Homework 6

ANOVA

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1.1 Find Bonferroni's CV for a family of 7 two-sided t tests, for a family of 8 two-sided t-tests, for a family of 9 two-sided t tests, and for a family of 1 two-sided t tests. Show the calculation of df and the levels. For accuracy, if you use the online calculator, retain at least five digits after the decimal for upper Q tail probability as input.

```r
bonferroni_cv <- function(df, m, alpha=0.05){
  alpha_prime <- alpha/m
  alpha_prime_two_sided <- alpha_prime/2
  return(qt(alpha_prime_two_sided, df, lower.tail=FALSE))
}

df <- 40-4
cat("Critical Values:",
    "\n7 :", bonferroni_cv(df, 7),
    "\n8 :", bonferroni_cv(df, 8),
    "\n9 :", bonferroni_cv(df, 9),
    "\n10:", bonferroni_cv(df, 10))
```

```
Critical Values:
7 : 2.852558
8 : 2.904552
9 : 2.950055
10: 2.990487
```

1.2 What is Scheffe's CV for two-sided t tests?

```
scheffe_cv <- function(N, k, alpha=0.05){
  df1 <- k-1
  df2 <- N-k
  cv <- sqrt(df1*qf(alpha, df1, df2, lower.tail=FALSE))
  return(cv)
}

N <- 40
cat("Critical Value:", scheffe_cv(N, 4))
```

```
Critical Value: 2.93237
```

Scheffe's CV is given by $CV = \sqrt{(k-1)F_{k-1,N-k,\alpha}}$. My work and the resulting critical value are shown above.

1.3 Compare to the Scheffe CV to the Bonferroni CVs. For how many pre-planned comparisons should you use Bonferroni method and for how many should you use Scheffe?

Scheffe's method results in a critical value of 2.93237, which is lower than the Bonferroni method for 9 comparisons but higher than the Bonferroni CV for 8 comparisons. As a result, we would prefer the Bonferroni method for a family of 8 or fewer pre-planned comparisons in the study design, since it would provide us greater statistical power. For 9 comparisons or more, we can use the Scheffe method.

2 Consider the following contrasts in an experiments with four groups and n = 15: μ1—μ2, μ2—μ3, μ3—μ4 and μ1—μ4. Consider only the following methods in this problem: Bonferroni, Holm, HSD, LSD, Fisher-Hayter, SNK, REGWQ, Dunnett and Scheffe.

2.1. Suppose you are interested in whether these comparisons are significant in two-sided tests. Among the listed methods, which are applicable? Why are the remaining methods not applicable? Among the applicable methods, which can be eliminated without calculating CV? Explain your answers.

Dunnet's method is only suitable for a family of treatment-control comparisons. It does not appear that all our contrasts are of this type, so Dunnet's method is not applicable.

This study design cannot be balanced because the sample size is 15 and there are 4 groups. As a result, we can rule out Tukey's HSD (but can consider Kramer's HSD), Fisher's LSD, and SNK.

The options we are left with are the Fisher-Hayter Procedure, Kramer's HSD, the Holm procedure, REGWQ, Bonferroni, and Scheffe. Bonferroni will always work, and it has the fewest limitations. Scheffe's method can work, but we would expect it to be less powerful because it controls for contrasts that we are not testing. If our contrasts are planned, we do not need to control for the family of all pairwise contrasts.

The most powerful of our remaining options is likely to be REGWQ. If we are interested in simultaneous confidence intervals, we would choose HSD, Bonferroni, or Scheffe. The Holm procedure would allow us to perform one-sided tests (without splitting our alpha value). Fisher-Hayter does not appear to offer any particular advantages over the other applicable tests. After all the above considerations, we most likely just want the most powerful test that is applicable (e.g., controls correctly for FWER). We could calculate critical values for the remaining tests and choose that way.

2.2 If you would like to build simultaneous CIs for the four contrasts. Among the listed methods, which are applicable? Why are the remaining methods not applicable? Among the applicable methods, which can be eliminated without calculating CV? Explain your answer. For the ultimate candidates, calculate their critical values (FWER controlled at 0.05) and decide which one should be used.

Of the applicable methods described above, we can construct simultaneous CIs using Bonferroni, HSD, and Scheffe's method. To check which would be the most powerful in this design, I calculated CVs for Scheffe's method, Bonferroni, and Kramer's HSD.

```r
hsd_cv <- function(N, k, alpha=0.05){
  df <- N - k
  return(qtukey(p=1-alpha, nmeans=k, df=df)/sqrt(2))
}

N <- 15
k <- 4
df <- N - k

cat("Bonferroni:", bonferroni_cv(df, k), "\nScheffe:", scheffe_cv(N, k), "\nHSD:", hsd_cv(
```

```
Bonferroni: 2.980872
Scheffe: 3.280595
HSD: 3.009548
```

We likely want to choose the most powerful test that correctly controls for FWER, so we would select Bonferroni in this case.