M1C03 Lecture 27

The Division Algorithm and the Well-Ordering Principle

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Announcement(s)

- Quiz due Friday
- Assignment 4 due Friday

Overview

Well-ordering principle.

The division algorithm.

Reference: Lakins, section 6.1.

Some simple questions

What is the largest integer q such that $2q \le 11$? What is the remainder 11 - 2q?

What is the largest integer q such that $3q \leq -7$? What is the remainder -7 - 3q?

Some simple questions

What is the smallest $0 \le r = 11 - 2q$ where q can be any integer?

What is the smallest $0 \le r = -7 - 3q$ where q can be any integer?

Division algorithm

Given integers a and b with b > 0.

The *quotient* of a by b is the largest integer q such that $bq \leq a$.

The *remainder* is r = a - bq. Note that $0 \le r < b$.

By definition of q and r, a=bq+r.

Example: a = 11, b = 2.

Example: a = -7, b = 3.

The division algorithm

Theorem

Let $a,b \in \mathbb{Z}$ with b>0. There exists a unique pair of integers q and r with $0 \le r < q$ such that

$$a=qb+r.$$

Example

The square of any integer is equal to 3k or 3k+1, $k\in\mathbb{Z}.$

The well-ordering principle

The Well-Ordering Principle: Let S be a non-empty subset of the set of non-negative integers, $\mathbb{Z}_{\geq 0}$.

Then S has a smallest element m.

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Proof of the division algorithm (existence)

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Proof of the division algorithm (uniqueness)