1. balení = 200 kusů ; nekompletní/počet = $1/3$; objednávka = 90

$\mathbb{P}(\text{kompletní} > 70) = ?$

$K \sim Bi(n, p)$ ✓, $n = 90$, $p = 2/3$ ✓

$\mathbb{P}(K > 70) =$ ~~[illegible]~~ $1 - \sum_{i=1}^{70} \mathbb{P}(K_i) \ldots$ ✓

pomocí CLV

$\sum_{i=71}^{90} P(K=i)$ by bylo snazší vysčítat

$K_i \sim Alt(p)$ ✓ $p = 2/3$ ✓, $n = 90$ $\mu = p = \frac{2}{3}$ $\sigma^2 = p(1-p) = \frac{2}{9}$ ✓

$\mathbb{P}(\sum_{i=1}^{90} K_i > 70) \sim \mathbb{P}(N(n \cdot \mu, n \cdot \sigma^2) > 70) = 1 - F(71)$?

✓ $= 1 - \Phi\left(\frac{71 - 90 \cdot \frac{2}{3}}{\sqrt{90 \cdot \frac{2}{9}}}\right) = 1 - \Phi\left(\frac{11}{2\sqrt{5}}\right) = 1 - \Phi(2{,}4597)$

$P(K \leq 70)$

$\in 1 - (0{,}99286 ; 0{,}99305) \in (0{,}00695 ; 0{,}00714)$

$\approx \Phi\left(\frac{70 - n\mu}{\sqrt{n\sigma^2}}\right)$ $\sim 0{,}7\%$ ✓

2.

$$\mathbb{F}(x) = \begin{cases} \frac{1}{a-x} + 1, & x \geq 2 \\ 0, & x < 2 \end{cases}$$

(i) $a = ?$; $\mathbb{F}(x) = \frac{1}{a-x} + 1$, $x \in (2; \infty)$

proč?

$$\mathbb{F}(2) = 0 \iff \frac{1}{a-2} + 1 = 0 \iff 1 = -1(a-2)$$

$$\iff -1 = a - 2 \iff a = 1 \checkmark$$

$$„\mathbb{F}(\infty) = 1" \iff \frac{1}{a-\infty} + 1" = 1 \iff „\frac{1}{a-\infty} = 0"$$

$$\iff \lim_{x \to -\infty} „\frac{1}{x}" = 0 \iff a < \infty$$

$$a \in \{1\} \cap (-\infty; \infty) \iff a = 1$$

$$\mathbb{F}(x) = \begin{cases} \frac{1}{1-x} + 1, & x \geq 2 \\ 0, & x < 2 \end{cases} \checkmark$$

(ii) $f(x) = \mathbb{F}(x)' = \frac{d\left(\frac{1}{1-x} + 1\right)}{dx} = \begin{cases} +\frac{1}{(1-x)^2}, & x \geq 2 \\ 0, & \text{jinde} \end{cases}$!

(iii) $\mathbb{E}X = \int_2^\infty x \cdot \frac{1}{(1-x)^2}\,dx = \left| \begin{matrix} v = x & v' = 1 \\ u' = \frac{1}{(1-x)^2} & u = \frac{1}{1-x} \end{matrix} \right| = \left[\left[\frac{x}{1-x}\right]_2^\infty - \int_2^\infty \frac{1}{1-x}\,dx \right]_2^{+\infty}$

$= \left[\frac{x}{1-x}\right]_2^\infty \left| \begin{matrix} a = 1-x & \infty \to -\infty \\ \frac{da}{dx} = -1 & 2 \to -1 \end{matrix} \right| - \left[-\ln|a|\right]_{-1}^{-\infty}$

PP neměli jste pro neurč. int.!

a ani substituci

$= \left[\frac{x}{1-x} + \ln|1-x|\right]_2^\infty = \lim_{x\to\infty} \left(\frac{x}{1-x} + \cancel{\lim_{x\to\infty}} \ln|1-x|\right) - \frac{2}{1-2} - \ln|1|$

obecně neplatí!!

$= \lim_{x\to\infty} \frac{x(1)}{x\left(-1+\frac{1}{x}\right)} + \lim_{x\to\infty} \ln|1-x| + 2 + 0 = \lim_{x\to\infty} \frac{1}{-1+\frac{1}{x}} + \ln|1-x| + 2$

$= \infty \checkmark$

$\mathrm{var}X =$ v poznámkách je rozptyl definován pokud střední hodnota leží $-\infty < X < \infty$

takže v našem případě není :-)

(iv) $\mathbb{P}(X>5) = 1-\mathbb{F}(5) = 1-\left(\frac{1}{1-5}+1\right) = \cancel{1}+\frac{1}{4}-\cancel{1} = \frac{1}{4}$

$\mathbb{P}(|X-2|\le 1) = \left| \begin{array}{l} x-2<0: \ -x+2\le 1 \iff x\ge +1 \\ x-2>0: \ x-2\le 1 \iff x\le 3 \end{array} \right|$

$= \mathbb{P}(+1\le X\le 3) = \mathbb{F}(3)-\mathbb{F}(1) = \mathbb{F}(3)$

$= \frac{1}{1-3}+1 = -\frac{1}{2}+1 = \frac{1}{2}$

$u_{0,25} = \left| \mathbb{F}(u_{0,25}) = \frac{1}{4} \right| \iff \frac{1}{1-u_{0,25}}+1 = \frac{1}{4} \iff 1 = -\frac{3}{4}(1-u_{0,25})$

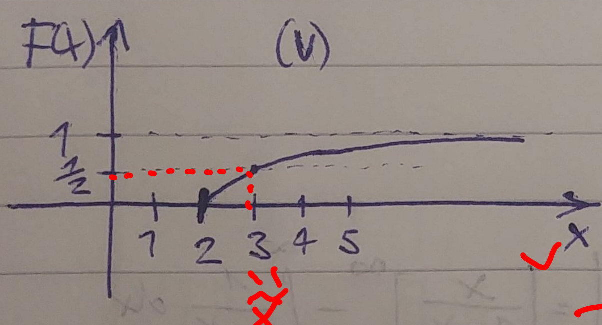
$\iff -\frac{4}{3} = 1-u_{0,25} \iff u_{0,25} = \frac{7}{3}$

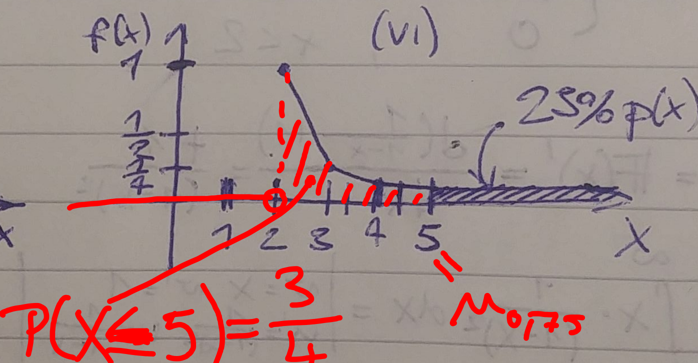
$u_{0,5} = ? \iff \mathbb{F}(u_{0,5}) = \frac{1}{2} \iff \frac{1}{1-u_{0,5}}+1 = \frac{1}{2} \iff 1 = -\frac{1}{2}(1-u_{0,5})$

$\iff -2 = 1-u_{0,5} \iff u_{0,5} = 3$

$u_{0,75} = ?,\ \mathbb{F}(u_{0,75}) = \frac{3}{4} \iff \frac{1}{1-u_{0,75}}+1 = \frac{3}{4} \iff 1 = -\frac{1}{4}(1-u_{0,75})$

$\iff -4 = 1-u_{0,75} \iff u_{0,75} = 5$





3. $\mathbb{P}(H|S) = \frac{1}{3} \quad \mathbb{P}(H|M) = \frac{2}{3} \quad \mathbb{P}(M) = \frac{2}{3}$

(m) $\mathbb{P}(I) = \mathbb{P}(I|S)\cdot\mathbb{P}(S) + \mathbb{P}(I|M)\cdot\mathbb{P}(M)$ ✓ :)

$= \frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3} = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$ ✓

(mm) $\mathbb{P}(M|H) = \frac{\mathbb{P}(H|M)\cdot\mathbb{P}(M)}{\mathbb{P}(H)} = \frac{\mathbb{P}(H|M)\cdot\mathbb{P}(M)}{\mathbb{P}(H|S)\cdot\mathbb{P}(S) + \mathbb{P}(H|M)\cdot\mathbb{P}(M)}$ anebo $1-\mathbb{P}(I)$

$= \frac{\frac{2}{3}\cdot\frac{2}{3}}{\frac{1}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{2}{3}} = \frac{4}{\cancel{9}\,5}$

4. p_Y

y_i	0	1	2	3
$p_Y(y)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

✓

p_Z:

z	0	1	2
$p_Z(z)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

✓

(a)

Y, Z nezávislé $\iff p(y_i, z_j) = p_Y(y_i)\cdot p_Z(z_j) \ \forall y_i, z_j \in Y, Z$

$p(0,0) = p_Y(0)\cdot p_Z(0) \iff 0 = \frac{1}{8}\cdot\frac{1}{4} \iff 0 = \frac{1}{32}$ neplatí

Y, Z nejsou nezávislé ✓

(b) $EY = \sum_{\forall y_i \in Y} y_i \cdot p_Y(y_i) = 0\cdot\frac{1}{8} + 1\cdot\frac{3}{8} + 2\cdot\frac{3}{8} + 3\cdot\frac{1}{8} = \frac{12}{8} = \frac{3}{2}$ ✓

$\mathrm{var}\,Y = \sum_{\forall y_i \in Y} y_i^2 \cdot p_Y(y_i) - EY = 0^2\cdot\frac{1}{8} + 1^2\cdot\frac{3}{8} + 2^2\cdot\frac{3}{8} + 3^2\cdot\frac{1}{8} - \frac{3}{2} = \frac{24-12}{8} = \frac{3}{2}$ ✓

$EZ = \sum_{\forall z_i \in Z} z_i \cdot p_Z(z_i) = 0\cdot\frac{1}{4} + 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} = \frac{4}{4} = 1$ ✓

$\mathrm{var}\,Z = \sum_{\forall z_i \in Z} z_i^2 \cdot p_Z(z_i) - EZ = 0^2\frac{1}{4} + 1^2\cdot\frac{1}{2} + 2^2\frac{1}{4} - 1 = \frac{2+4-4}{4} = \frac{1}{2}$ ✓

$\mathrm{cov}(Y,Z) = \sum_{\forall y_i z_j \in (Y,Z)} y_i \cdot z_j \cdot p(y_i, z_j) - EYEZ = \cancel{0\cdot 0\cdot 0} + \cancel{0\cdot 1\cdot 0} + \cancel{0\cdot 2\cdot \frac{1}{8}}$
$+ \cancel{1\cdot 0\cdot 0} + 1\cdot 1\cdot \frac{1}{4} + 1\cdot 2\cdot\frac{1}{8}$
$+ \cancel{2\cdot 0\cdot \frac{1}{8}} + 2\cdot 1\cdot\frac{1}{4} + \cancel{2\cdot 2\cdot 0}$
$+ \cancel{3\cdot 0\cdot\frac{1}{8}} + \cancel{3\cdot 1\cdot 0} + \cancel{3\cdot 2\cdot 0} - \frac{3}{2}$

$= \frac{1}{4} + \frac{2}{8} + \frac{2}{4} - \frac{3}{2} = \frac{1+1+2-6}{4} = -\frac{2}{4} = -\frac{1}{2}$ ✓

:) $E(Y,Z) = (EY, EZ) = \left(\frac{3}{2}, 1\right)$ ✓; $\mathrm{var}(Y,Z) = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$ ✓

(bb) $\mathbb{P}(Y \geq 1 \wedge Z \leq 0) = \sum_{i=1}^{3}\sum_{j=0}^{0} p(y_i, z_j) = \frac{1}{4}$ ✓

(bc) $\mathbb{P}(|Y-Z| = 1)$ $|\ Y - Z = 1 \iff Y = Z+1 \vee Y - Z = -1 \iff Y = Z - 1\ |$

$= 0 + 0 + \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$ ✓

(c) $\rho(Y,Z) = \frac{\mathrm{cov}(Y,Z)}{\sqrt{\mathrm{var}\,Y}\cdot\sqrt{\mathrm{var}\,Z}} = \frac{-\frac{1}{2}}{\sqrt{\frac{3}{2}}\cdot\sqrt{\frac{1}{2}}}$ ✓ $= \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

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