Homework 3

Multilevel Modeling

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1

In a univariate random intercept, random slope model, test the effect of SEX on student-rated popularity (POPULAR). Now test the effect of SEX on teacher-rated popularity (TEACHPOP). Interpret each set of results in isolation. [12]

```
studentpopularity <- lme(POPULAR~SEX,
                    random=~SEX|SCHOOL,
                    data=popular,
                    method='ML')

teacherpopularity <- lme(TEACHPOP~SEX,
                    random=~SEX|SCHOOL,
                    data=popular,
                    method='ML')

stargazer(studentpopularity, teacherpopularity, header=FALSE,
          column.labels = c("Student Popularity", "Teacher Popularity")
          )
```

```
getVarCov(studentpopularity)
```

Random effects variance covariance matrix (Intercept) SEX (Intercept) 0.93043 -0.13916 SEX -0.13916 0.26892 Standard Deviations: 0.96459 0.51858

```
getVarCov(teacherpopularity)
```

Table 1

	Dependent variable:	
	POPULAR Student Popularity	TEACHPOP Teacher Popularity
	(1)	(2)
SEX	0.843***	0.231***
	(0.059)	(0.030)
Constant	4.890***	4.370***
	(0.099)	(0.088)
Observations	2,000	2,000
Log Likelihood	−2,164.809	−1,917.054
Akaike Inf. Crit.	4,341.617	3,846.108
Bayesian Inf. Crit.	4,375.223	3,879.714
Note:		*p<0.1; **p<0.05; ***p<0.01

```
Random effects variance covariance matrix
            (Intercept)       SEX
(Intercept)    0.740310 -0.013167
SEX           -0.013167  0.023267
  Standard Deviations: 0.86041 0.15254
```

I fit a model with a random intercept and random slope to test whether the student's sex predicts their popularity. There is a positive, significant slope, which means that women (coded as 1) are more popular than men (coded as 0) in this dataset.

I fit a model with a random intercept and a random slope to test whether the student's sex predicts their teacher's popularity. There is a positive, significant slope, which means that female students (coded as 1) are more likely to view their teachers as popular.

2

In a joint multivariate model that allows all intercepts and slopes to covary, but does not allow the level-1 residuals to covary across variables, assess the same effects you did in (1). Report and interpret the results. What is better about the analysis in (2) vs. (1)? [12]

```r
allpopularity <- lme(popst~0+conss+sexs+const+sext,
                     random=list(school=~0+conss+sexs+const+sext),
                     weights=varIdent(form=~1|dv),
                     data=popularmv,
                     method='ML',
                     control=lmeControl(maxIter=300,msMaxIter=300,niterEM=100,msMaxEval=500
summary(allpopularity)
```

```
Linear mixed-effects model fit by maximum likelihood
  Data: popularmv
       AIC      BIC    logLik
  7947.589 8048.294 -3957.794

Random effects:
 Formula: ~0 + conss + sexs + const + sext | school
 Structure: General positive-definite, Log-Cholesky parametrization
         StdDev    Corr
conss    0.9653866 conss  sexs   const
sexs     0.5131059 -0.274
const    0.8606287  0.949 -0.066
sext     0.1675852 -0.351  0.985 -0.113
Residual 0.6265977

Variance function:
 Structure: Different standard deviations per stratum
 Formula: ~1 | dv
 Parameter estimates:
     pops      popt
1.0000000 0.9072412
Fixed effects:  popst ~ 0 + conss + sexs + const + sext
         Value  Std.Error   DF  t-value p-value
conss 4.892015 0.09863185 3897 49.59874       0
sexs  0.838267 0.05880989 3897 14.25384       0
const 4.367215 0.08795587 3897 49.65234       0
sext  0.233945 0.03085772 3897  7.58142       0
 Correlation:
      conss  sexs   const
sexs  -0.303
const  0.910 -0.057
sext  -0.188  0.473 -0.180
```

```
Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max 
-3.04648211 -0.64753693 -0.08155832  0.58439975  3.52777096 

Number of Observations: 4000
Number of Groups: 100 
```

```r
getVarCov(allpopularity)
```

```
Random effects variance covariance matrix
          conss      sexs     const      sext
conss  0.931970 -0.135800  0.788120 -0.056783
sexs  -0.135800  0.263280 -0.028977  0.084707
const  0.788120 -0.028977  0.740680 -0.016351
sext  -0.056783  0.084707 -0.016351  0.028085
  Standard Deviations: 0.96539 0.51311 0.86063 0.16759 
```

To test whether sex predicts student popularity and teacher popularity, I developed a multivariate, multilevel model with school as a clustering variable. Sex is a significant predictor of both student ($T = 14.25, p < 0.05$) and teacher popularity ($T = 7.58, p < 0.05$). The model indicates that student popularity is 0.08 higher for women, and teacher popularity is 0.23 higher for women. Both student and teacher popularity is higher for women.

While the results are effectively the same, this approach is better because we now have a single model, which is more parsimonious than fitting two separate models, and allows us to estimate the covariance between predictors for each response variable. The covariance matrix demonstrates that teacher and student popularity scores covary (0.79), but the slope estimates do not covary (0.08). This indicates that teacher and student popularity are likely strongly correlated, but the effect of sex on these two response variables is substantially different.

3

Now test the effect of the level-2 predictor teacher experience (TEXP) on both student-rated and teacher-rated popularity (without SEX in the model). Report and interpret the results. [12]

```r
model.texp <- lme(popst~0+conss+texps+const+texpt,
                    random=list(school=~0+conss+const),
                    weights=varIdent(form=~1|dv),
```

```r
                     data=popularmv,
                     method='ML',
                     control=lmeControl(maxIter=300,msMaxIter=300,niterEM=100,msMaxEval=500
summary(model.texp)
```

```
Linear mixed-effects model fit by maximum likelihood
  Data: popularmv
       AIC      BIC   logLik
  8755.499 8812.146 -4368.75

Random effects:
 Formula: ~0 + conss + const | school
 Structure: General positive-definite, Log-Cholesky parametrization
         StdDev    Corr
conss    0.7036685 conss
const    0.7154833 0.975
Residual 0.7989844

Variance function:
 Structure: Different standard deviations per stratum
 Formula: ~1 | dv
 Parameter estimates:
    pops     popt
1.000000 0.733103
Fixed effects:  popst ~ 0 + conss + texps + const + texpt
         Value  Std.Error   DF   t-value p-value
conss 3.972376 0.17390867 3897 22.841736       0
texps 0.093439 0.01105138 3897  8.454972       0
const 3.448803 0.17422796 3897 19.794773       0
texpt 0.072368 0.01107078 3897  6.536807       0
 Correlation:
      conss  texps  const
texps -0.909
const  0.929 -0.844
texpt -0.844  0.929 -0.909

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-3.03937640 -0.66257772 -0.05985166  0.66542747  3.03319082

Number of Observations: 4000
```

```
Number of Groups: 100
```

```
getVarCov(model.texp)
```

```
Random effects variance covariance matrix
        conss   const
conss 0.49515 0.49087
const 0.49087 0.51192
  Standard Deviations: 0.70367 0.71548
```

To test whether teacher experience predicts student popularity and teacher popularity, I developed a multivariate, multilevel model with school as a clustering variable. Teacher experience is a significant predictor of both student ($T = 8.45, p < 0.05$) and teacher popularity ($T = 6.54, p < 0.05$). The model indicates that student popularity increases by 0.09 for every unit of teacher experience, and teacher popularity increases by 0.07 for every unit of teacher experience. Both students and teachers are rated as more popular when the teacher has more experience.

4

You have now learned at least two ways to formally test the hypothesis that the effects in (3) are equal (the deviance test and a multiparameter test). Use both of these methods to test the hypothesis of equal slopes. Report and interpret the results. Are the p-values the same? Close? Report them to as many decimal places as possible. [12]

```
library(car)
```

```
Loading required package: carData
```

```
linearHypothesis(model.texp,c('texps=texpt'),verbose=TRUE)
```

```
Hypothesis matrix:
            conss texps const texpt
texps=texpt     0     1     0    -1
```

```
Right-hand-side vector:
*rhs*
    0

Estimated linear function (hypothesis.matrix %*% coef - rhs)
texps=texpt
 0.02107154


Linear hypothesis test

Hypothesis:
texps - texpt = 0

Model 1: restricted model
Model 2: popst ~ 0 + conss + texps + const + texpt

  Df  Chisq Pr(>Chisq)
1
2  1 25.626  4.144e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The χ^2 test is reported above, which results in an extremely small p-value ($\chi^2 = 25.626, p = 0.0000004144$). I couldn't figure out how to run a multiparameter test on a multivariate model in R, so I performed the contrast in SPSS (sadly…) using the contrast matrix [0, 1, 0, -1]. Results are reported below:

Contrast Estimates

Contrast	Estimate	Std. Error	df	Test Value	t	Sig.	95% Lower	95% Upper
L1	.021042	.004143	102.200	0	5.079	.000	.012824	.029261

The multiparameter test was significant, $t = 5.079, p = 0.000002$.

Either test would provide sufficient evidence to reject the null hypothesis that the slopes are equal. Both p-values are extremely small, but the deviance test yields a smaller p-value.

5 Power Analysis

Using MLPowSim, conduct an a priori power analysis for the slope point estimates in the following multilevel model, with conjectured parameter values as indicated:

Limit your attention to a potential data set with 40 clusters of size 10 each, and maximum likelihood (ML) estimation. Assume the predictors are each standard normal both at level-1 and level-2. What is the approximate power for detecting each slope at .05 ? Speculate about why these power estimates are so different even though both population values are .2. [12]

```
output
```

```
   N  n zLb0 zpb0 zUb0 sLb0 spb0 sUb0  zLb1  zpb1  zUb1  sLb1  spb1  sUb1  zLb2
1 40 10    0    0    0 0.05 0.05 0.05 0.313 0.348 0.383 0.341 0.345 0.349 0.983
  zpb2  zUb2  sLb2  spb2  sUb2
1 0.99 0.997 0.992 0.993 0.993
```

In a dataset with 40 clusters of 10 observations, the power to detect the random slope β_{1j} is 0.349, and the power to detect the fixed slope β_{2j} is a comfortable 0.993. The statistical power of the two slopes is dramatically different because we need to estimate additional parameters to calculate a random effect. This is true even though the effect size is the same for the two variables. We should probably look to have more than 10 observations for each cluster in a mixed effects model.

Extra Credit

Compose a poem (any type: haiku, limerick, sonnet, ballad, free verse… whatever you like) describing how multilevel modeling makes you feel. [+3]

I wrote a haiku:
A model's a thought
Novelty unconsidered
Layers provide nuance

But I couldn't resist...

Multi-level modeling and Multi-level marketing have the same abbreviation, MLM. Write a poem that plays on this polysemy.

MLM, two different paths to explore,
One leads to sales, the other to much more,
Multi-level modeling, statistics in tow,
Building models that can accurately show.

Patterns within data, from individuals to groups,
Accounting for differences, with techniques that snoop,
Hierarchies and levels, all in one place,
To explain phenomena, with a clearer trace.

Multi-level marketing, a different domain,
Involves selling products, for a monetary gain,
Recruiting new members, to join the chain,
Building a team, to share in the gain.

Both MLMs, with different aims in sight,
One aims for understanding, with insight so bright,
The other aims for profit, with sales so tight,
Two different paths, both with their own light.