M1C03 Lecture 20

Union, Intersection, Complement, and Cartesian Product

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Announcement(s)

1. Assignment 3 due Friday.
2. Quiz 3 due Friday.

Overview

Proving two sets are equal.

De Morgan's laws for sets.

Cartesian product.

Reference: Lakins, 4.2.

Recap

Definition: $A \subseteq B$ if $(\forall x)(x \in A \implies x \in B)$.

Definition: $A = B$ if $(\forall x)(x \in A \Longleftrightarrow x \in B)$.

Note: $A = B$ is equivalent to $A \subseteq B$ and $B \subseteq A$.

Definition: Let A and B be sets. The *union of A and B* is

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$

Definition: Let A and B be sets. The *intersection of A and B* is

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

Definition: Let A and B be sets. The *complement of A in B* is

$$B - A = \{x \mid x \in B \text{ and } x \notin A\}.$$

The *complement of A* (in the universe $\mathcal{U}$) is

$$\overline{A} = \mathcal{U} - A.$$

An identity

Theorem

Let A and B be subsets of a universe $\mathcal{U}$. Then,

$$B - A = B \cap \overline{A}.$$

Proof "by double inclusion"

Proof.

Given two sets X and Y. Want to show $X = Y$.

Proof that $X \subseteq Y$: Fix x arbitrary and assume $x \in X$. We want to show $x \in Y$. ...

Proof that $Y \subseteq X$: Fix x arbitrary and assume $x \in Y$. We want to show $x \in X$. ...

Thus, since $X \subseteq Y$ and $Y \subseteq X$, we have shown that $X = Y$. □

Rolling dice

Recall our example with one red die and one blue die from last week.

a) Consider the set of rolls that are doubles and satisfy $r + b = 7$. Describe rolls that are not in this set.
b) Describe the set of rolls that are not doubles or satisfy $r + b \neq 7$.

Theorem (Lakins, Theorem 4.2.6)

Let A and B be subsets of a universe $\mathcal{U}$.

1. $\overline{A \cap B} = \overline{A} \cup \overline{B}$.
2. $\overline{A \cup B} = \overline{A} \cap \overline{B}$

de Morgan's laws

de Morgan's laws

Cartesian product

Definition: Let A and B be sets. The *Cartesian product* of A and B is

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}.$$