第二章：随机信号与系统：

1. 随机信号（序列）通过线性时不变系统：时域频域的分析方法

$$y(t)=x(t)*h(t)=\int_{-\infty}^{\infty}h(\tau)x(t-\tau)d\tau，\ Y(\omega)=H(j\omega)X(\omega)$$

$$S_y(\omega)=\int_{-\infty}^{\infty}R_y(\tau)e^{-j\omega\tau}d\tau=\left|H(j\omega)\right|^2 S_x(\omega)$$

2. 平稳随机序列的参数模型：ARMA 模型，AR 模型，MA 模型：自相关函数，功率谱密度函数，三种模型之间的联系，模型的建立

$$y(n)=\sum_{k=0}^{q}b_k w(n-k)-\sum_{k=1}^{p}a_k y(n-k) \qquad \text{滑动平均分量+自回归分量}$$

3. 随机信号的复表示：正弦型信号的复表示，任意信号的复表示

第三章：信号检测：判断是否存在信号或者存在哪个信号问题：假设检验问题处理

1. 几种准则下的判决规则都具有如下似然比检验形式：$\lambda(x)=\dfrac{f(x|H_1)}{f(x|H_0)}\underset{H_0}{\overset{H_1}{\gtrless}}th$

最大后验概率准则：$P(H_1|x)\underset{H_0}{\overset{H_1}{\gtrless}}P(H_0|x)$，$th=\dfrac{P(H_0)}{P(H_1)}$

最小错误概率准则：$\bar{P}_e=P(H_0)P(D_1|H_0)+P(H_1)P(D_0|H_1)\xrightarrow{R0,R1}\min$，$th=\dfrac{P(H_0)}{P(H_1)}$

$$P(D_1|H_0)=\int_{R_1}f(x|H_0)dx$$

贝叶斯准则：

$$\bar{C}=P(H_0)\left[C_{00}P(D_0|H_0)+C_{10}P(D_1|H_0)\right]+P(H_1)\left[C_{01}P(D_0|H_1)+C_{11}P(D_1|H_1)\right]\to\min$$

$$th=\frac{P(H_0)(C_{10}-C_{00})}{P(H_1)(C_{01}-C_{11})}$$

极小极大准则：

$$\bar{C}(p,p_1)=C_{00}\,p+C_{11}(1-p)+(C_{10}-C_{00})\alpha(p_1)p+(C_{01}-C_{11})\beta(p_1)(1-p)\ge\bar{C}(p,p)=\bar{C}_{\min}(p)$$

$$p_1=\arg\max_{p}\bar{C}_{\min}(p),\quad \frac{\partial\bar{C}(p,p_1)}{\partial p}=0,\quad th=\frac{p_1(C_{10}-C_{00})}{(1-p_1)(C_{01}-C_{11})}$$

纽曼-皮尔逊准则：　$\min P(D_0|H_1)\quad s.t.\ P(D_1|H_0)=\alpha$，$th$ 由给定的虚警概率确定。

2. 对于 M 种假设的假设检验问题：

$$\bar{C}=\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}C_{ij}P\left(D_i\left|H_j\right.\right)P\left(H_j\right)\overset{R_0,R_1,\cdots,R_{M-1}}{\rightarrow}\min$$

当 $C_{ii}=0, C_{ij}=1, i\neq j$：似然比检验形式 $\lambda\left(x\right)=\dfrac{f\left(x\left|H_i\right.\right)}{f\left(x\left|H_j\right.\right)}\overset{H_i}{\geq}\dfrac{P\left(H_j\right)}{P\left(H_i\right)}$，$j=0,\cdots,M-1,\neq i$

3. 多样本的假设检验问题：

$$\lambda(\mathbf{x})=\frac{f(\mathbf{x}|H_1)}{f(\mathbf{x}|H_0)}=\frac{f\left(x_1,x_2,\cdots x_N\left|H_1\right.\right)}{f\left(x_1,x_2,\cdots x_N\left|H_0\right.\right)}\underset{H_0}{\overset{H_1}{\gtrless}}\frac{P(H_0)(C_{10}-C_{00})}{P(H_1)(C_{01}-C_{11})}=th$$

4. 判决性能的计算

根据判决规则，确定检验统计量，计算检验统计量在两种假设下的概率密度函数。

$$P\left(D_1\left|H_1\right.\right)=\int_{th}^{+\infty}f\left(\lambda\left|H_1\right.\right)d\lambda$$

5. 复合的假设检验问题

$$\lambda\left(\mathbf{x}\right)=\frac{f(\mathbf{x}|H_1)}{f(\mathbf{x}|H_0)}=\frac{\int_{(\mathbf{\Theta})}f\left(\mathbf{x}|\mathbf{\Theta},H_1\right)f_1\left(\mathbf{\Theta}\right)d\mathbf{\Theta}}{\int_{(\mathbf{\Phi})}f\left(\mathbf{x}|\mathbf{\Phi},H_0\right)f_0\left(\mathbf{\Phi}\right)d\mathbf{\Phi}}\underset{H_0}{\overset{H_1}{\gtrless}}\frac{\left(C_{10}-C_{00}\right)P\left(H_0\right)}{\left(C_{01}-C_{11}\right)P\left(H_1\right)}=th$$

6．高斯白噪声中已知信号的检测： 根据似然比判决设计最佳接收机，计算系统性能。

$$f\left(x\left(t\right)\left|H_i\right.\right)=\lim_{N\to\infty}f\left(x_1,x_2,\cdots,x_N\left|H_i\right.\right)=F\exp\left\{-\frac{1}{N_0}\int_0^T\left[x\left(t\right)-s_i\left(t\right)\right]^2dt\right\}$$

$$\lambda\left(x\left(t\right)\right)=\underset{H_0}{\overset{H_1}{\gtrless}}th\Rightarrow\int_0^T\left[s_1\left(t\right)-s_0\left(t\right)\right]x\left(t\right)dt\underset{H_0}{\overset{H_1}{\gtrless}}\frac{N_0}{2}\ln th+\frac{1}{2}\int_0^T\left[s_1^2\left(t\right)-s_0^2\left(t\right)\right]dt$$

通信接收机（最小错误概率准则）：

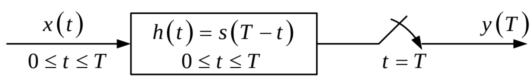
$$P_e=1-\Phi\left(\sqrt{\left(1-\rho\right)E/N_0}\right),\quad \rho=\frac{1}{E}\int_0^T s_0\left(t\right)s_1\left(t\right)dt,\quad E=\frac{1}{2}\left(E_0+E_1\right)$$

雷达接收机（NP 准则）：$P_D=1-\Phi\left(\Phi^{-1}\left(1-\alpha\right)-\sqrt{2E_1/N_0}\right)$

7. 匹配滤波器

$t=t_0$ 时刻的瞬时输出信噪比 $SNR_o=\dfrac{\left|s_o\left(t_0\right)\right|^2}{E\left\{n_o^2\left(t\right)\right\}}\leq\dfrac{E}{N_0/2}$ 。$H\left(j\omega\right)=S^*\left(\omega\right)e^{-j\omega t_o}$ 。

$$y\left(T\right)=\int_0^T x\left(t\right)s\left(t\right)dt$$



8. 信号的分集接收：设计最佳接收机，计算系统性能

$$\lambda(\mathbf{x}(t))=\frac{f\left(x_1(t),\cdots,x_M(t)\middle|H_1\right)}{f\left(x_1(t),\cdots,x_M(t)\middle|H_0\right)}=\prod_{i=1}^{M}\frac{f\left(x_i(t)\middle|H_1\right)}{f\left(x_i(t)\middle|H_0\right)}=\prod_{i=1}^{M}\lambda\left(x_i(t)\right)\underset{H_0}{\overset{H_1}{\gtrless}}th$$

9. 高斯色噪声中已知信号的检测

预白化方法：

$$\xrightarrow[n(t)\text{为色噪声}]{x(t)=s(t)+n(t)}\boxed{\begin{matrix}h_1(t)\\ H_1(j\omega)\end{matrix}}\xrightarrow[n_1(t)\text{为白噪声}]{s_1(t)+n_1(t)}\boxed{\begin{matrix}h_2(t)\\ H_2(j\omega)\end{matrix}}\xrightarrow{\text{输出}}$$

$$H(j\omega)=H_1(j\omega)H_2(j\omega)=\frac{1}{S_n^+(\omega)}\frac{S^*(\omega)}{S_n^-(\omega)}e^{-j\omega T}=\frac{S^*(\omega)}{S_n(\omega)}e^{-j\omega T}$$

卡亨南-洛维展开：$\int_0^T R_n(t_1-t_2)f_j(t_2)dt_2=\lambda_j f_j(t_1)$，$0\le t_1\le T$

实信号下，$x(t)=\sum_k x_k f_k(t)$，$x_k=\int_0^T x(t)f_k(t)dt$，$x_k$ 间互不相关。

$$\lambda(x(t))=\lim_{N\to\infty}\lambda(x_1,x_2,\cdots,x_N)\underset{H_0}{\overset{H_1}{\gtrless}}th$$

$$\int_0^T x(t)\eta_1(t)dt-\int_0^T x(t)\eta_0(t)dt\underset{H_0}{\overset{H_1}{\gtrless}}th',\quad th'=\ln th+\frac{1}{2}\int_0^T s_1(t)\eta_1(t)dt-\frac{1}{2}\int_0^T s_0(t)\eta_0(t)dt$$

10. 高斯白噪声中随机相位信号的检测：设计最佳接收机，计算系统性能

$$\lambda(x(t))=\frac{f\left(x(t)\middle|H_1\right)}{f\left(x(t)\middle|H_0\right)}=\frac{\int_0^{2\pi}f\left(x(t)\middle|\theta,H_1\right)f(\theta)d\theta}{f\left(x(t)\middle|H_0\right)}=\exp\left(-\frac{A^2T}{2N_0}\right)I_0\left(\frac{2Aq}{N_0}\right)\underset{H_0}{\overset{H_1}{\gtrless}}th$$

判决规则：$q=\sqrt{a^2+b^2}\underset{H_0}{\overset{H_1}{\gtrless}}th'$ ------门限 th' 由 α 确定（NP 准则）

$$a=q\sin\theta_0=\int_0^T x(t)\cos\omega_c t dt$$

$$b=q\cos\theta_0=\int_0^T x(t)\sin\omega_c t dt$$

计算性能：$f\left(a,b\middle|\theta,H_i\right)\to f\left(q,\theta_0\middle|\theta,H_i\right)\to f\left(q\middle|\theta,H_i\right)\to f\left(q\middle|H_i\right)$

非相干频移键控信号的检测：设计最佳接收机，计算系统性能

$$\lambda\left(x(t)\right)=\frac{f\left(x(t)\middle|H_1\right)}{f\left(x(t)\middle|H_0\right)}=\frac{\int_0^{2\pi}f\left(x(t)\middle|\theta,H_1\right)f\left(\theta\right)d\theta}{\int_0^{2\pi}f\left(x(t)\middle|\phi,H_0\right)f\left(\phi\right)d\phi}=\frac{I_0\left(\dfrac{2Aq_1}{N_0}\right)}{I_0\left(\dfrac{2Aq_0}{N_0}\right)}\underset{H_0}{\overset{H_1}{\gtrless}}1$$

$$q_i=\left[\left(\int_0^T x(t)\sin\omega_i t\,dt\right)^2+\left(\int_0^T x(t)\cos\omega_i t\,dt\right)^2\right]^{1/2},\quad i=1,2,\quad q_i>0$$

第四章：参量估计

1. 估计准则

最大后验概率估计：$\hat{\theta}_{MAP}=\underset{\theta}{\arg\max}\, f\left(\theta\,|\,\mathbf{x}\right)=\underset{\theta}{\arg\max}\ln f\left(\theta\,|\,\mathbf{x}\right)$

最大似然估计：$\hat{\theta}_{ML}=\underset{\theta}{\arg\max}\, f\left(\mathbf{x}\,|\,\theta\right)=\underset{\theta}{\arg\max}\ln f\left(\mathbf{x}\,|\,\theta\right)$

最小均方误差估计：$E\left\{e^2\left(\hat{\theta}\right)\right\}=\int_{(\theta)}\int_{(\mathbf{x})}\left(\theta-\hat{\theta}\right)^2 f\left(\theta,\mathbf{x}\right)d\theta d\mathbf{x}\to\min$，$\hat{\theta}_{MS}=\int_{(\theta)}\theta f\left(\theta\,|\,\mathbf{x}\right)\mathrm{d}\theta$

线性最小均方误差估计：$E\left\{\left[\theta-\left(a+\sum_{k=1}^{N}b_k x_k\right)\right]^2\right\}\to\min$

$$\hat{\theta}_{LMS}=a+\mathbf{b}^T\mathbf{x}=E\left\{\theta\right\}+Cov\left\{\theta,\mathbf{x}\right\}Cov^{-1}\left\{\mathbf{x},\mathbf{x}\right\}\left[\mathbf{x}-E\left\{\mathbf{x}\right\}\right]$$

正交条件（充要）：$E\left\{\left(\theta-\hat{\theta}_{LMS}\right)\mathbf{x}^T\right\}=0$

最小平均绝对误差估计：$E\left\{\left|e\left(\hat{\theta}\right)\right|\right\}\to\min$，$\int_{-\infty}^{\hat{\theta}_{ABS}}f\left(\theta\middle|\mathbf{x}\right)d\theta=\int_{\hat{\theta}_{ABS}}^{\infty}f\left(\theta\middle|\mathbf{x}\right)d\theta$

贝叶斯估计：$E\left\{c\left(\hat{\theta}\right)\right\}\to\min$

最小二乘估计：线性观测模型下 $\hat{\theta}_{LS}=\underset{\hat{\theta}}{\arg\min}\left[\mathbf{x}-\mathbf{H}\hat{\boldsymbol{\theta}}\right]^T\left[\mathbf{x}-\mathbf{H}\hat{\boldsymbol{\theta}}\right]$，

$$\hat{\boldsymbol{\theta}}_{LS}=\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{x}=\mathbf{H}^{\#}\mathbf{x}$$

2. 多参量估计

$$\left[\frac{\partial}{\partial\theta_i}\ln f\left(\boldsymbol{\theta}\,|\,\mathbf{x}\right)\right]\Bigg|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_{MAP}}=0,\quad\left[\frac{\partial}{\partial\theta_i}\ln f\left(\mathbf{x}\,|\,\boldsymbol{\theta}\right)\right]\Bigg|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_{ML}}=0,\ i=1,2,\cdots,M$$

$$E\left\{\left[\boldsymbol{\theta}-\mathbf{a}-\mathbf{B}\mathbf{x}\right]^T\left[\boldsymbol{\theta}-\mathbf{a}-\mathbf{B}\mathbf{x}\right]\right\}\to\min$$

$$\hat{\boldsymbol{\theta}}_{LMS}=E\left\{\boldsymbol{\theta}\right\}+Cov\left\{\boldsymbol{\theta},\mathbf{x}\right\}Cov^{-1}\left\{\mathbf{x},\mathbf{x}\right\}\left[\mathbf{x}-E\left\{\mathbf{x}\right\}\right],\qquad E\left\{\left(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_{LMS}\right)\mathbf{x}^T\right\}=0$$

$$\hat{\boldsymbol{\theta}}_{LS}=\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{x}=\mathbf{H}^{\#}\mathbf{x}$$

3. 计算估计性能：$E\left\{\hat{\theta}\right\},E\left\{\left(\hat{\theta}-E\{\hat{\theta}\}\right)^2\right\},E\left\{(\theta-\hat{\theta})^2\right\}$

无偏性：$E\left\{\hat{\boldsymbol{\theta}}\right\}=\boldsymbol{\theta}$ 或 $E\left\{\hat{\boldsymbol{\theta}}\right\}=E\left\{\boldsymbol{\theta}\right\}$

有效性：无偏估计量的均方误差达到最小。对于确定单参量无偏估计量 $\hat{\theta}$，有

$$E\left\{\left(\hat{\theta}-\theta\right)^2\right\}=\left\{E\left\{\left[\frac{\partial}{\partial\theta}\ln f\left(\mathbf{x}\,|\,\theta\right)\right]^2\right\}\right\}^{-1}=-\left\{E\left\{\frac{\partial^2}{\partial\theta^2}\ln f\left(\mathbf{x}\,|\,\theta\right)\right\}\right\}^{-1}$$

第五章：波形估计

1. 连续维纳滤波器

$$E\left\{e^2\left(t\right)\right\}=E\left\{\left[g\left(t\right)-y\left(t\right)\right]^2\right\}=E\left\{\left[g\left(t\right)-\int_{-\infty}^{\infty}h\left(\tau\right)x\left(t-\tau\right)d\tau\right]^2\right\}\xrightarrow{h(t)}\min$$

线性最小均方误差估计的正交条件：$E\left\{e\left(t\right)x\left(\tau'\right)\right\}=0,\ \begin{cases}-\infty<\tau'<\infty & \text{非因果}\\ -\infty<\tau'\le t & \text{因果}\end{cases}$

维纳一霍夫方程：$R_{gx}\left(\eta\right)=\int_{-\infty}^{\infty}h\left(\lambda\right)R_x\left(\eta-\lambda\right)d\lambda,\ \begin{cases}-\infty<\eta<\infty & \text{非因果}\\ 0\le\eta<\infty & \text{因果}\end{cases}$

求解：$H\left(j\omega\right)=\dfrac{S_{gx}\left(\omega\right)}{S_x\left(\omega\right)}$ --------物理不可实现

$$H\left(s\right)=\frac{1}{S_x^+\left(s\right)}\left[\frac{S_{gx}\left(s\right)}{S_x^-\left(s\right)}\right]^+$$ ------- 物理可实现

$$E\left\{e^2\left(t\right)\right\}_{\min}=E\left\{e\left(t\right)g\left(t\right)\right\}=R_g\left(0\right)-\int_{-\infty}^{\infty}h\left(\lambda\right)R_{gx}\left(\lambda\right)d\lambda$$

2. 离散维纳滤波器

$$E\left\{e^2\left(k\right)\right\}=E\left\{\left[g\left(k\right)-y\left(k\right)\right]^2\right\}=E\left\{\left[g\left(k\right)-\sum_{i=-\infty}^{\infty}h\left(i\right)x\left(k-i\right)\right]^2\right\}\to\min$$

线性最小均方误差估计的正交条件：$E\left\{e\left(k\right)x\left(j\right)\right\}=0,\ \begin{cases}-\infty<j<\infty & \text{非因果}\\ -\infty<j\le k & \text{因果}\end{cases}$

维纳一霍夫方程：$\sum_{i=-\infty}^{+\infty} h(i) R_x(l-i) = R_{gx}(l), \begin{cases} -\infty < l < \infty & \text{非因果} \\ 0 \le l < \infty & \text{因果} \end{cases}$

求解：$H(z) = \frac{S_{gx}(z)}{S_x(z)}$-------物理不可实现

$$H(z) = \frac{1}{S_x^+(z)} \left[\frac{S_{gx}(z)}{S_x^-(z)} \right]^+ \text{------ 物理可实现}$$

对应有限样本的维纳滤波器：$\sum_{i=0}^{N-1} h(i) R_x(l-i) = R_{gx}(l), l = 0,1,\cdots, N-1$

$$\mathbf{R}_x \mathbf{h} = \mathbf{r}_{gx}，\ \mathbf{h} = \mathbf{R}_x^{-1} \mathbf{r}_{gx}$$