Dual decomposition

Problem Formulation

- Singleton Factors $\theta_i(x_i) \rightarrow$ single variable
- Large factors $\theta_F(x_F) \rightarrow$ multiple variables

$$MAP(\theta) = \max_x \left(\sum_{i=1}^{n} \theta_i(x_i) + \sum_F \theta_F(x_F) \right)$$

1. $$\sum_{i=1}^{n} \max \theta_i(x_i) + \sum_F \max_{x_F} \theta_F(x_F)$$ local decision making

Divide and conquer

2. Force Agreement by introducing costs

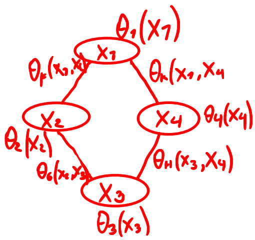
$$= \max_x \left(\sum_{i=1}^{n} \left(\theta_i(x_i) + \underbrace{\sum_{F: i \in F} \lambda_{Fi}(x_i)}_{i \text{ slave}} \right) + \underbrace{\sum_F \left(\theta_F(x_F) - \sum_{i \in F} \lambda_{Fi}(x_i) \right)}_{F \text{ slave}} \right)$$

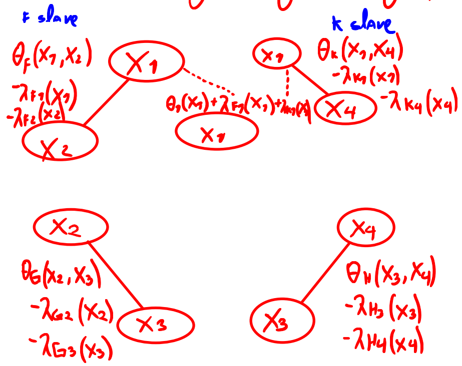
Agree with i slaves

$$L(\lambda) = \sum_{i=1}^{n} \max_{x_i} \left(\theta_i(x_i) + \sum_{F: i \in F} \lambda_{Fi}(x_i) \right) + \sum_F \max_{x_F} \left(\theta_F(x_F) - \sum_{i \in F} \lambda_{Fi}(x_i) \right)$$

$\theta_i^{-\lambda}$ penalty messages $\theta_F^{-\lambda}$

$L(\lambda)$ is upper bound on $MAP(\theta)$ for any setting of λ's





Divide and Conquer

- Slaves don't have to be factors in original model
  - Subsets of factors that admit tractable solution to local maximization task

$$X_1 - X_2 - X_3$$

$\theta_F(x_1,x_2)+\theta_G(x_2,x_3)$ FG slave

$-\lambda_{FG1}(x_1)-\lambda_{FG2}(x_2)-\lambda_{FG3}(x_3)$

$$X_1 - X_4 - X_3$$

$\theta_K(x_1,x_4)+\theta_H(x_3,x_4)$

$-\lambda_{KH1}(x_1)-\lambda_{KH3}(x_3)-\lambda_{KH4}(x_4)$

Large Factors

Singleton slaves

X_1

$\theta_1(x_1)+\lambda_{FG1}(x_1)+\lambda_{KH1}(x_1)$

X_2

$\theta_2(x_2)+\lambda_{FG2}(x_2)$

⋮

1. Divide the factors in set of disjoint regions
2. Other tractable classes