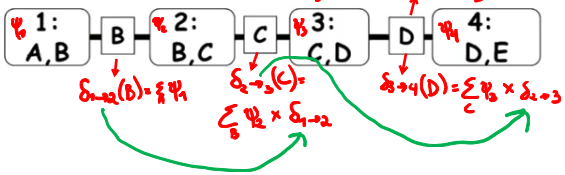
Message Passing in Trees

Pairwise potentials $\delta_{3\to2}(C)=\sum_B \psi_3 \delta_{4\to3}(D)=\sum_E \psi_4$



CORRECTNESS

$\beta_3(C,D)=\psi_3 \times \delta_{2\to3} \times \delta_{3\to4}$

$=\psi_3 \times (\sum_B \psi_2 \times \delta_{1\to2}) \times \sum_E \psi_4$

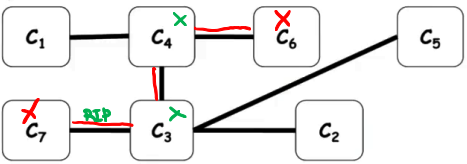
$=\psi_3 \times (\sum_B \psi_2 \times \sum_A \psi_1) \times \sum_E \psi_4$

Product of factors
marginalize out unnecessary variables
Variable elimination
Legal order of operations

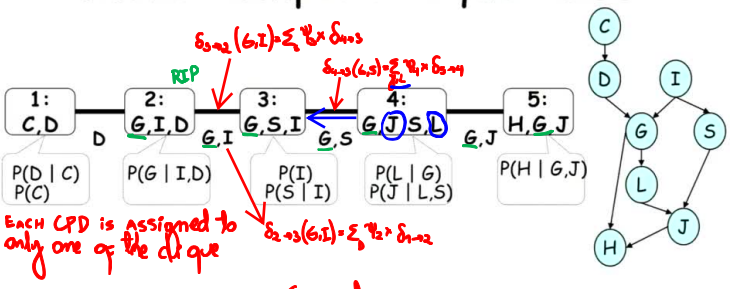
Clique Tree
- Undirected tree each that:
  - Nodes are clusters $C_i \subseteq \{X_1,\dots,X_n\}$
  - Edge between C_i and C_j associated with sepset $S_{i,j}=C_i \cap C_j$ equals

Family Preservation
- Given set of factors Φ, we assign each ϕ_k to a cluster $C_{\alpha(k)}$ s.t. $Scope[\phi_k] \subseteq C_{\alpha(k)}$
- For each factor $\phi_k \in \Phi$, there exists a cluster C_i s.t. $Scope[\phi_k] \subseteq C_i$

Running Intersection Property
- For each pair of clusters C_i, C_j and a variable $X \in C_i \cap C_j$ there exists a unique path between C_i and C_j for which all cluster and sepsets contain X



More Complex Clique Tree



Clique Tree Message Passing Correctness

- If X is eliminated when we pass the message $C_i \to C_j$
- Then X does not appear in the C_j side of the tree
- RIP relates to the order variables are eliminated

Daphne Koller