1. balení = 200 kusů, nekompletní / počet = $\frac{1}{3}$, objednávka = 90

$\mathbb{P}(\text{kompletní} > 70) = ?$

$K \sim Bi(n, p)$, $n = 90$, $p = \frac{2}{3}$

$\mathbb{P}(K > 70) = \cancel{\ldots}\ 1 - \sum_{i=1}^{70} \mathbb{P}(K_i) \ldots$

pomocí CLV

$K_i \sim Alt(p)$, $p = \frac{2}{3}$, $n = 90$ $\mu = p = \frac{2}{3}$ $\sigma^2 = p(1-p) = \frac{2}{9}$

$\mathbb{P}(\sum_{i=1}^{90} K_i > 70) \sim \mathbb{P}(N(n \cdot \mu, n \cdot \sigma^2) > 70 = 1 - F(71)$

$= 1 - \Phi\left(\frac{71 - 90 \cdot \frac{2}{3}}{\sqrt{90 \cdot \frac{2}{9}}}\right) = 1 - \Phi\left(\frac{11}{2\sqrt{5}}\right) = 1 - \Phi(2{,}4597)$

$\in 1 - (0{,}99286; 0{,}99305) = (0{,}00695; 0{,}00714)$

$\sim 0{,}7\%$

2.
$$\mathbb{F}(x) = \begin{cases} \frac{1}{a-x} + 1, & x \geq 2 \\ 0, & x < 2 \end{cases}$$

(i) $a = ?\ ; \mathbb{F}(x) \in \frac{1}{a-x} + 1,\ x \in (2; \infty)$

$$\mathbb{F}(2) = 0 \iff \frac{1}{a-2} + 1 = 0 \iff 1 = -1(a-2)$$
$$\iff -1 = a - 2 \iff a = 1$$
$$\mathbb{F}(\infty) = 1 \iff \frac{1}{a-\infty} + 1 = 1 \iff \frac{1}{a-\infty} = 0$$
$$\iff \lim_{x \to -\infty} \frac{1}{x} = 0 \iff a < \infty$$
$$a \in 1 \cap (-\infty; \text{"}\infty - 1\text{"}) \iff a = 1$$
$$\mathbb{F}(x) = \begin{cases} \frac{1}{1-x} + 1, & x \geq 2 \\ 0, & x < 2 \end{cases}$$

(ii) $f(x) = \mathbb{F}(x)' = \dfrac{d\left(\frac{1}{1-x} + 1\right)}{dx} = \dfrac{+1}{(1-x)^2}$

(iii) $$\mathbb{E}X = \int_2^\infty x \cdot \frac{1}{(1-x)^2}\, dx = \left| \begin{matrix} v = x & v' = 1 \\ u' = \frac{1}{(1-x)^2} & u = \frac{1}{1-x} \end{matrix} \right| = \left[\frac{x}{1-x}\right]_2^\infty - \int_2^\infty \frac{1}{1-x}\, dx$$
$$= \left[\frac{x}{1-x}\right]_2^\infty \left| \begin{matrix} a = 1-x & \infty \to -\infty \\ \frac{da}{dx} = -1 & 2 \to -1 \end{matrix} \right| - \left[-\ln|a|\right]_{-1}^{-\infty}$$
$$= \left[\frac{x}{1-x} + \ln|1-x|\right]_2^\infty = \lim_{x \to \infty} \frac{x}{1-x} + \lim_{x \to \infty} \ln|1-x| - \frac{2}{1-2} - \ln|1|$$
$$= \lim_{x \to \infty} \frac{x(1)}{x(-1+\frac{1}{x})} + \lim_{x \to \infty} \ln|1-x| + 2 + 0 = \lim_{x \to \infty} \overbrace{\frac{1}{-1+\frac{1}{x}}}^{\to -1} + \overbrace{\ln|1-x|}^{\to \infty} + \overbrace{2}^{\to 2}$$
$$= \infty$$

$\mathrm{var}X =$ v poznámkách je rozptyl definován pokud střední hodnota leží $-\infty < X < \infty$

(iv) $\mathbb{P}(X>5) = 1 - \mathbb{F}(5) = 1 - \left(\frac{1}{1-5} + 1\right) = \cancel{1} + \frac{1}{4} - \cancel{1} = \frac{1}{4}$

$\mathbb{P}(|X-2| \leq 1) = \left| \begin{array}{l} x-2<0: \ -x+2 \leq 1 \iff x \geq +1 \\ x-2>0: \ x-2 \leq 1 \iff x \leq 3 \end{array} \right|$

$= \mathbb{P}(+1 \leq X \leq 3) = \mathbb{F}(3) - \mathbb{F}(1) = \mathbb{F}(3)$

$= \frac{1}{1-3} + 1 = \frac{-1}{2} + 1 = \frac{1}{2}$

$u_{0,25} = \left| \mathbb{F}(u_{0,25}) = \frac{1}{4} \right| \iff \frac{1}{1-u_{0,25}} + 1 = \frac{1}{4} \iff 1 = -\frac{3}{4}(1-u_{0,25})$

$\iff \frac{-4}{3} = 1 - u_{0,25} \iff u_{0,25} = \frac{7}{3}$

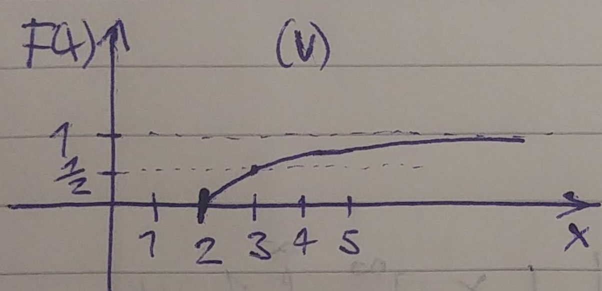
$u_{0,5} = ? \iff \mathbb{F}(u_{0,5}) = \frac{1}{2} \iff \frac{1}{1-u_{0,5}} + 1 = \frac{1}{2} \iff 1 = -\frac{1}{2}(1-u_{0,5})$

$\iff -2 = 1 - u_{0,5} \iff u_{0,5} = 3$

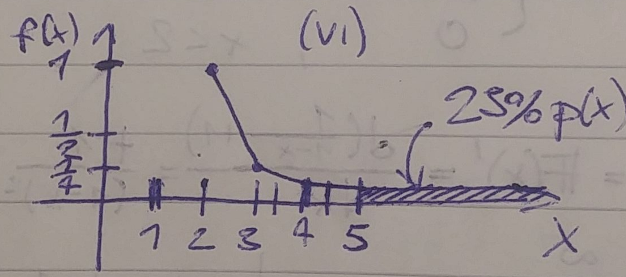
$u_{0,75} = ?, \ \mathbb{F}(u_{0,75}) = \frac{3}{4} \iff \frac{1}{1-u_{0,75}} + 1 = \frac{3}{4} \iff 1 = -\frac{1}{4}(1-u_{0,75})$

$\iff -4 = 1 - u_{0,75} \iff u_{0,75} = 5$

(v)



(vi)



3. $\mathbb{P}(H|S) = \frac{1}{3}$ $\quad \mathbb{P}(H|M) = \frac{2}{3}$ $\quad \mathbb{P}(M) = \frac{2}{3}$

(M) $\mathbb{P}(I) = \mathbb{P}(I|S)\cdot\mathbb{P}(S) + \mathbb{P}(I|M)\cdot\mathbb{P}(M)$

$= \frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3} = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$

(mn) $\mathbb{P}(M|H) = \frac{\mathbb{P}(H|M)\cdot\mathbb{P}(M)}{\mathbb{P}(H)} = \frac{\mathbb{P}(H|M)\cdot\mathbb{P}(M)}{\mathbb{P}(H|S)+\mathbb{P}(H|M)}$

$= \frac{\frac{2}{3}\cdot\frac{2}{3}}{\frac{1}{3}+\frac{2}{3}} = \frac{4}{9}$

4. p_Y

y_i	0	1	2	3
$p_Y(y_i)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

p_Z:

z	0	1	2
$p_Z(z)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(a)

Y, Z nezávislé $\iff p(y_i, z_j) = p_Y(y_i)\cdot p_Z(z_j)\ \forall y_i, z_j \in Y, Z$

$p(0,0) = p_Y(0)\cdot p_Z(0) \iff 0 = \frac{1}{8}\cdot\frac{1}{4} \iff 0 = \frac{1}{32}$

Y, Z nejsou nezávislé

(b) $EY = \sum_{\forall y_i \in Y} y_i \cdot p_Y(y_i) = 0\cdot\frac{1}{8} + 1\cdot\frac{3}{8} + 2\cdot\frac{3}{8} + 3\cdot\frac{1}{8} = \frac{12}{8} = \frac{3}{2}$

$\operatorname{var} Y = \sum_{\forall y_i \in Y} y_i^2 \cdot p_Y(y_i) - EY = 0^2\cdot\frac{1}{8} + 1^2\cdot\frac{3}{8} + 2^2\cdot\frac{3}{8} + 3^2\cdot\frac{1}{8} - \frac{3}{2} = \frac{24-12}{8} = \frac{3}{2}$

$EZ = \sum_{\forall z_i \in Z} z_i \cdot p_Z(z_i) = 0\cdot\frac{1}{4} + 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} = \frac{4}{4} = 1$

$\operatorname{var} Z = \sum_{\forall z_i \in Z} z_i^2 \cdot p_Z(z_i) - EZ = 0^2\cdot\frac{1}{4} + 1^2\cdot\frac{1}{2} + 2^2\cdot\frac{1}{4} - 1 = \frac{2+4-4}{4} = \frac{1}{2}$

$\operatorname{cov}(Y,Z) = \sum_{\forall y_i z_j \in (Y,Z)} y_i \cdot z_j \cdot p(y_i, z_j) - EYEZ = \cancel{0\cdot 0\cdot 0} + \cancel{0\cdot 1\cdot 0} + \cancel{0\cdot 2\cdot \frac{1}{8}}$
$+ \cancel{1\cdot 0\cdot 0} + 1\cdot 1\cdot\frac{1}{4} + 1\cdot 2\cdot\frac{1}{8}$
$+ \cancel{2\cdot 0\cdot\frac{1}{8}} + 2\cdot 1\cdot\frac{1}{4} + \cancel{2\cdot 2\cdot 0}$
$+ \cancel{3\cdot 0\cdot\frac{1}{8}} + \cancel{3\cdot 1\cdot 0} + \cancel{3\cdot 2\cdot 0} - \frac{3}{2}$

$= \frac{1}{4} + \frac{2}{8} + \frac{2}{4} - \frac{3}{2} = \frac{1+1+2-6}{4} = -\frac{2}{4} = -\frac{1}{2}$

$E(Y,Z) = (EY, EZ) = (\frac{3}{2}, 1)$; $\operatorname{var}(Y,Z) = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

(bb) $\mathbb{P}(Y \geq 1 \wedge Z \leq 0) = \sum_{i=1}^{3}\sum_{j=0}^{0} p(y_i, z_j) = \frac{1}{4}$

(bc) $\mathbb{P}(|Y-Z| = 1)$ $\mid Y-Z = 1 \iff Y = Z+1 \vee Y-Z = -1 \iff Y = Z-1 \mid$

$= 0 + 0 + \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

(c)

$\rho(Y,Z) = \frac{\operatorname{cov}(Y,Z)}{\sqrt{\operatorname{var} Y}\cdot\sqrt{\operatorname{var} Z}} = \frac{-\frac{1}{2}}{\sqrt{\frac{3}{2}}\cdot\sqrt{\frac{1}{2}}} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

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