M1C03 Lecture 18

Factorial and Binomial Coefficients

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Announcement(s)

1. Test 1 Friday evening.
2. Test 1 details are on Avenue.
3. No quiz this week.

Overview

Reference: Lakins, 3.2 and 4.1.

The n rooks problem

Let n be a positive integer. How many ways are there to arrange n rooks on a $n \times n$ chessboard so that no two rooks are attacking?

The n rooks problem

Factorial and binomial coefficients

Definition: Define $0! = 1$. For all positive integers n, define $n! = (n-1)! \cdot n$.

$$n! = 1 \cdot 2 \cdots (n-1) \cdot n.$$

Definition: For all non-negative integers n, k with $k \leq n$, define

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Binomial coefficients

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

The binomial theorem

Theorem

Let x be a variable. For all non-negative integers n,

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

The binomial theorem

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Let x be a variable. For all non-negative integers n,

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Counting subsets of a fixed size

Theorem

For all non-negative integers n and k with $k \leq n$, the number of subsets of size k of a set of size n is $\binom{n}{k}$.

Theorem

For all non-negative integers n, if X is a set with n elements, then $\mathcal{P}(X)$ is a set with 2^n elements.