M1C03 Lecture 15

The Principle of Mathematical Induction

Jeremy Lane

Oct 20, 2021

Announcement(s)

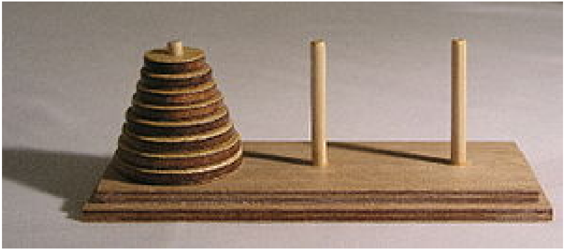
1. Quiz 5 and Assignment 2 are due Friday.
2. Test 1 details are on Avenue.
3. Test review on Thursday.

Overview

We continue with *The Principle of Mathematical Induction*.

Reference: Lakins, Section 3.1.

Last time



Goal: Move the stack to the third peg.

Rules:

1. You can only move one piece at a time (by taking it from the top of a stack and putting it on another peg).

2. You cannot put a piece on top of a smaller piece.

Theorem

For all positive integers n, it is possible to win the game with n pieces in the stack.

The Principle of Mathematical Induction

Let $P(n)$ be a statement that depends on an arbitrary positive integer n.

IF

1. $P(1)$ AND
2. For all positive integers n, if $P(n)$, then $P(n+1)$

THEN for all positive integers n, $P(n)$.

Theorem

For all positive integers n, it is possible to win the game with n pieces in the stack.

Exercise

Theorem

For all positive integers n, the fewest number of moves needed to win the game with n pieces is $2^n - 1$.

Example

Theorem

Every positive integer is larger than 5.

Proof.

We give a proof by induction.

Induction Step: Assume $5 < n$. Since $0 < 1$, we know that $n < n + 1$ (order property 1).

Since $5 < n$ and $n < n + 1$, it follows by the transitive property of $<$ that $5 < n + 1$.

Thus, it follows by induction that $5 < n$ for all positive integers n. $\square$

Example

$$1 + 2 + 3 + \cdots + n$$

Theorem

For all positive integers n, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Theorem

For all positive integers n, $4^n - 1$ is divisible by 3.

Challenge

You are allowed to cut a pizza in n straight lines. What is the largest possible number of pieces of pizza?