M1C03 Lecture 24

Injective, Surjective, and Bijective Functions

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Announcement(s)

1. Test 1 results are out
2. Quiz due Friday
3. Assignment 4 posted

Overview

Injective, surjective, and bijective functions

Reference: Notes on functions (Avenue), Lakins Chapter 5.

Example

What are some examples of functions in real-life?

Injective Functions

A function $f\colon X \to Y$ is *injective* if for all $x_1, x_2 \in X$, if $f(x_1) = f(x_2)$, then $x_1 = x_2$.

Functions on the alphabet

Let

$$\mathcal{L} = \{\text{“}a\text{”}, \text{“}b\text{”}, \ldots, \text{“}z\text{”}\}, \qquad \mathcal{U} = \{\text{“}A\text{”}, \text{“}B\text{”}, \ldots, \text{“}Z\text{”}\}, \qquad \mathcal{A} = \mathcal{L} \cup \mathcal{U}.$$

Let Cap be the correspondence that converts a letter to upper case.

Describe all subsets $X \subseteq \mathcal{A}$ such that $Cap\colon X \to \mathcal{A}$ is injective.

Surjective Functions

A function $f : X \to Y$ is *surjective* if for all $y \in Y$, there exists $x \in X$ such that $f(x) = y$.

Functions on the alphabet

Let

$$\mathcal{L} = \{\text{“}a\text{”}, \text{“}b\text{”}, \ldots, \text{“}z\text{”}\}, \qquad \mathcal{U} = \{\text{“}A\text{”}, \text{“}B\text{”}, \ldots, \text{“}Z\text{”}\}, \qquad \mathcal{A} = \mathcal{L} \cup \mathcal{U}.$$

Let Cap be the correspondence that converts a letter to upper case.

Describe all subsets $Y \subseteq \mathcal{A}$ such that $Cap\colon \mathcal{A} \to Y$ is surjective.

Functions on binary sequences

Recall

$$B_4 = \left\{ \begin{array}{cccccccc} 0000, & 1000, & 0100, & 0010, & 0001, & 1100, & 1010, & 1001, \\ 0101, & 0011, & 0110, & 1110, & 1101, & 1011, & 0111, & 1111 \end{array} \right\}$$

Which of the following functions are injective, surjective?

- $n\colon B_4 \to B_4$ bit flip.
- $r\colon B_4 \to B_4$ right shift.
- $t\colon B_4 \to B_3$ removes the leftmost digit.
- $a\colon B_3 \to B_4$ appends 0 as the leftmost digit.

Functions of real numbers

Which of the following functions are injective, surjective?

- $f\colon [0,\infty) \to [0,\infty)$, $f(x) = \sqrt{x}$.
- $g\colon \mathbb{R} \to \mathbb{R}$, $g(x) = x^2$.

Bijective functions

A function $f : X \to Y$ is *bijective* if it is surjective and injective.

Function composition

Let $f\colon X \to Y$ and $g\colon A \to B$ be functions with $Y \subset A$. The *composition of $f\colon X \to Y$ and $g\colon A \to B$* is the function

$$g \circ f\colon X \to B$$

defined by the rule $g \circ f(x) = g(f(x))$.

Functions on the alphabet

Let

$$\mathcal{L} = \{\text{“}a\text{”}, \text{“}b\text{”}, \ldots, \text{“}z\text{”}\}, \qquad \mathcal{U} = \{\text{“}A\text{”}, \text{“}B\text{”}, \ldots, \text{“}Z\text{”}\}, \qquad \mathcal{A} = \mathcal{L} \cup \mathcal{U}.$$

Let $Cap\colon \mathcal{A} \to \mathcal{A}$ be the function that converts a letter to upper case.

Let $Low\colon \mathcal{A} \to \mathcal{A}$ be the function that converts a letter to upper case.

What is $Cap \circ Low(\text{“}a\text{”})$?

What is $Low \circ Cap(\text{“}a\text{”})$?

Functions on binary sequences

- $n\colon B_4 \to B_4$ bit flip.
- $r\colon B_4 \to B_4$ right shift.
- $t\colon B_4 \to B_3$ removes the leftmost digit.
- $a\colon B_3 \to B_4$ appends 0 as the leftmost digit.

Compute:

- $n \circ n(1010)$
- $a \circ t(1010)$
- $t \circ a(111)$
- $r \circ l(1010)$
- $l \circ r(1111)$
- $r \circ (r \circ r)(1010)$