M1C03 Final Exam MC Practice

December, 2021

Some notes:

- These questions are select MULTIPLE choices. On the exam all the MC questions will be select ONE.
- Focus on understanding and using the definitions and carefully applying logic to those definitions when needed.
- Take your time and do scratchwork if necessary. Some of these questions are more complex than others.
- Try the problems by yourself first. If you are stuck on a problem, ask about it on Piazza.
- If you want extra practice writing proofs, try carefully writing a proof for some of the multiple choice items that are true.

Let ABC be a triangle.
Select ALL statements that are true.

1. $\angle ABC + \angle BCA + \angle CAB = \pi$
2. If $\angle ABC = \angle BAC$, then $AB = AC$.
3. If $AB = AC$, then $\angle ABC = \angle BCA$.
4. If $\angle ABC = \pi/2$, then $(AB)^2 + (AC)^2 = (BC)^2$.

2. Geometry

Let AB and CD and EF be lines such that AB intersects CD at M and CD intersects EF at N.

Select ALL statements that are true.

1. If $\angle AMN = \angle DNM$, then AB is parallel to EF.
2. If $\angle AMN + \angle DNM = \pi$, then AB is parallel to EF.
3. If AB intersects EF, then $\angle AMN + \angle DNM \neq \pi/2$.
4. If $\angle BMD = \angle FND$, then AB is parallel to EF.
5. If $\angle AMD > \angle FNC$, then AB is parallel to EF.

3. Logic

Let P and Q be statements.
Select ALL statements that are true.

1. $P \wedge \neg Q$ is logically equivalent to $\neg(\neg P \vee Q)$.
2. If $P \implies Q$, then $Q \implies P$.
3. If $P \implies Q$, then $\neg Q \implies \neg P$.
4. If $P \implies Q$, then Q is true.
5. If $P \implies \neg Q$ and Q is true, then P is false.
6. If P is false, then $P \implies Q$ is true.
7. If P is true and Q is true, then $P \implies Q$ is true.

Select ALL statements that are true.

1. $(\forall x \in \mathbb{R})(\exists y \in \mathbb{Z})(x \leq y)$.
2. $(\exists x \in \mathbb{R})(\forall y \in \mathbb{Z})(x \leq y)$.
3. $(\exists y \in \mathbb{Z})(\forall x \in \mathbb{R})(x \leq y)$.
4. $(\forall y \in \mathbb{Z})(\exists x \in \mathbb{R})(x \leq y)$.
5. $(\forall x \in \mathbb{R})(\exists p, q \in \mathbb{Z},\ q \neq 0)(x = \frac{p}{q})$
6. $(\forall p \text{ prime})(\forall a, b \in \mathbb{Z})(p|ab \implies p|a \vee p|b)$.
7. $(\exists n, a, b \in \mathbb{Z})(n|ab \wedge (n \not| a \wedge n \not| b))$.
8. If p prime and $p = ab$, $ab \in \mathbb{Z}$, then $a = 1$ or $b = 1$.
9. $(\forall x \in \emptyset)(\pi = 3)$.

5. Integers and number theory

Let m, n, p, q be integers.
Select ALL statements that are true.

1. If $m|n$ and $n|p$, then $m|p$.
2. If m^2 is even, then m is even.
3. If $p|mn$, then $p|m$ or $p|n$.
4. If $1 = pm + qn$, then $\gcd(m, q) = \gcd(m, n) = \gcd(p, q) = \gcd(p, n) = 1$
5. If $m = qn + p$, then $\gcd(m, n) = \gcd(n, p)$.
6. Every integer is even or odd.
7. No integer is both even and odd.
8. If n is not divisible by 3, then $n = 3k + 1$ for some $k \in \mathbb{Z}$.

Let $A \subset \mathbb{R}$.
Select ALL statements that are TRUE.

1. If A is bounded above, then A has a supremum.
2. If A has a supremum, then it is unique.
3. If A has a maximum element, then A has a supremum.
4. If A has a supremum, then A has a maximum element.
5. If A has a supremum, then A is bounded.
6. If $1000 \notin A$, then A is bounded above.
7. If $\forall a \in A$, $5 \leq a$, then A is bounded below.
8. If A is not bounded, then $\mathbb{R} - A$ is bounded.
9. If A is closed, then A is bounded.
10. If A is bounded, then A is closed.
11. If A is bounded and the supremum and infimum of A are both contained in A, then A is closed.

Select ALL statements that are TRUE.

1. $\sqrt{2}$ is irrational.
2. $\sqrt{3}$ is irrational.
3. $\sqrt{4}$ is irrational.
4. $\sqrt{5}$ is irrational.
5. If a is irrational and b is irrational, then $a + b$ is irrational.
6. If a is irrational and b is irrational, then $a \cdot b$ is irrational.
7. If a is irrational and b is rational, then $a + b$ is irrational.
8. If a is irrational and b is rational, then $a \cdot b$ is irrational.

8. Functions

Consider the function $f \colon \mathbb{N} \to \mathbb{N}$, $f(n) = n + 1$. Select ALL statements that are TRUE

1. f is injective.
2. f is surjective.
3. f is bijective.
4. f is invertible.
5. There exists a function $g \colon \mathbb{N} \to \mathbb{N}$ such that $f \circ g = I_{\mathbb{N}}$.
6. There exists a function $g \colon \mathbb{N} \to \mathbb{N}$ such that $g \circ f = I_{\mathbb{N}}$.

9. Sets

Let $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2\}$, $C = \{3, 4\}$, $D = \{1, 4\}$.
Select ALL statements that are TRUE.

1. $(B \cup C) - D = \{2, 3\}$
2. $B \cup C \subset A$
3. $A \cap D \subset B$
4. $\mathcal{P}(B) \subseteq \mathcal{P}(A)$
5. $(1, 3) \in A \times B$

10. Sets

Suppose we are given open sets $A_i \subseteq \mathbb{R}$ for all $i \in \mathbb{N}$.
Select ALL statements that are NOT NECESSARILY TRUE.

1. $\bigcup_{i=1}^{\infty} A_i$ is open.
2. $\bigcup_{i=1}^{\infty} \overline{A}_i$ is closed.
3. $\bigcap_{i=1}^{\infty} A_i$ is open.
4. $\bigcap_{i=1}^{\infty} \overline{A}_i$ is closed.

11. Cardinality

Select ALL statements that are TRUE.

1. $|\mathbb{N}| = |\mathbb{R}|$
2. $|\mathbb{Z}| = |\mathbb{Q}|$
3. $|\mathbb{Q}| = |\mathbb{R}|$
4. $|\mathbb{Q}| = |(0, 1)|$
5. $|\{1, 2, 3, 4\}| = |(0, 4)|$
6. $|\mathbb{Q}| = |\mathbb{R} - \mathbb{Q}|$.
7. $|\{1, 2, 3, 4\}| = |\mathcal{P}(\{1, 2\})|$.
8. $|\mathbb{N}^2| = |\mathbb{N}|$.
9. $|\{0, 1\} \times \{0, 1, 2\}| = 5$

12. Cardinality

Select ALL sets that are countable.

1. $\mathcal{P}(\mathcal{P}(\emptyset))$
2. $\mathbb{R}$
3. $\mathbb{Q}$
4. $\emptyset$
5. $(0, 1)$
6. The set B_∞ of infinite binary sequences (e.g., 101000111010...)
7. The set of infinite binary sequences such that only finitely many digits in each sequence are 1's (the rest are 0's).
8. $\mathbb{R} - \mathbb{Q}$.