Machines, Logic and Quantum Physics

Reading Report

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Abstract

The article [1] was published Dr. David Deutsch and Artur Ekert for the University of Oxford on 1999. The article proposes new considerations on the current mathematical abstractions that stem from our knowledge of the laws of physics, namely quantum mechanics. It makes strong enphasis on the quantum interference phenomenom that is unique on quantum mechanics. This process violates the rule of additivity in probability theory and calls for a reform in our notions, both on logic and mathematics.

Hypothesis and Evidence

The authors state that there are quantum physical properties that challenge our current notions of mathematics and impel us to modify those notions.

Evidence is given by presenting an example of such property, namely, the constructive and destructive quantum interferences for calculating the probability amplitudes and outcome of a quantum algorithm.

Contribution

A first contribution of the article is the idea that nowadays mathematics must be updated to reflect the theoretical and empirical science of quantum mechanics. This contribution was derived from a previous Deutsch's contribution, the Church-Turing physical principle.

The authors show clearly how quantum interference can be solid ground for proposing new logical gates such as the $\sqrt{\text{not}}$. Instead of adding probabilities, quantum interference is based on probability amplitudes that can cancel out or added. This explains why two intermediate events can occur simultaneously, even though just one is measured. So, there are many alternative ways an output event can occur.

A second contribution is that it shows how to use quantum interference to amplify correct outcomes and suppress incorrect ones inside a quantum algorithm. The use of the $\sqrt{\text{not}}$ gate is explained in Deutsch's first algorithm for determining if a function is balanced or constant. Shor's algorithm also uses interference to show factoring is tractable on a quantum computer while on a classical probabilistic computer grow exponentially. The author compares the roles of probability in the quantum Turing machine vs the classical probabilistic Turing machine and show its exponential power.

A third contribution is that the notion of a *proof* must be viewed from the expressive power of the computation itself regardless of its mathematical translation that would be impossible to check step-by-step.

Limitations and Weaknesses

The rationale for justifying new mathematical abstractions is based on some weak assumptions (no free launch):

1. Conjectural Galilean mathematical derivations are intrinsically better than axiomatic Aristotelean mathematical derivations [1].

[1] even though Eugene Wigner explains its benefits

2. Mathematics and logic are based on our perception of physical phenomena [2] rather than having its origin in a world of abstract concepts [3].

However, further on the mathematical impossibility is exemplified with the $\sqrt{\text{not}}$ concrete example in detail.

Future Work

The translation of exponential quantum mechanical processes into current mathematics is considered by Deutsch and Ekert impossible (based on a talk R. Feynman gave). Still, extending the expressive power of mathematics to include a detailed description of the processes is highly desirable. Probably, a proof for including the factoring (and many other) quantum tractable problems into the *BPP* class will be of great use for understanding the mathematical expressions to develop.

New gates besides the $\sqrt{\text{not}}$ shown can be developed from different quantum mechanical physical properties. Furthermore, a combination of those properties can also produce new mathematical abstractions.

Also, at the time of this writing there wasn't any fully-fledged real implementation of a quantum computer. This work is of cardinal importance to future generations of quantum computer devices.

References

[1] D. Deutsch, A. Ekert, and R. Lupacchini, "Machines, logic and quantum physics," *Bulletin of Symbolic Logic*, vol. 6, no. 3, pp. 265–283, 2000.

[2] see also Alan Schoenfeld work on mathematical problem solving

[3] proposed by Plato and Penrose explains