M1C03 Lecture 8

Conditional statements

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Announcement(s)

1. Assignment 1 due Friday
2. Quiz 2 due Friday

Overview

- Finishing from last lecture
- Conditional statements
- Converse, contrapositive, and biconditional
- Tautology and contradiction
- Modus ponens (?)

Reference: Lakins, pp. 6 – 9.

Last time

The four basic connectives are defined by their truth tables:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	$\neg P$
T	F
F	T

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

Two expressions involving statements and connectives are *logically equivalent* if they have the same meaning (i.e. they have the same truth table).

Theorem (DeMorgan's Law)

1. $\neg(P \vee Q)$ *is logically equivalent to* $\neg P \wedge \neg Q$*.*
2. $\neg(P \wedge Q)$ *is logically equivalent to* $\neg P \vee \neg Q$*.*

Theorem

$P \implies Q$ *is logically equivalent to* $Q \vee \neg P$*.*

Sometimes it is helpful to clarify the meaning of an expression by adding brackets. For example $\neg P \wedge \neg Q$ above is better written as $(\neg P) \wedge (\neg Q)$.

Exercise

Show that:

- $\neg(P \implies Q)$ is logically equivalent to $\neg Q \wedge P$.
- $P \implies Q$ is logically equivalent to $\neg(\neg Q \wedge P)$

Implication

P	Q	$P \implies Q$	$\neg P \vee Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Example: If a person is in outer space without a space suit for longer than a minute, then they will be dead.

Complete the following truth table.

P	Q	$P \implies Q$	$Q \implies P$
T	T	T	
T	F	F	
F	T	T	
F	F	T	

Example: If the Blue Jays win a game, then Jeremy is happy.

Complete the following truth table.

P	Q	$\neg P$	$\neg Q$	$P \implies Q$	$\neg Q \implies \neg P$
T	T	F	F		
T	F	F	T		
F	T	T	F		
F	F	T	T		

Example: If the Blue Jays win a game, then Jeremy is happy.

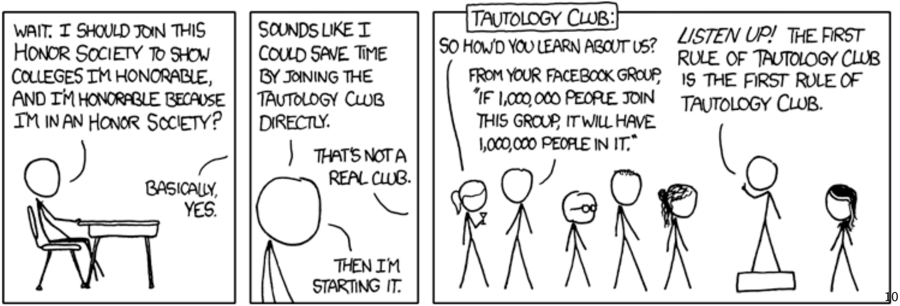
Biconditional

The *biconditional* is the expression $(P \implies Q) \wedge (Q \implies P)$. We write $P \iff Q$.

P	Q	$P \implies Q$	$Q \implies P$	$P \iff Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Example: The Blue Jays win a game if and only if Jeremy is happy.

P	$\neg P$	$P \vee \neg P$
T	F	
T	F	
F	T	
F	T	



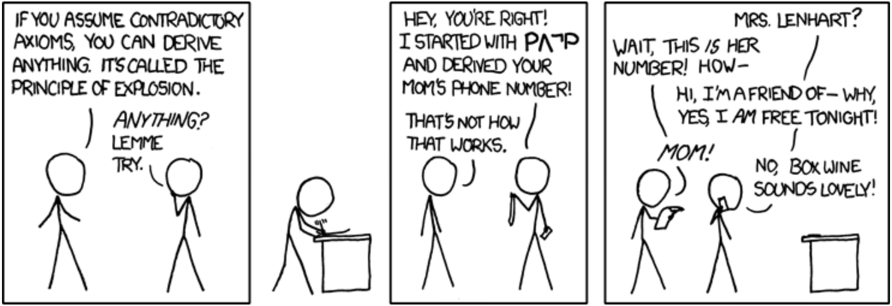
Contradiction

Complete the following truth table.

P	$\neg P$	$P \wedge \neg P$
T	F	
T	F	
F	T	
F	T	

Example: $1 = 2$.

Example: Two distinct lines are parallel and they intersect.



$P \land \neg P$	Q	$(P \land \neg P) \implies Q$
F	T	
F	F	

Modus Ponens

Modus ponens is the latin name for the type of deductive argument we use over and over in direct proofs.

Theorem (Modus Ponens)

If P is true and we know that $P \implies Q$, then Q is true.

P	Q	P	$P \implies Q$	Q
T	T			
T	F			
F	T			
F	F			