M1C03 Lecture 22.5

Open/closed subsets of $\mathbb{R}$

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Announcement(s)

1. Test 1 results are in, they will be posted soon.
2. Quiz due Friday.

Overview

Open and closed subsets of $\mathbb{R}$.

Reference: Lakins, 4.2, 4.3 (exercises).

Examples

- $\bigcap_{j=1}^{\infty}(-1/j, 1/j) = \{0\}.$
- $\bigcup_{j=1}^{\infty}[1/j, 1 - 1/j] = (0, 1).$

Open and closed sets in $\mathbb{R}$ [Lakins, Exercise 4.2.24, 4.3.11, 4.3.12]

Definition: A set $S \subseteq \mathbb{R}$ is *open* if for all $x \in S$, there exists a real number $r > 0$ such that $(x - r, x + r) \subseteq S$.

Definition: A set $S \subseteq \mathbb{R}$ is *closed* if $\overline{S} = \mathbb{R} - S$ is open.

Example: $\mathbb{R}$ is an open set. $\emptyset \subset \mathbb{R}$ is a closed set.

Example

$\emptyset \subset \mathbb{R}$ is an open set. $\mathbb{R}$ is a closed set.

Example

The interval $(0,1) \subset \mathbb{R}$ is an open set. It is not closed.

Example

The interval $[0, 1] \subset \mathbb{R}$ is a closed set. It is not open.

Warning!

Closed $\neq$ not open. Open $\neq$ not closed.

- $\mathbb{R}$ and $\emptyset$ are open and closed.
- $(0, 1)$ is open and not closed.
- $[0, 1]$ is closed and not open.
- $[0, 1)$ is not open and not closed.

Exercises

Theorem (Lakins, Exercise 4.2.24)

Let J be an index set and let A_j be a family of sets indexed by J with $A_j \subset \mathbb{R}$.

- *If for all $j \in J$, A_j is open, then $\bigcup_{j \in J} A_j$ is open.*
- *If for all $j \in J$, A_j is closed, then $\bigcap_{j \in J} A_j$ is closed.*

However, the following statements are FALSE:

- If for all $j \in J$, A_j is open, then $\bigcap_{j \in J} A_j$ is open.
- If for all $j \in J$, A_j is closed, then $\bigcup_{j \in J} A_j$ is closed.