M1C03 Lecture 5

The Parallel Postulate

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Announcement(s)

1. Homework 1 will be posted this weekend

Overview

1. Parallel lines
2. Proof by contradiction
3. The parallel postulate
4. Sum of interior angles of a triangle

Postulates

The axioms for constructing geometric figures with a *compass* and *straightedge*.

[P1] A straight line may be drawn from any one point to another point.

[P2] A line segment can be extended any distance beyond each endpoint.

[P3] A circle can be constructed from any given point and radius.

[P4] All right angles are equal to one another.

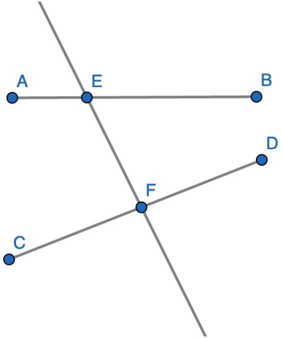
[P5] Today!

Parallel lines

Definition: Two lines are *parallel* if they can be extended infinitely in both directions without intersecting.

Proposition

- If a line intersects two straight lines and the alternate angles are equal, then the two straight lines are parallel.



Parallel lines

Proposition

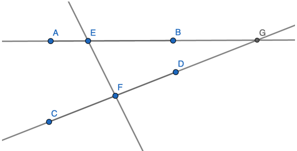
If a line intersects two straight lines and the alternate angles are equal, then the two straight lines are parallel.

Proof.

We give a proof by contradiction. Suppose a line (EF) intersects two straight lines (AB and CD) so that the alternate angles are equal, but *the two lines are not parallel*.

1. Since the two lines are not parallel, they eventually intersect. Label the intersection point G.
2. By assumption, the exterior angle $\angle AEF$ equals the opposite interior angle $\angle EFD$.
3. But the exterior angle must be greater than the opposite interior angles, so this is impossible!

Since we have reached a contradiction, our assumption that the two lines are not parallel must be false. Thus, the two lines are parallel. □



Proof by contradiction

Until now, the proofs we saw were "direct proofs." In a *direct proof* one starts from the assumptions of the claim and works logically towards the conclusion.

The previous proof is an example of proof by contradiction. The strategy in a proof by contradiction is slightly different.

Suppose we want to show a claim is true. In other words, we want to show that if the assumptions are true, then the conclusion is true. We know that the conclusion is either TRUE or FALSE; there are no other possibilities. If we can rule out the possibility that the conclusion is FALSE, then by process of elimination it must be TRUE.

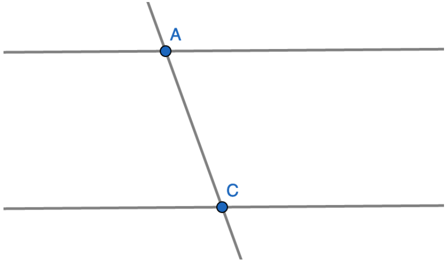
In a *proof by contradiction* we start by assuming what we are given AND that *the conclusion of our claim is FALSE*. Then we use these assumptions to reach a new conclusion that is logically impossible. This impossibility rules out the possibility that the conclusion of our claim is FALSE.

This is a trickier method of proof. We will revisit it.

The Parallel Postulate

Proposition

- If a line intersects two parallel straight lines, then the alternate angles are equal.



To prove this, we need the last postulate:

[P5] If two lines are parallel to the same line, then they are parallel to each other.

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Proposition

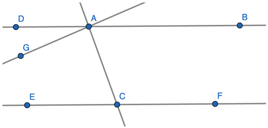
If a line intersects two parallel straight lines, then the alternate angles are equal.

Proof.

Suppose for the sake of contradiction that $\angle DAC \neq \angle FCA$.

1. (exercise) Draw the line GA so that $\angle GAC = \angle FCA$. GA is different from AB.
2. Then GA is parallel to EF.
3. (P5) Since AB is also parallel to EF, we have that AB is parallel to GA.
4. Thus we have constructed two lines AB and GA that intersect and are parallel, which is impossible!

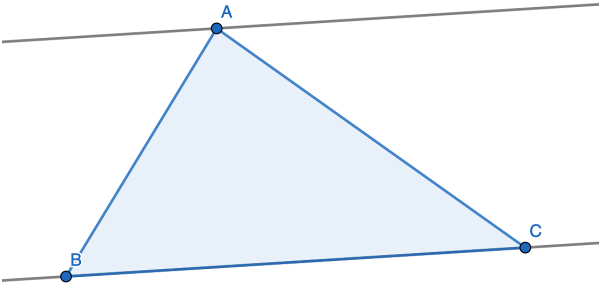
Since we have reached a contradiction, our assumption that $\angle DAC \neq \angle FCA$ must be false. Thus, $\angle DAC = \angle FCA$. □



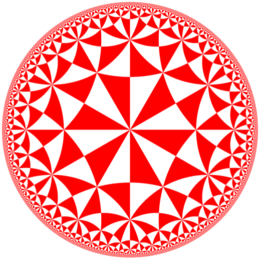
Proposition

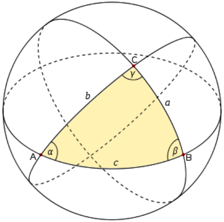
The sum of the interior angles of a triangle equals the sum of two right angles.

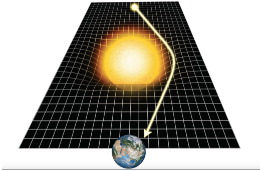
Hint: use the following diagram.



The parallel postulate has an important history that resulted in modern geometry and general relativity!







Work on understanding the parallel postulate was also related to efforts to better understand the foundations of Euclidean Geometry and math as a whole.