Probabilistic Modelling of Football Results

Background - Football

- Goals are very rare events (often times the final score is 0-0), but still the main measure of success in a match.
- Goal-scoring models include Expected Goals (xG), and Poisson.
- Poisson focuses more on match/outcome results, e.g. probability of wins, draws, losses.
    - Limited in-depth analysis of a team’s performance.

Research Question

- **Can we model football results using a Poisson distribution with informative, team-specific parameters?**

- Do win-streaks influence a team's results?

Methodology

Assumptions

- Number of goals scored by a team in a match is a Poisson variable.
  - Each goal is independent of another.
  - Match duration is constant.
- Number of goals scored by a team depends only on their offensive strength and the opponent's defensive strength.
- Given the model's parameters, matches are independent of each other.
- A team's strengths/weaknesses do not change throughout the season.
- We focused on a single season: 2014-15 English Premier League.

Model

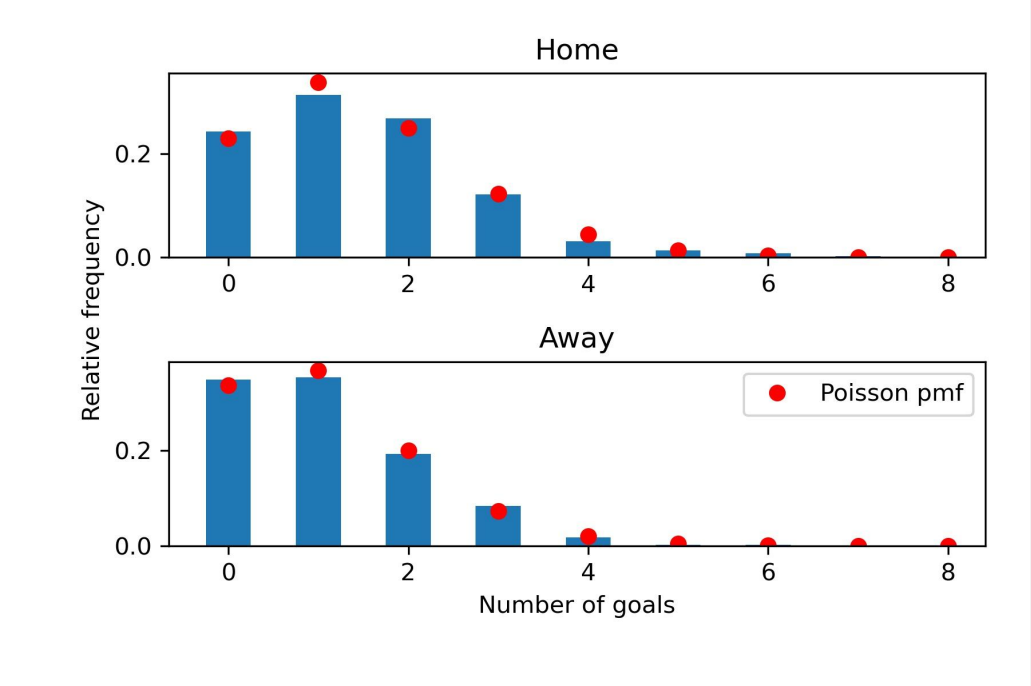
Match between teams i and j, with i playing at home, and a final score of ***n-m***

$$X \sim \text{Pois}(O_{H,i} D_{A,j})$$
$$Y \sim \text{Pois}(O_{A,j} D_{H,i})$$

$$P(X = n) = \frac{(O_{H,i} D_{A,j})^n \exp(-O_{H,i} D_{A,j})}{n!}$$
$$P(Y = m) = \frac{(O_{A,j} D_{H,i})^m \cdot \exp(-O_{A,j} D_{H,i})}{m!}$$

Statistical Significance

- Chi-Squared goodness of fit test
- P-values: 0.603 and 0.75 for home and away respectively.



Model Implementation

- Our main goal is to estimate the model parameters numerically
  - Use gradient descent to maximise log-likelihood of the data given our model parameters
  - Independence assumptions allow us to look at each match separately

$$P(X = n) = \frac{(O_{H,i}D_{A,j})^n \exp(-O_{H,i}D_{A,j})}{n!}$$

$$P(Y = m) = \frac{(O_{A,j}D_{H,i})^m \cdot \exp(-O_{A,j}D_{H,i})}{m!}$$

Model Implementation

$$O_{H,i} \leftarrow O_{H,i} + \gamma * \left(\frac{n}{O_{H,i}} - D_{A,j} \right)$$

$$D_{H,i} \leftarrow D_{H,i} + \gamma * \left(\frac{m}{D_{H,i}} - O_{A,j} \right)$$

$$O_{A,j} \leftarrow O_{A,j} + \gamma * \left(\frac{m}{O_{A,j}} - D_{H,i} \right)$$

$$D_{A,j} \leftarrow D_{A,j} + \gamma * \left(\frac{n}{D_{A,j}} - O_{H,i} \right)$$

Momentum

$$X \sim \text{Poisson}(O_{H,i} D_{A,j} \mathbf{s^{d_i}})$$

$$s \leftarrow s + \gamma * \left(\frac{n d_i}{s} - d_i O_{H,i} D_{A,j} s^{d_i - 1} \right)$$

$$s \leftarrow s + \gamma * \left(\frac{m d_j}{s} - d_j O_{A,j} D_{H,i} s^{d_j - 1} \right)$$

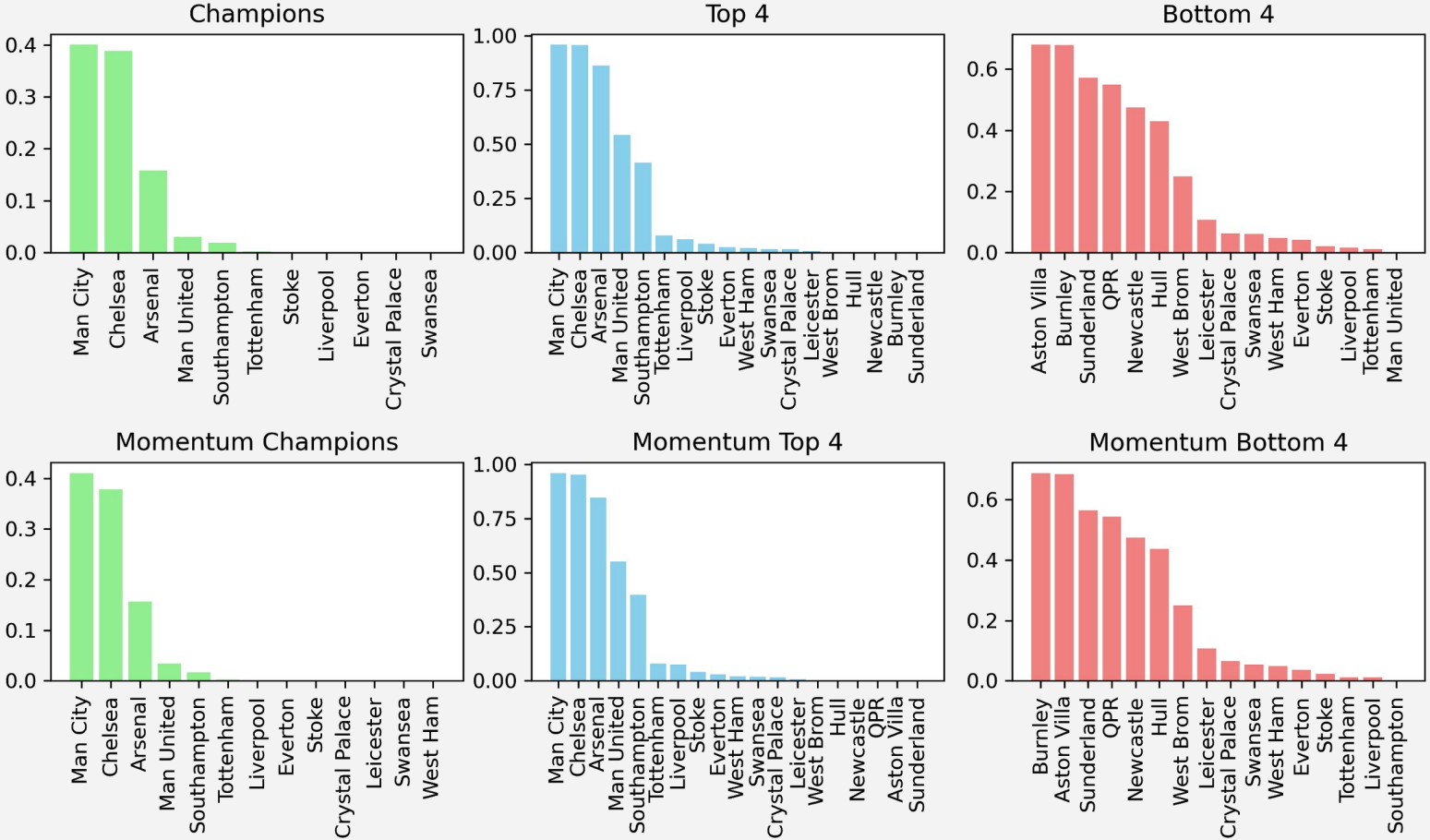
Results

Estimated parameters:

- Momentum parameters were very similar

$$s = 1.0033$$

	Team	Points	W	D	L	GD	GF	GA	O_H	D_H	O_A	D_A
1	Chelsea	87	26	9	3	41	73	32	1.53	0.48	1.78	1.00
2	Man City	79	24	7	7	45	83	38	1.87	0.76	1.91	1.08
3	Arsenal	75	22	9	7	35	71	36	1.74	0.74	1.47	0.98
4	Man United	70	20	10	8	25	62	37	1.74	0.77	1.03	0.99
5	Tottenham	64	19	7	12	5	58	53	1.33	1.24	1.35	1.28
6	Liverpool	62	18	8	12	4	52	48	1.30	1.03	1.09	1.24
7	Southampton	60	18	6	14	21	54	33	1.58	0.66	0.83	0.89
8	Swansea	56	16	8	14	-3	46	49	1.15	1.12	0.95	1.19
9	Stoke	54	15	9	14	3	48	45	1.36	1.11	0.80	1.02
10	Crystal Palace	48	13	9	16	-4	47	51	0.90	1.40	1.31	1.04
11	Everton	47	12	11	15	-2	48	50	1.17	1.07	1.03	1.26
12	West Ham	47	12	11	15	-3	44	47	1.09	0.92	0.94	1.26
13	West Brom	44	11	11	16	-13	38	51	1.03	1.31	0.71	1.09
14	Leicester	41	11	8	19	-9	46	55	1.22	1.12	0.91	1.45
15	Newcastle	39	10	9	19	-23	40	63	1.13	1.36	0.70	1.58
16	Aston Villa	38	10	8	20	-26	31	57	0.79	1.26	0.66	1.38
17	Sunderland	38	7	17	14	-22	31	53	0.69	1.35	0.75	1.10
18	Hull	35	8	11	19	-18	33	51	0.82	1.20	0.70	1.17
19	Burnley	33	7	12	19	-25	28	53	0.61	1.05	0.70	1.37
20	QPR	30	8	6	24	-31	42	73	1.03	1.22	0.95	2.12


Champions
Man City
Chelsea
Arsenal
Man United
Southampton
Tottenham
Stoke
Liverpool
Everton
Crystal Palace
Swansea
Top 4
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Chelsea
Arsenal
Man United
Southampton
Tottenham
Liverpool
Stoke
Everton
West Ham
Swansea
Crystal Palace
Leicester
West Brom
Hull
Newcastle
Burnley
Sunderland
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Tottenham
Liverpool
Southampton

Observations

- Momentum had very little impact on overall performance
- As expected, goal-scoring proficiency has a great impact on a team's performance, according to our model
    - Manchester City were favoured
    - Southampton performed much better than their final standing suggests
    - Aston Villa over-performed - should have been relegated

Conclusion

Final Remarks

- Model relies heavily on goal-scoring proficiency, and less so on results and playstyle (because of assumptions).
- The model is able to tell whether a team is performing or not, but does not give the underlying reasons.
- Momentum has a very small impact on a team's performance.
- Somewhat expensive parameter calculation.
- Possible application to betting.

Possible Extensions

- Look at more data, and train for a longer time period to account for computational costs.
- Team dynamics such as player choices, injuries,
- Compare with more complex models that use in-game statistics.
- Compare the Premier League with other competitions.
    - The 2014-2015 English Premier League was somewhat one-sided.

Questions?