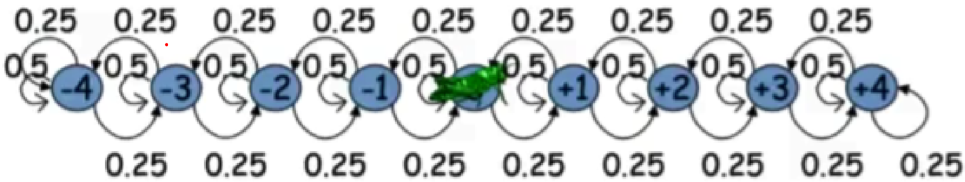
Markov Chain Monte Carlo

Method for sampling from a distribution p that is intractable to sample from

Markov Chain



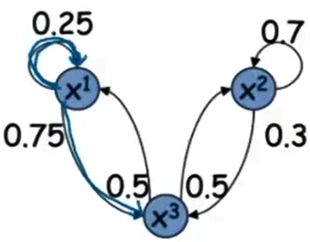
$T(x \to x')$ $\quad \sum_{x'} T(x \to x') = 1$

Temporal Dynamics

$$p^{(t+1)}(X^{(t+1)} = x') = \sum_{x} p^{(t)}(X^{(t)} = x)\, T(x \to x')$$

$$p^{(t)}(x') \approx p^{(t+1)}(x') = \sum_{x} p^{(t)}(x)\, T(x \to x')$$

$$\pi(x') = \sum_{x} \pi(x)\, T(x \to x')$$



$\pi(x^1) = 0.25\pi(x^1) + 0.5\pi(x^3)$

$\pi(x^2) = 0.7\pi(x^2) + 0.5\pi(x^3)$

$\pi(x^3) = 0.75\pi(x^1) + 0.3\pi(x^2)$

$\pi(x^1) + \pi(x^2) + \pi(x^3) = 1$

$\pi(x^1) = 0.2$

$\pi(x^2) = 0.5$

$\pi(x^3) = 0.3$

Regular Markov Chains

- A Markov chain is regular if there exists k such that for every x, x' the probability of getting from x to x' in exactly k steps is > 0
- Theorem: A regular Markov chain converges to a unique distribution regardless of start state
- Sufficient conditions for regularity
  - Every two states are connected
  - For every state there is a self-transition