Presentation 3

Fluid Properties: Density and Viscosity

Definition of Fluid Viscosity

- The resistance of a fluid which is being deformed by either shear stress or tensile stress
- Dynamic viscosity (μ)
- Kinematic viscosity ($\nu=\mu/\rho$) – ratio of viscous and inertial forces
- Dynamic viscosity (μ) Units

$$\frac{kg}{m \times s}, Pa \times s, P \text{ (Poise)}, mP, \text{etc.}$$

Definition of Fluid Density

- The total mass per unit volume
- (Mass of Liquid + Mass of Solids)/volume
- Units

$$\frac{kg}{m^3}, \quad \frac{lb}{ft^3}, \quad \text{etc.}$$

Density, Viscosity, and Hydraulic Conductivity

- Fluid density and viscosity have a direct effect on hydraulic conductivity

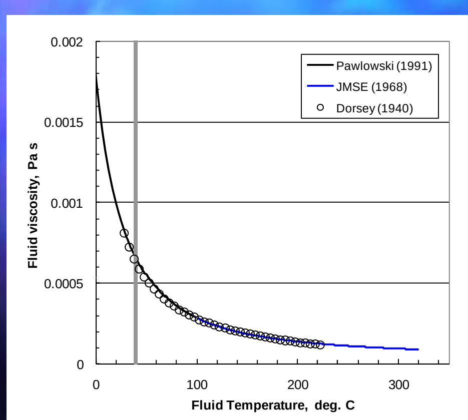
$$K = \frac{k\rho g}{\mu}$$

Fluid Viscosity

- In general, fluid viscosity is a function of temperature and solute concentrations:

$$\mu = f(T, C)$$

Fluid Viscosity – relation to temperature



Pawlowski (1991)

$$\mu = 10^{-3}\left[1 + 0.015512(T - 293.15)\right]^{-1.572}$$

JSME (1968)

$$\mu = 241.4 \times 10^{-3} \cdot 10^{\left(\frac{247.8}{T - 140}\right)}$$

Dorsey (1940)

$$\upsilon = 10^{-4} \times \frac{0.332}{(T - 260.15)}$$

A General Form of Fluid Viscosity

$$\mu = \mu_o\left(T^{\circ C}\right) + \sum_{k=1}^{N} \frac{\partial \mu}{\partial C_k}\left(C_k - C_{o,k}\right),$$ Hughes and Sanford (2004)

$$\mu_o\left(T^{\circ C}\right) = \left(239.4 \times 10^{-7}\right) 10^{\left(\frac{248.37}{T + 133.15}\right)}$$

$$TDS = \sum_{k=1}^{N} C_k$$

Fluid Viscosity in SEAWAT

$$\mu = \mu\left(T^{\circ C}\right) + \sum_{k=1}^{n} \frac{\partial \mu}{\partial C_k}\left(C_k - C_{o,k}\right)$$

$$\mu\left(T^{\circ C}\right) = A_1 \cdot A_2^{\left(\frac{A_3}{T + A_4}\right)}$$

$$\mu\left(T^{\circ C}\right) = A_1 \cdot \left[A_2 + A_3\left(T + A_4\right)\right]^{A_5}$$

$$\mu\left(T^{\circ C}\right) = A_1 \cdot T^{A_2}$$

A_1	A_2	A_3	A_4	A_5	Equation (in report)	Reference
2.394×10^{-5}	10	248.37	133.15	--	18	Voss (1984)
1×10^{-3}	1	1.5512×10^{-2}	-20	-1.572	19	Pawlowski (1991)
0.168	-1.0868	--	--	--	20	Guo and Zhao (2005)*

*Relation is for oil viscosity as a function of temperature (between 5 and 170 degrees Celsius). Relation is not applicable to water.

Fluid Density

- In general, fluid density is a function of temperature, pressure and solute concentrations:

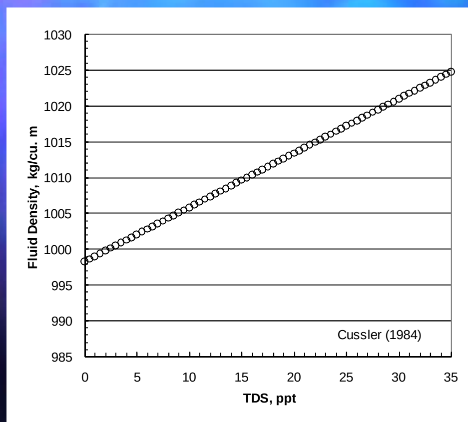
$$\rho = f(T, P, C)$$

Fluid Density - isothermal

- Under isothermal conditions, and assuming the fluid is incompressible, fluid density is a function of solute concentrations:

$$\rho = f(C)$$

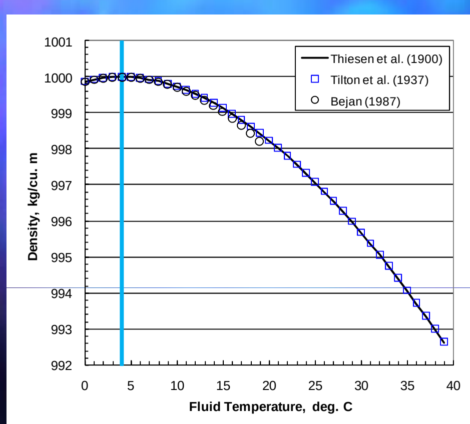
Fluid Density – isothermal (20°C)



$$\rho = \rho_o + \frac{\partial \rho}{\partial C}\left(C - C_o\right)$$

$$\frac{\partial \rho}{\partial C} = 756,\ C = \text{TDS mass fraction}$$

Fluid Density – non-isothermal



Thiesen et al. (1900)

$$\rho = 1000\cdot\left(1 - \left(\frac{(T - 3.98)^2}{503570}\right)\cdot\left(\frac{T + 283}{T + 67.26}\right)\right)$$

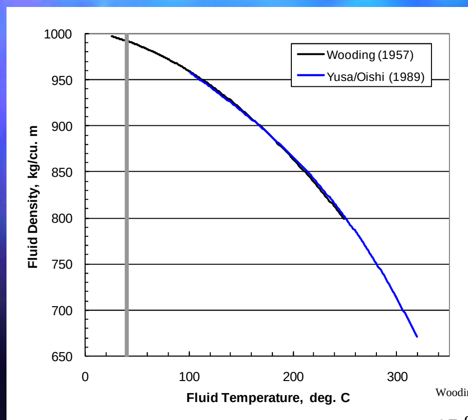
Tilton et al. (1937)

$$\rho = 1000\cdot\left(1 - \left(\frac{(T - 3.98)^2}{508929.2}\right)\cdot\left(\frac{T + 288.9414}{T + 68.12963}\right)\right)$$

Bejan (1987)

$$\rho = 1000\cdot\left(1 - 8\times 10^{-6}\,(T - 3.98)^2\right)$$

Fluid Density – non-isothermal



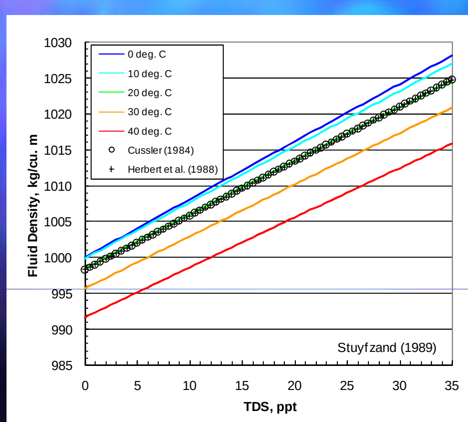
Wooding (1957)

$$\rho = 996.9\cdot\left(1 - 3.17\times10^{-4}(T - 298.15) - 2.56\times10^{-6}(T - 298.15)^2\right)$$

Yusa and Oishi (1989)

$$\rho = 1758.4 + 10^3 T\cdot\left(-4.8434\times10^{-3} + T\left(1.0907\times10^{-5} - 9.8467\times10^{-9}T\right)\right)$$

Fluid Density – non-isothermal



Stuyfzand (1989)

$$\rho = 1000\cdot\left(1 + 0.805C - 6.5\times10^{-6}(T - 4 + 220C)^2\right)$$

Herbert et al. (1988)

$$\rho = \rho_o\left(\frac{\rho_s}{\rho_o}\right)^C$$

Fluid density – General linear form

$$\rho = \rho_o + \sum_{k=1}^{n} \frac{\partial \rho}{\partial C_k}(C_k - C_{o,k}) + \frac{\partial \rho}{\partial T}(T - T_o) + \frac{\partial \rho}{\partial P}(P - P_o)$$

$$TDS = \sum_{k=1}^{n} C_k$$

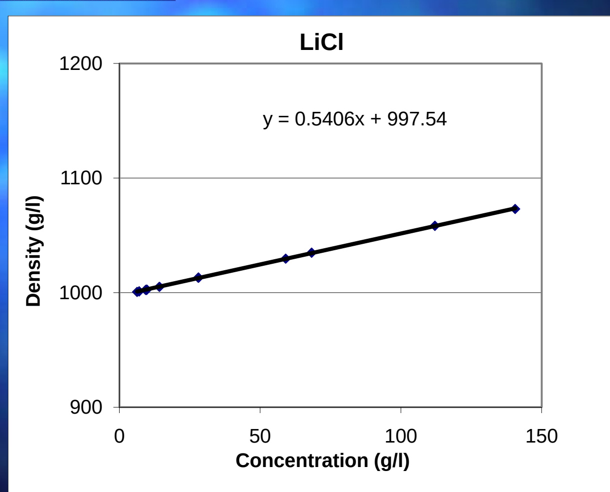
Assuming $C_{o,k} = 0$

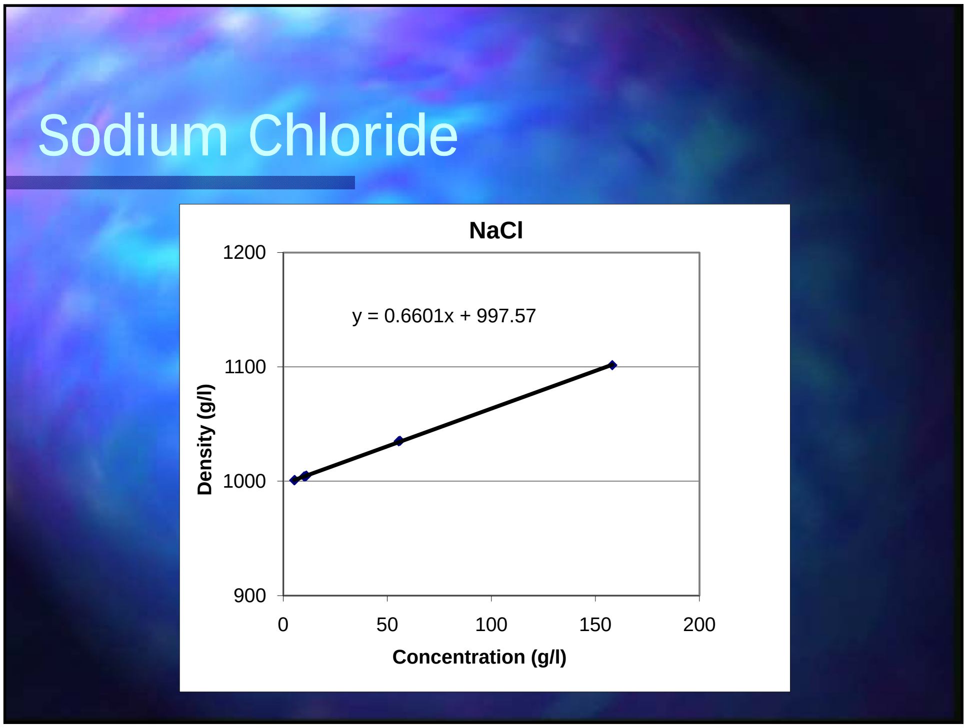
$$\rho = \rho_o + \sum_{k=1}^{n} \frac{\partial \rho}{\partial C_k} C_k$$

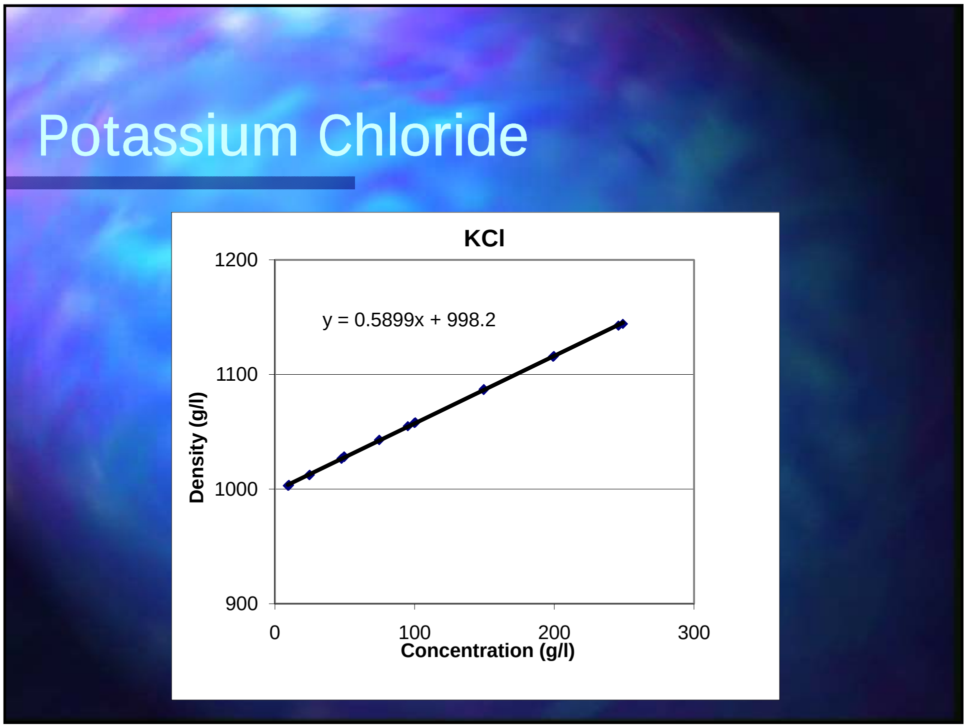
Lab Measurements of Fluid Density and Concentraions

Baxter, et al. 1911

Lithium Chloride

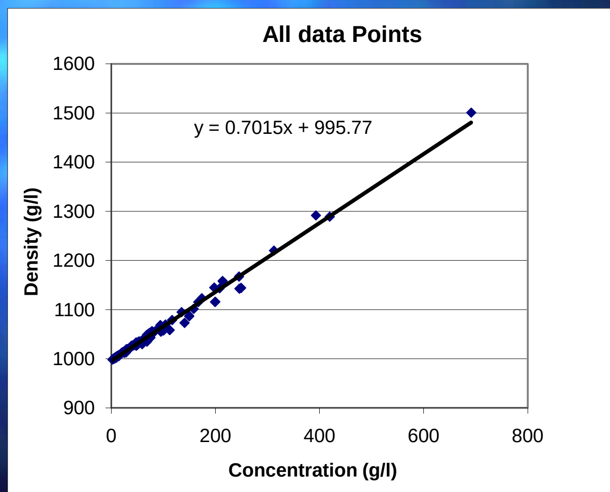



Sodium Chloride
NaCl
y = 0.6601x + 997.57
Density (g/l)
1200
1100
1000
900
0
50
100
150
200
Concentration (g/l)


Potassium Chloride
KCl
y = 0.5899x + 998.2
Density (g/l)
1200
1100
1000
900
0
100
200
300
Concentration (g/l)

All Data

Data source: Baxter et al. (1911)



All Data

Salts	Molecular weight	# samples	Regression Equation	Density Slope
LiCl	42.4	12	y = 0.5406x + 997.54	0.5406
NaCl	58.46	8	y = 0.6601x + 997.57	0.6601
KCl	74.56	22	y = 0.5899x + 998.2	0.5899
LiBr	86.86	18	y = 0.6972x + 997.56	0.6972
NaBr	102.92	13	y = 0.7278x + 998.48	0.7278
KBr	119.02	16	y = 0.6941x + 997.5	0.6941
LiI	133.86	9	y = 0.725x + 997.01	0.725
NaI	149.92	9	y = 0.7495x + 997.39	0.7495
KI	166.02	8	y = 0.7134x + 997.32	0.7134
Average		115		0.677511
All Data			y = 0.7015x + 995.77	0.7015

Data Source: Baxter, et al., 1911

Density Calculation in SEAWAT

$$\rho = \rho_f + E \cdot C$$

where

$$E = \frac{\Delta\rho}{\Delta C} = \frac{\rho_{\max} - \rho_{\min}}{C_{\max} - C_{\min}}$$

A Special Case of Density Slope

$$E = \frac{\Delta\rho}{\Delta C} = \frac{\rho_{seawater} - \rho_f}{C_{TDS} - 0}$$

e.g.

$$E = \frac{1025 - 1000}{35} = 25/35 = 0.7143$$

VDF File (cont.)

Example VDF file

```
asr.vdf - WordPad
File  Edit  View  Insert  Format  Help

        1         1         1         0                     MTDNCONC MFNADVFD NSWTCPL IWTABLE
       0.        0.                                         DENSEMIN DENSEMAX
    1000.       0.7                                         DENSEREF DENSESLP
    0.001                                                   FIRSTDT

For Help, press F1                                                    NUM
```

DENSEREF and DENSESLP

Concentration		
units	**freshwater**	**seawater**
mg/L	0	35,000
g/L	0	35
lbs/ft^3	0	2.188
relative	0	1

Density		
units	**Freshwater (DENSEREF)**	**seawater**
g/cm^3	1	1.025
kg/m^3	1000	1025
lbs/ft^3	62.42	63.98

$$DENSESLP = \frac{\Delta \rho}{\Delta C}$$

Example:
use concentration in mg/L and density in kg/m^3

$$DENSESLP = \frac{1025 - 1000 \ kg/m^3}{35000 - 0 \ mg/L} = 7.14 \ x10^{-4}$$

Density length units must be same as length units used for model grid, hydraulic conductivity, etc.

INPUT of CONCENTRATIONS

VERSION	TDS	Chloride
SEAWAT 1.2	Yes	No
SEAWAT 2.1	Yes	No
SEAWAT 2000 SEAWAT v. 4	Yes	Yes

Using Chloride Concentration

- SEAWAT 2000 or version 4
- Chloride/TDS ratio is constant
- Appropriate Density Slope

Chloride vs. TDS

- TDS is preferred to be used for density calculation
- Chloride concentration may be used for density calculation if the ratio of chloride concentration to TDS is relatively uniform (55%)

Composition of “typical” seawater

Major Solutes	Concentration (mg/Kg)	Percentage (%)
Chloride	19350	55.06
Sodium	10760	30.62
Sulfate	2710	7.71
Magnesium	1290	3.67
Calcium	411	1.17
Potassium	399	1.13
Bicarbonate	142	0.4
Bromide	67	0.19
Others	20	0.04

Field Measurements of Chloride and TDS Concentration (Brevard, FL)

Well Name	Sample ID	Cl (mg/l)	TDS (mg/l)	Cl/TDS (%)
UF-1D	S-12	59	532	11.09
	S-13	64	536	11.94
	S-14	71	536	13.25
	S-15	70	524	13.36
	S-16	65	580	11.21
	S-17	70	568	12.32
	S-18	56	518	**10.81**
	S-40	672	1372	48.98
	S-25	882	2078	42.44
	S-33	2717	5446	49.89

Data Source: BFA, 2001

Well Name	Sample ID	Cl (mg/l)	TDS (mg/l)	Cl/TDS (%)
UF-2S	S-1	124	556	22.30
	S-2	183	668	27.40
	S-3	390	1136	34.33
	S-4	691	1772	39.00
	S-5	833	2044	40.75
	S-6	845	1984	42.59
UF-3D	S-32	49	412	11.89
	S-33	52	384	13.54
	S-34	64	388	16.49
	S-35	55	400	13.75
	S-36	55	388	14.18
	S-37	52	376	13.83
	S-27	74	896	**8.26**
	S-25	134	694	19.31
	S-23	2010	4372	45.97

How to estimate TDS

- Specific conductance
- Regression Analysis of Chloride data

How to Predict Chloride Concentrations

- Multi-species Solute Transport
- Proportional Calculation
- Standard MODFLOW/MT3D Approach