Dual Decomposition Algorithm

$\bar{\theta}_i^{\lambda}(x_i) = \theta_i(x_i) + \sum_{F: i \in F} \lambda_{Fi}(x_i)$ $\quad$ $\bar{\theta}_F^{\lambda} = \theta_F(x_F) - \sum_{i \in F} \lambda_{Fi}(x_i)$

x_i $\quad$ F

1. Initialize all λ's to be 0
2. Repeat for $t = 1, 2, \ldots$
   - Locally optimize all slaves $\quad x_F^* = \arg\max_{x_F} \bar{\theta}_F^{\lambda}(x_F)$ $\quad x_i^* = \arg\max_{x_i} \bar{\theta}_i^{\lambda}(x_i)$
   - For all F and all $i \in F$
     - If $x_F^*(i) \neq x_i^*$ then

$\alpha_t > 0$

$\lambda_{Fi}(x_i^*) := \lambda_{Fi}(x_i^*) - \alpha_t$ ← decrease preference

$\lambda_{Fi}(x_{Fi}^*) := \lambda_{Fi}(x_{Fi}^*) + \alpha_t$ ← agree

If slaves agree is a map assignment

Otherwise solve the decoding problem x^*

- Heuristics
  - Decomposition into spanning trees
  - Voting of slaves
  - Weighted average
  - Score is easy to evaluate