Product → Summation

$$P_\Phi(X) \propto \prod_K \phi_K(D_K)$$

$$\text{Argmax} \prod_K \phi_K(D_K)$$

$\downarrow \log \phi_K$

$$\text{Argmax} \underbrace{\sum_K \theta_K(D_K)}_{\theta(X_1,\dots,X_m)}$$

a^1	b^1	8
a^1	b^2	1
a^2	b^1	0.5
a^2	b^2	2

$\log_2$

a^1	b^1	3
a^1	b^2	0
a^2	b^1	-1
a^2	b^2	1

$$\max_D \max_C \max_B \max_A \left(\theta_1(A,B) + \theta_2(B,C) + \theta_3(C,D) + \theta_4(D,E)\right)$$

$$\max_D \max_C \max_B \left(\theta_2(B,C) + \theta_3(C,D) + \theta_4(D,E) + \underbrace{\max_A \theta_1(A,B)}_{\lambda_1(B)}\right)$$

Factor Summation

a^1	b^1	3
a^1	b^2	0
a^2	b^1	-1
a^2	b^2	1

b^1	c^1	4
b^1	c^2	1.5
b^2	c^1	0.2
b^2	c^2	2

a^1	b^1	c^1	3+4=7
a^1	b^1	c^2	3+1.5=4.5
a^1	b^2	c^1	0+0.2=0.2
a^1	b^2	c^2	0+2=2
a^2	b^1	c^1	-1+4=3
a^2	b^1	c^2	-1+1.5=0.5
a^2	b^2	c^1	1+0.2=1.2
a^2	b^2	c^2	1+2=3

Factor Maximization

max-marginalization

a^1	b^1	c^1	7
a^1	b^1	c^2	4.5
a^1	b^2	c^1	0.2
a^1	b^2	c^2	2
a^2	b^1	c^1	3
a^2	b^1	c^2	0.5
a^2	b^2	c^1	1.2
a^2	b^2	c^2	3

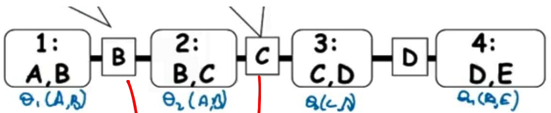
a^1	c^1	7
a^1	c^2	4.5
a^2	c^1	3
a^2	c^2	3

$$\lambda_4(e) = \max_{a,b,c,d} \theta(a,b,c,d,e)$$

Best value that I can get if we mandate $E=e$

= max marginal

Daphne Koller



max-sum instead of sum-product

$\lambda_{1\to2}(B) = \max_A \theta_1$ $\quad$ $\lambda_{2\to3}(C) = \max_B(\theta_2 + \lambda_{1\to2})$

$\theta_1(A,B)$

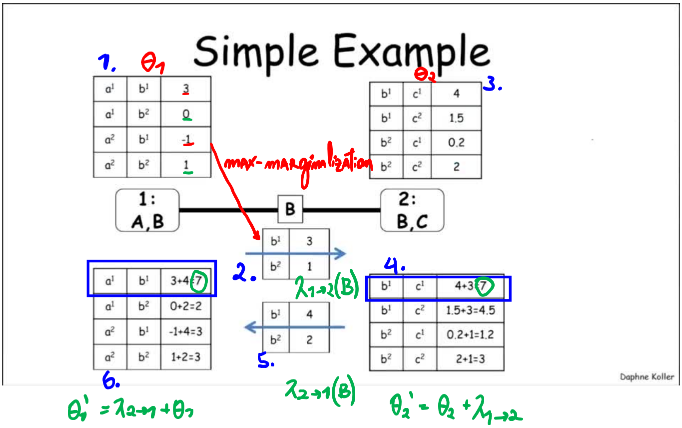
a^1	b^1	3
a^1	b^2	0
a^2	b^1	-1
a^2	b^2	1

$\theta_2(B,C)$

b^1	c^1	4
b^1	c^2	1.5
b^2	c^1	0.2
b^2	c^2	2

Map assignment

a^1	b^1	c^1	$3+4 = \boxed{7}$
a^1	b^1	c^2	$3+1.5 = 4.5$
a^1	b^2	c^1	$0+0.2 = 0.2$
a^1	b^2	c^2	$0+2 = 2$
a^2	b^1	c^1	$-1+4 = 3$
a^2	b^1	c^2	$-1+1.5 = 0.5$
a^2	b^2	c^1	$1+0.2 = 1.2$
a^2	b^2	c^2	$1+2 = 3$



Map assignment = $a_1\ b_1$

$\beta_i(C_i) = \theta_i(C_i) + \underbrace{\sum_k \lambda_{k\to i}}_{\text{All incomming messages}}$ $\qquad W_i = \{X_1, \dots, X_n\} - C_i$

7

$B_i(C_i) = \max_{W_i} \theta(C_i, W_i)$ → max assignment to variables not assigned to the clique
max-marginal

Belief of that clique

- Calibration: cliques agree on shared variables

$$\max_{C_i \to S_{i,j}} \beta_i(C_i) = \max_{C_j \to S_{i,j}} \beta_j(C_j)$$

Assignment

a^1	b^1	3+4=7
a^1	b^2	0+2=2
a^2	b^1	-1+4=3
a^2	b^2	1+2=3

$b^1 = 7$
$b^2 = 3$

$\beta_1 = \theta_1 + \lambda_{2\to1}$

Assignment

$b_1 = 7$
$b^2 = 3$

b^1	c^1	4+3=7
b^1	c^2	1.5+3=4.5
b^2	c^1	0.2+1=1.2
b^2	c^2	2+1=3

$\beta_2 = \theta_2 + \lambda_{1\to2}$

- The same clique tree algorithm used for sum-product can be used for max-sum
- As in sum-product, convergence is achieved after a single up-down pass
- Result is a max-marginal at each clique C:
  - For each assignment c to C, what is the score of the best completion to c