Towards a General Modelling Framework of Resource Competition in Cognitive Development

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Abstract

Although multiple differences between typically and atypically developing children have been robustly observed in empirical studies, we lack a unified theoretical framework on how these observed differences in cognitive phenotypes might arise. Here, we introduce a first step towards such a framework based on an extension of the mutualistic network model, which conceptualizes cognitive development in terms of reciprocal interactions between cognitive processes that promote each other's growth. We extend the mutualistic network model with dynamic mechanisms of competition for limited resources, such as time and environmental factors. By doing so, the modelling framework defines both an empirical and theoretical research agenda. We show that this relatively simple model generates patterns of development that potentially explain five established phenomena in cognitive development: (1) developmental phases, (2) slower cognitive development in atypical cohorts, (3) lack of early indicators for atypical development, (4) developmental regression in the autistic profile, and (5) "quasi-autism" caused by extreme environmental deprivation. Theoretically, the presented framework fits a generic movement towards formal theory construction. The model can easily be reproduced and developed further through our openly available code, and we offer several paths forward in doing so.

Keywords: (a)typical cognitive development, mutualistic network model, resource competition,regulatory mechanisms, shared limited resources

"There is no comfortable theorem assuring that increased diversity and complexity beget enhanced community stability; rather, as a mathematical generality, the opposite is true. The task, then, is to elucidate the devious strategies which make for stability in enduring natural systems. There will be no one simple answer to these questions." — Robert May (1973)

1 Introduction

Current developmental theories focus on interaction patterns between genetic and environmental factors to unravel the observed stochasticity in individual cognitive profiles. Specifically, recent theoretical developments conceptualize typical and atypical cognitive phenotypes as resulting from an active process unfolding throughout development (Karmiloff-Smith, 2009). Instead of simply executing the instructions coded for by genes, this process is characterized by limiting alternative developmental pathways as the individual matures (Constantino et al., 2021). In other words, not only typical profiles but atypical phenotypes, too, are the result of a general process of ontogenetic adaptation, where the range of paths open to a child becomes increasingly constrained as development furthers (Johnson et al., 2015).

Previously, it has been hypothesized that reciprocal 'mutualistic' interactions between developing abilities are key to typical cognitive development (van der Maas et al., 2006), resulting in often-replicated positive manifold: evenly distributed positive correlations between performance on different cognitive tasks, such as vocabulary, math, and executive functioning (van der Maas et al., 2006). The positive manifold, however, is not uniform across the population. Some instances of atypical development are, for instance, characterized by specific deficits, such as seen in dyslexia, in which performance on one cognitive domain is (far) beneath what is to be expected based on the performance in other cognitive domains (Lyon et al., 2003). Similarly, autism is characterized by so-called uneven cognitive profiles in which some skills are near or even better than typical performance, and others show deficits (Brunsdon and Happé, 2014). In addition, the autistic population has often been reported to show a higher incidence of extraordinary talents (sometimes referred to as savant skills; Baron-Cohen et al., 2009; Happé, 2018; Treffert, 2009; Bal et al., 2022).

In the literature, uneven cognitive profiles have posed a significant challenge to existing accounts of developmental disorders (Happé, 2018). To our knowledge, no developmental theoretical model can account for (a) typical development (i.e., all cognitive abilities are around levels expected in typical cohorts), (b) specific deficits (i.e., one cognitive domain performs far underneath typical performance, such as seen in dyslexia), (c) uneven cognitive profiles (i.e., some cognitive abilities are better or near levels seen in typical development whereas other are below typical performance, such as seen in autism), and (d) intellectual disabilities (i.e., all cognitive abilities are far beneath typical performance).

In this paper, we argue that resource competition is a formative force in cognitive development and aim to present a modeling framework that incorporates resource competition to accommodate all of these cognitive phenotypes. We approach this task by (a) developing a dynamic mutualistic network model that incorporates resource competition and (b) evaluating whether this extended mutualism model can better explain the noted cognitive profiles associated with typical and atypical development.

The outline of the paper is as follows. First, we explain how the mutualistic network model represents the cognitive system. Although the mutualistic network model assumes that cognitive development is limited by the availability of resources, we argue that the model is incomplete, as it does not represent the fact that cognitive abilities are *in competition* over these resources. To lift this limitation, we draw from the ecology literature to investigate how components of a system interact under conditions of shared, limited resources, with a particular focus on a well-established mathematical model of resource competition (Revilla, 2009). Subsequently, we extend the mutualistic network model of cognitive development by integrating it with this resource competition model. We investigate the conditions under which the resulting integrated model leads to typical vs. atypical development. Finally, we discuss the implications of the model for our understanding of which mechanisms may drive the over- or underdevelopment of specific abilities in some phenotypes while constraining or advancing the universal growth of all cognitive abilities in other phenotypes.

2 Mutualistic network model

Traditionally, the positive manifold has been explained by an underlying factor of general intelligence, the so-called g-factor, which has been hypothesized to determine performance on a wide variety of cognitive tasks (Spearman, 1961). The limited explanatory power of the g-factor, however, led researchers to look for different explanations of the positive manifold (Savi et al., 2021).

Van der Maas et al. (2017, 2006) proposed that the positive manifold does not have to result from one underlying g-factor but may arise from reciprocal coupling among different lower-level cognitive abilities, i.e., growth in one cognitive ability may promote growth in another cognitive ability and vice versa. For instance, increasing language comprehension may positively influence growth in abstract reasoning ability and vice versa. Through this process of reciprocal interaction, the positive manifold arises naturally (van der Maas et al., 2006).

In the mutualistic network model, intelligence is not seen as a cause of performance but as the result of coupled developmental processes that unfold over time. Dynamical systems theory can be used to model this process as it describes how variables change over time as a function of themselves and other variables in the system via a set of (coupled) differential or difference equations. This type of model has been introduced in developmental psychology by van Geert (1991). As such, the mutualistic network model can be represented using a differential equation that encodes how abilities grow together over time. The mutualistic network model consists of two parts. In the first part, the development of individual abilities is represented in terms of autocatalytic, logistic growth. The second part represents the mutualistic interactions between these developing abilities. Together these two parts result in the following equation for the growth of cognitive abilities:

$$\frac{dx_i}{dt} = a_i x_i (1 - \frac{x_i}{K_i}) + a_i \sum_{j=1}^{X} M_{ij} x_i x_j / K_i \tag{1}$$

where, x_i represents one of the X cognitive abilities. The first part, the logistic growth, contains two parameters, namely a_i, which is the rate of growth of the cognitive ability (determining the steepness of the logistic growth function), and the carrying capacity K_i (determining the asymptote or upper limit of the logistic growth function). As the carrying capacity K is the limiting factor, it is considered the long-term sum over limited resources (van Geert, 1991).

In the second part, the presence and strength of the dynamic couplings are determined by the interaction matrix M, which represents the extent to which any given ability promotes growth in any other ability. It is important to note that the growth rates and carrying capacities are sampled independently from each other, so the cognitive abilities are initially uncorrelated. However, they become correlated over time due to their reciprocal coupling, as shown analytically and through simulations by van der Maas et al. (2006).

The dynamic coupling parameters in the M matrix are expected to be positive, but the exact values might vary, and different choices of M lead to somewhat different patterns of correlations. If all elements of M are positive and equal, we obtain correlation matrices consistent with the one-factor model, whereas if the elements of M are sampled from a normal distribution with a positive mean (such that most but not all elements are positive), we end up with correlation matrices typical for the higher-order factor model (van der Maas et al., 2006). Readers can find the annotated R script in the online repository.[1] We refer readers unfamiliar with dynamical systems modeling to the Appendix for a tutorial on how to simulate data from the mutualistic network model.

2.1 Modelling atypical development

Thomas et al. (2012) showed that qualitative and quantitative differences in the architecture of the mutualistic network model (e.g., missing cross-domain coupling and lower growth rates) could produce atypical development, including slower cognitive development and cognitive profiles with specific deficits. These modelling predictions are also empirically demonstrated via latent growth models (e.g., McArdle et al., 2000; Ferrer and McArdle, 2004; Kievit et al., 2017). For instance, Ferrer et al. (2010) found that, in contrast to typical readers, people with dyslexia show no reciprocal coupling between WISC-R IQ scores and reading ability. Similarly, Quinn et al. (2020) found reciprocal coupling between vocabulary knowledge and reading comprehension in typically developing children but not in children with learning disabilities. Accordingly, the mutualism model can account for robust phenomena associated with typical and atypical cognitive development

[1]The R script is uploaded to the OSF repository `https://osf.io/uy2ra`

(see Appendix for further detail). Yet, despite its theoretical and empirical explanatory success, we note three important limitations to the mutualistic network model as a framework for understanding atypical development.

First, although the model can produce some phenomena associated with atypical cognitive development, this production is largely limited to a one-to-one mapping between the atypicalities specified in the model (e.g., a slower growth rate) and the atypicalities in cognitive development the model produces (e.g., slower cognitive development). In other words, in previous work, the atypical development of cognitive abilities produced by the model was largely built into the code directly. As such, although the model can certainly *represent* atypical development, it is questionable whether it also *explains* it (see van Dongen et al. (2022) for a perspective on when a formal model explains a phenomenon).

Second, the mutualistic network model incorporates finite resources as a limiting factor via the carrying capacity, but it does not incorporate competition for these resources. If the development of individual cognitive processes depends on finite resources, this implies that cognitive processes can be in competition with one another, a dynamic that almost certainly impacts both typical and atypical development. Accordingly, any model of cognitive development will be strengthened by incorporating competition for the allocation of finite, shared resources. In Table 1 we list several internal (e.g., individual neurobiology) and external (e.g., financial) resources that support (or limit) cognitive development.

Earlier work by van Geert and colleagues (e.g., van Geert, 1991, 1994; Den Hartigh et al., 2016) has advocated representing these aspects of development through approaches that model cognitive growth under competition over limited resources (see the Appendix for an overview of these models and simulation code). All these developmental models, however, share the assumption of direct competition between cognitive abilities, while we argue that the main competition between cognitive abilities is not direct but instead is competition for limited resources such as time and money.

Third, there are robust phenomena associated with cognitive development that are not accounted for by previous extensions of the mutualism model, namely uneven cognitive profiles and a higher incidence of extraordinary talents typically associated with the autistic profile (sometimes referred to as savant skills; Baron-Cohen et al., 2009; Happé, 2018; Treffert, 2009; Bal et al., 2022). Given these explanatory shortcomings, there is a need for further extensions of cognitive development models to expand the range of typical and atypical features of development that these models can account for.

Table 1: Examples of resources, each of which is finite and plausibly implicated in cognitive development. In line with van Geert (1994), we divide resources into internal (e.g., individual neurobiology) and external (e.g., financial).

Internal Resources	
Biological constraints	Resources resulting from the underlying neural system, such as neuronal speed, which may in part be genetic.
Cognitive abilities	Lower-level cognitive abilities, such as working memory, may serve as resources for higher-level cognitive abilities, such as mathematical skills.
Time investment	A natural limit of cognitive development is due to time investment: Spending more time practising particular cognitive abilities may come at the expense of developing other cognitive abilities.
Motivation	Motivation is crucial in developing abilities, e.g., we might be forced by our environment to spend time on math exercises (e.g., in school) but improve little in performance because we are distracted or bored.
External resources	
Financial	These include material resources, such as nutrition, and possibilities for extracurricular activities (e.g., piano lessons).
Education	The educational environment can provide learning opportunities and high-quality practice regimes.
Motivation	The environment can provide a motivational structure via reinforcement (e.g., by rewarding children for learning achievement.
Parenting	Parenting styles provide different ranges of (typical) input and support, with childhood adversity even limiting cognitive development.

3 Modelling Resource Competition

Current models in the developmental domain lack a mechanism through which multiple cognitive abilities indirectly compete for limited resources.[2] As limited resources are plausibly implicated in driving the differences between typical and atypical development, the question arises of how this factor should be included in developmental models.

To address this question, we turn to the literature in ecology, which has developed modeling frameworks for competition over limited resources, particularly between species in a shared environment. Accordingly, ecologists have done considerable work in modeling the dynamics of how systems behave under conditions of resource competition. This may shed light on how to model similar competition among cognitive processes.

One well-studied ecological model is the resource competition model (Tilman et al., 1982; Revilla, 2009). The simplest version of the resource competition model states that the growth of a species is determined by its current size x_i and the sum over all available resources r_j, where μ_{ij} determines how much species i benefits from the availability of resource j. If no resource is available, the species will die out with the speed of the death rate d_i (i.e., population size will move towards zero). The death rate is assumed to be independent of resource uptake.

The growth of resource r_j consists of two parts. The first part is the growth rate, described by a concave function, which is determined by r (i.e., the steepness of the concave function) and r_{max} (i.e., the asymptote of the concave function). The second part is the depletion rate, determined by the sum of all species multiplied by their consumption rate b_{ij}. Hence, the more abundant a species is, the more resources it consumes. Two equations specify these dynamics:

$$\frac{dx_i}{dt} = x_i((\sum_{j=1}^{R} \mu_{ij} r_j) - d_i) \tag{2}$$

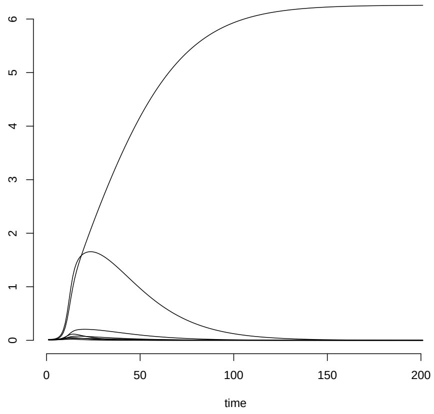
$$\frac{dr_j}{dt} = r(r_{max} - r_j) - r_j(\sum_{i=1}^{X} b_{ij} x_i), \tag{3}$$

[2]van Geert (1991) presents models in which growth in one cognitive ability inhibits growth in another, but no mechanism explains what would happen if resources grow, since resources are simply defined by the difference between the level of one cognitive ability and that of another. In the OSF repository, we provide the reader with an annotated R file to simulate data from the models presented in van Geert (1991).

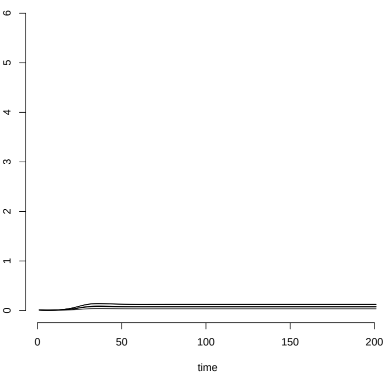
where x_i represents the population size of a particular species and in r_j represents the size of a particular resource.

To illustrate the effect of competition over resources on growth, we simulate data from this model under a scenario with eight species and one resource (Figure 1a and 1b). As depicted in Figure 1a, if we allow for differences in the death rate (d) and in how much a given species benefits from the resource (μ), then one of the species "wins," growing at the expense of all the others. The species that wins is the one that benefits most from the resource availability (i.e., highest μ).

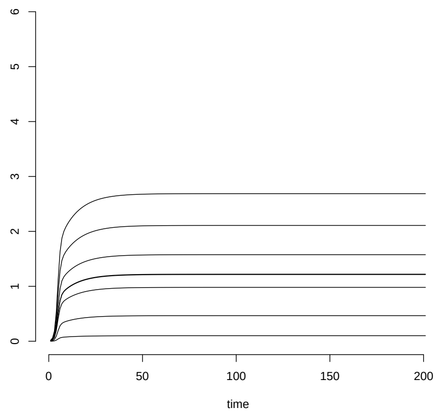
Figure 1b shows that if we do not allow for such differences (i.e., set the parameter values of the growth rate μ and the death rate d equal across species), competition prevents any species from benefiting from the resource. In contrast, when these parameters are equal but there are multiple resources (i.e., a scenario with limited competition) or species benefit more from the availability of one resource (i.e., higher μ's), most species show growth. This growth, however, is relatively limited and is entirely determined by the initial values. In the Appendix, we provide a tutorial on implementing the resources competition model in R (de Boer and Pagie, 1983).



(a) Competitive exclusion



(b) Lack of population growth



(c) Unequal population sizes

Figure 1: Characteristic patterns produced by the competition resource model with eight species. To manipulate the degree of competition, we varied the number of finite resources between one (full competition) and eight (limited competition). In panel (a) the system can be seen to reach an equilibrium state of competitive exclusion, i.e., one species takes all resources while other species decline. The species that takes over has the highest average μ_{ij} (i.e., benefits most from the availability of resources). Panel (b) represents the same simulation, but here parameter values are equal across species. The competition over resources now causes all species to attain very low equilibrium states. Panel (c) represents a simulation with sufficient resources. Here, we observe an equilibrium state of coexistence of species, but due to competition over resources, the equilibrium values are uneven and low.

3.1 The competitive exclusion principle

The simulation results presented in Figure 1 demonstrate that when species differ in their growth and death rates, some species will thrive at the expense of others. In particular, the number of co-existing species is always less than the number of resources. The biodiversity of many species that use a small number of resources is, apparently, an unstable state (Huisman and Weissing, 2001). This dominance of one or a small number of species over all other species is referred to as the *competitive exclusion principle* (Levin, 1970).

As May (1973) famously stated: "There is no comfortable theorem assuring that increased diversity and complexity beget enhanced community stability; rather, as a mathematical generality, the opposite is true. The task, then, is to elucidate the devious strategies which make for stability in enduring natural systems. There will be no one simple answer to these questions." As May describes, ecological models predict a world dominated by a few prevailing species rather than the biodiversity we observe in nature.

Translated into a cognitive context, this finding has important implications. This is because, following the plausible assumption that cognitive abilities feature competition over limited resources too, we should actually expect hyperspecialization or savant skills (Figure 1a). That is, we should expect to live in a world in which one or a few cognitive abilities take over all resources while other abilities fail to develop, i.e., a world in which savants are the rule rather than the exception.

From this perspective, therefore, the uneven distribution of abilities associated with some profiles of atypical development (e.g., hyperspecialization) needs no further explanation: they follow directly from competition for limited resources. In fact, the phenomenon that now requires explanation is that development does not always follow this pattern of hyperspecialization. Thus, the question is no longer "what mechanisms *produce* uneven cognitive profiles in *atypical development*?" but rather "what mechanisms *prevent* uneven cognitive profiles in *typical development*?" In other words, an entirely new and unexpected puzzle is raised: "How is a diverse and balanced profile of well-developed cognitive functions possible in the first place?"

3.2 The foundations of stable coexistence in cognitive development

If resource competition plays an essential role in cognitive development, there may be several mechanisms that prevent cognitive development from "breaking down" under limited resources, and that maintain a diverse set of abilities and skills.

A first option is that most cognitive abilities do not compete over finite resources because they operate in parallel. Such a case may arise when cognitive abilities simultaneously benefit from the same resources. For example, engaging in conversations is a time investment that promotes language development and social skills; practicing soccer benefits muscle development and hand-eye coordination, and playing an instrument benefits auditory processing and fine motor skills. A plausible mechanism that supports such processes would involve the same information being processed in parallel in different parts of the system, independently of each other, so that they do not interfere. In this context, it is interesting to note that people with autism have been reported to have difficulties with parallel processing (Johnson, 2017).

A second option would be that most cognitive abilities compete, but sufficient resources are available to develop all cognitive abilities to sufficient levels. For instance, in such a situation, there is enough time to develop both math and language skills, to learn how to tie shoes and bake cookies, and so on. The degree to which abilities compete for resources may also depend on the rate with which they develop. For instance, developmental deficits may cause a person to take a long time to learn something (i.e., low μ's) so that more resources are needed than for someone without developmental problems. Such a situation would naturally lead to more competition between abilities.

A third option is that there may be internal and external factors that intervene to diversify how we allocate our resources , so-called *regulatory mechanisms*. The ecology literature provides potential insight into the mechanisms that diversify the allocation of resources. Here, we consider three approaches that have been identified as supporting the stable coexistence of multiple species under limited resources, approaches that could be added to a model of cognitive development.

First, one mechanism is to add sufficient *self-regulation* to the components of the model. In some ecological models, researchers have proposed that when a species becomes too abundant, the death rate will be higher (e.g., due to easier disease outbreaks; Gavina et al., 2018). When coupled with less self-regulation on scarce species, strong self-regulation on abundant species prevents one species from dominating completely. Second, another form of regulation is to impose elementary developmental differences within a species (e.g., juveniles are not able to reproduce, consume more resources than adults, and are more easy prey from other species; de Roos et al., 2003). Due to these developmental differences, species that grow more quickly will also be more vulnerable to other prey and limited resources because they consist mainly of juveniles. We call this mechanism *growth-dependent resource depletion*. Third, one mechanism is to allow for mutualistic relations between species to increase coexistence; when we add beneficial effects, we reduce competition and thus increase biodiversity (Hale et al., 2020; Bascompte, 2019; Dean, 1983).

3.3 Regulatory mechanism in cognitive development

In this section, we translate these regulatory mechanisms from the ecological modeling literature to cognitive development. In the following section, we integrate these regulatory mechanisms into the resource competition model.

Self-regulation. In typical development, a constant time investment in developing an ability tends to produce decreasing payoff (also known as the law of diminishing returns). The extent to which people are driven by growth itself (i.e., practicing to get better rather than to reach a given standard) will lead to self-regulation of the skill in question: time investments will fall as the ability growth levels off, and the remaining time will become available for the growth of other abilites. Developmental problems may arise if the child continues to invest time when returns are diminishing, or even invests more time in an attempt to maintain the growth rate. For instance, we know that extreme fixation (Richler et al., 2010) and processing styles with extreme attention to detail (Shah and Frith, 1993; Jolliffe and Baron-Cohen, 1997; O'riordan et al., 2001) are more common in people with autism.

Growth-dependent resource depletion. Differences in the rate of resource consumption can depend on the stage of development of a cognitive ability. This may result from external and internal mechanisms. Externally, we expect and require children to develop many skills up to a certain level, but this level is rarely the maximum level. Thus, society practices a 'satisficing' criterion (Simon, 1956): if one's reading ability is good enough, one is not pushed to improve it further. This means that the external demand for further developing a skill falls off once a given standard of adequacy is reached. At that point, the demand for growth vanishes, and time should be invested in other skills for which there is a demand. A possible problem may arise when people do not satisfice but optimize; i.e., when a person continues to strive for excellence and perfection even when performance is already at a satisfactory level. Such continued optimization of a skill may both harm the development of other abilities and induce stronger competition between them, leading to hyperspecialization.

Mutualistic interactions. In section 1, mutualistic interactions between cognitive abilities were proposed as an essential driver for even cognitive development. Although resources only affect the internal growth of abilities, if one of the abilities grows as a result of resource investment, this can still positively affect other abilities due to mutualistic interactions.

4 The combination of regulatory mechanisms with competition for limited resources

As noted in the preceding sections, the mutualistic network model provides a plausible framework for typical development, explaining the positive manifold and other representative empirical phenomena, but struggles to explain atypical development. In contrast, the competition model adequately represents the dynamic allocation of finite resources and, through this mechanism, can account for atypical developmental patterns (i.e., hyperspecialisation, Figure 1a; intellectual disabilities, Figure 1b; uneven cognitive profiles, Figure 1c). It fails, however, to accommodate key empirical phenomena such as the positive manifold. In this section, we therefore develop a modeling approach that combines the constraining mechanism of allocating finite resources with a subset of regulatory mechanisms in cognitive development. We review various regulatory mechanisms as potential extensions of the resource competition model and investigate whether either one or a combination of these mechanisms can provide a framework that can explain typical and atypical development in a single model.

4.1 Implementing regulatory mechanism in the resource competition model

We extend the resource competition model in three ways. First, by incorporating *mutualistic interactions* between cognitive abilities. Second, by implementing *self-regulation* through the use of a logistic growth function. And third, by adopting *growth-dependent resource depletion* through the assumption that resources are consumed to a maximal extent during the growth of cognitive abilities. This aligns with the plausible assumption that the initial growth phase requires a higher resource allocation, while maintaining the ability at a certain level necessitates a lower resource demand.

In our proposed model, the cognitive system within a person is represented as a collection of abilities and skills that grow depending on the amount of resources they consume and that promote each other's growth. Take the example of a child learning to play an instrument. The child's musical ability will grow if the child spends more time practicing the instrument, if the parents motivate the child to practice more, and if the quality of the teacher or teaching material is better (i.e., all examples of more external resources). How much the child's musical ability benefits from these external resources depends on how talented the child is and how well other cognitive abilities are developed, such as attention, memory, auditory and visual processing, and fine motor skills (i.e., all examples of more internal resources). The growing musical abilities of the child may benefit (some of) these other cognitive abilities (i.e., there may be mutual interactions between abilities). However, the development of other abilities will also be constrained by skill development, as the time and money spent on one skill cannot be spent on developing other skills: time spent playing the violin cannot be spent practicing soccer (i.e., competition between skills for limited resources).

Mathematically formulated, the extended resource model—which we refer to as mutualistic resource competition model—can be characterized through the following differential equations:

$$\frac{dx_i}{dt} = x_i(\sum_{j=1}^{R} \mu_{ij} r_j \overbrace{(1 - \frac{x_i}{K_i})}^{\text{Logistic growth}} - d_i) + \overbrace{\sum_{j=1}^{X} M_{ij} x_i x_j / K_i}^{\text{Mutualism}} \tag{4}$$

$$\frac{dr_j}{dt} = r(r_{max} - r_j) - r_j \sum_{i=1}^{X} b_{ij} \begin{cases} \frac{dx_i}{dt}, & \text{growth-dependent depletion} \\ x_i, & \text{otherwise} \end{cases} \tag{5}$$

In these equations, x_i represents one out of X cognitive abilities, and r_i represents one out of R resources. Equation 4 describes the growth function of the abilities and contains two possible extensions to the competition resource model, namely a logistic growth part and a mutualistic part. In the logistic growth part, the availability of resources determines the steepness of the logistic growth function. That is, when resources are available (i.e., $\sum_{j=1}^{R} \mu_{ij} r_j$ is positive), cognitive abilities will grow. However, once the cognitive ability exceeds the carrying capacity K_i of that specific ability ($\frac{x_i}{K_i}$ becomes larger than one and thus $1 - \frac{x_i}{K_i}$ becomes negative), there is a decrease in growth until the carrying capacity is reached again.

Hence, the growth of a cognitive ability is limited either (1) by a lack of resources or (2) by reaching the carrying capacity. The decline rate, d_i, represents the rate at which an individual forgets acquired cognitive abilities or skills. The second mutualistic part is similar to the mutualism model presented in Equation 1 and represents the reciprocal couplings between cognitive abilities (van der Maas et al., 2006).

Equation 5 describes the growth or decline of the resources, which mainly follows the original resource competition model in Equation 4, except that the resources are either consumed by cognitive abilities based on their current level or based on their rate of change (i.e., growth-dependent resource depletion). The former situation is consistent with the resource competition model. The latter determines that no resources will be consumed when the rate of change is zero (e.g., because the cognitive ability has reached its carrying capacity). In contrast, in regimes characterized by rapid growth ($\frac{dx_i}{dt}$ has a high value), the cognitive ability in question will consume more resources. In this way, the model can encode the principle that developing a cognitive ability costs more resources than maintaining the cognitive ability at a certain level.

4.2 Simulating typical development

We use simulation studies to evaluate the impact of three modifications to the resource competition model proposed above (mutualistic interactions, self-regulation, and growth-dependent resource depletion). We do this for eight cognitive abilities and one resource (see Appendix for a tutorial on how to reproduce and modify the model code). The results of implementing these mechanisms, either separately or in combination with each other, are depicted in Figure 2. These simulations indicate that these three mechanisms separately result in a relatively limited diversity of cognitive abilities (see Figures 2a, 2b, and 2c). A combination of logistic growth and growth-dependent resource depletion, as seen in Figures 2g, allows for the coexistence of eight cognitive abilities on a single resource but fails to show a positive manifold. The combination of all three modifications result in both the coexistence of numerous cognitive abilities and the positive manifold. We, thus, conclude that we need all three mechanisms to best represent typical cognitive development.

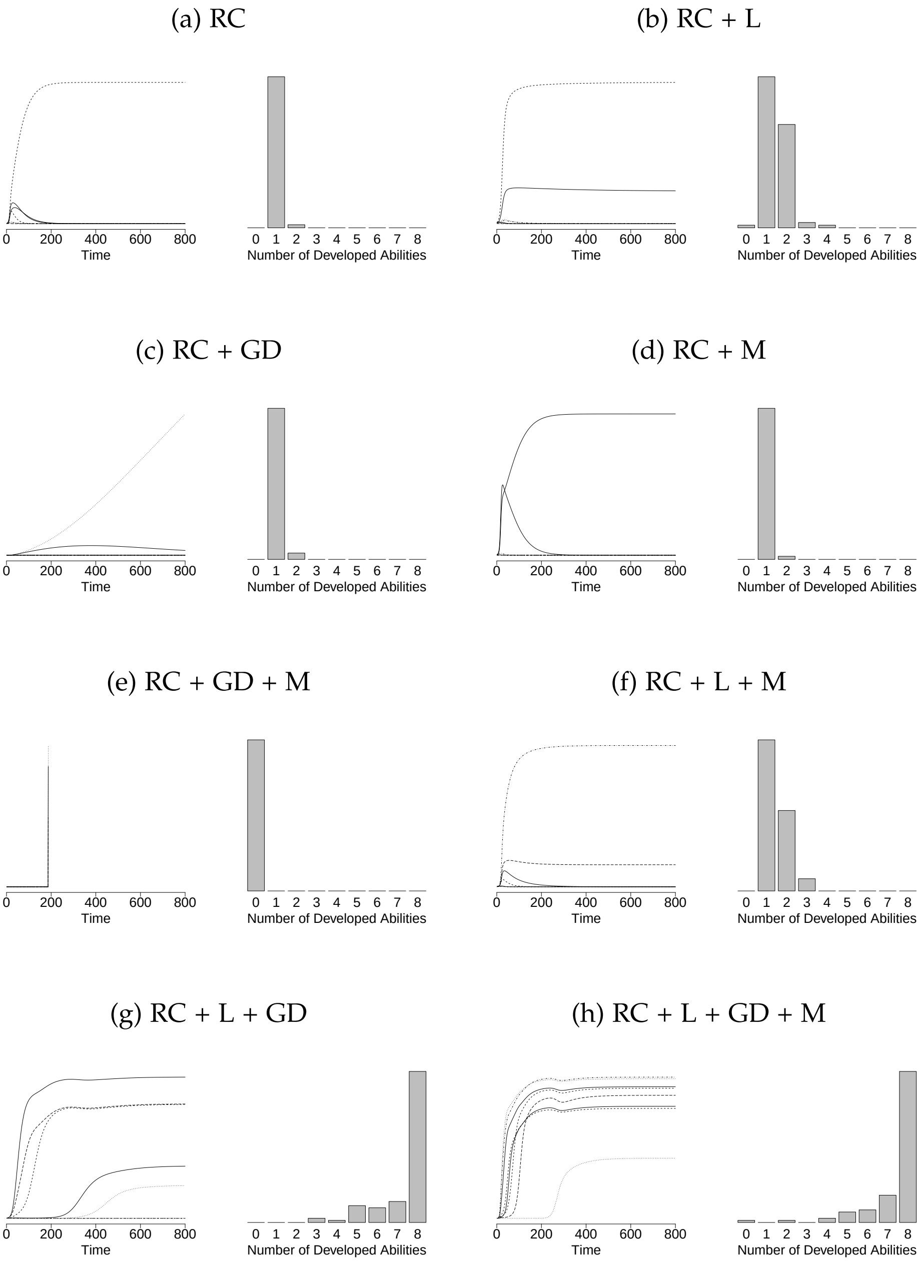


Figure 2: The effect of three extensions of the resource competition model (RC), namely logistic growth (L), growth-dependent resource depletion (GD), and mutualistic interactions (M). We present their effects separately and in combination with each other. For each combination, these figures show one randomly chosen trajectory of 800 iterations and a histogram (N = 100) displaying the frequency of the number of developed cognitive abilities after 8,000 iterations. Data were simulated for eight cognitive abilities and one resource.

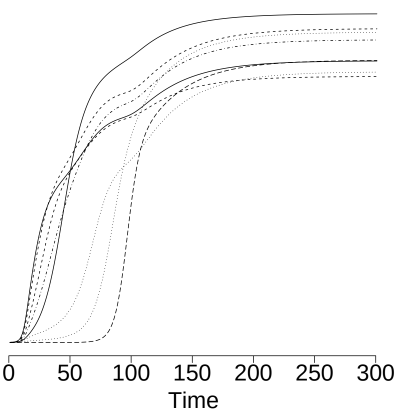
4.3 Simulating atypical development

We also executed the model under different parameter values to demonstrate how the proposed model can capture characteristics of cognitive development. The simulation results in Figure 3 illustrate that the model can account for the four characteristic types of development identified earlier.

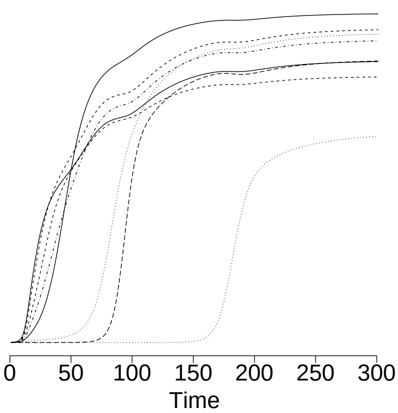
The first situation depicts typical development, in which there is limited competition for resources and where all cognitive abilities develop until sufficient levels (Figure 3a). In the second simulation, we set one of the μ's to a low value, corresponding to a specific deficit that stems from one underlying cognitive impairment (Figure 3b). A third situation, where we sampled μ's from a wide distribution, resulted in uneven cognitive profiles where only a couple of cognitive abilities grow, and the rest remains undeveloped (Figure 3c). Lastly, we simulate a situation of intellectual disabilities, where all cognitive abilities benefit minimally for the presence of the resource (i.e., low μ's; Figure 3d).

Besides accounting for four cognitive profiles, the model can reproduce, and thus potentially explain, five phenomena that have been found and replicated across different studies and cohorts of people. The first phenomenon is that cognitive development is characterized by sequential growth. The current simulation shows that growth in different cognitive abilities emerges at various points and that the growth rate is unstable over development. The growth of cognitive abilities at different times in the trajectory occurred naturally during the simulation. The cognitive abilities that grow first are the ones that benefit most from the available resources. Once these abilities are developed, they require fewer resources, so more resources remain available for other cognitive abilities to grow. To the best of our knowledge, we present the first model in which it naturally occurs that many cognitive abilities lay dormant until later in development.

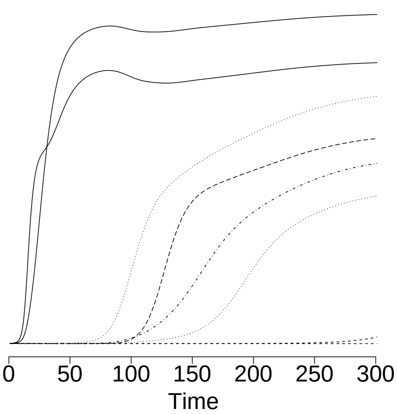
This suggests that temporal dispersion across developmental stages is a way of dealing with resource-limited growth, so that cognitive abilities do not compete for the same resources at the same time. A prediction about sequential growth that results from the model is that the growth of cognitive abilities follows each other in rapid succession due to their mutualistic relations. Thus, the full development of one cognitive ability (i.e., the rate of change is zero) has a cascading effect on the other cognitive abilities in the system by reducing competition for resources.



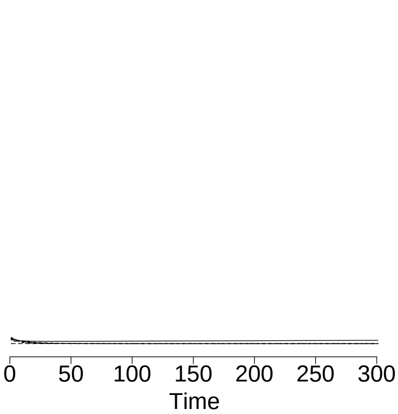
(a) Typical development



(b) Specific deficits



(c) Uneven cognitive profile



(d) Intellectual disabilities

Figure 3: Characteristic patterns produced by the final resource model. Data were simulated for eight cognitive abilities and one resource. To manipulate the degree of competition, we varied the extent to which cognitive abilities benefited from the availability of the resource (by adapting the μ's.

The second and third phenomena we observe are the generally slower cognitive development when the beneficial effects of resource levels are low (i.e., μ's are low and therefore competition is high) compared to typical development, and the lack of early predictors of atypical development. Thus, although cognitive development is slower with limited resources, these effects do not show immediately (i.e., the onset of the trajectories of specific deficits and uneven cognitive profiles look similar to the trajectory of even development). This finding is in line with the fact that developmental disorders are rarely diagnosed at very early stages of development; for instance, in autism diagnostics, it is well-established that the toddler age is the earliest state in which atypicalities can be detected (Pierce et al., 2011; Mundy, 2018). A recent review of early markers for developmental disorders reported some evidence for early indicators starting from around 12 months and weak evidence for those under 12 months (Micai et al., 2020).

A fourth phenomenon produced in our simulations resulting in uneven cognitive profiles (Figure 3c) is the loss of previously established behavior and skills, evidenced by a drop in the level of a cognitive ability. In the empirical literature, this phenomenon has been documented as developmental regression (Karmiloff-Smith, 2009) and is associated with the autistic profile. We also see, as specified in the autism literature, that there is a degree of recovery that takes place after the regression.

Lastly, we observe a correlation between resource levels and cognitive performance. Thus, our model predicts that in groups with fewer resources, e.g., lower socio-economic status or genetic endowment, we should see more uneven cognitive profiles. This finding may represent the phenomenon of "quasi-autism", where children growing up in orphanages under extremely adverse conditions show a higher incidence rate of autistic features compared to the general population (Rutter et al., 1999). At the population level, the observed correlation between resource levels and cognitive performance could provide a potential explanation for the Flynn effect (Flynn, 1984), as it allows for a larger resource pool–mostly external due to growth in wealth–as a driver of improvements in cognitive performance.

5 Discussion

This paper presents an integrated modeling framework that accounts for typical and atypical development of cognitive abilities. The central components of the model are a) mutualism, implemented in a dynamic network model, in which cognitive abilities facilitate each other's growth, and b) resource competition, in which cognitive abilities compete for finite resources in development. The combination of these principles provides a common explanatory framework that accommodates phenomena representative of typical development (e.g., developmental stages and the positive manifold; van der Maas et al. (2006, 2017), and at the same time, can reproduce characteristic atypical profiles documented in the literature (e.g., intellectual disabilities, hyperspecialization, uneven cognitive abilities, and specific deficits).

Although we have identified several possible societal and psychological mechanisms that plausibly serve to mitigate resource competition, to our knowledge, dedicated empirical research has yet to address this topic explicitly. Given the plausibility of the basic model ingredients of mutualism and resource competition, such research projects invite an important and straightforward inroad to further understanding atypical development profiles.

5.1 Model extensions

The fact that the simple mechanisms of mutualism and resource competition offer a possible explanation for such a wide range of phenomena is, in our view, remarkable. In addition, because both principles seem highly plausible on independent grounds, we argue that any account of development should incorporate them. Naturally, the present paper has constructed but one model representing these principles. We have focused on what could be considered the *simplest nontrivial case*, as the model represents mutualism and resource competition in the simplest possible way.

There are many ways in which the mechanisms governing resources are more complex than represented in our model. For example, a country's educational system is likely to shape how these resources play out in various ways, for example by making schooling mandatory with the intention of allocating resources optimally, and thus forcing diversity in skill development. Furthermore, in the current model, resources are independent of each other, whereas a better assumption might be that resources themselves form a dynamic network of interacting components (Den Hartigh et al., 2016). So that the levels of the resources increase or decrease as a consequence of interactions with other resources.

Clearly, many extensions of the model are possible and could be the subject of further investigation; primary examples include the addition of a dynamical model for the interaction between resources and model structures that allow for multiplier effects that represent the interaction between genetic factors and the environment (Dickens and Flynn, 2001; van der Maas et al., 2019). Furthermore, in the current model, the number of cognitive abilities is predetermined and the emergence of new cognitive abilities and skills remains unexplored. Van Geert (1991), for instance, coined the term allopetric growth as a principle for the emergence of new cognitive abilities, where emergence occurs in relative isolation, so that new abilities do not compete for resources with already established cognitive abilities.

5.2 Model implications

Although the focus of this paper has been on *cognitive* development, the general idea that developmental processes compete for resources may have broader applicability. For example, in the psychopathology literature, the network theory of mental disorders posits that interactions between symptoms may constitute the backbone of psychopathology (Cramer et al., 2010; Borsboom, 2017).

It is plausible that some of these interactions are governed by competition for limited resources. For instance, low energy levels are a characteristic symptom of depression and may represent a limited resource that constrains behavioral processes that typically have positive effects, such as exercise and social interaction. Similarly, money is a very concrete resource constraint that undoubtedly plays a role in symptom networks characterizing substance abuse (Rhemtulla et al., 2016) as money spent on drugs cannot be spent on, for example, a healthy diet or a productive family life. Finally, recent research has shown that in some slowly developing mental disorders, such as schizophrenia, measurable cognitive decline may precede the appearance of positive symptoms by more than a decade (Jonas et al., 2022).

As we have shown, such patterns of cognitive decline may arise from resource competition. In addition, cognitive abilities may themselves function as resources for other processes (such as reasoning and planning); if so, declines in cognitive ability may promote resource competition between processes that depend on it and, as such, produce a cascade of interactions between problems occurring at higher levels of analysis.

Second, the general modeling framework of cognitive development allows us to study and evaluate architectural risk factors in early developmental phases that may be relevant for atypical outcomes that can affect well-being in later childhood or adolescence. In particular, high levels of resource competition may provide a novel focus for research that aims to understand such processes. A primary and directly measurable finite resource is time. Competition for time is a basic fact for any set of processes that cannot benefit from the same activity at the same time. While this observation will come as no surprise to either scientists or laypeople, its consequences may be much more far-reaching than one might initially suspect. As we have shown, systematic imbalances in time allocation can produce profiles associated with hyperspecialization (characteristically associated with autism) and specific deficits (characteristically associated with problem areas such as dyslexia and dyscalculia). Because time allocation is measurable and, in some cases, actionable through regulatory mechanisms, early detection of this type of resource competition can play a critical role in mitigating its effects later in development. One straightforward approach that has become available through new sources of digital data is to measure children's time allocation in educational online games (Klinkenberg et al., 2011) to link structural imbalances in time allocation to later developmental processes. In addition, interventions can explicitly focus on mitigating competition for time resources.

Third, the modeling approach we have followed in this paper is based on analogical abduction (Haig, 2013): by constructing an analogy between the development of species and the development of cognitive abilities, we were able to borrow well-studied formal modeling approaches from population biology and apply them to cognitive development. While analogical abduction has been argued to be a common and important process in theory formation, it is rarely explicitly pursued as a methodological strategy.

The model development as described in this paper may serve as a template for how analogical abduction can be used as an ingredient in the burgeoning literature on theory construction methodology (Borsboom et al., 2021; van Rooij and Baggio, 2021; Haslbeck et al., 2021) and, as such, may contribute to addressing the "theory crisis" in psychology (Fried, 2021; Eronen and Bringmann, 2021; Oberauer and Lewandowsky, 2019). The process of analogical abduction is, in our experience, highly profitable for theory generation, especially when formal models adapted from the source domain (here: population biology) can be directly implemented in the target domain (here: cognitive development).

To facilitate this process, the appendix can be used as a tutorial on how to implement the modeling framework in the Grind package (de Boer and Pagie, 1983). This means that the interested reader can directly reproduce many of the previous dynamical system models in cognitive development, all the modeling results reported in this paper, and extend the model to develop their own theoretical variations.

5.3 Conclusion

The focus of this paper has been creative: we have concentrated on the construction of a model rather than validating or testing it. As such, the model should be regarded as speculative. At the same time, the fact that the model has been formalized means that unambiguous predictions can be directly derived and tested in dedicated research.

Importantly, not all implications can be anticipated in advance, and some may be very surprising even to those who construct the model. For example, in this project, we deliberately set out to construct a model that would accommodate atypical development, and as such, these phenomena were explicitly targeted. However, we were surprised to find that the model principles also imply (and thus provide a tentative explanation for) the occurrence of developmental stages in which abilities emerge in a sequential manner.

This illustrates the value of formal models as "thinking tools" that allow us to gauge implications of our explanatory principles that we cannot foresee due to our own cognitive limitations. This provides an important bridge between verbal theory and empirical research. As such, we hope that the proposed model will provide fertile ground for further development, validation, and testing.

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References

Bal, V. H., Wilkinson, E., and Fok, M. (2022). Cognitive profiles of children with autism spectrum disorder with parent-reported extraordinary talents and personal strengths. *Autism*, 26(1):62–74.

Baron-Cohen, S., Ashwin, E., Ashwin, C., Tavassoli, T., and Chakrabarti, B. (2009). Talent in autism: hyper-systemizing, hyper-attention to detail and sensory hypersensitivity. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 364(1522):1377–1383.

Bascompte, J. (2019). Mutualism and biodiversity. *Current Biology*, 29(11):R467–R470.

Borsboom, D. (2017). A network theory of mental disorders. *World Psychiatry*.

Borsboom, D., van der Maas, H. L., Dalege, J., Kievit, R. A., and Haig, B. D. (2021). Theory Construction Methodology: A Practical Framework for Building Theories in Psychology. *Perspectives on Psychological Science*, 16(4):756–766.

Brunsdon, V. E. and Happé, F. (2014). Exploring the 'fractionation' of autism at the cognitive level. *Autism*, 18(1):17–30.

Constantino, J. N., Charman, T., and Jones, E. J. (2021). Clinical and translational implications of an emerging developmental substructure for autism. *Annual review of clinical psychology*, 17:365–389.

Cramer, A. O., Waldorp, L. J., van der Maas, H. L., and Borsboom, D. (2010). Comorbidity: A network perspective.

de Boer, R. J. and Pagie, L. (1983). Great integrator differential equations.

de Roos, A. M., Persson, L., and McCauley, E. (2003). The influence of size-dependent life-history traits on the structure and dynamics of populations and communities. *Ecology Letters*, 6(5):473–487.

Dean, A. M. (1983). A simple model of mutualism. *The American Naturalist*, 121(3):409–417.

Den Hartigh, R. J., Van Dijk, M. W., Steenbeek, H. W., and van Geert, P. L. (2016). A dynamic network model to explain the development of excellent human performance. *Frontiers in Psychology*, 7:532.

Dickens, W. T. and Flynn, J. R. (2001). Heritability estimates versus large environmental effects: the IQ paradox resolved. *Psychological review*, 108(2):346–369.

Eronen, M. I. and Bringmann, L. F. (2021). The Theory Crisis in Psychology: How to Move Forward. *Perspectives on Psychological Science*, 16(4):779–788.

Ferrer, E. and McArdle, J. J. (2004). An experimental analysis of dynamic hypotheses about cognitive abilities and achievement from childhood to early adulthood. *Developmental psychology*, 40(6):935.

Ferrer, E., Shaywitz, B. A., Holahan, J. M., Marchione, K., and Shaywitz, S. E. (2010). Uncoupling of reading and iq over time: Empirical evidence for a definition of dyslexia. *Psychological science*, 21(1):93–101.

Flynn, J. R. (1984). The mean iq of americans: Massive gains 1932 to 1978. *Psychological bulletin*, 95(1):29.

Fried, E. I. (2021). Lack of Theory Building and Testing Impedes Progress in The Factor and Network Literature. *https://doi.org/10.1080/1047840X.2020.1853461*, 31(4):271–288.

Gavina, M. K. A., Tahara, T., Tainaka, K.-i., Ito, H., Morita, S., Ichinose, G., Okabe, T., Togashi, T., Nagatani, T., and Yoshimura, J. (2018). Multi-species coexistence in lotka-volterra competitive systems with crowding effects. *Scientific reports*, 8(1):1–8.

Goh, B. (1979). Stability in models of mutualism. *The American Naturalist*, 113(2):261–275.

Haig, B. (2013). Analogical modeling: a strategy for developing theories in psychology. *Frontiers in Psychology*, 4.

Hale, K. R., Valdovinos, F. S., and Martinez, N. D. (2020). Mutualism increases diversity, stability, and function of multiplex networks that integrate pollinators into food webs. *Nature communications*, 11(1):1–14.

Happé, F. (2018). Why are savant skills and special talents associated with autism? *World Psychiatry*, 17(3):280.

Haslbeck, J., Ryan, O., Robinaugh, D. J., Waldorp, L. J., and Borsboom, D. (2021). Modeling psychopathology: From data models to formal theories. *Psychological Methods*.

Huisman, J. and Weissing, F. J. (2001). Fundamental unpredictability in multispecies competition. *The American Naturalist*, 157(5):488–494.

Johnson, M. H. (2017). Autism as an adaptive common variant pathway for human brain development. *Developmental Cognitive Neuroscience*, 25:5–11.

Johnson, M. H., Gliga, T., Jones, E., and Charman, T. (2015). Annual research review: Infant development, autism, and adhd–early pathways to emerging disorders. *Journal of Child Psychology and Psychiatry*, 56(3):228–247.

Jolliffe, T. and Baron-Cohen, S. (1997). Are people with autism and asperger syndrome faster than normal on the embedded figures test? *Journal of Child Psychology and Psychiatry*, 38(5):527–534.

Jonas, K., Lian, W., Callahan, J., Ruggero, C. J., Clouston, S., Reichenberg, A., Carlson, G. A., Bromet, E. J., and Kotov, R. (2022). The Course of General Cognitive Ability in Individuals With Psychotic Disorders. *JAMA Psychiatry*.

Karmiloff-Smith, A. (2009). Nativism versus neuroconstructivism: rethinking the study of developmental disorders. *Developmental psychology*, 45(1):56.

Kievit, R. A., Lindenberger, U., Goodyer, I. M., Jones, P. B., Fonagy, P., Bullmore, E. T., in Psychiatry Network, N., and Dolan, R. J. (2017). Mutualistic coupling between vocabulary and reasoning supports cognitive development during late adolescence and early adulthood. *Psychological Science*, 28(10):1419–1431.

Klinkenberg, S., Straatemeier, M., and van der Maas, H. L. J. (2011). Computer adaptive practice of Maths ability using a new item response model for on the fly ability and difficulty estimation. *Computers Education*, 57(2):1813–1824.

Levin, S. A. (1970). Community equilibria and stability, and an extension of the competitive exclusion principle. *The American Naturalist*, 104(939):413–423.

Lyon, G. R., Shaywitz, S. E., and Shaywitz, B. A. (2003). A definition of dyslexia. *Annals of dyslexia*, 53(1):1–14.

McArdle, J. J., Hamagami, F., Meredith, W., and Bradway, K. P. (2000). Modeling the dynamic hypotheses of gf–gc theory using longitudinal life-span data. *Learning and Individual Differences*, 12(1):53–79.

Micai, M., Fulceri, F., Caruso, A., Guzzetta, A., Gila, L., and Scattoni, M. L. (2020). Early behavioral markers for neurodevelopmental disorders in the first 3 years of life: An overview of systematic reviews. *Neuroscience & Biobehavioral Reviews*, 116:183–201.

Mundy, P. (2018). A review of joint attention and social-cognitive brain systems in typical development and autism spectrum disorder. *European Journal of Neuroscience*, 47(6):497–514.

Oberauer, K. and Lewandowsky, S. (2019). Addressing the theory crisis in psychology. *Psychonomic Bulletin and Review*, 26(5):1596–1618.

O'riordan, M. A., Plaisted, K. C., Driver, J., and Baron-Cohen, S. (2001). Superior visual search in autism. *Journal of Experimental Psychology: Human Perception and Performance*, 27(3):719.

Pierce, K., Carter, C., Weinfeld, M., Desmond, J., Hazin, R., Bjork, R., and Gallagher, N. (2011). Detecting, studying, and treating autism early: the one-year well-baby check-up approach. *The Journal of pediatrics*, 159(3):458–465.

Quinn, J. M., Wagner, R. K., Petscher, Y., Roberts, G., Menzel, A. J., and Schatschneider, C. (2020). Differential codevelopment of vocabulary knowledge and reading comprehension for students with and without learning disabilities. *Journal of educational psychology*, 112(3):608.

R Core Team (2021). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria.

Revilla, T. (2009). *Multispecies resource competition*. PhD thesis, PhD dissertation. University of Groningen, The Netherlands.

Rhemtulla, M., Fried, E. I., Aggen, S. H., Tuerlinckx, F., Kendler, K. S., and Borsboom, D. (2016). Network analysis of substance abuse and dependence symptoms. *Drug and Alcohol Dependence*, 161:230–237.

Richler, J., Huerta, M., Bishop, S. L., and Lord, C. (2010). Developmental trajectories of restricted and repetitive behaviors and interests in children with autism spectrum disorders. *Development and psychopathology*, 22(1):55–69.

Rohr, R. P., Saavedra, S., and Bascompte, J. (2014). On the structural stability of mutualistic systems. *Science*, 345(6195):1253497.

Rutter, M., Andersen-Wood, L., Beckett, C., Bredenkamp, D., Castle, J., Groothues, C., Kreppner, J., Keaveney, L., Lord, C., O'Connor, T. G., et al. (1999). Quasi-autistic patterns following severe early global privation. *The Journal of Child Psychology and Psychiatry and Allied Disciplines*, 40(4):537–549.

Savi, A. O., Marsman, M., and van der Maas, H. L. (2021). Evolving networks of human intelligence. *Intelligence*, 88:101567.

Shah, A. and Frith, U. (1993). Why do autistic individuals show superior performance on the block design task? *Journal of child Psychology and Psychiatry*, 34(8):1351–1364.

Simon, H. A. (1956). Rational choice and the structure of the environment.

Soetaert, K. (2009). *rootSolve: Nonlinear root finding, equilibrium and steady-state analysis of ordinary differential equations*. R package 1.6.

Soetaert, K. and Petzoldt, T. (2010). Inverse modelling, sensitivity and monte carlo analysis in R using package FME. *Journal of Statistical Software*, 33(3):1–28.

Soetaert, K., Petzoldt, T., and Setzer, R. W. (2010). Solving differential equations in R: Package deSolve. *Journal of Statistical Software*, 33(9):1–25.

Spearman, C. (1961). " general intelligence" objectively determined and measured.

Stone, L. (2020). The stability of mutualism. *Nature communications*, 11(1):2648.

Thomas, M. S., Baughman, F. D., and Karaminis, T. (2012). Modelling developmental disorders. In *Current issues in developmental disorders*, pages 103–134. Psychology Press.

Tilman, D., Kilham, S. S., and Kilham, P. (1982). Phytoplankton community ecology: the role of limiting nutrients. *Annual review of Ecology and Systematics*, 13(1):349–372.

Treffert, D. A. (2009). The savant syndrome: an extraordinary condition. a synopsis: past, present, future. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 364(1522):1351–1357.

van der Maas, H. L., Dolan, C. V., Grasman, R. P., Wicherts, J. M., Huizenga, H. M., and Raijmakers, M. E. (2006). A dynamical model of general intelligence: the positive manifold of intelligence by mutualism. *Psychological review*, 113(4):842.

van der Maas, H. L., Kan, K.-J., Marsman, M., and Stevenson, C. E. (2017). Network models for cognitive development and intelligence. *Journal of Intelligence*, 5(2):16.

van der Maas, H. L., Savi, A. O., Hofman, A., Kan, K.-J., and Marsman, M. (2019). The network approach to general intelligence. In McFarland, D. J., editor, *General and Speci c Mental Abilities*. Cambridge Scholars Publishing.

van Dongen, N., van Bork, R., Finnemann, A., van der Maas, H., Robinaugh, D., Haslbeck, J., de Ron, J., and Borsboom, D. (2022). Improving psychological explanations.

van Geert, P. (1991). A dynamic systems model of cognitive and language growth. *Psychological review*, 98(1):3.

van Geert, P. (1994). *Dynamic systems of development: Change between complexity and chaos.* Harvester Wheatsheaf.

van Rooij, I. and Baggio, G. (2021). Theory Before the Test: How to Build High-Verisimilitude Explanatory Theories in Psychological Science. *Perspectives on Psychological Science*, 16(4):682–697.

Appendix

An overview of developmental models

In this section we present the main models we found in the literature that represent cognitive development as a dynamical network model (see Table 2 for an overview). All these models (i.e., van Geert, 1991; 1994, Den Hartigh et al., 2016, 2018; Zwerwer Den Hartigh, 2022; van der Maas, 2017; 2006) describe development as a system of coupled logistic equations and incorporate resources as a limiting factor in cognitive development. They incorporate limited resources either indirectly via the carrying capacity (e.g., van der Maas, 2006) or directly where the level of one cognitive ability impacts the growth rate of another cognitive ability (e.g., van Geert, 1991; 1994). The results of these models reproduce some of the phenomena discussed in the current manuscript (e.g., lack of early indicators). However, none of these models directly represent the allocation of resources over different cognitive abilities.

To the best of our knowledge, our model is the first to propose a model of resource competition in dynamic networks where cognitive abilities compete for finite resources. It extends the models of van Geert (1991) and van der Maas et al. (2006) by explicitly modeling the dynamics of the resources (i.e., resources are depleted by the growth of cognitive abilities). It extends the model of talent development by Den Hartigh et al. (2016, 2018; Zwerwer Den Hartigh, 2022), in which resources are modeled dynamically, by relating development to a broader context than talent development, and thus aims to explain both typical and atypical cognitive development.

Table 2: Overview of the main dynamical systems models in cognitive development.

Authors	Description	Equation(s)	Output	Direct or indirect interactions between cognitive abilities
van der Maas et al. (2006)	Logistic growth equations coupled via an interaction matrix	$\frac{dx_i}{dt} = a_i x_i(1 - \frac{x_i}{K_i}) + a_i \sum_{j=1}^{X} M_{ij} x_i x_j / K_i$	Positive manifold, heritability effects, lack of early indicators	Direct interactions between cognitive abilities.
van Geert (1991, 1994)	Logistic growth equations, where the growth rate of one cognitive ability depends on the level of another cognitive ability	$x_{i_{t+1}} = (a_i - c_i x_{j_t}) x_{i_t} - a_i \frac{x_{i_t}^2}{K})$	Lack of early indicators	Direct interactions between cognitive abilities.
Den Hartigh et al. (2016)	Logistic growth equations coupled via an interaction matrix	$\frac{dx_i}{dt} = (a_i x_i(1 - \frac{x_i}{K_i}) + \sum_{j=1}^{X} M_{ij} x_i x_j)(1 - \frac{x_i}{K_i})$	Lack of early indicators and (hard-coded) developmental phases	No interactions between cognitive abilities specified.
de Ron et al. (2023)	Logistic growth equations coupled via an interaction matrix combined with resource competition dynamics	$\frac{dx_i}{dt} = a_i x_i(\sum_{j=1}^{R} \mu_{ij} r_j (1 - \frac{x_i}{K_i}) - d_i) + a_i \sum_{j=1}^{X} M_{ij} x_i x_j / K_i$ $\frac{dr_j}{dt} = r(r_{max} - r_j) - r_j(\sum_{i=1}^{X} b_{ij} \frac{dx_i}{dt})$	Lack of early indicators, developmental phases, and positive manifold	Both direct interaction between cognitive abilities and indirect interactions via shared, limited resources

Simulation code

In this section, we provide the R code to simulate data from the presented models and to generate all the figures in the current paper. All modeling code can also be found on OSF (`https://osf.io/uy2ra/`). We use the Grind package (i.e., Great integrator differential equations; de Boer and Pagie, 1983), a modeling framework to easily analyze and visualize models consisting of (coupled) differential or difference equations in the programming language R (R Core Team, 2021).

First, we have to set up the Grind environment. Grind is not available on CRAN, but can be downloaded from GitHub. To do this, we use the 'install_github' function from the package 'remotes'. Once installed, we can load all the Grind functionality into our R environment using the 'library(Grind)' command:

```
# Installation of Grind:
install.packages("remotes")
remotes::install_github("hansschepers/grindr")
library(Grind)
```

Grind relies on three other packages that we need to install and load as well, namely deSolve (Soetaert et al., 2010), rootSolve (Soetaert, 2009) and FME (Soetaert and Petzoldt, 2010). The deSolve and rootSolve packages are used for the numerical integration of the differential equations. FME is only used in cases where the parameters of a model are to be estimated from data.

```
# Load packages:
library(deSolve)
library(rootSolve)
library(FME)
```

Mutualistic network model (Equation 1)

There are three components needed to simulate code from any dynamical system: (1) differential equations that the model consists of, (2) the parameter values, and (3) the initial values of all the variables in the model. Grind uses default names for these model components: The differential equations must be defined in the function 'model()', the parameter values in a vector 'p', and the initial values of the variables in a vector 's'.

```
# Define the model:
model <- function(t, state, parms){
  with(as.list(c(state, parms)),{
    x <- state[1:X]
    dx <- a*x*(1-x/K) + a*(x * M %*% x)/K
    return(list(dx))
  })
}

# Determine the number of cognitive abilities:
X <- 16

# Choose parameter values:
M <- matrix(0.05, X, X); diag(M) <- 0
a <- rnorm(X, 0.2, 0.05)
K <- rnorm(X, 20, 5)

# Set parameters and initial values:
p <- c(a = a, K = K, M = M)
s <- c(x = runif(X, 0.1, 0.5))
```

After defining all these components, as shown in the code above, we can use the 'run()' function to numerically integrate the model and return the trajectory of all the variables in the model. By default the trajectory will be 100 iterations, but you can specify any number of iterations with the 'tmax' argument. We can plot the trajectory of a single run directly in Grind by setting 'timeplot = TRUE' (see Figure 4 for the resulting plot). However, we can also save the modelling results by setting 'table = TRUE' so that we can use our preferred plotting package in R to visualize the trajectory.

```
# Run the model:
run(main = 'Mutualism model', timeplot = TRUE, legend = FALSE)
```

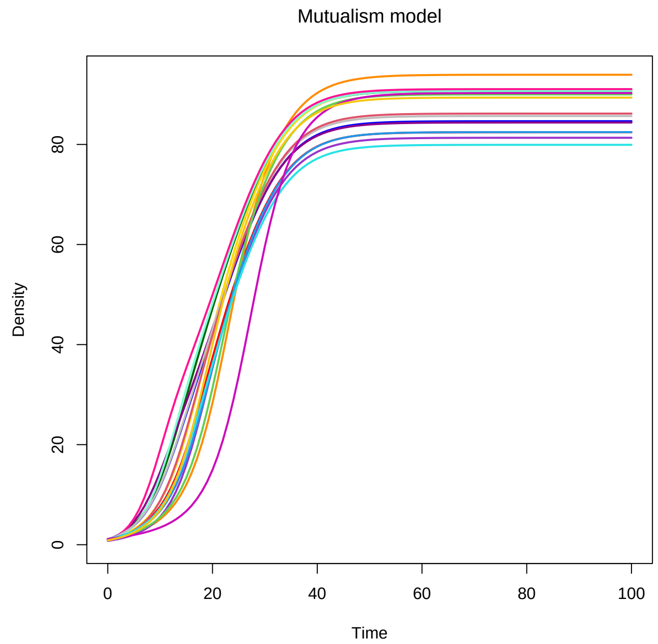


Figure 4: Modelling output when running the mutualistic network model in Grind for 100 iterations.

We can check if the mutualistic network model can account for the positive manifold as is done in van der Maas et al. (2006) by running the simulation many times (for now, 1000 samples should suffice), sampling parameter values and initial values, and running the model until it reaches the asymptote(for now, the default of 100 iterations should be fine), and then computing the correlation coefficient of the last values of all the cognitive abilities. The 'run()' function automatically returns the last values of all variables in the model.

```
# Specify the number of iterations and set up data frame:
nr_of_pp <- 1000
data <- matrix(0, nr_of_pp, X)

# Make a for loop to check positive manifold:
for(i in 1:nr_of_pp) {
  a <- rnorm(X, 0.2, 0.05)
  K <- rnorm(X, 20, 5)
  x <- rnorm(X, 1, 0.1)
  s <- x; p <- c(a = a, K = K)
```

```
  data[i,] <- run(timeplot = FALSE)
}
```

We can plot the frequency of the correlation coefficients of our 1000 samples in a histogram, where we expect that all correlation coefficients to be above zero. Figure 5 depicts the results when we run the following code:

```
# Plot the results in a histogram:
hist(cor(data)[cor(data)<1], main = 'Positive manifold',
xlab = 'Intercorrelations', xlim = c(0,0.8))
```

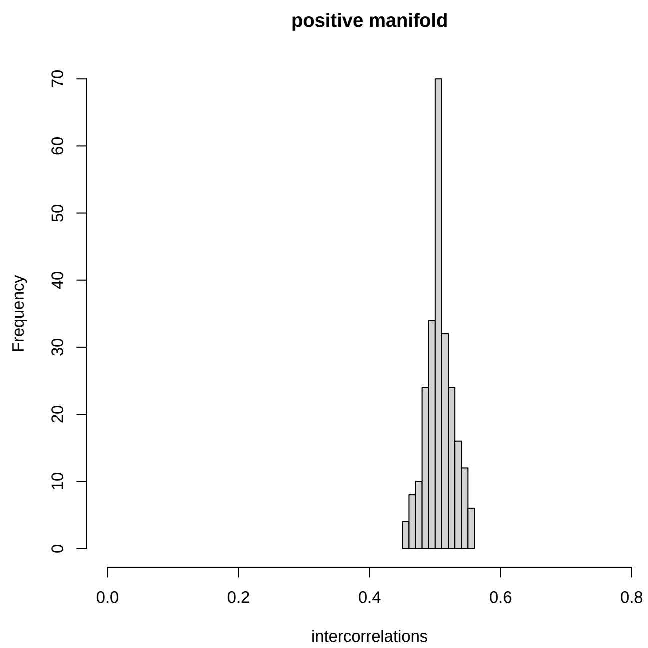


Figure 5: Histogram of the intercorrelations between cognitive abilities when running the mutualistic network model a 1000 times.

5.3.1 Modelling atypical development with the mutualistic network model

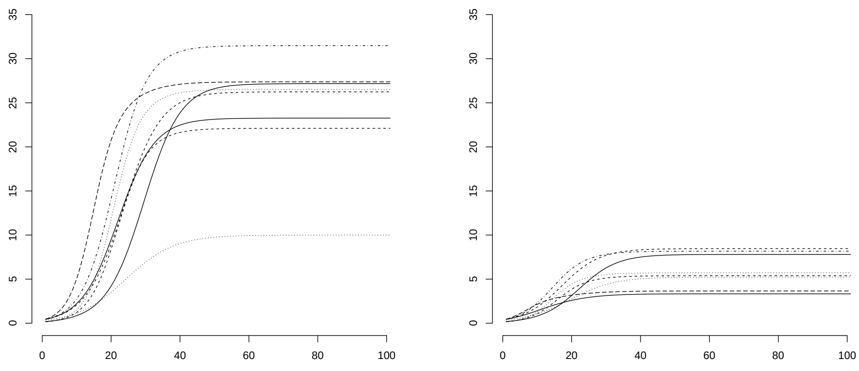
To demonstrate how specific parameter settings in the mutualistic network model can capture characteristics of atypical development, we simulate data where (1) we set one of the carrying capacities to a low value compared to the other carrying capacities and remove the reciprocal coupling with that cognitive ability, and (2) we sample the carrying capacities from a normal distribution with a low mean and specify only weak cross-domain couplings between cognitive abilities. The first situation results in specific deficits, as depicted in Figure 6a, where the early deficit in a cognitive domain stems from one underlying cognitive impairment. The second situation results in overall intellectual disabilities, as depicted in Figure 6b, where all cognitive abilities stay at a low level of development. To do this, we can re-run the mutualistic network model specified above (via the 'run' function), whileonly redefining the components in Grind that we wish to change.

```
# Modeling specific deficit:
K <- c(rnorm(X-1, 20, 5), 10)
M[,X] <- M[X,] <- 0
result <- run(timeplot = FALSE, table = TRUE)

# Plot the trajectory:
matplot(result[,2:(X+1)], type = "l",
        ylab = "", bty = "n", col = "black", ylim = c(0,80))

# Modelling intellectual disabilities:
K <- rnorm(X, 5, 2)
M <- matrix(0.02, X, X); diag(M) <- 0
result <- run(timeplot = FALSE, table = TRUE)

# Plot the trajectory:
matplot(result[,2:(X+1)], type = "l",
        ylab = "", bty = "n", col = "black", ylim = c(0,80))
```



(a) Specific deficits

(b) Intellectual Disabilities

Figure 6: Here, we show two examples of how to model atypical development in the mutualistic network framework. (a) We can model a cognitive profile with a specific deficit by setting one of the carrying capacities to a low value and removing the reciprocal coupling with that cognitive ability. (b) We can model a cognitive profile of intellectual disabilities by setting all the carrying capacities to low values and specifying only weak cross-domain couplings in the M matrix. See the main text for criticism of this 'built-in' solution.

5.3.2 Network Model of talent development by Den Hartigh et al.

Here we show how to simulate data from the talent development model (Den Hartigh et al., 2016) with Grind. Compared to the code for the mutualism model, there are two important differences. First, in the talent development model, the logistic growth function is multiplied by another logistic function. This is done to limit growth to the ultimate carrying capacity C. If the positive interactions in the M matrix are too strong, the model will grow exponentially, which is a problem well-known and still actively researched in the ecological literature (Goh, 1979; Stone, 2020; Rohr et al., 2014).

Second, the talent development model focuses on the development of one skill only. Thus, one of the nodes reflects the ability of interest and the other nodes are resources that support or inhibit each other and the ability through a direct or an indirect connection. Thus, while in the simulations of the mutualistic network model the cross-domain couplings in the M matrix are all positive and equal, in the talent development the interaction matrix M contains negative values. Each pair of nodes is connected with a probability of 25%, and the strength of the connection is drawn from a Gaussian distribution (M = 0, sd = 0.02).

```
# Define the model:
model <- function(t, state, parms) {
  with(as.list(c(state, parms)),{
    x <- state[1:X]
    dx <- (r*x*(1 - x/K) + (x * M %*% x)) * (1 - x/C)
    return(list(dx))
  })
}

# Determine the number of nodes:
X <- 10

# Set the interaction parameters:
M <- matrix(rnorm(X^2, 0, 0.02), X, X); diag(M) <- 0
M <- matrix(sample(0:1, X^2, replace = TRUE, prob = c(0.75, 0.25)), X, X) * M

# Set parameter values:
r <- rnorm(X, 0.05, 0.01)
K <- rnorm(X, 1, 0.15)
C <- runif(X, 10, 25)
```

In the talent development model, the first four nodes are initialized and the rest of the nodes emerge later. When simulating 500 time steps (i.e., set tmax = 500), the time of emergence of the different nodes is drawn from a uniform distribution from 1 to 350. The 'after' argument of the 'run' function in Grind allows the user to define events to occur during the simulation. For the talent development model we use the after statement to sample initial values (from a uniform distribution from 0.01 to 0.05) at their designated time of emergence.

```
T_emergence <- round(runif(X - 4, 1, 350))

# Run the model:
p <- c(r = r, K = K, M = M, C = C)
s <- c(x = c(runif(4, 0, 0.05), rep(0, X-4)))
```

```
run(tmax = 500, timeplot = TRUE, legend = FALSE,
    after = "if (t %in% T_emergence)
             state[4 + which(T_emergence == t)] <- runif(1, 0.01, 0.05)")
```

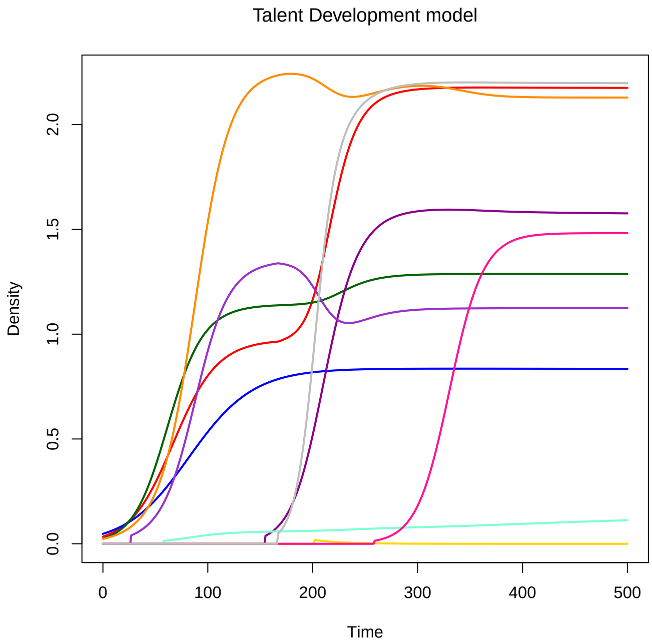


Figure 7: Modelling output when running the talent development model for 10 nodes.

5.3.3 Network Model of Coupled Growers by van Geert (1991)

Van Geert (1991) uses difference equations instead of differential equations to describe cognitive growth. In order for the Grind package to work with difference equations, we need to select method = "euler" in the 'run' function. When setting the method argument to Euler we also need to set the stepsize. For difference equations the step size must be one (i.e., set tstep = 1).

```
# Define the model:
model <- function(t, state, parms) {
  with(as.list(c(state,parms)), {
    dx1 <- (r + c1 * x2) * x1 - r * x1^2 / K
    dx2 <- (r + c2 * x1) * x2 - r * x2^2 / K
    return(list(c(dx1, dx2)))
  })
```

```
}

par(mfrow = c(2, 2))

# Set parameter values and run the model:
p <- c(K = 1, r = 0.4, c1 = -0.05, c2 = -0.15)
s <- c(x1 = 0.01, x2 = 0.01)
run(method = "euler", tstep = 1)

# Set parameter values and run the model:
p <- c(K = 1, r = 0.05, c1 = -0.1, c2 = 0.099)
s <- c(x1 = 0.01, x2 = 0.01)
run(tmax = 150, method = "euler", tstep = 1)

# Set parameter values and run the model:
p <- c(K = 1, r = 0.05, c1 = -0.1, c2 = -0.09)
s <- c(x1 = 0.0125, x2 = 0.01)
run(tmax = 500, method = "euler", tstep = 1)

# Set parameter values and run the model:
p <- c(K = 1, r = 0.05, c1 = -0.1, c2 = -0.09)
s <- c(x1 = 0.0126, x2 = 0.01)
run(tmax = 500, method = "euler", tstep = 1)
```

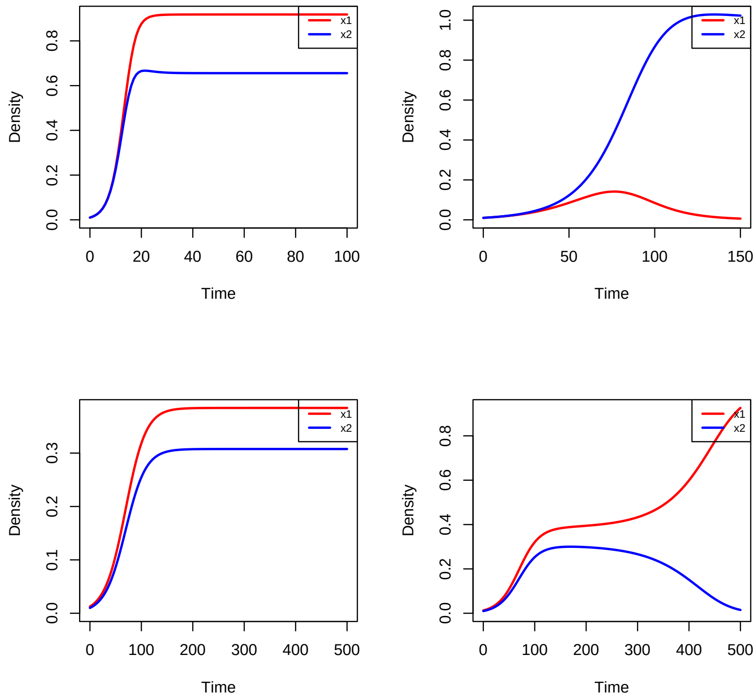


Figure 8: Output when we run the code above in Grind. The current figure is similar to Figure 15 in van Geert (1991)

Figure 15 shows the output of running the specified code. In line with Figure 15 of van Geert (1991), if we simulate under parameters such that the growth rate exceeds a threshold level (about 0.1) and the two cognitive abilities grow sufficiently fast, both cognitive abilities evolve towards a steady state. For further details we refer to van Geert 1991.

Resource competition model (Equation 2 and 3)

In this section, we show how to simulate data from the standard resource competition model. Now that we have two different differential equations, be aware that the model function should return a list of differential equations in the same order as specified in the function itself. We start by including one resource and eight cognitive abilities.

```
# Define the model:
model <- function(t, state, parms){
  with(as.list(c(state,parms)),{
    x <- state[1:X]
```

```
    r <- state[(X+1):(X+R)]
    dx <- x * (apply(t(mu) * r, 2, sum) - d)
    dr <- rj*(rmax-r) - r * apply(b * x, 2, sum)
    return(list(c(dx, dr)))
  })
}

# Determine the number of cognitive abilities and resources:
X <- 16
R <- 1

# Set parameter values:
mu <- matrix(runif(nr_var*nr_r, 0.1, 0.3),nr_var,nr_r)
d <- runif(nr_var, 0.05, 0.2)
rj <- 0.1
rmax <- runif(nr_r, 1, 5)
b <- matrix(runif(nr_var*nr_r, .05, .2), nr_var, nr_r)

# Set initial values:
x <- runif(X, 0, 0.02)
r <- runif(R, 1, 2)

# Run model:
p <- c(mu = mu, d = d, rj = rj, rmax = rmax, b = b)
s <- c(x = x, r = r)

result <- run(timeplot = TRUE, tmax = 200, legend = FALSE,
               main = "Resource Competition Model")
```

Running the above code, we get a plot of the trajectories of both the resources and the cognitive abilities (Figure 9). In line with the competitive exclusion principle, we see that after only 200 iterations, one cognitive ability outcompetes all others. However, it might be difficult to distinguish which lines represent the resources and which are the cognitive abilities, as the plot does not show a legend (see the argument 'legend = FALSE'). If we wish, we can plot the resources and cognitive abilities separately by saving the modeling data and using the matplot function to visualize the trajectories (or any other preferred plotting function).

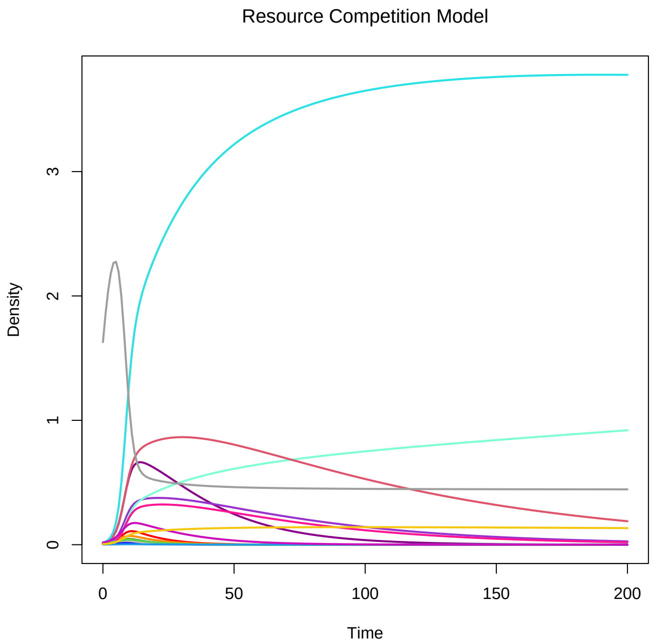


Figure 9: Grind modeling output when running the resource competition model with eight cognitive abilities and one resource.

Mutualistic resource competition model (Equation 4 and 5)

In this section we combine the previous mutualistic network model with the resource competition model. Figure 10 depicts a graphical representation of the mutualism resource model with three cognitive abilities and two resources and Table 3 shows the distributions from which the parameter values in our model simulations are drawn and their interpretation in cognitive development. The parameter values are arbitrarily chosen to produce reasonable modeling output and should therefore only be interpreted relative to each other. For instance, the decline rate is lower than the growth rate, which assumes an asymmetry between learning and forgetting something. While this difference is meaningful, the actual values should not be interpreted directly.

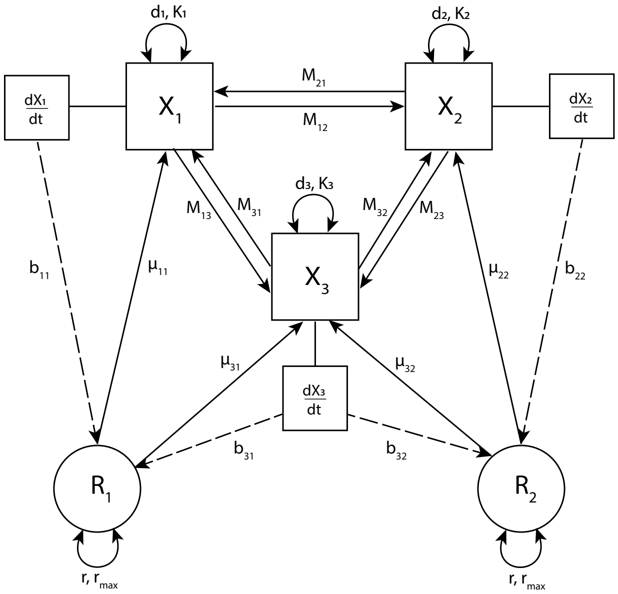


Figure 10: A graphical representation of the mutualism resource model with three cognitive abilities and two resources. Solid, directed lines represent positive relations, dashed directed lines represent negative relations, and undirected lines represent properties. Boxes represent cognitive abilities or properties of cognitive abilities, and circles represent resources. Cognitive abilities grow depending on the availability of resources (i.e., competition resource model) and the growth of other cognitive abilities in the model (i.e., mutualistic network model).

```
# Determine the number of cognitive abilities and resources:
X <- 8
R <- 1

# Set parameter values:
mu <- matrix(runif(X*R, 0.05, 0.2), X, R)
d <- runif(X, 0.05, 0.2)
rj <- 0.1
rmax <- runif(R, 1, 5)
b <- matrix(runif(X*R, .05, .2), X, R)
K <- rnorm(X, 20, 2)
M <- matrix(runif(X*X, 0, 0.005), X, X); diag(M) <- 0

# Set initial values:
x <- runif(X, 0, 0.02)
r <- runif(R, 1, 2)
```

To make it easy to turn on or off the different modelling components discussed in section 4.1, we decided to make a separate function that takes as arguments these three model components (logistic growth, growth-dependent resource depletion and mutualistic interactions) and returns the correct model specification:

```
# Modelling function:
resourceCompetition <- function(logistic        = c(TRUE, FALSE),
                                growthDependent = c(TRUE, FALSE),
                                mutualism       = c(TRUE, FALSE)) {

  model <- function(t, state, parms){
    with(as.list(c(state,parms)),{

      x <- state[1:X]
      r <- state[(X+1):(X+R)]

      resource  <- apply(t(mu) * r, 2, sum)
      mutualism <- if (mutualism) (x * M %*% x) / K else 0
      logistic  <- if (logistic) (1 - x/K) else 1

      dx <- x * (resource * logistic - d) + mutualism

      depletion <- if (growthDependent) apply(b * c(dx), 2, sum)
                   else apply(b * x, 2, sum)
      dr <- rj * (rmax - r) - r * depletion

      return(list(c(dx, dr)))
    })
  }
}
```

Now that we specified the one function to produce all combination of model extensions, we can easily reproduce all subfigures of Figure 2. For instance, the code underneath run the trajectory of subfigure 2h.

```
model <- resourceCompetition(logistic = TRUE,
                             growthDependent = TRUE,
                             mutualism = TRUE)
run(timeplot = TRUE, table = FALSE, tmax = 400)
```

Table 3: Overview of the parameters in our proposed model and their interpretation in terms of cognitive development

Par.	Range	Name	Function
μ	Uni(.1, .3)	Growth rate	To what extent an individual benefits from the availability of a certain resource
d	Uni(.05, .2)	Decline rate	How easily an individual forgets a certain learned cognitive skill
r	0.1	Resource recovery rate	How quickly the resource replenishes
r_{max}	Uni(1, 5)	Resource capacity	To what value the resource replenishes
b	Uni(.05, .2)	Consumption rate	How much of the resources is used by the cognitive ability
M	Uni(.01, .05)	Coupling strength	Reciprocal coupling between cognitive abilities (i.e., how much growth in one ability stimulates growth in another ability)
K	Norm(20, 5)	Carrying capacity	Specifies the upper limit on the growth of a cognitive ability (i.e., the level of growth if all available resources were invested in this specific ability)