M1C03 Lecture 7

Logical connectives and truth tables

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Announcement(s)

1. Assignment 1 due Friday
2. Quiz 2 due Friday

Overview

- The four basic connectives
- Truth tables
- Logically equivalent statements

Reference: Lakins, pp. 2 – 8.

Logical connectives

Consider the statements "it is raining" and "today is Wednesday."

We can combine them to produce new statements:

- It is raining *and* today is Wednesday.
- It is raining *or* today is Wednesday.
- It is *not* raining.
- *If* it is raining, *then* today is Wednesday.

Logical connectives combine one or more statements to produce a new statement.

Logical connectives

There are four basic connectives:

- *Conjunction* (and)
- *Disjunction* (inclusive or)
- *Negation* (not)
- *Implication* (if, then), sometimes referred to as *conditional*

We represent statements with capital letters. e.g. $P =$"it is raining" and $Q =$"today is Wednesday."

We represent the basic connectives above with the symbols $\wedge$, $\vee$, $\neg$, and $\implies$.

- $P \wedge Q$: "It is raining *and* today is Wednesday."
- $P \vee Q$: "It is raining *or* today is Wednesday."
- $\neg P$: "It is *not* raining."
- $P \implies Q$: "*If* it is raining, *then* today is Wednesday."

Truth tables

Truth tables tell us **precisely** what statements formed using logical connectives mean.

They do this by describing the truth value of the new statment for every possible combination of truth values of the statements that it was built from.

The truth tables of the four basic connectives are:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	$\neg P$
T	F
F	T

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

These truth tables are **the definition** of what the four basic connectives mean.

WARNING: the logical meaning of disjunction and implication is defined above. It might not agree with what your expectations. Be careful. When in doubt, use the truth table.

Truth tables for complex expressions

Truth tables become very useful when we encounter expressions with more complex combinations of logical connectives.

Example: Consider $(\neg P) \vee Q$. In our running example, this would be "It is not raining or today is Wednesday."

To determine exactly what $(\neg P) \vee Q$ means, we write down it's truth table.

P	Q	$\neg P$	$(\neg P) \vee Q$
T	T		
T	F		
F	T		
F	F		

Exercise

Complete the following truth table.

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T					
T	F					
F	T					
F	F					

Here are the four fundamental truth tables in case you forgot them:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	$\neg P$
T	F
F	T

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

Logical equivalence

If two expressions have the same truth table, then they have the same logical meaning. In this case, we say that the expressions are *logically equivalent*.

Here are some important examples of logical equivalences.

Theorem (DeMorgan's Law)

1. $\neg(P \vee Q)$ *is logically equivalent to* $\neg P \wedge \neg Q$.
2. $\neg(P \wedge Q)$ *is logically equivalent to* $\neg P \vee \neg Q$.

Theorem

$P \implies Q$ *is logically equivalent to* $Q \vee \neg P$.

Useful fact: If you apply the same connective to two logically equivalent expressions, then the resulting expressions are logically equivalent. For example, if A is logically equivalent to B, then $\neg A$ is logically equivalent to $\neg B$.