M1C03 Lecture 23

Functions

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Announcement(s)

1. Test 1 results are out
2. Quiz due Friday
3. Assignment 4 posted

Overview

Definition and examples of functions

Reference: Notes on functions (Avenue), Lakins Chapter 5

Functions

A *function* consists of three things:

- a set X called the *domain*,
- a set Y called the *codomain*, and
- a *correspondence* (or rule, or formula) that assigns to every element of the domain a unique element of the codomain.

Functions on the alphabet

Let

$$\mathcal{L} = \{\text{“}a\text{”}, \text{“}b\text{”}, \ldots, \text{“}z\text{”}\}, \qquad \mathcal{U} = \{\text{“}A\text{”}, \text{“}B\text{”}, \ldots, \text{“}Z\text{”}\}, \qquad \mathcal{A} = \mathcal{L} \cup \mathcal{U}.$$

Let Cap be the correspondence that converts a letter to upper case.

Let Low be the correspondence that converts a letter to lower case.

Functions on binary sequences

Consider the set of binary sequences of length 4,

$$B_4 = \left\{ \begin{array}{cccccccc} 0000, & 1000, & 0100, & 0010, & 0001, & 1100, & 1010, & 1001, \\ 0101, & 0011, & 0110, & 1110, & 1101, & 1011, & 0111, & 1111 \end{array} \right\}.$$

- (bit flip) $n\colon B_4 \to B_4$ flips each digit in the sequence.
- (right shift) $r\colon B_4 \to B_4$ shifts every digit to the right. The leftmost digit becomes 0 and the rightmost digit disappears.
- (left shift) $l\colon B_4 \to B_4$ shifts every digit to the left. The rightmost digit becomes 0 and the leftmost digit disappears.

- $t\colon B_4 \to B_3$ removes the leftmost digit.
- $a\colon B_3 \to B_4$ appends 0 as the leftmost digit.

Functions of real numbers

Consider the correspondences:

- $f(x) = \sqrt{x}$ (where $\sqrt{x}$ means the non-negative square root of x), and
- $g(x) = x^2$.

Example

Let $\mathcal{C} = \{\text{“}black\text{”}, \text{“}red\text{”}, \text{“}blue\text{”}, \text{“}yellow\text{”}, \text{“}white\text{”}, \text{“}turquoise\text{”}\}$. Let $\mathcal{S}$ be the set of students in the classroom.

Let s be the correspondence that assigns a colour in $\mathcal{C}$ to students in the classroom that are wearing clothes of that colour.

Does this define a function $s\colon \mathcal{C} \to \mathcal{S}$? Why or why not?

Example

Let P be the set of polynomials of any degree in a variable x.

What are some examples of functions whose domain and codomain involves P?

Example

Let $M_{m\times n}(\mathbb{R})$ be the set of $m \times n$ matrices (a $m \times n$ square array of numbers).

What are some examples of functions whose domain and codomain involves $M_{m\times n}(\mathbb{R})$?

Example

What are some examples of functions in real-life?