Homework 2

Multilevel Modeling

Langdon Holmes

Question 1

```r
library(haven) # read .sav file
library(lme4)
library(stargazer) # LaTeX tables
library(performance) # ICC
df <- read_sav("../data/mlm-homework-2.sav")
```

Use the "build-up" stepwise strategy of model building to construct a model for the educational data, using language test scores (LANGPOST) as the dependent variable and (potentially, depending on how it goes) percentage minority (PERCMINO) and SES (SES) as predictors. To anchor your analysis, use the null model (random effects ANOVA) as your simplest model.

```r
# we can use the constant 1 as a predictor to specify a null model
up.null <- lme4::lmer(langpost ~ 1 + (1|schoolnr), data = df)
cat("An adjusted ICC of", round(icc(up.null)$ICC_adjusted, 2), "indicates that MLM is appr
```

```
An adjusted ICC of 0.23 indicates that MLM is appropriate.
```

```r
up.1 <- lmer(langpost ~ 1 + percmino + (1|schoolnr), data = df)
up.2 <- lmer(langpost ~ ses + percmino + (1|schoolnr), data = df)
up.3 <- lmer(langpost ~ ses + percmino + (ses|schoolnr), data = df)
up.4 <- lmer(langpost ~ ses*percmino + (ses|schoolnr), data = df)
```

All models are shown in Table 1. The estimate for the slope in the null model is 40, which is roughly equal to the mean of the language post-test scores. Not very informative

Adding percentage minority results in an improvement to model fit, and a low p-value for the coefficient, which indicates a significant effect of minority percentage on language scores. This effect is negative, which means that schools with a higher percentage of minority students perform less well on the language test.

Adding in socioeconomic status also improves model fit, and the effect is also significant. This time, the effect is positive, meaning that students with higher socioeconomic status have higher language scores. We can interpret this coefficient to mean that for every unit increase in socioeconomic status, we can expect language score to improve by about 0.3. Adding in socioeconomic status also reduces the estimated effect of minority percentage, because socioeconomic status explains some of that variation.

Then I tried estimating a random slope for socioeconomic status that varies across schools. This had a negative impact on model fit, and changed our parameter estimates very little.

Even so, I decided to add an interaction effect between socioeconomic status and minority percentage to see if this could help to explain the variation in schools. I found that this interaction effect was significant, but quite small. This model seems to recover the effect of minority percentage on language post-test scores, but the overall fit as measured by AIC and BIC is worse.

```
stargazer(up.null, up.1, up.2, up.3, up.4, header=FALSE,
          column.labels = c("Null", "+percmino", "+ses", "+ses slope", "+ses*percmino")
            )
```

I would select the third model, which includes socio-economic status of the student as a level-1 predictor and percentage minority of the school as a level-2 predictor. There is also a random intercept for school. While the interaction between socioeconomic status and percentage minority was significant, it reduced overall model fit as measured by AIC. Adding a random slope for socioeconomic status had little to no effect on model fit. Compared to model 3, the random slope resulted in a slight improvement in Log Likelihood and AIC, but a slight deterioration in BIC. Unless this slope was critical to my research question, I would prefer to use model 3, which is more parsimonious.

Question 2

Now use the “tear down” stepwise strategy. To anchor your analysis, use as the most complex model one with random intercepts, in which SES serves as a level-1 predictor with random slopes and PERCMINO as a level-2 predictor of both intercepts and slopes. Do you settle on the same model as in #1? (I’m not leading you; I really don’t know!) [15]

Table 1

	Dependent variable:				
			langpost		
	Null	+percmino	+ses	+ses slope	+ses*percmino
	(1)	(2)	(3)	(4)	(5)
ses			0.295***	0.300***	0.280***
			(0.017)	(0.018)	(0.021)
percmino		−0.147***	−0.096***	−0.087***	−0.154***
		(0.028)	(0.027)	(0.028)	(0.044)
ses:percmino					0.003**
					(0.001)
Constant	40.362***	41.467***	33.038***	32.833***	33.379***
	(0.428)	(0.441)	(0.640)	(0.728)	(0.780)
Observations	2,287	2,287	2,287	2,287	2,287
Log Likelihood	−8,126.541	−8,116.429	−7,978.204	−7,974.398	−7,978.109
Akaike Inf. Crit.	16,259.080	16,240.860	15,966.410	15,962.800	15,972.220
Bayesian Inf. Crit.	16,276.290	16,263.800	15,995.080	16,002.940	16,018.100
Note:					*p<0.1; **p<0.05; ***p<0.01

```
down.1 <- lmer(langpost ~ ses*percmino + (ses|schoolnr), data = df)
down.2 <- lmer(langpost ~ ses + percmino + (ses|schoolnr), data = df)
down.3 <- lmer(langpost ~ ses + percmino + (1|schoolnr), data = df)
```

For the “tear down” strategy (see Table 2), I started with a model that includes random intercepts, SES, PERCMINO, the interaction between them, and a random slope. This is the same model I tried at the end of the build up strategy.

I removed the interaction effect first, which resulted in an improvement in overall model fit according to any metric. This also results in a much smaller coefficient estimate for minority percentage, but I am not sure at this point which of these estimates is more accurate.

Finally, I tried removing the random slope, which again improved BIC, but resulted in worse model fit as measured by Log Likelihood and AIC.

```
stargazer(down.1, down.2, down.3, header=FALSE,
          column.labels = c("Max", "-ses*percmino", "-ses slope")
          )
```

In this case, I would again choose the model with a fixed slope because I do not think that the minor improvements in model fit would justify the inclusion of additional parameters The Bayesian information criterion supports my approach in both cases. This makes sense, since the BIC penalizes model complexity, which feels appropriate for an exploratory analysis such as this.

NOTE: Subsequent analyses show that the interaction between percentage minority and socioeconomic status is interesting and probably worth modeling. In the future, I may place more stock in the significance of parameter estimates and less stock in BIC as a single measure of model fit.

Question 3

Now fit the model with a random intercept and a random slope. Use the website to plot and interpret the significant cross-level interaction effect. Leave the “df” boxes blank, and remember that the web page does not understand scientific notation (i.e., if you see 0.193383E-02, enter 0.00193383 instead). If Rweb is not working, you can simply copy and paste the generated code directly into R. Include and interpret: [20]

```
library(effects)
model.1 <- lmer(langpost ~ ses*percmino + (ses | schoolnr), data = df)
```

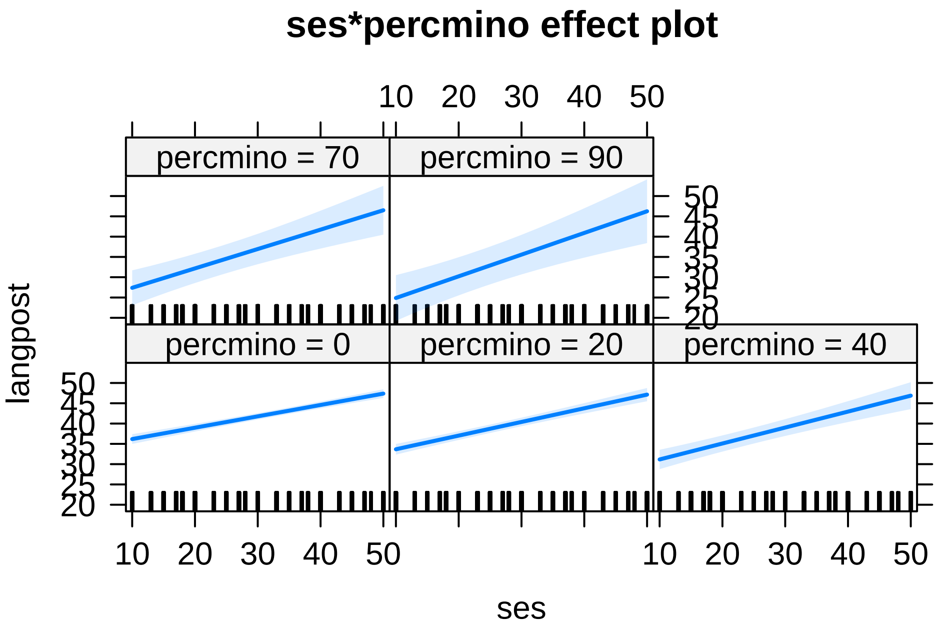
Table 2

	Dependent variable:		
		langpost	
	Max	-ses*percmino	-ses slope
	(1)	(2)	(3)
ses	0.280***	0.300***	0.295***
	(0.021)	(0.018)	(0.017)
percmino	−0.154***	−0.087***	−0.096***
	(0.044)	(0.028)	(0.027)
ses:percmino	0.003**		
	(0.001)		
Constant	33.379***	32.833***	33.038***
	(0.780)	(0.728)	(0.640)
Observations	2,287	2,287	2,287
Log Likelihood	−7,978.109	−7,974.398	−7,978.204
Akaike Inf. Crit.	15,972.220	15,962.800	15,966.410
Bayesian Inf. Crit.	16,018.100	16,002.940	15,995.080
Note:		*p<0.1; **p<0.05; ***p<0.01	

```
Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
Model failed to converge with max|grad| = 0.0370203 (tol = 0.002, component 1)
```

```
Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, : Model is nearl
 - Rescale variables?
```

```
plot(effect(c("ses*percmino"), model.1, KR=T))
```



```
summary(model.1)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: langpost ~ ses * percmino + (ses | schoolnr)
   Data: df

REML criterion at convergence: 15956.2

Scaled residuals:
    Min      1Q  Median      3Q     Max 
-3.4429 -0.6256  0.0843  0.7177  3.0229 
```

```
Random effects:
 Groups   Name        Variance  Std.Dev. Corr 
 schoolnr (Intercept) 28.977298 5.38306       
          ses          0.005574 0.07466  -0.88
 Residual             56.521111 7.51805       
Number of obs: 2287, groups:  schoolnr, 131

Fixed effects:
             Estimate Std. Error t value
(Intercept)  33.379351   0.779925  42.798
ses           0.280045   0.020576  13.611
percmino     -0.153779   0.044438  -3.461
ses:percmino  0.002824   0.001427   1.979

Correlation of Fixed Effects:
            (Intr) ses    percmn
ses         -0.857              
percmino    -0.479  0.430       
ses:percmin  0.357 -0.474 -0.777
optimizer (nloptwrap) convergence code: 0 (OK)
Model failed to converge with max|grad| = 0.0370203 (tol = 0.002, component 1)
Model is nearly unidentifiable: very large eigenvalue
 - Rescale variables?
```

```
# Variances for gammas can be found by squaring the standard error.
print("Covariance of intercept and slope (for tau_10):")
```

```
[1] "Covariance of intercept and slope (for tau_10):"
```

```
as.matrix(Matrix::bdiag(VarCorr(model.1)))
```

```
            (Intercept)          ses
(Intercept)  28.9772985 -0.354289229
ses          -0.3542892  0.005574296
```

```
print("Variance Covariance Matrix:")
```

```
[1] "Variance Covariance Matrix:"
```

```
vcov(model.1)
```

```
4 x 4 Matrix of class "dpoMatrix"
               (Intercept)           ses      percmino  ses:percmino
(Intercept)   0.6082826014 -1.375507e-02 -1.660930e-02  3.979571e-04
ses          -0.0137550698  4.233518e-04  3.929310e-04 -1.391987e-05
percmino     -0.0166092977  3.929310e-04  1.974698e-03 -4.926732e-05
ses:percmino  0.0003979571 -1.391987e-05 -4.926732e-05  2.037613e-06
```

a

Text output (interpret only the “simple intercepts and simple slopes” and “regions of significance” sections).

```
CASE 3 TWO-WAY INTERACTION SIMPLE SLOPES OUTPUT

Your Input
========================================================
  w1(1)       = 0
  w1(2)       = 45
  w1(3)       = 90
  x1(1)       = 0
  x1(2)       = 27
  x1(3)       = 45
  Intercept   = 33.379351
  x1 Slope    = 0.280045
  w1 Slope    = -0.153779
  w1x1 Slope  = 0.002824
  alpha       = 0.05

Asymptotic (Co)variances
========================================================
  var(g00) 0.6082826
  var(g10) 0.00042335
  var(g01) 0.0019747
  var(g11) 0.00000204
  cov(g00,g01) -0.0166093
  cov(g10,g11) -0.00001392
  cov(g00,g10) -0.01375507
  cov(g01,g11) -0.00004927
```

```
Region of Significance on w (level-2 predictor)
========================================================
  w1 at lower bound of region = -11538.7709
  w1 at upper bound of region = -45.6588
  (simple slopes are significant *outside* this region.)

Simple Intercepts and Slopes at Conditional Values of w
========================================================
  At w1(1)...
    simple intercept = 33.3794(0.7799), z=42.7982, p=0
    simple slope     = 0.28(0.0206), z=13.6106, p=0
  At w1(2)...
    simple intercept = 26.4593(1.7641), z=14.9984, p=0
    simple slope     = 0.4071(0.0574), z=7.0907, p=0
  At w1(3)...
    simple intercept = 19.5392(3.6897), z=5.2957, p=0
    simple slope     = 0.5342(0.1201), z=4.4482, p=0

Simple Intercepts and Slopes at Region Boundaries for w
========================================================
  Lower Bound...
    simple intercept = 1807.8(513.1292), z=3.5231, p=0.0004
    simple slope     = -32.3054(16.4808), z=-1.9602, p=0.05
  Upper Bound...
    simple intercept = 40.4007(2.4983), z=16.171, p=0
    simple slope     = 0.1511(0.0771), z=1.9602, p=0.05

Region of Significance on x (level-1 predictor)
========================================================
  x1 at lower bound of region = 33.1067
  x1 at upper bound of region = 3327.9384
  (simple slopes are significant *outside* this region.)

Simple Intercepts and Slopes at Conditional Values of x
========================================================
  At x1(1)...
    simple intercept = 33.3794(0.7799), z=42.7982, p=0
    simple slope     = -0.1538(0.0444), z=-3.4606, p=0.0005
  At x1(2)...
    simple intercept = 40.9406(0.4173), z=98.1102, p=0
    simple slope     = -0.0775(0.0283), z=-2.7417, p=0.0061
  At x1(3)...
```

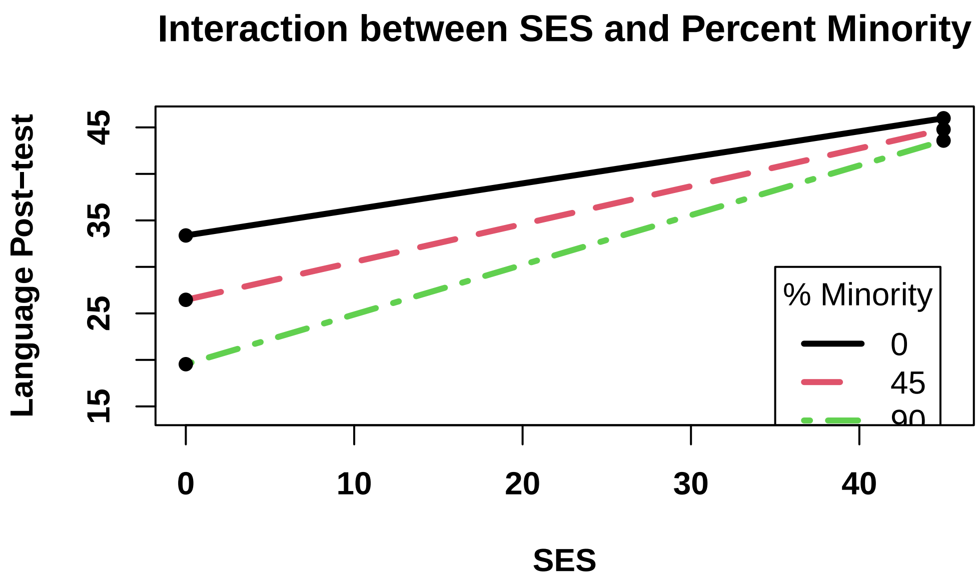
```
    simple intercept = 45.9814(0.4771), z=96.3791, p=0
    simple slope     = -0.0267(0.0408), z=-0.654, p=0.5132

Simple Intercepts and Slopes at Region Boundaries for x
========================================================
  Lower Bound...
    simple intercept = 42.6507(0.4019), z=106.121, p=0
    simple slope     = -0.0603(0.0308), z=-1.9602, p=0.05
  Upper Bound...
    simple intercept = 965.3519(67.8067), z=14.2368, p=0
    simple slope     = 9.2443(4.716), z=1.9602, p=0.05
```

The region of significance shows for which values of a parameter the slope is significant. Our level-2 predictor, percmino, is significant for all possible values from [0,100] because the upper bound of the region of significance is below 0. The level-1 predictor, socioeconomic status, overlaps with its region of significance. The lower bound of the region of significance for SES is 33.11, which means that the slope estimate is not significant for SES values above this lower bound. In practical terms, this means that there is not a significant effect of SES on language post-test scores for SES values above 33.11. This analysis is confirmed by examining the Simple Intercepts and Slopes at Conditional Values of SES. While the intercept estimate is significant for all conditional values of SES, the slope estimate is not significant at the maximum value of SES, 45.

b

A plot of the simple regression of LANGPOST on SES at three conditional values of PERCMINO: the minimum observed (0%), middle (45%), and maximum observed (90%).

```
xx <- c(0,45)   #  <-- change to alter plot dims
yy <- c(14.2508,45.9814)   #  <-- change to alter plot dims
leg <- c(35,30)   #  <-- change to alter legend location
x <- c(0,45)   #  <-- x-coords for lines
y1 <- c(33.3794,45.9814)
y2 <- c(26.4593,44.7799)
y3 <- c(19.5392,43.5785)
plot(xx,yy,type='n',font=2,font.lab=2,xlab='SES',ylab='Language Post-test',main='Interacti
lines(x,y1,lwd=3,lty=1,col=1)
lines(x,y2,lwd=3,lty=5,col=2)
lines(x,y3,lwd=3,lty=6,col=3)
points(x,y1,col=1,pch=16)
points(x,y2,col=1,pch=16)
points(x,y3,col=1,pch=16)
legend(leg[1],leg[2],legend=c('0','45','90'),lwd=c(3,3,3),lty=c(1,5,6),col=c(1,2,3), title
```



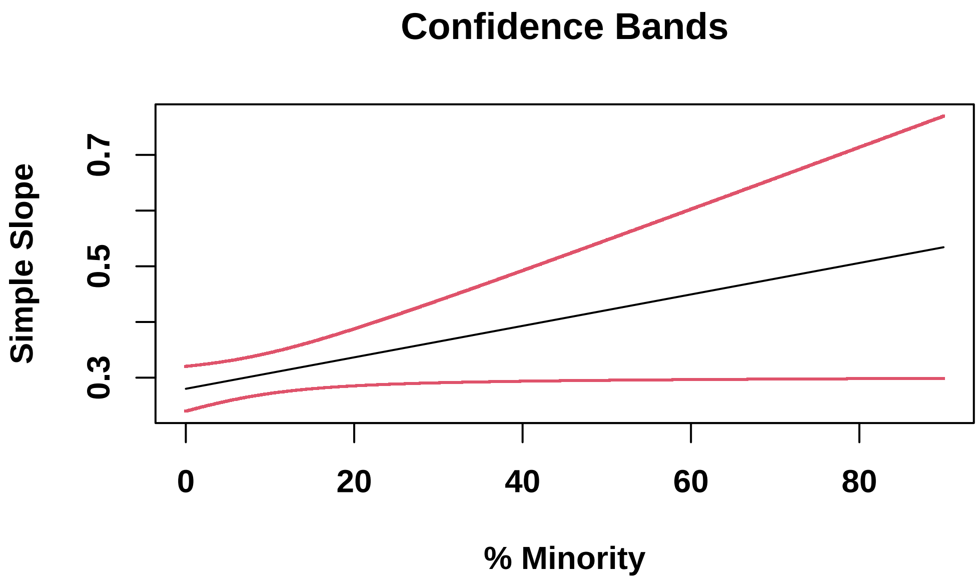
This plot illustrates how the effect of SES on post-test scores is moderated by the minority composition of the school. The socioeconomic status of students attending schools with a lower percentage of minority students has a less pronounced effect on language post-test scores. Students attending schools with a higher percentage of minority students perform less well across the board, but this effect is less pronounced for high socioeconomic status students at these schools. In practical terms, the (negative) effect of attending a high minority percentage school is more pronounced for low SES students.

c

A plot of the confidence bands around the simple slope of LANGPOST regressed on SES. The x-axis of this plot should extend from the minimum to maximum observed values of PERCMINO.

```
z1=0  #supply lower bound for w1 here
z2=90   #supply upper bound for w1 here
z <- seq(z1,z2,length=1000)
fz <- c(z,z)
```

```
y1 <- (0.280045+0.002824*z)+(1.9602*sqrt(0.0004233518+(2*z*-0.00001391987)+((z^2)*0.000002
y2 <- (0.280045+0.002824*z)-(1.9602*sqrt(0.0004233518+(2*z*-0.00001391987)+((z^2)*0.000002
fy <- c(y1,y2)
fline <- (0.280045+0.002824*z)
plot(fz,fy,type='p',pch='.',font=2,font.lab=2,col=2,xlab='% Minority',ylab='Simple Slope',
lines(z,fline)
f0 <- array(0,c(1000))
lines(z,f0,col=8)
abline(v=-11538.7709,col=4,lty=2)
abline(v=-45.6588,col=4,lty=2)
```



The confidence band indicates a 95% confidence interval for the estimate of the effect of SES on post-test scores. At lower values of percentage minority, the slope is increasing. At the highest values of percentage minority, the confidence bands extend to a zero or near-zero slope, indicating that we should be less confident about the slope estimate for schools with a high minority percentage. The confidence band is roughly symmetrical, so it is equally likely that the slope is even steeper for schools with a high minority percentage. In practical terms, we can be more confident about the effect of SES on language post-test scores for schools with a lower percentage of minority students.

Question 4

Einstein allegedly claimed that you never really know a subject until you can explain it to your grandmother. Please pretend I am your grandmother, and that I just asked you what group mean centering and grand mean centering are. Explain these concepts to the best of your ability. Assume that your "grandmother" has no quantitative training, speaks fluent English, and is genuinely curious. [10]

Hi grandma! I am taking a boat load of statistics courses, and I am really excited to share something I learned with you. In statistics, it is sometimes helpful to carefully alter our data before we do statistics on it. Of course, we have to be careful not to change our data in a way that would make our results nonsense, so we are sure to go about it carefully. If we carefully change our data, we can sometimes make our statistical methods work better, or maybe the data just makes more sense if we change it a little bit. My new favorite trick is centering, which is when we subtract a certain value from all the measurements in our dataset. Since you were a teacher, you know that student learning outcomes are better at some schools than others. So what if we are interested in doing statistics that focuses on individual students' performance in different schools? We couldn't compare the performance of a student at La Canada high school to the performance of a student at Pasadena high school directly, because Pasadena high school usually has better performing students. It turns out, we can use group mean centering to look at individual student performance. For this, we just subtract the average performance of Pasadena school students from each students' performance score, and we do the same for La Canada students. Then we get a performance score that shows how a student's performance relates to other students in their same school. But that's not all! Imagine we instead wanted to measure how much a student's performance is affected by the length of their commute to school. Our statistics methods will tell us the expected performance of a student with a zero-length commute, but that's kind of silly because no one lives at a public high school. If we subtract the average commute length from our data, then our baseline estimate is for the average commute time, and we can see how much increases in commute time affect student performance a little more clearly. That one is called grand mean centering. I know it may seem weird to change your data, but it can actually be really useful!