M1C03 Lecture 19

Union, Intersection, Complement, and Cartesian Product

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Announcement(s)

1. Content > Tests > Test 1.
2. Read the Scantron instructions! Bring a pencil, eraser.

Some advice:

1. Take your time.
2. Read everything carefully.
3. Remember and use the definitions.
4. For proofs:
    1. Understand the statement you are trying to prove (definitions!).
    2. Write the Given/Goal.
    3. Do scratch work and remember some of the problem solving tips (e.g. work backwards, cases, etc.)
    4. Write down the proof.
    5. Try to justify each step.
5. If there is still time, re-read the test and your answers.

Overview

There are several important operations on sets.

Intersection, union, and complement are intimately related to the logical connectives: ‘and’, ‘or’, and ‘not’.

For example, the de Morgan’s laws from logic lead to de Morgan’s laws for sets.

There is also Cartesian product (and power set, but we covered that already).

Reference: Lakins, 4.2.

Recap

"Definition": A set is an unordered collection of distinct things.

A set can only tell us whether a thing is contained in it or not.

A set is not an ordered list! A thing cannot be contained in a set multiple times!

Enumerative description: $\{a, b, c\}$

Constructive/set-builder description: $\{x \in \mathbb{Z} \mid (\exists k)(n = 2k)\}$.

Definition: $A \subseteq B$ if $(\forall x)(x \in A \implies x \in B)$.

Definition: $A = B$ if $(\forall x)(x \in A \iff x \in B)$.

Note: $A = B$ is equivalent to $A \subseteq B$ and $B \subseteq A$.

Rolling dice

Suppose we have two 6-sided dice. One is red and one is blue.

Rolling dice

Describe the set of all rolls that are 'doubles' or satisfy $r + b = 7$.

Set union

Definition: Let A and B be sets. The *union of A and B* is

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$

Rolling dice

Describe the set of all rolls where r is even and b is odd.

Set intersection

Definition: Let A and B be sets. The *intersection of A and B* is

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

Rolling dice

Describe the set of all rolls that are not doubles.

Rolling dice

Describe the set of all rolls where the blue die is 2 and the red die is not 3.

Set complement

Definition: Let A and B be sets. The *complement of A in B* (also called the *difference of B and A*) is

$$B - A = \{x \in B \mid x \notin A\}.$$

The complement of A in the universe $\mathcal{U}$ is simply the *complement of A*. Denote $\overline{A} = \mathcal{U} - A$.

Rolling dice

a) Consider the set of rolls that are doubles and satisfy $r + b = 7$. Describe rolls that are not in this set.
b) Describe the set of rolls that are not doubles or satisfy $r + b \neq 7$.

de Morgan's laws

Theorem (Lakins, Theorem 4.2.6)

Let A and B be subsets of a universe $\mathcal{U}$.

1. $\overline{A \cap B} = \overline{A} \cup \overline{B}$.
2. $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Cartesian product

Definition: Let A and B be sets. The *Cartesian product* $A \times B$ is the set of pairs

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}.$$