frac{1}{1 + e^{-z}} \right) \\ &= \frac{1}{1 + e^{-z}} - \frac{1}{(1 + e^{-z})^2} \\ &= \frac{1 + e^{-z}}{(1 + e^{-z})^2} - \frac{1}{(1 + e^{-z})^2} \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{e^{-z}}{(1 + e^{-z})^2} \end{aligned}$$

$$\therefore f'(z) = a(1 - a)$$