Logical Reversibility of Computation

Reading Report

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Abstract

The article written by Charles H. Bennett [1] presented a *proof by construction* for the existence of a reversible 3-tape Turing machine emulator for each irreversible general-purpose 1-tape Turing machine. A physical avatar of this machine would be able to dissipate less energy than kT[1] per step.

Hypothesis and evidence

The author states that for every logically irreversible one-tape Turing machine, a logically reversible 3-tape Turing machine emulator could be built. Furthermore, this new logically reversible machine could serve to build thermodynamically reversible computers in the physical world that would dissipate far less energy that current computers.

In his dissertation, the author first present a *proof* of equivalence using a logically reversible n-tape Turing machines using a quadruple representation rather than quintuples. Then, he demonstrates that in a 3-stage machine, only two extra tapes need to be added in order to allow reversibility.

Evidence for reducing memory requirements is also presented. Firstly, the author shows that an injective function can be reversed without the need of keeping storage. Moreover, the author presents a clever algorithm to reduce the v^2 steps required to emulate a v -step reversible operation into just $\log(v)$ at expenses of time increase in proportion to v^2.

Finally, an example taken from the physical world is used for comparison, namely the biological RNA/DNA replication process.

Contribution

The article makes three concrete contributions: (1) the *proof* for a reversible Turing machine that emulates any other general-purpose Turing machine, (2) a couple of ways to cut temporary memory consumption and (3) an example of logical reversibility in nature's chemical systems and how could they be translated into computers.

Firstly, the *proof* of reversibility and equivalence of this emulator is constructed in the following order:

1. Definition of a deterministic n -tape Turing machine that satisfy several requirements, the most important of those is the use of non-overlapping domains
2. Reduction from the n -tape machine to just 3 tapes for `working`, `history` and `output`
3. Algorithm for building a 3-tape 3-stage logically reversible Turing machine and its *proof of correctness*

However, this 3-stage machine would need v^2 steps to emulate a v -steps irreversible computation. So, the author presents two memory reduction techniques:

1. For injective functions that produce a unique output from the input and can be reversed by mapping, a stage expansion from 3 to 7 is presented to allow erasing unwanted states
2. A clever algorithm is built upon the observation that each segment has been preformed reversibly and uses only $\log(v)$ steps

[1] where k is the Boltzmann constant and T is the room's temperature

That proven, the author now proceeds to suggest how this machine could be translated into more energy-efficient computers. A lower bound in energy dissipation was initially calculated for classical computers by Landauer [2] at $kT \ln 2$ ($3x10^{-21}$ joules) per bit forgotten. If the machine's model does not need to forget its previous state it would be able to beat that bound. DNA replication through the RNA polymerase can be seen as a thermally activated chemical copying Turing machine.

Given the previous example and considering the reversibility of the proposed Turing machine, a computer can be built to operate close to the thermal equilibrium that dissipates less than $kT \ln(2)$[2] per bit. However, an external force must be applied to prune branches that lead to wrong random solutions.

Limitations and weaknesses

There is a unit mismatch between Landauer's work -which is in bits- and Bennett's works -which is in states-. Thus, the author does not provide an updated lower bound for energy consumption in this new set of computers, considering all the operations of a reversible 3-stage Turing machine. Maybe this bound can not be set wthout a concrete implementation, but probably both -bits and states- will be in the same order of magnitude.

The source of the energy for pruning the branches of a computer built on those principles and how to direct this energy is not clear.

Future work

Several energy sources must be tested to determine pros and cons of each in terms of mechanisms for pruning the branches and to stay in the thermal equilibrium. Then, by studying these and other properties, we will be able to update the lower bound in terms of the machine's states.

Also, Bennett surmises that biological systems limit their speed to the kinetic maximum to escape from harmful effects of radiation and other physical phenomenon. Computers are also vulnerable to these effects at some degree. Future work could include mechanisms to counteract these effects without neglecting energy efficiency.

References

[1] C. H. Bennett, "Logical reversibility of computation," *IBM J. Res. Dev.*, vol. 17, no. 6, pp. 525–532, Nov. 1973.

[2] R. Landauer, "Irreversibility and heat generation in the computing process," *IBM journal of research and development*, vol. 5, no. 3, pp. 183–191, 1961.

[2] where 2 is the mean branching factor