Numerical Simulation of Flood Wave Modification Due to Bank Storage Effects

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Abstract. The modification of a flood wave due to bank storage effects can be calculated by using numerical methods. The dynamic equations describing one-dimensional open channel flow and the equation for two-dimensional transient groundwater flow are solved simultaneously, coupled by an expression for flow through the wetted perimeter of the channel. Numerical experiments indicate that flood waves may be modified considerably by bank storage, particularly in the lower segments of a long reach, and that the degree of modification is influenced markedly by the hydraulic conductivity of the aquifer.

INTRODUCTION

The numerical simulation of open channel flow and of groundwater flow has each received considerable attention in recent years. The coupling of the two systems, however, has been neglected. In the simulation of open channel flow, groundwater discharge to the channel is treated as an unknown component of lateral inflow. In the simulation of groundwater movement the hydraulic head in the channel is predefined and the effect of groundwater flow on stream elevation is neglected. The purpose of this paper is to demonstrate the modification of a flood hydrograph due to bank storage effects by routing a flood wave through a reach of channel with a permeable bed by using a mathematical model of the coupled groundwater and surface water systems.

The application of mathematical methods to the study of stream–aquifer interaction has previously been confined to calculating the response of the groundwater system to changes in river elevation. Many important problems encountered in the field have been investigated by solving analytically or numerically the one-dimensional groundwater flow equation in conjunction with appropriate boundary conditions [*Wenzel and Sand,* 1942; *Stallman and Papadopulos,* 1966; *Hantush,* 1963; *Cooper and Rorabaugh,* 1963; *Rorabaugh,* 1960; *Singh,* 1969; *Pinder et al.,* 1969; *Hornberger et al.,* 1970]. These analyses assume that the stream elevation changes only as a function of time and that horizontal groundwater flow occurs only normal to the stream.

To study the modification of a flood hydrograph due to bank storage effects, a more complex mathematical model is necessary. In this situation the time and space distribution of stream elevation must be determined by solving the differential equations which describe one-dimensional open channel flow and include the effect of groundwater flow normal to the channel through its wetted perimeter. This flow through the wetted perimeter of the channel is a function of the hydraulic head in the aquifer as well as the elevation of the stream. To compute the head in the aquifer the mathematical model must simulate transient horizontal two-dimensional unconfined groundwater flow because flow in the aquifer is not necessarily normal to the stream. The interdependence of the open channel flow and the groundwater flow models necessitates the simultaneous solution of the differential equations describing each system. The equation coupling the two systems is Darcy's law, which describes flow through the wetted perimeter of the channel.

THEORETICAL DEVELOPMENT

The partial differential equations describing one-dimensional unsteady flow in open channels of constant cross section are [*Stoker,* 1956, p. 8]

$$z\frac{\partial v}{\partial l} + v\frac{\partial z}{\partial l} + \frac{\partial z}{\partial t} = \frac{q_l}{b} + \frac{q_g}{b} \qquad (1)$$

and

$$v\frac{\partial v}{\partial l} + g\frac{\partial z}{\partial l} + v\frac{q_l + q_a}{bz} + \frac{\partial v}{\partial t} = g(S_0 - S_f) \qquad (2)$$

where (Figure 1)

b, channel width, L;
g, gravitational acceleration, L/T^2;
l, space coordinate;
q_a, flow into the channel per unit length through its wetted perimeter, L^2/T;
q_l, lateral inflow per unit length over the channel banks and from tributaries, L^2/T;
S_f, friction slope, dimensionless;
S_0, slope of the channel bottom, dimensionless;
t, time;
v, velocity of flow, L/T;
z, the depth of flow, L.

Several numerical techniques are available for solving these hyperbolic, first order, nonlinear partial differential equations (an excellent review of these methods is presented in *Amein and Fang* [1969]). In addition to equations 1 and 2 initial and boundary conditions are required. The initial conditions are the depth and velocity of flow in the channel. In the problems considered here the boundary conditions are a description of either stage or discharge as a function of time at the upstream boundary and a definition of the stage–discharge relationship at the downstream boundary.

In this initial phase of our research an explicit finite difference staggered-net method for solving equations 1 and 2 provided satisfactory results. This approach was developed by *Isaacson et al.* [1956] and later modified by *Ragan* [1966]. Our computer program for solving these equations is a modification of one provided by M. E. Jennings using the approach described by Ragan.

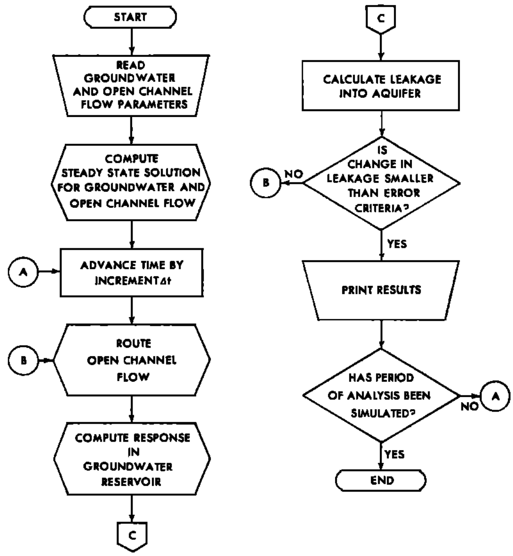

Fig. 2. Flow chart for iterative solution of aquifer–stream system.

The equation for horizontal, two-dimensional flow of groundwater below the channel is

$$\nabla \cdot T\nabla h = S\frac{\partial h}{\partial t} + \frac{q_a}{b + 2z} + W(x, y, t) \qquad (3)$$

and elsewhere, where the aquifer is unconfined, the equation is

$$\nabla \cdot Km\nabla h = S_y\frac{\partial h}{\partial t} + W(x, y, t) \qquad (4)$$

In equations 3 and 4

h, hydraulic head, L;
m, saturated thickness of the aquifer, L; a linear function of h;
K, hydraulic conductivity, L/T;
S, storage coefficient, dimensionless;
S_y, specific yield, dimensionless;
T, transmissivity, L^2/T;
$W(x, y, t)$, vertical discharge from the aquifer per unit area, L/T.

Because there are no sources or sinks in the problems considered here, $W(x, y, t)$ is zero.

Boundary conditions for equations 3 and 4

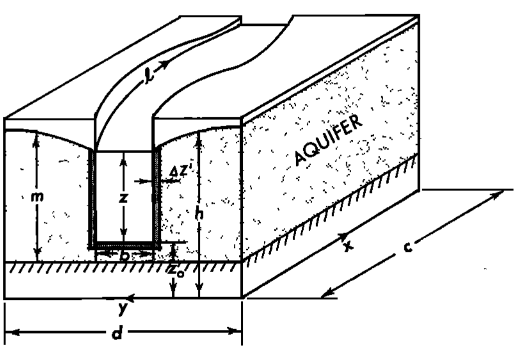

Fig. 1. Schematic representation of aquifer–stream system.

used in the bank storage problem are (Figure 5)

$$\frac{\partial h}{\partial x}(0, y, t) = 0$$

$$\frac{\partial h}{\partial x}(c, y, t) = 0$$

$$\frac{\partial h}{\partial y}(x, 0, t) = 0$$

$$\frac{\partial h}{\partial y}(x, d, t) = 0$$

The initial condition is

$$h(x, y, 0) = f(x, y)$$

where $f(x, y)$ is the equilibrium solution in the aquifer for the aquifer–stream system with steady flow in the stream.

The finite difference method used to solve equations 3 and 4 is the iterative alternating direction implicit technique [*Douglas and Rachford*, 1956]. The finite difference approximations to these equations can be found in *Bredehoeft and Pinder* [1970].

To couple equations 1, 2, and 3, another expression describing the movement of water between the two systems must be defined. As indicated above, this expression is Darcy's law and is written in finite difference form for vertical flow from the aquifer to the stream as

$$\frac{q_g}{b + 2z} = -K_p \frac{z_+ z_0 - h}{\Delta z'}$$

where K_p is the hydraulic conductivity of the bottom sediments of the channel, L/T; $\Delta z'$ is the thickness of the bottom sediments along the wetted perimeter of the channel, L; and z_0 is the elevation of the stream bottom measured from the same datum as h, L.

Equations 1, 2, and 3 or 4 give us three equations in the three unknowns v, z, and h; the remaining parameters are defined. To solve these equations simultaneously an iterative procedure as indicated in the flow chart (Figure 2) is used. After defining the necessary physical parameters for the aquifer and stream systems, the initial conditions are calculated. The open channel flow equations 1 and 2 are solved for steady flow conditions; the groundwater equations 3 and 4 are then solved for steady flow by using the calculated stream elevations. The open channel flow equations must now be solved once again because the groundwater inflow term q_g has been modified by the new head distribution in the aquifer. This cycle (iteration) is continued (solving equations 1 and 2 and then equations 3 and 4) until the change in the groundwater inflow term q_g between successive calculations is within a predetermined error tolerance. This solution to the aquifer–stream system is the initial condition, $f(x, y)$ indicated above, used to study the transient response of both the aquifer and the stream to the introduction of a flood wave at the upstream boundary.

To proceed with the solution of the transient problem the procedure indicated above is repeated for each time interval Δt. The equations for open channel flow and groundwater flow are solved repeatedly for the same point in time until the change in leakage is less than the error tolerance. When this occurs the simulation proceeds to the new time $t + \Delta t$.

In general, two to five iterations are necessary for convergence to a solution.

Error analysis. The accuracy of the groundwater model and streamflow model could be determined separately, but the only check on the combined system is a mass balance calculation based on the amount of fluid entering and leaving each system. The groundwater model was verified by using the solution for one-dimensional flow in an aquifer when the stream elevation is defined as an arbitrary function of time [*Pinder et al.*, 1969]. The open channel flow model was verified by computing a mass balance at each node along its length and by studying the variation in discharge at each node at steady flow conditions. As a check on the combined aquifer–stream system, the change in the quantity of fluid in the aquifer was compared with the change in the quantity of fluid in the stream at the end of each time step Δt.

BANK STORAGE AND MODIFICATION OF THE FLOOD HYDROGRAPH

In discussing the stream–aquifer system the following questions will be considered:

1. How is a flood wave modified by leakage to a hydraulically connected floodplain aquifer?
2. How does the width and permeability of

this aquifer influence the effect observed in 1?

3. How does the flow of water through the wetted perimeter of the channel vary along its length and with time?

4. How does the flood wave affect the head distribution in the floodplain aquifer?

Modification of the flood wave. To study the modification of a flood wave due to bank storage, an input discharge hydrograph was routed through a reach of channel with and without leakage. In one routing the channel was assumed to be impervious and in the other routing leakage through the wetted perimeter of the channel to a floodplain aquifer was permitted. A comparison of stream discharge for the two simulations at the same point in space and time indicates the effect of bank storage on the flood wave.

Figure 3 shows the modification of a flood hydrograph caused by bank storage effects. The flood plain aquifer in this problem extends 130,000 feet along the length of the channel and is 1400 feet across the valley; it is surrounded by impermeable material on all sides. The hydraulic conductivity of the aquifer is 0.01 ft/sec and the initial saturated thickness ranges from 220 feet at the upstream boundary to 90 feet at the downstream boundary. The stream flows along the axis of the valley through a straight channel with constant cross section and a slope of 0.001. The ratio of hydraulic conductivity of the stream bed to its thickness along the wetted perimeter of the stream is 4 ft/sec/ft and, therefore, is not a limiting factor in the amount of water entering the aquifer. The channel is 100 feet wide and the initial depth of flow is 20 feet. The steady flow discharge of this stream prior to the introduction of the flood wave was 18,000 cubic feet per second.

A comparison of Figures 3*a* and 3*b* demonstrates that the effect of bank storage is cumulative and may be of considerable importance in regulating flood discharge in the lower reaches of the stream. In Figure 3*a* the discharge is being monitored at a point 50,000 feet below the upstream boundary, and the modification of the discharge hydrograph is much less than indicated in Figure 3*b* at a point 90,000 feet farther downstream. Bank storage in this particular problem attenuates the flood wave, decreasing the peak discharge and extending the hydrograph base time. It is apparent that this attenuation will generate a long-term recession curve similar to but independent of regional groundwater discharge to the stream.

To illustrate the influence of the floodplain aquifer width on the modification of the hydro-

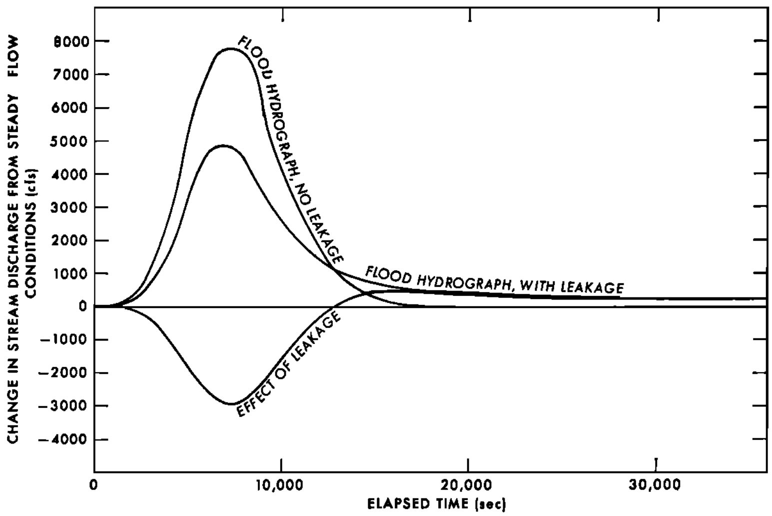

Fig. 3*a*. Attenuation of a flood wave at a point 50,000 feet from the upstream boundary for a stream with and without leakage to the aquifer. Hydraulic conductivity of aquifer, 0.01 ft/sec; width of aquifer, 1400 feet.

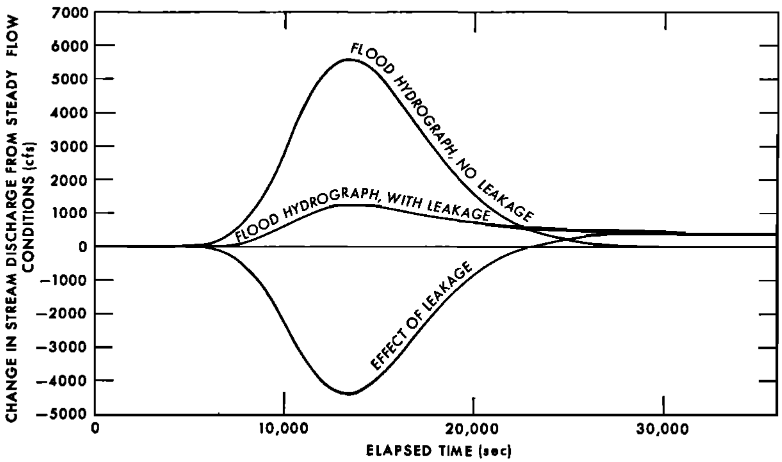

Fig. 3*b*. Attenuation of a flood wave at a point 140,000 feet from the upstream boundary for a stream with and without leakage to the aquifer. Hydraulic conductivity of aquifer, 0.01 ft/sec; width of aquifer, 1400 feet.

graph, the valley width was increased from 1400 feet to 5000 feet. A comparison of curve *b*, Figure 4 (width of 5000 feet) and Figure 3*b* (width of 1400 feet) indicates that increasing the width of the aquifer had a minor influence on bank storage in this problem. The reason for the low sensitivity of stream discharge to the increase in aquifer width is indicated in Figure 5 (the contours represent change in head from the initial conditions in the aquifer $f(x, y)$). It is apparent that, in a problem of this kind, where the river elevation is increased for a relatively short period of time, the bank storage effect decreases rapidly with distance from the channel. An extension of the aquifer beyond the area of primary dynamic response will have a minor effect on bank storage. The extent of this area would be a function of the

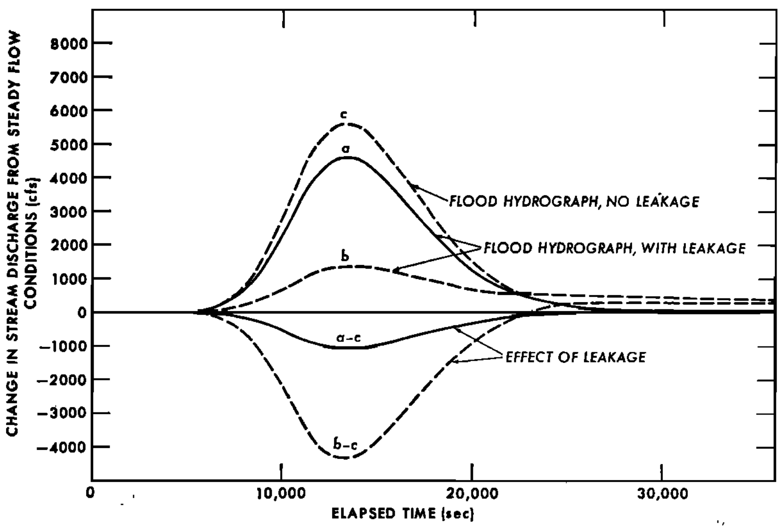

Fig. 4. Influence of aquifer width and hydraulic conductivity on change in stream discharge. Observation point is 140,000 feet from upstream boundary. Curve *a*, width of aquifer, 5000 feet; hydraulic conductivity, 0.001 ft/sec. Curve *b*, width of aquifer, 5000 feet; hydraulic conductivity, 0.01 ft/sec.

stream and aquifer parameters as well as the upstream boundary condition.

To study the importance of aquifer hydraulic conductivity on modification of the flood wave, the hydraulic conductivity of the aquifer in the preceding problem was decreased from 0.01 to 0.001 ft/sec. As may have been anticipated, this change altered the response of the aquifer–stream system considerably. Curves *a* and *b* in Figure 4 were generated by using hydraulic conductivities of 0.001 and 0.01, respectively; the remaining physical parameters were the same for both. With a decrease in hydraulic conductivity, less fluid was able to enter and leave the floodplain during the hydrograph base time resulting in a smaller modification of the flood wave.

Bank storage. The attenuation of the flood wave as it moves downstream causes a decrease in the rate of flow of water through the wetted perimeter of the channel. This is due to the lower hydraulic head gradient between the stream and the aquifer in the lower reach. As a result bank storage decreases downstream. The variation in flow through the wetted perimeter of the channel as a function of distance downstream and time since the beginning of the simulation is presented in Figure 6. The time interval between each curve is 4500 seconds. According to these curves the maximum rate of flow from the stream occurs as the flood wave enters the channel reach and the flow rate decreases rapidly over time. After 13,500 seconds (see curve 3) water begins to return to the stream near the upstream boundary while flow into bank storage continues downstream. The excess head distribution in the aquifer that develops from this pattern of flow from the channel is indicated in Figure 5. Because a higher groundwater mound is created upstream than downstream, the horizontal groundwater component parallel to the stream is larger than for the initial steady state system.

If lateral inflow were introduced along the channel so that stream discharge increased

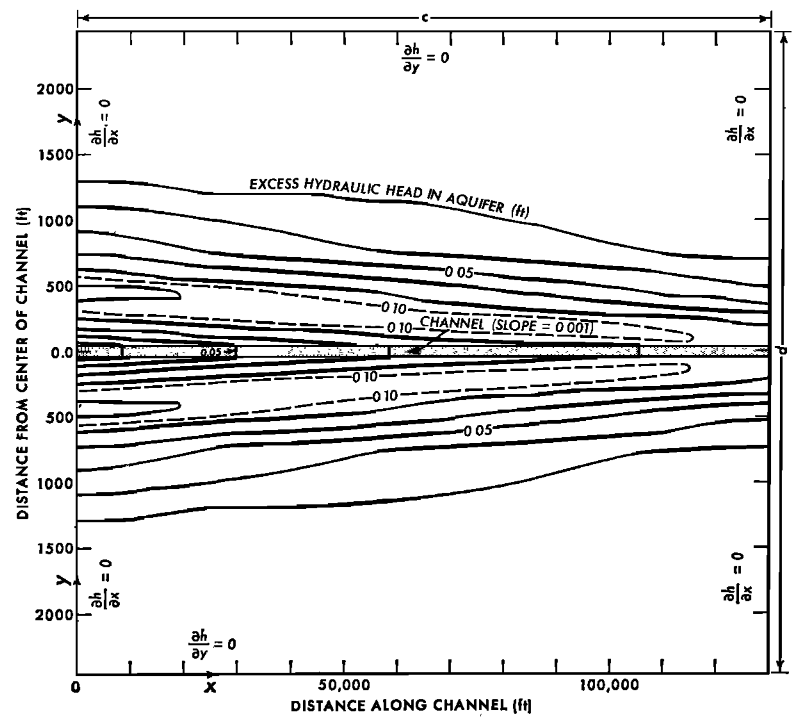

Fig. 5. Hydraulic head distribution in an alluvial aquifer in response to a flood wave. Elapsed time, 21.25 hours; hydraulic conductivity, 0.01 ft/sec; contour interval, 0.02 foot.

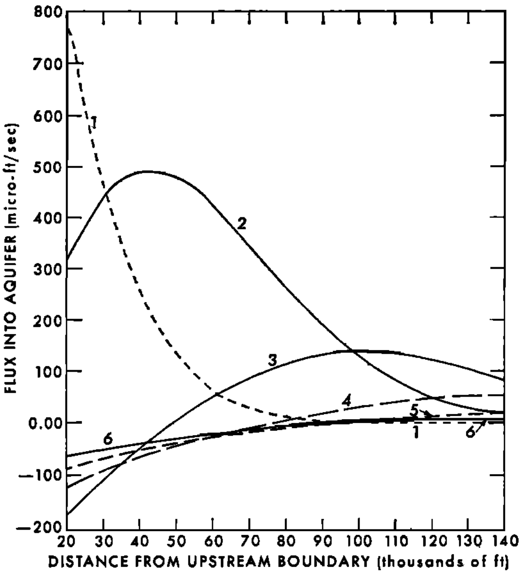

Fig. 6. Flow into bank storage along channel. Hydraulic conductivity, 0.01 ft/sec; slope of channel bottom, 0.001; width of aquifer, 1400 feet; time interval between curves, 4500 sec.

downstream, the spacial and temporal distribution of flow into the aquifer would be changed and the above discussion would not hold.

DIGITAL MODEL OF THE STREAM–AQUIFER SYSTEM

The results presented in this paper were generated by using a computer program that solves the open channel flow equations explicitly and the groundwater flow equations implicitly. Because small time steps are required for stability in the explicit technique, the open channel flow equations were solved several times for each solution of the groundwater equations. The computer program is presently being modified to use an implicit finite difference technique for the open channel flow in the hope that both sets of equations can be solved for a time step of the same size.

Although no attempt was made to optimize the efficiency of the code used in this model, the program is remarkably fast. Calculation time on the CDC 6600 required to simulate 95 groundwater model time steps was approximately 90 seconds. (The groundwater model used 225 nodes and the open channel flow model used 16 nodes.)

DISCUSSION OF RESULTS

The problems presented here were selected to demonstrate the response of the stream–aquifer system to the propagation of a flood wave along a stream hydraulically connected to a floodplain aquifer and having no lateral inflow over the channel banks or from tributaries. A relatively simple system was selected to show the effect of selected parameters. The numerical techniques employed, however, are flexible and more complex problems can be treated easily.

The model presented here should be considered as a research tool. Currently a more flexible program is being developed to be applied to field problems.

The following general conclusions may be drawn from the problems considered here:

1. Provided the appropriate physical parameters can be determined, the dynamics of the stream–aquifer system can be simulated by using available numerical techniques.
2. Bank storage attenuates a flood wave and this modification of the wave may be considerable in the lower segment of a long reach.
3. The extension of hydrograph base time by bank storage effects may generate a recession curve similar in appearance to one due to regional groundwater flow.
4. The length of the channel reach and the hydraulic conductivity of the floodplain aquifer have a considerable influence on the modification of a flood wave by bank storage.
5. The response of a floodplain aquifer to the propagation of a flood wave along a hydraulically connected channel decreases rapidly with distance from the stream.

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