Running head: LEARNING FROM HUMAN ACTION

Learning from others:

The consequences of psychological reasoning for human learning

Patrick Shafto

University of Louisville

Noah D. Goodman & Michael C. Frank

Stanford University

Abstract

Models of learning usually assume that data are randomly collected facts about the world. From early childhood, however, human beings learn not only from the collection of facts, but from social situations as well: from observation of other people, from communication, and from explicit teaching. In these situations, the data are the result of human actions—actions that come about because of people's goals and intentions. To interpret the implications of others' actions correctly, learners must understand the people generating the data. We provide a Bayesian analysis of learning from knowledgeable others, which formalizes how a learner may reason from a person's actions and goals to infer their knowledge about the world. We illustrate this framework using two examples from causal learning and conclude by discussing the implications for cognition, social reasoning, and cognitive development.

Learning from others:

The consequences of psychological reasoning for human

learning

Children are often compared to scientists, but even a perfect scientist, using experiments alone, would struggle to rediscover all of human knowledge in the span of one lifetime. How then are children able to acquire a good fraction of this knowledge in just a few years? The answer must be that children do not rediscover everything—they use their ability to reason intuitively about other people to learn what others already know. It is the goal of this paper to sketch a formal analysis of learning from knowledgeable others, by using the tools of Bayesian inference and a careful examination of the kinds of goals that give rise to human actions. We begin by motivating the need of a learner to consider the particular goals of people in their environment.

Imagine, while living in Paris, you decide to search for the best cup of coffee in the city. As you wander, you find yourself a good distance away from your neighborhood. You observe three different pieces of evidence: first, a man wearing a baseball cap and an "I heart Paris" t-shirt (obviously a tourist) turns into cafe #1, buys a coffee, and looks down at his cup. Second, Véronique, a woman from your neighborhood, enters cafe #2 to get a coffee, and looks down at her cup. Third, Madeleine, another woman from your neighborhood, goes into cafe #3 and buys a cup. Madeleine sees you, and she nods at the coffee.

Which cafe would you think has the best coffee? You know very little about the coffee at cafe #1, because the tourist likely chose the cafe at random. Cafe #2 was visited by a local, but maybe Véronique was just strapped for time and grabbing a cuppa wherever she could. Without knowing her motivations, you can't tell whether she has gone out of her way to go to this particular cafe (though you may guess at her

motivations, and hence guess her beliefs about this cafe). On the other hand, in Cafe #3, Madeleine telegraphed her intentions to you: She was there for the coffee, and she congratulated you for figuring out a local secret. Although nothing is certain (for example, Madeleine could have terrible taste), cafe #3 seems likely to have the best coffee, and cafe #2 is likely to have better coffee than #1. In this paper, we propose a formal framework for understanding why reasoning from randomly chosen, goal-directed, and communicative actions lead to qualitative differences in learning from superficially similar observations.

Traditional approaches to understanding how people learn about the world so quickly and robustly have focused on the nature of human representations and a priori biases about the physical world, investigating the biases that allow rapid and accurate learning from observed data. In concept learning, the problem has been characterized as "carving nature at its joints" and the debates have been over the types of representations that support these abilities (Gelman, 1996; Keil, 1989; Mandler, 1992; Medin & Schaffer, 1978; Nosofsky, 1984; Posner & Keele, 1968; Rosch & Mervis, 1975; Rosch, 1978). Similarly, in causal learning, the learning problem has been viewed as one of discovering the laws that govern physical world (Michotte, 1963), and debates have been over the representational and inferential mechanisms that support these abilities (Cheng, 1997; Gopnik, Glymour, Sobel, Schulz, & Danks, 2004; Griffiths & Tenenbaum, 2005; Rescorla & Wagner, 1972). These approaches have been very successful in describing reasoning in situations where the data are (assumed to be) observed objectively. For example, in a syllogistic reasoning problem—if A, then B; observing A implies B—the conditions of observation of A are assumed to be irrelevant to the truth of B. A is simply a given.

Much of the evidence observed in human learning does not have this character. Often evidence is provided *by someone, for some purpose.* A potter throwing a pot, a friend fiddling with their iPod, a parent demonstrating how to tie a shoe, and a teacher conveying a mathematical concept are all creating evidence with a particular goal in mind.

The goals of the pot thrower and the iPod fiddler have to do with the world—they are trying to get matter or artifacts to conform to their desires. The goals of the parent and teacher have to do with the observer; they want the observer to learn a particular fact, skill, or concept. In each of these cases, it is possible for people to learn a tremendous amount from only a small number of data points.

Yet for many traditional formal approaches, learning based on limited data is nearly impossible except in the most circumscribed domains (Gold, 1967; Savage, 1951; Wolpert & Macready, 1997; Zinkevich, 2003). Most famously, Gold (1967) proved that in any formal language that is sufficiently broad to express an infinite range of possible sentences (so that the sentences could not possibly be enumerated one by one), language learning is impossible. Strikingly, this proof suggests that human languages are unlearnable! The conflict between between human intuition and formal analysis creates a puzzle. How is human learning so quick and successful when formal learning frameworks suggest that it should be slow and fatally conservative?

We believe that the key to this puzzle lies in the assumptions that traditional learning theory makes about the *conditions of observation.* For instance, Gold's proof assumes that the datapoints for learning are selected by an adversary. Imagine trying to learn which cafe is good when everyone is deliberately trying to mislead you! When this assumption is relaxed even slightly and data are assumed to be sampled randomly, the language learning problem studied by Gold is found to be considerably less difficult (Horning, 1969). Nonetheless, learning from randomly sampled data—as opposed to data from an adversary who wants to "fool" the learner (in Gold's words)—can still be quite difficult. Paris is a big city. Imagine trying to learn where the best coffee is by randomly sampling cafes. It would take a very long time. Yet most models adopt this sort of random-sampling assumption (Anderson, 1991; Love, Medin, & Gureckis, 2004; Goodman, Tenenbaum, Feldman, & Griffiths, 2008; Kruschke, 1992; Medin & Schaffer, 1978;

Nosofsky, 1984; Nosofsky, Palemeri, & McKinley, 1994; Pothos & Chater, 2002).

Research on human learning paints a very different picture of how datapoints are selected (Bandura, Ross, & Ross, 1961; Bruner, 1966; Csibra & Gergely, 2009; Gergely & Csibra, 2003; Meltzoff & Moore, 1977; Tomasello, 1999; Tomasello, Carpenter, Call, Behne, & Moll, 2005; Vygotsky, 1978). A wide variety of approaches have pointed to people, and their intentions, as an important factor in learning, highlighting that data are chosen rather than random (Bruner, 1966; Vygotsky, 1978) and that observed data are often the consequence of goal-directed actions (Bandura et al., 1961; Gergely & Csibra, 2003; Meltzoff & Moore, 1977) or intentional communication/teaching (Coady, 1992; Csibra & Gergely, 2009; Harris, 2002; Tomasello et al., 2005). For instance, Csibra & Gergely (2009) suggest that young children's interpretation of observed data changes fundamentally based on whether the demonstrator engages the child with ostensive cues—saying the child's name, using child-direct speech, shifting gaze between the child and the object of the demonstration—prior to the demonstration. According to their account, these cues lead children to assume that the demonstrated data are not randomly sampled instances but purposefully sampled to support broad generalizations.

Psychological intuitions about what makes human learning so effective remain exactly that—intuitions—until we can formalize and test whether they actually lead to faster, more robust learning. Indeed, one explanation for the disagreement is that there has not been a framework in which these different proposals can be formalized and their implications tested. We describe a computational theory of learning from other people's actions with the goal of reconciling the formal literature on learning with psychological accounts about learning from other people. Our approach is inspired by classic work on attribution theory that characterizes social attributions as inferences about unseen traits or goals (Kelley, 1967; Jones & Davis, 1965), and builds on recent work that formalizes "theory of mind" as the inverse of rational decision making (Baker, Saxe, & Tenenbaum,
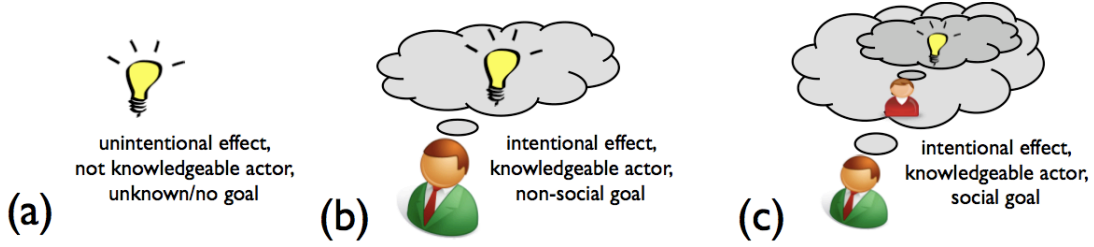
Figure 1. Schematic of three social learning contexts. (a) When learning from physical evidence, learners reason about the implications of the data for different hypotheses. (b) When learning from goal-directed action, learners reason about the actor's goal with respect to the world. (c) When learning from communicative action, learners reason about which belief the actor intends the learner to have.

2009). Bayesian methods describe how learners can work backward from an agent's actions to her goals and beliefs; when the learner has reason to believe that the agent is knowledgeable, the learner can then use this information to strengthen inferences about the structure of the world. We use three different learning situations to illustrate how learning may differ as a consequence of this kind of intuitive psychological reasoning: 1) learning from physical evidence, 2) learning from goal-directed action, and 3) learning from communicative action (see Figure 1). We discuss how to formalize different kinds of goal-directed actions in our framework, as well as how different assumptions about the agent's goals may afford qualitatively different inferences by learners. We conclude by discussing implications for theories of learning, cognitive development, and the relationship between social and cognitive psychology.

A framework for modeling goal-directed actions

We want to formalize how an actor's goal can be used by learners to guide inference: Why does observing Madeleine's nod lead to a strong inference about the quality of the

coffee, while observing a tourist look down while drinking the coffee does not? We can formalize this inference in a way that depends on actors knowledgeability, as a relation between actions that learners observe, a, the effects of those actions, e, the actor's goal, g, and the learner's hypotheses about the world, h.

The objective of the learner is to infer the posterior probability of a hypothesis—in this case, given a set of actions, events, and goals. In our example, what should we believe about the coffee? Using Bayes' rule, the posterior probability of the hypothesis, $P(h|a,e,g)$, can be factored into three terms: the prior probability of the hypothesis, $P(h)$, the likelihood of the action given the goal and assuming the hypothesis is true, $P(a|g,h)$, and the likelihood of the effect given the action and hypothesis, $P(e|a,h)$. We can then use Bayes' rule to write down the relationship between these factors:

$$P(h|a,e,g) = \frac{P(e|a,h)P(a|g,h)P(h)}{\sum_{h'} P(e|a,h')P(a|g,h')P(h')}. \tag{1}$$

The prior, $P(h)$, represents the learners' expectations entering the situation. Is good coffee very common in Paris, or rare? The likelihood of the action given the goal and hypothesis, $P(a|g,h)$, represents the degree to which the action is consistent with the hypothesis and the actor's goal. For example, the tourist's action, to purchase coffee at this shop, is consistent with the goal of drinking coffee and either hypothesis: The coffee could be either good or bad. Finally, the likelihood of the effect $P(e|a,h)$ depends on the underlying structure of the world; given the hypothesis is true, to what degree is the effect likely to follow from the action? Here, the tourist will likely obtain coffee of unknown providence. The degree to which we believe the hypothesis depends on our prior beliefs, the choice of actions in light of the goal and hypothesis, and the effects of the chosen actions. The denominator in the fraction is the sum over all possible hypotheses—often referred to as the "normalizing constant"—which ensures that the posterior probability of each hypothesis reflects its probability relative to all other possibilities.

One of the strengths of Bayesian models is that they can instantiate and reason about richly structured, theory-like knowledge in the learner's prior beliefs (Tenenbaum, Griffiths, & Kemp, 2006; Tenenbaum, Kemp, & Shafto, 2007) and integrate these with information from data. In the formulation above, this structure is manifest in the terms $P(h)$, the prior, and $P(e|a, h)$, the likelihood of evidence given an action and hypothesis.

In this paper, however, we are concerned with the third term, $P(a|g, h)$. This term is fundamentally *psychological*; it encodes beliefs about the actor's knowledge and different assumptions about the goals underlying her actions. We will formalize how, for example, the actions of minimally goal-directed (and likely naïve) actions of the tourist conveys meaning that is different from the actions Véronique and Madeleine; how these actor's goals lead learners to different interpretations of superficially similar actions. We are specifying how people's intuitive theories of psychology can be leveraged to facilitate learning.

In contrast to the psychological formulation above, typical Bayesian models of learning (and standard models of learning more generally) assume that the evidence is independent of actions by any agents, $P(e|a, h) = P(e|h)$, or that the actions (aka *interventions*) are generated uniformly at random, $(P(a|g, h) \propto 1)$. In the first case, we recover the usual formulation of Bayesian learning:

$$P(h|e, g) = \frac{P(e|h)P(h)}{\sum_{h'} P(e|h')P(h')}. \tag{2}$$

In the second case, we recover the standard formulation of Bayesian causal learning, where effects depend on actions, $P(e|a, h)$, but the actions themselves are random. Under either formalization of learning, there is no role for other people in learning; actions are taken as given.

To develop our understanding of the implications of psychological reasoning—the $P(a|g, h)$ term—we build on the idea that actions are chosen by a knowledgeable person

to produce intended goals (see for instance Gergely & Csibra, 2003; Dennett, 1987). This idea is formalized via the Luce choice rule (Luce, 1959),

$$P(a|g,h) = \frac{P(g|a,h)}{\sum_{a'} P(g|a',h)}. \qquad (3)$$

Intuitively, the left-hand-side asks which actions should be chosen, given the actor's goals and beliefs. The right-hand-side provides the answer: Actions should be preferred to the degree that they are likely to lead to desired goal, given the actor's beliefs and the choice of that action.

Thus, to formalize learning in social situations, we must specify what the goals are, and how likely different choices of actions are to lead to the desired goal. Critically, in our framework, different kinds of goals will lead to different choices of actions; actions are purposeful. If the learner is aware of the actor's goals, then she can use this information, together with the choice of actions, to infer what the actor knows about the world. Indeed, even a guess about the actor's goal is often sufficient for the learner to infer what the actor knows.

People may have a great variety of different goals. Returning to our initial example, a goal may be merely to bring about an effect. I may walk to a far off neighborhood because I want some good coffee. Alternately, my goal may be to communicate or teach someone something: that *this* is good coffee (Clark, 1996; Grice, 1975; Sperber & Wilson, 1984). In the first case, my goal is to affect a state of the (physical) world. In the second case, my goal is to affect a change in another person's beliefs. These two different kinds of goals have different consequences for learning. In the following section, we introduce two scenarios from related experimental work, which we use to investigate the distinct consequences of rational and communicative goals.
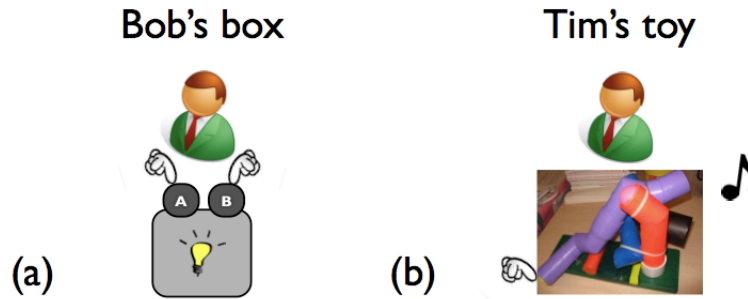
Figure 2. Two causal learning scenarios. (a) "Bob's Box": The learner observes a box with two buttons. Bob (in green) presses both buttons simultaneously and a light illuminates. (b) "Tim's Toy": The learner observes a complex toy with a non-obvious number of causal properties. Tim pulls a knob in the purple tube—eliciting a squeak—then does not perform any additional actions.

Implications of goal-directed actions for learning

We illustrate the effects of intuitive psychological reasoning on learning with two simple causal learning problems: inferring the causal structure of a device from observed data, and inferring the number of causal properties of a novel object from observed data. The first situation, taken from Goodman, Baker, & Tenenbaum (2009), we call "Bob's Box." This situation involves a learner observing a box with two buttons on the top. Bob presses the buttons simultaneously, and a light illuminates (see Figure 2a). The learner must infer the causal structure—which buttons are necessary to cause the effect—one button, the other, both, or neither.

The second situation, taken from Bonawitz, Shafto, et al. (2011), we call "Tim's Toy." This situation involves a learner observing a complex toy with an unknown number of causal properties. Tim reaches into the purple tube and pulls a knob eliciting a squeak, and then stops (see Figure 2b). With Tim's Toy, the learner observes a set of cause-effect

relationships and must infer the causal structure of the toy.

In the following sections, we develop the intuitions that underlie the predictions of our computational framework (see Figure 3 for quantitive predictions; see Appendix for full mathematical details). We consider actors who choose actions randomly, actors who choose actions to bring about effects, and actors who choose actions to communicate knowledge about the world to others (e.g. to teach). To highlight the role of social inference in each of these scenarios, we assume that all hypotheses are equally likely and that causal relationships are deterministic. These assumptions are not meant to represent people's beliefs about these particular causal learning problems, but are instead made for pedagogical and computational simplicity and to highlight the dramatic effects social inference can have on learning.

Bob's Box: Inferring the causes of an effect

Consider Bob's Box (Figure 2a). Possible hypotheses, h, include that Button A causes the light, Button B causes the light, Buttons A and B together cause the light, or neither button causes the light. With these details in place, we can ask, what would a learner infer from watching Bob press both buttons?

Physical evidence. As can be seen the top left panel of Figure 3, based on physical evidence alone three hypotheses are all equally likely. This reflects the fact that the observed actions and effects are ambiguous (i.e. the evidence is confounded). Given that Bob pressed both A and B and elicited the effect, all hypotheses are possible: A could be the cause of the effect, as could B, or A and B together could be the cause of the effect.

Goal-directed action. As the middle-left panel in Figure 3 shows, knowing that Bob knows how the toy works and that his goal is to turn on the light allows the learner to disambiguate the possible hypotheses. After all, if Bob was knowledgeable about the toy and interested in turning on the light, he would definitely press both buttons if the true
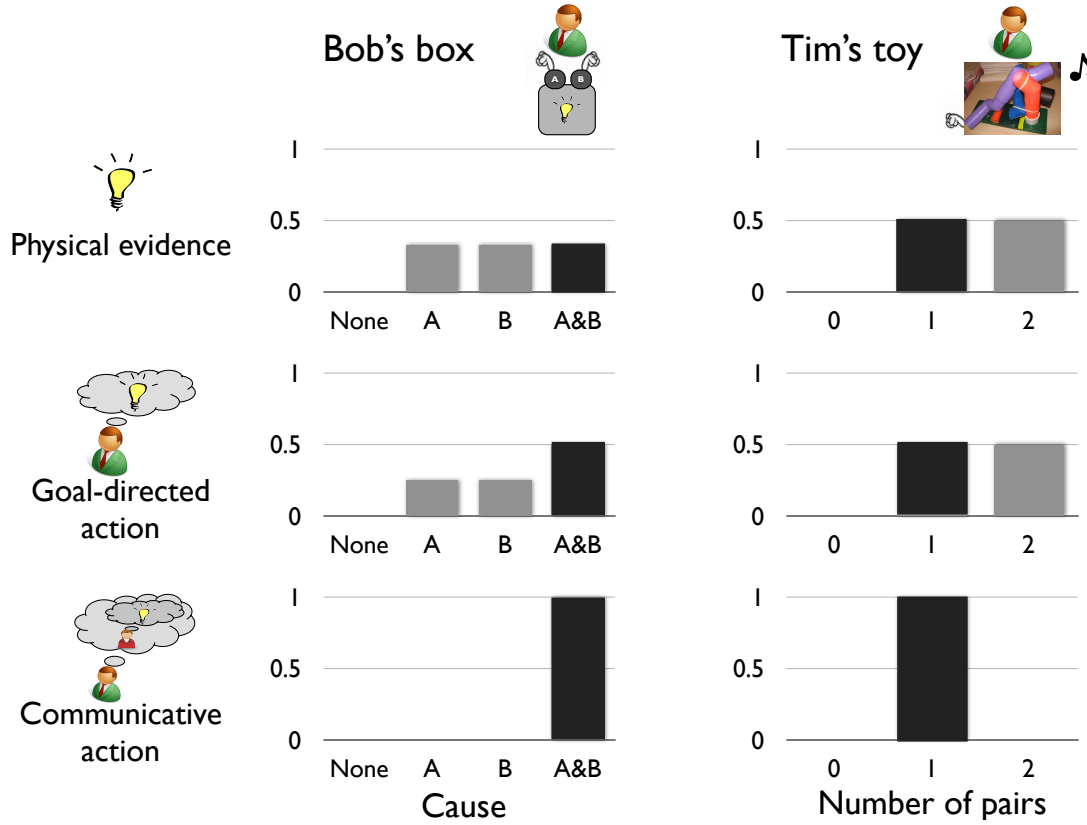
Figure 3. Learning predictions for the two causal learning scenarios (columns) in three different situations (rows). The x-axes list possible hypotheses; the y-axes indicate probability. Note that within each column, learners are assumed to see the same evidence, varying only the social context. Throughout, we assume effects must have a cause and causal relationships are deterministic. The black bar indicates the true state of the world in each column. In the left column, both goal-directed action and communicative action contexts lead to stronger inferences than learning from evidence alone. In the right column, communicative action contexts lead to stronger inferences than either goal-directed action or evidence alone. Thus, we see qualitative dissociations in learning across social contexts.

hypothesis was A and B. Of course, there is also some chance that, if the true hypothesis was A alone (or B alone), he would press both buttons—that action would still elicit the

effect—but in those situations this is not his only option, so those actions would be less probable (see Equation 3; more actions that lead to the effect results in a larger denominator and lower overall probability). Because the learner knows Bob's goal, the learner infers that choice of actions, $P(a|g, h)$, are most consistent with the hypothesis that A and B are both necessary, leading to the inference that A and B is the most likely hypothesis, though both A alone and B alone remain plausible. This is an important qualitative shift; unlike physical evidence, goal-directed action supports learning about the true hypothesis from this single observation.

Communicative action. As the bottom-left panel in Figure 3 shows, communicative action leads to the strongest inferences about the true hypothesis. Bob has looked at you, drawn your attention to the toy, then pressed both A and B. Why should this differ from simply goal-directed action? As above, the action A and B leading to the effect remains consistent with the hypotheses A, B, and A and B. However, Bob is teaching you how the toy works; he is inviting you to think about how the toy works. If the true cause was A, Bob could press A and B; however, Bob is choosing based on how his action will affect the learner's beliefs. This means that Bob's choice (Equation 3) depends on the learner's inferences (Equation 2). Because this action would lead the learner to be confused, Bob would choose to demonstrate A alone only if the true hypothesis was A alone. For the learner, this means that, given Bob's intention to teach, his actions are only consistent with the hypothesis that A and B are both necessary to elicit the effect, leading to a strong inference that A and B is the correct hypothesis (see Equation 2).

Tim's Toy: Learning the number of latent causes

Next, consider Tim's Toy (Figure 2b), a complicated collection of tubes and coils and things that appear to be knobs and buttons. How many cause-effect relationships are there? Possible hypotheses include that there are no cause-effect relationships, that there

is one cause-effect relationship, that there are two cause effect relationships, or even more. For purposes of demonstration, we consider only zero, one, or two relationships, with equal prior probabilities.

Physical evidence. The top-right panel of Figure 3 shows that physical evidence alone leaves considerable uncertainty about the true hypothesis. Given the observation that one action leads to an effect, the hypothesis of no action-effect pairs is ruled out. However, the learner cannot be certain whether that is the only action-effect pair, and is thus uncertain about two remaining hypotheses.

Goal-directed action. The middle-right panel of Figure 3 shows that goal-directed actions do not change learner's inferences; why? Imagine Tim states, "I love squeaking, I'm going to squeak my toy" and then pulls a knob, eliciting a squeak. As with physical evidence, the hypothesis of no cause-effect pairs is ruled out. However, Tim's intent to bring about squeaking does not provide information about whether other (relatively unloved) effects do or do not exist.

Communicative action. The bottom-right panel of Figure 3 shows that communicative action does lead to confident inferences about the true hypothesis. Tim clearly intends to teach you about the toy and chooses to demonstrate that pulling the knob causes squeaking but demonstrates nothing else. As in previous scenarios, the hypothesis that there are no action-effect relationships is ruled out. If the true hypothesis was a single action-effect pair, then Tim could not perform any additional demonstrations. Alternately, if the true hypothesis was two action-effect pairs, Tim's demonstrations would be consistent with the hypothesis, but would be inconsistent with his goal—to teach you how the toy works—and this choice would be less probable (see Equation 3; recall that the goal $P(g|a,h)$ is for the learner to infer the correct hypothesis, i.e. $P(g|a,h)$ is described by Equation 2). Tim's reasoning about the learner's inferences leads Tim to discount alternative, but possible, choices of evidence. Therefore, from the learner's perspective,

Tim's choice to demonstrate only a single action-effect pair is only be sensible if there really was only one cause-effect relationship, leading to the inference that there are no other pairs to be discovered.

Empirical evidence

Recent studies have tested the predictions of the computational framework. Specifically, the Bob's Box example was based on a recent study by Goodman et al. (2009) which investigated the different implications of goal-directed action and physical evidence for learning. Similarly, the Tim's Toy example was based on a recent study by Bonawitz, Shafto et al. (2011) which investigated the different implications of communicative action, goal-directed action, and physical evidence for learning. We review those studies and their findings, as well as convergent findings that support the predictions of our computational framework.

Goodman et al. (2009) investigated adults' inferences from goal-directed action and physical evidence. As in our Bob's box example, participants were presented with scenarios in which a person interacted with a toy that had two buttons and a light. The person was either knowledgeable about the toy and decided to turn the light on (goal-directed action) or not knowledgeable. In both cases, the person pressed both buttons and the light turned on. Participants were asked what made the light turn on. Results showed that people in the goal-directed action condition inferred that both buttons A and B were necessary to elicit the effect, while participants in the physical evidence condition were relatively unsure about the cause of the light.

This distinction between physical evidence and goal-directed action provides an explanation for pervasive over-imitation results found in the developmental literature (Goodman et al., 2009). A variety of recent studies have shown that, when learning, children appear to be overly faithful to actions produced by demonstrators, even repeating

actions that are clearly superfluous to eliciting the desired outcome (Horner & Whiten, 2005; Lyons, Young, & Keil, 2007; Meltzoff, 1995). While researchers have offered a variety of explanations for such findings, the explanations rely on ad-hoc mechanisms (e.g. automatic causal encoding) to explain behavior. Our framework offers a different account, in which learners leverage intuitive psychological inferences to support learning in otherwise ambiguous situations. From our perspective, rather than being a strange anomaly in the human cognitive system, over-imitation is a sensible approach to learning when surrounded by knowledgeable others (cf. Krueger & Funder, 2004).

Bonawitz, Shafto et al. (2011) investigated children's inferences from communicative action, goal-directed action, and physical evidence. As in our Tim's Toy example, children observed scenarios in which a person interacted with a novel, complex-looking toy. In one study, the person was either knowledgeable about the toy and engaged the child via ostensive cues (see Csibra & Gergely, 2009), or not knowledgeable. In both conditions, the child observed that pulling a purple knob led to squeaking, either as a result of an intentional or accidental (communicative action and physical evidence, respectively) demonstration. Children were then given the toy and allowed to play with it. If the communicative action led to the inference that only one cause-effect relationship was present, then they should engage in less exploratory play. The results, including the number of actions tried and the number of non-demonstrated causal relations discovered by children, confirmed that communicative action led to decreased exploration relative to than children who observed physical evidence. A subsequent study contrasted goal-directed action and communicative action, showing that communicative action led to decreased exploration relative to goal-directed action. Together, these results confirm the prediction that communicative action did in fact lead to a robust inference that only the demonstrated causal relationship exists.

Other recent studies have also investigated the implications of teaching and

communicative actions. Shafto & Goodman (2008) showed that adults draw different inferences in concept learning when data are selected by a knowledgeable teacher, as opposed to a naïve demonstrator. Their results suggest that learners infer that teachers are selecting diverse examples, in turn supporting the inference that narrower concepts are more probable than concepts that include examples not chosen by the teacher. Similarly, recent results in mathematics learning suggest that selection of non-diverse mathematics problems (e.g. $2 + 1 = 3$ and $3 + 2 = 5$, but not $3 = 2 + 1$) may be the cause of erroneous inferences about the concept of equality (e.g. McNeill, 2008).

Although the framework described above is only beginning to be applied to language learning, there are a number of examples that support the predictions about learning from communicative actions. Xu & Tenenbaum (2007) describe an example where they contrast examples sampled by an ignorant learner (much like our tourist choosing a bad cafe) versus a knowledgeable teacher. The learner observed as the objects were chosen and labeled. Consistent with the predictions of our framework, they found that learners generalized conservatively when the examples were chosen by the teacher, and generalized more broadly when the examples were chosen by an ignorant learner.

Ideas about speakers' goals and intentions have also been influential in discussions of language acquisition. Many theorists have proposed that an understanding of speakers' communicative intentions is key in understanding (Clark, 1996) and acquiring language (Bloom, 2002; Clark, 2003). Frank, Goodman, and Tenenbaum (2009) used a Bayesian model to capture this intuition in the context of associative word learning tasks. This model, which attempted to jointly infer speakers' communicative intentions and meanings of words, performed better than simple associative models across a range of tasks, including learning from corpus data and fitting human performance.

In addition, in recent work, Frank, Goodman, and colleagues (2009, under review) used a model related to the communicative model described above to capture ideas from

Gricean pragmatics. This work demonstrated that the basic principles underlying the communicative model effectively captured the idea that the Gricean maxim "be informative." They compared the predictions of this model to adult data on both word production and word learning and found a tight quantitative correspondence. While this work is relatively new, it suggests that the kind of framework we have discussed here can in fact be applied productively to a range of aspects of language acquisition and processing.

Summary: Implications of goal-directed actions for learning

In each of our scenarios, we held constant the data that the learner observed and derived predictions from the computational framework. We considered the inferences that were afforded in three different scenarios: learning from physical evidence, learning from goal-directed action, and learning from communicative action. In each case, we showed how actors' goals affected the inferences that could be drawn from the very same evidence. We outlined recent research suggesting that both children and adults use other people's goals to support learning. Furthermore, our computational framework suggested that these inferences were rational consequences of considering actions as being goal-directed. As in our coffee-finding example, intuitive psychological reasoning provided information that can dramatically affect learning.

We have focused on two kinds of social goals: goal-directed actions and communicative action. The strength of our framework is in showing that these goals can be formalized. Data that results from other people's actions are ultimately a consequence of their goals. The goal may be a state of the world, as in goal-directed actions. The goal may also be a belief in another person.

These are two examples of others' goals that affect inference, but they are not the only possibilities. For instance, a person may be interested in deceiving or lying, instead of teaching or (cooperatively) communicating. These kinds of goals can be straightforwardly

formalized within this framework (Shafto, Eaves, Navarro, & Perfors, in press; Warner, Stoess, & Shafto, 2011). Considerable work remains in simply expanding on what is currently unknown—we do not yet have definitive lists of the kinds of goals that are relevant—and then cataloging and organizing the myriad goals that people may have, and their respective implications for learning.

For the purposes of exposition, we have assumed that the people being observed are knowledgeable, but this assumption is not necessary. Given a model of how knowledgeable and naïve (i.e. not knowledgeable) individuals choose actions, it is straightforward to formalize how learners could infer who is knowledgeable (and similarly, who is well-intentioned). Indeed, Shafto et al. (in press) propose a model of this problem, and bring it to bear on research on the development of epistemic trust (Corriveau & Harris, 2009; Corriveau, Fusaro, & Harris, 2009; Koenig & Harris, 2005; Pasquini, Corriveau, Koenig, & Harris, 2007). In this literature, children's success in choosing reliable informants is attributed to their ability to monitor who is knowledgeable. By contrasting the predictions of two models that formalize inference about informants' knowledge or their knowledge and intent, Shafto et al. (in press) argue that 4-year-olds' behavior is best explained by joint inference about knowledge and intent, as opposed to the standard interpretation—inference about knowledge alone. A similar approach finds that 3-year-olds' behavior is, in contrast, best explained by inference about knowledge alone, suggesting developmental changes in behavior on these tasks. Additionally, recent research has begun to explore the relationship between epistemic trust and attachment theory. Corriveau, Harris, et al. (2009) showed for example that children with avoidant attachment patterns are equally likely to trust information provided by a stranger as their caregiver, unlike children with secure attachment patterns, who trust their caregiver more than a stranger. Together these results suggest that inferences about people's knowledge and intent can be formalized and that our formal approach hold promise in unifying and

explaining phenomena across development and social learning more broadly.

Conclusions

How do people learn and reason about the world so rapidly and effectively? Traditional computational approaches to learning have focused on prior biases and knowledge representation as the explanation. We have argued that intuitive psychological reasoning plays a critical role in explaining the success of human learning. Under our approach, data are not inert observations with a fixed, context-free meaning, but carry with them information about people's beliefs and intentions. We have presented a formal analysis that explains the effects of intuitive psychological reasoning on learning as a consequence of rational inference about people's goals and beliefs. This analysis explains why two social learning contexts—learning from goal-directed action and learning from communicative action—differ in the inferences that they afford and differ from learning in non-social contexts.

We have focused on very simple causal learning situations for the purposes of exposition, but the framework naturally applies to non-causal learning domains, and to situations of considerably greater complexity. Our recent work provides applications of these ideas to concept learning (Goodman et al., 2009; Gweon, Tenenbaum, & Schulz, 2010; Lucas, Griffiths, Xu, & Fawcett, 2009; Shafto & Goodman, 2008; Shafto, Goodman, & Griffiths, under revision; Warner et al., 2011), referential ambiguity (Frank, Goodman, & Tenenbaum, 2009a,b), and causal learning in children (Bonawitz, Shafto, et al., 2011, Buchbaum, Griffiths, Gopnik, & Shafto, 2011).

Theories of cognitive development have almost uniformly claimed that other people are important for learning; however, theorists have differed in what role they assign for other people in learning, and how that role addresses basic problems of learning. Proposals have variously suggested that merely allowing learners to explore is enough to

support learning (Bruner, 1966; Vygotsky, 1978), that some kind of modeling or imitative learning is key (Bandura et al., 1961; Meltzoff & Moore, 1977), or that reasoning about communicative intent is critical (Csibra & Gergely, 2009; Tomasello et al., 2005).

Our framework shows that these different proposals lead to very different predictions about learning, and suggests a synthesis of these different views that opens the door for continued exploration of how intuitive psychological reasoning affects learning. The most powerful goals are not ubiquitous in learning—not every action is intended to be communicative. So, while leveraging the strongest social contexts will lead to more powerful learning, identifying when different assumptions are appropriate is critical as well.

Our approach suggests that intuitive psychological reasoning potentially provides a very strong lever by which learners may capitalize on others' knowledge to learn about the world (see also Coady, 1992; Csibra & Gergely, 2009; Harris, 2002; Tomasello et al., 2005). The key difference from previous formal approaches is how one views other people. If other people are viewed as random or even malicious in choosing their actions, then learning will likely be very hard. However, if people are viewed as approximately rational, goal-directed agents, or as knowledgeable and helpful teachers, then the learning problem becomes much more tractable. These assumptions make it possible for learners to learn what others already know, rather than rediscovering all knowledge from the ground up.

APPENDIX: Computational details

Here we review the computations that form the basis for Figure 2.

Bob's Box: Inferring the causes of an effect

Possible hypotheses, h, include that Button A causes the light, Button B causes the light, Buttons A and B together cause the light, or neither button causes the light; we assume that the prior probabilities, $P(h)$, are all equal to 1/4. For simplicity, we will consider actions, a, and effects, e, in pairs, focusing on a subset of the total possibilities. Specifically, we consider observations including doing nothing leading to no effect, pressing A leading to the light, pressing B leading to the light, and pressing A and B leading to the light. We assume that causes deterministically bring about their effects; that is, $P(e|a,h)$ is 1 if a is a cause of e according to hypothesis h, and zero otherwise.

Physical evidence. Assuming the actor chose randomly, the probability of choosing the action "press A and B", $P(a|g,h)$, is 1/4, because the chosen action is one of four possibilities. Because the effect is deterministic, $P(e|a,h) = 1$, and prior beliefs in the hypotheses are the same, $P(h) = 1/4$, the numerator of Equation 1 is $(1/4) \times (1/4) = .0625$. But both of the other viable hypotheses (Button A only and Button B only) have the same probability! Thus, by Equation 1, the probability of each hypothesis is 1/3.

Goal-directed action. If Bob knows how the machine works, we assume that he has chosen his action rationally to bring about the effect via Equation 3. If A and B together cause the light, the probability of pressing both buttons is 1 because that is the only action that would lead to the goal. Thus, the learner can infer that the probability of A and B jointly causing the light is proportional to $P(e|a,h)P(a|h,g)P(h) = 1 \times 1 \times (1/4) = 1/4 = .25$. In contrast, the probability that A alone is the cause would be proportional to $1 \times (1/2) \times (1/4) = 1/8 = .125$ (the probability

of B alone is the same). Thus, the probability that A and B jointly cause the light is

$\frac{.25}{.25+.125+.125+0} = .5$.

   Communicative action. In this case, Bob chooses his action to maximize the

learner's belief in the correct hypothesis. Recall, if the action was chosen randomly,

$P(a|g, h) = 1/4$, the total probability that A and B were the cause would be 1/3, as would

the probability of A alone or B alone. At first glance, the actions A and B appear

ambiguous; however, the learner must reason about why the teacher chose these actions,

as opposed to other possibilities. What if the true cause was A alone? Then, the teacher

could have pressed only A, or A and B, to elicit the effect. However, pressing A alone

would lead the learner unambiguously to the inference that A alone is the cause; there is

no other explanation. So, $P(h|a, e, g)$ for the hypothesis A alone would be 1 if the teacher

pressed A, and the $P(h|a, e, g)$ for the hypothesis A and B would be 1/3 (it could be A

alone, B alone, or A and B). Up to this point, the reasoning is based completely on the

assumption that actions were chosen randomly.

   In communicative situations, teachers and learners are yoked—each reasons about

the other—suggesting that the learner's inferences are the *input* to the teacher's choices. If

the teacher's goal is for the learner to infer the correct hypothesis, and A alone is the

cause, then the probability of pressing A is $\frac{1}{1+1/3} = .75$. Similarly, the probability of

pressing A and B is $\frac{1/3}{1+1/3} = .25$. On the other hand, if A and B jointly cause the effect,

then the probability of choosing A and B is 1 because that is the only action that will lead

to the effect. In a communicative situation, learners who observe that pressing A and B

leads to the effect will infer that A is less likely to be the cause,

$P(e|a, h)P(a|h, g)P(h) = 1 \times .25 \times (1/4) = .0625$ (the same as probability that B is the

cause), and A and B is more likely to be the cause,

$P(e|a, h)P(a|h, g)P(h) = 1 \times 1 \times (1/4) = .25$. Thus, the probability of the hypothesis A

and B after one step is $\frac{.25}{.25+.0625+.0625+0} \approx .67$.

Of course, because this is a recursive inference, one could continue reasoning about how the teacher and learner would update their actions. The result of this process is that the only reason a teacher would choose to press A and B was if A and B were both necessary to elicit the effect (i.e. $P(h|a, e, g) = 1$, see Figure 3).

Tim's Toy: Learning the number of latent causes

Possible hypotheses include that there are no cause-effect relationships, that there is one cause-effect relationship, that there are two cause effect relationships, or even more. For purposes of demonstration, we consider only zero, one, or two relationships, with equal prior probabilities, $P(h) = 1/3$. The action-effect pairs we consider include no interventions leading to no effect, one intervention leading to one effect, or two interventions leading to two effects.

Physical evidence. Assuming that actions are chosen randomly, the probability of a single action is $P(a|g, h) = 1/3$. Thus, given one action that elicits an effect, the probability of one cause-effect pair is proportional to $P(e|a, h)P(a|g, h)P(h) = 1 \times (1/3) \times (1/3) \approx .11$. This is the same as the probability of two cause-effect pairs, thus $P(h|a, e, g)$ for each is .5.

Goal-directed action. Imagine Tim's goal is to elicit an effect; in this case the probability of choosing one action would have been $P(a|g, h) = .5$ because either one or two actions would have lead to the goal. The probability of inferring that there is only one cause-effect relationship would be proportional to $P(e|a, h)P(a|g, h)P(h) = 1 \times .5 \times .33 = .165$. Of course, the probability of two causal relationships would be the same, because there is no need to elicit the other, given the goal; the $P(h|a, e, g) = .5$ for one hypothesis and two hypotheses.

Communicative action. If the learner observed zero actions and the choice was randomly sampled, then all hypotheses remain equally probable: $P(h|a, e, g) = 1/3$ for all

hypotheses. If the learner observed one action leading to an effect, then the possibility of zero cause-effect pairs is ruled out; however, the learner remains uncertain whether there are one or two pairs, $P(h|a, e, g) = 1/2$ for both hypotheses. If the learner observed two actions leading to two effects, the only possibility is that there are two pairs, $P(h|a, e, g) = 1$ for the hypothesis that there are two cause-effect pairs.

Because this is a communicative situation, the learner's inferences are inputs to the teacher's inference. If there is truly one cause-effect pair, the teacher chooses between doing nothing leading to no effect or one action with an effect. The probability of choosing zero actions is $\frac{1/3}{1/3+1/2} = .4$, and the probability of choosing one action is $\frac{1/2}{1/3+1/2} = .6$. Similarly, if there truly are two pairs, then the teacher chooses between zero, one, or two actions. The probability of choosing zero actions is $\frac{1/3}{1/3+1/2+1} \approx .18$, the probability of choosing one action is $\frac{1/2}{1/3+1/2+1} \approx .27$, and the probability of choosing two actions is $\frac{1}{1/3+1/2+1} \approx .55$.

Because teachers and learners are yoked, the learner also reasons about the teacher. Given the observation of one action leading to an effect, the learner must infer whether there is one or two cause-effect pairs. The probability of one pair would be proportional to $P(e|a, h)P(a|g, h)P(h) = 1 \times .6 \times .33 \approx .198$, and the probability of the hypothesis that there are two pairs would be proportional to $P(e|a, h)P(a|g, h)P(h) = 1 \times .27 \times .33 \approx .089$. The hypothesis that there is only one cause-effect pair is more probable, $P(h|a, e, g) = \frac{.198}{.198+.089} \approx .69$. Again, because this is a recursive inference, one could continue reasoning about how the teacher and learner would update their actions. The result of this process for this case is that the only reason a teacher would choose one action is if there was only one cause-effect pair, i.e. $P(h|a, e, g) = 1$.

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