e.g. 1、2 在 ipad上.

e.g.3.

(1) $E\{e^{j\phi_i}e^{-j\phi_k}\} = \delta_{ik}$

$i\neq k$时 $E\{\cdot\cdot\}=E\{\cdot\}E\{\cdot\}$

$E\{e^{j\phi_i}\}=\int_0^{2\pi}\frac{1}{2\pi}e^{j\phi_i}d\phi_i = 0$

(2) $A_i\in\mathbb{R}$

$x_n = w_n + \sum_{i=1}^{L} A_i e^{jw_i n} e^{j\phi_i}$

$\vec{e}_n = \begin{bmatrix} e^{-jw_1 n} \\ \vdots \\ e^{-jw_L n}\end{bmatrix}$ $\quad \vec{\phi} = \begin{bmatrix} A_1 e^{j\phi_1} \\ \vdots \\ A_L e^{j\phi_L}\end{bmatrix} \in \mathbb{C}^{L\times 1} = \mathrm{diag}(A_1\cdots A_L)\begin{bmatrix} e^{j\phi_1} \\ \vdots \\ e^{j\phi_L}\end{bmatrix}$ $\cancel{= \mathrm{diag}(e^{j\phi_1}\cdots e^{j\phi_L})\begin{bmatrix} A_1 \\ \vdots \\ A_L\end{bmatrix}}$

notice: $e_n^\dagger e_n = L$

$x_n = w_n + e_n^\dagger \phi$

共轭的是被减

$R_x(m) = \cancel{E\{x(m)x^*(\}} E\{x_{n+m}x_n^*\}$

$= E\{(w_{n+m}+e_{n+m}^\dagger\phi)(w_n^* + \phi^\dagger e_n)\}$

$= \sigma_w^2\delta(m) + \cancel{L\sum_{i=1}^{L}|A_i|^2} E\{e_{n+m}^\dagger \phi\phi^\dagger e_n\}$

(w_n与ϕ独立, $Ew_n=0$. ∴交叉项=0)

$E\{\phi\phi^\dagger\} = \Lambda\cdot E\left\{\begin{bmatrix} e^{j\phi_1} \\ \vdots \\ e^{j\phi_L}\end{bmatrix}[e^{-j\phi_1}\cdots e^{-j\phi_L}]\right\}\cdot\Lambda$

$= \Lambda[\delta_{ik}]\Lambda$ (由(1))

$= \Lambda I\Lambda$

$= \Lambda^2 = \mathrm{diag}(A_1^2\cdots A_L^2)$

$R_x(m) = \sigma_w^2\delta(m) + e_{n+m}^\dagger\Lambda^2 e_n$

$= \sigma_w^2\delta(m) + \sum_{i=1}^{L} A_i^2 e^{jw_i m}$

$e_{n+m}^\dagger e_n = \sum_{i=1}^{L} e^{jw_i m}$

(3) $H(z) = a_0 + a_1 z^{-1} + \cdots + a_m z^{-m}$

$h(n) = a_0\delta(n) + a_1\delta(n-1) + \cdots + a_m\delta(n-m)$

$y_n = x_n * h(n) = \sum_{k=0}^{m} a_k x_{n-k} = \vec{a}^\dagger\vec{x}_n$

$\vec{a} = \begin{bmatrix} a_0 \\ \vdots \\ a_m\end{bmatrix}$ $\quad \vec{x}_n = \begin{bmatrix} x_n \\ \vdots \\ x_{n-m}\end{bmatrix} \in \mathbb{C}^{(m+1)\times 1}$

$E\{|y_n|^2\} = \cancel{E\{y_n^\dagger y_n\}}$

$\cancel{= E\{x_n^\dagger a a^\dagger x_n\}}$

$= E\{y_n y_n^\dagger\}$

$= E\{a^\dagger x_n x_n^\dagger a\}$

$= a^\dagger E\{x_n x_n^\dagger\} a$

$\vec{x}_n = \vec{w}_n + \begin{bmatrix} e_n^\dagger \\ \vdots \\ e_{n-m}^\dagger\end{bmatrix}\phi$ $\quad E_n = [e_n \cdots e_{n-m}]$

$= \vec{w}_n + [e_n\cdots e_{n-m}]^\dagger\phi \quad \in \mathbb{C}^{L\times(m+1)}$

$= \vec{w}_n + E_n^\dagger\phi$

$E\{x_n x_n^\dagger\} = E\{w_n w_n^\dagger\} + E\{E_n^\dagger\phi\phi^\dagger E_n\}$

$= \sigma_w^2\mathrm{diag}(1,\cdots 1) + E_n^\dagger\Lambda^2 E_n$

$= \sigma_w^2 I + E_n^\dagger\Lambda^2 E_n$

$= \sigma_w^2 I + F^\dagger D_n^\dagger\Lambda^2 D_n F$

$= \sigma_w^2 I + F^\dagger\Lambda^2 F$

$\therefore E\{|y_n|^2\} = \sigma_w^2 a^\dagger a + a^\dagger F^\dagger\Lambda^2 F a$

$= \sigma_w^2\sum_{i=0}^{m}|a_i|^2 + \sum_{i=1}^{L}A_i^2\left|\sum_{k=0}^{m} a_k e^{-jw_i k}\right|^2$

$= \sigma_w^2\sum_{i=0}^{m}|a_i|^2 + \sum_{i=1}^{L}A_i^2\left|\sum_{k=0}^{m} a_k e^{-jw_i k}\right|^2$

$|(Fa)_i|^2$

$$E_n = [e_n \cdots e_{n-M}] \in \mathbb{C}^{L\times(M+1)}$$

$$= \begin{bmatrix} e^{-j\omega_1 n} & \cdots & e^{-j\omega_1(n-M)} \\ \vdots & & \vdots \\ e^{-j\omega_L n} & \cdots & e^{-j\omega_L(n-M)} \end{bmatrix}$$

$$= \begin{bmatrix} e^{-j\omega_1 n} & & \\ & \ddots & \\ & & e^{-j\omega_L n} \end{bmatrix} \begin{bmatrix} 1, & e^{j\omega_1}, & \cdots, & e^{j\omega_1 M} \\ \vdots & & & \vdots \\ 1, & e^{j\omega_L}, & \cdots, & e^{j\omega_L M} \end{bmatrix}$$

↑ Vandermonde

$$= D_n F$$

notice $D_n^{+} D_n = I$

Bayes

$$th = \frac{P(H_0)(C_{10}-C_{00})}{P(H_1)(C_{01}-C_{11})}$$

$C_{ij} = \delta_{ij}$

MAP. MEP

$$th = \frac{P(H_0)}{P(H_1)}$$

min max

$$th = \frac{P_0(C_{10}-C_{00})}{(1-P_0)(C_{01}-C_{11})}$$

NP

$$th = \mu = \lambda(th')$$

th'

$\alpha.\ \alpha_0$：虚警概

$\lambda($

设

if $P(H_0) = P(H_1) = \frac{1}{2}$

MAP $\Leftrightarrow$ Maximum Likelihood 准则

$$P(H_i|\vec{x}) = \frac{f(\vec{x}|H_i)P(H_i)}{f(\vec{x})}$$ 后验

$P(H_i)$ 先验

NP: Neyman-Pearson Criterion

纽曼(内曼、奈曼)-皮尔逊

雷达系统：希望降低 P_{fa}

通信系统： --- P_e.

单样本	多样本
x	$\vec{x}$

二元 $M=2$; H_0, H_1

非完全 Γ function

$$\Gamma(x,s) = \int_x^{+\infty} t^{s-1} e^{-t} dt$$

$$\gamma(x,s) = \int^{x} t^{s-1} e^{-t} \ldots$$

e.g.4 H_0 $x(t)=n(t)$

H_1 $x(t)=s(t)+n(t)$

$s(t),\ n(t)$ $\mu=0$ 窄带 Guass

(1)

$$\begin{cases} n(t)=a_n(t)\cos\omega_0 t-b_n(t)\sin\omega_0 t \\ s(t)=a_s(t)\cos\omega_0 t-b_s(t)\sin\omega_0 t \end{cases}$$

$x(t)=a(t)\cos\omega_0 t-b(t)\sin\omega_0 t$

$x=\sqrt{a^2+b^2}$ Rayleigh

$$\begin{cases} H_0: & a\sim N(0,\sigma_0^2),\ b\sim N(0,\sigma_0^2),\ \sigma_0^2=\sigma_n^2 \\ H_1: & a\sim N(0,\sigma_1^2)\ \ b\sim N(0,\sigma_1^2) \end{cases}$$

$\sigma_1^2=\sigma_0^2+\sigma_s^2$

Rayleigh:

$$f(x|H_i)=\frac{x}{\sigma_i^2}\exp\left\{-\frac{x^2}{2\sigma_i^2}\right\}$$

$$\therefore\ x^2\ \mathop{\gtrless}_{H_0}^{H_1}\ \ln\left(\frac{th(\sigma_n^2+\sigma_s^2)}{\sigma_n^2}\right)\frac{2(\sigma_s^2+\sigma_n^2)\sigma_n^2}{\sigma_s^2}\triangleq th'$$

see RMK)

$th'>0$ 时 $x\ \mathop{\gtrless}_{H_0}^{H_1}\ \sqrt{th'}$

RMK:

$$\lambda(x)=\frac{f(x|H_1)}{f(x|H_0)}>th$$

$$\ln\left[\frac{\sigma_0^2}{\sigma_1^2}\exp\left\{x^2\left(\frac{1}{2\sigma_0^2}-\frac{1}{2\sigma_1^2}\right)\right\}\right]>\ln th$$

$$x^2\frac{\sigma_1^2-\sigma_0^2}{2\sigma_1^2\sigma_0^2}>\ln\frac{th\sigma_1^2}{\sigma_0^2}$$

$$x^2\geqslant\frac{2\sigma_1^2\sigma_0^2}{\sigma_1^2-\sigma_0^2}\ln\frac{th\cdot\sigma_1^2}{\sigma_0^2}$$

不知,无法算

$$P(D_1|H_1)=P(\lambda\geqslant th|H_1)=\int_{th}^{+\infty}f(\lambda|H_1)\,d\lambda$$

$$=P(x\geqslant\sqrt{th'}|H_1)=\int_{\sqrt{th'}}^{+\infty}f(x|H_1)\,dx$$

$$\therefore P(D_1|H_1)=\exp\left\{-\frac{th'}{2\sigma_1^2}\right\}=P(D_1|H_0)^{\frac{\sigma_0^2}{\sigma_1^2}}$$

$$P(D_1|H_0)=\exp\left\{-\frac{th'}{2\sigma_0^2}\right\}$$

(2) $$\lambda(\vec{x})=\prod_{i=1}^{M}\lambda(x_i)\ \mathop{\gtrless}_{H_0}^{H_1}\ th$$

$$\therefore\ \sum_{i=1}^{M}x_i^2\ \mathop{\gtrless}_{H_0}^{H_1}\ \frac{2\sigma_1^2\sigma_0^2}{\sigma_1^2-\sigma_0^2}\ln\left[\frac{th\cdot(\sigma_1^2)^M}{\sigma_0^2}\right]\triangleq th'$$

$\|$

G

a_i,b_i iid

$$G=\sum_{i=1}^{M}(a_i^2+b_i^2)\sim\chi^2_{2M}$$ (标准化后)

$$\therefore f(G|H_i)=\frac{1}{\sigma_i^{2M}2^M\Gamma(M)}G^{M-1}\exp\left\{-\frac{G}{2\sigma_i^2}\right\}$$

$$P(D_1|H_0)=\int_{th'}^{+\infty}f(G|H_0)\,dG$$

$$=\int_{\frac{th'}{\sigma_n^2}}^{+\infty}\frac{G^{M-1}e^{-\frac{G}{2}}}{2^M\Gamma(M)}dG=\frac{1}{\Gamma(M)}\int_{\frac{th'}{2\sigma_n^2}}^{+\infty}\left(\frac{G}{2}\right)^{M-1}\times e^{-\frac{G}{2}}d\left(\frac{G}{2}\right)$$

$$=1-I\left(\frac{th'}{2\sqrt{M}\sigma_n^2},M-1\right)$$

同理:

$$P(D_1|H_1)=1-I\left(\frac{th'}{2\sqrt{M}\sigma_1^2},M-1\right)$$

Pearson 非完全Γ函数

不完全Γ函数的 Pearson 形式

e.g.5 $y=\sum_{i=1}^{n} x_i$, $x_i \overset{iid}{\sim} N(0,\sigma^2)$

$n \sim Poi(\lambda)$ $\quad P(n=k)=\frac{\lambda^k}{k!}e^{-\lambda}$, $k\in N$

$$\begin{cases} H_0: & n\geq 2 \\ H_1: & n\leq 1 \end{cases}$$

解

$$\lambda(y)=\frac{f(y|H_1)}{f(y|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} th$$

$\because x_i$ iid Guass

$y|n \sim N(0,(n+1)\sigma^2)$

$$f(y|H_1)=f(y|n\leq 1)$$

$$P(Y\leq y|n\leq 1)=\frac{P(Y\leq y, n\leq 1)}{P(n\leq 1)}$$

$$=\frac{\sum_{n\leq 1} P(n)P(Y\leq y|n)}{P(n\leq 1)}$$

$$\therefore f(y|H_1)=\frac{1}{P(n\leq 1)}\sum_{n\leq 1}P(n)f(y|n)$$

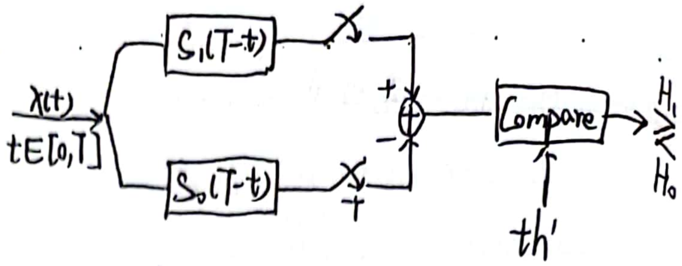
$$=\frac{1}{e^{-\lambda}(\lambda+1)}\left[e^{-\lambda}f(y|0)+\lambda e^{-\lambda}f(y|n=1)\right]$$

$$=\frac{1}{\lambda+1}\left[\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}+\frac{1}{\sqrt{2\pi}\sqrt{2}\sigma}e^{-\frac{x^2}{4\sigma^2}}\right]$$

$$f(y|H_0)=f(y|n\geq 2)$$

$$=\frac{1}{1-e^{-\lambda}(\lambda+1)}\cdot\sum_{k\geq 2}^{+\infty}\frac{\lambda^k}{k!}e^{-\lambda}f(y|n=k)$$

$\Rightarrow \lambda(y)$



匹配滤波器

M元通信系统:

$$G_i=\int_0^T x(t)s_i(t)\,dt, \quad i=0,\cdots,M-1$$

H_i: $x(t)=s_i(t)+n(t)$

$$Cov(G_kG_l|H_j)=E\{(G_k-\bar{G}_k)(G_l-\bar{G}_l)|H_j\}$$

$$\cancel{\bar{G}_k = E} \quad \bar{G}_k|H_j=E\int_0^T(s_j(t)+n(t))s_k(t)\,dt$$

$$=\int_0^T s_j(t)s_k(t)\,dt$$

$$(G_k-\bar{G}_k)|H_j=\int_0^T n(t)s_k(t)\,dt$$

$$\therefore Cov=E\left\{\int n(t_1)s_k(t_1)dt_1, \int n(t_2)s_l(t_2)dt_2\right\}$$

$$=\iint s_k s_l\, E(n(t_1)n(t_2))\,dt_1dt_2$$

WN: $R(t_1-t_2)=\delta(t$

$$=\int_0^T s_k(t)s_l(t)\,dt\cdot\frac{N_0}{2}$$

s_i 正交

$$=E\cdot\delta_{kl}\cdot\frac{N_0}{2}$$

e.g.6. noise 中类噪声检测问题

H_0: $x(t)=n(t)$

H_1: $x(t)=n(t)+s(t)$

$s \sim N(0,\frac{S_0}{2})$ $n\sim N(0,\frac{N_0}{2})$ 独立.

带限 $|w|<\Omega=2\pi B$

PSD: $\frac{S_0}{2}$. $\frac{N_0}{2}$ 设计似然比接收机.

解: H_0: $R_x(\tau)=R_n(\tau)$

H_1: $R_x(\tau)=R_n(\tau)+R_s(\tau)$

$R_s(\tau)=\frac{S_0\Omega}{2\pi}\cdot Sa(\Omega\tau)$

$R_n(\tau)=\frac{N_0\Omega}{2\pi}\cdot Sa(\Omega\tau)$

$\Delta t=\frac{\pi}{\Omega}$, $N=\frac{T}{\Delta t}=2BT$

$x_k|H_i \sim N(0,\sigma_i^2)\rightarrow f$

$\sigma_0^2=\frac{N_0\Omega}{2\pi}$ $\sigma_1^2=\frac{(N_0+S_0)\Omega}{2\pi}$

$$\lambda(\vec{x})=\prod_{k=1}^{N}\lambda(x_k)\ \overset{H_1}{\underset{H_0}{\gtrless}}\ th$$

$\cdot\frac{N_0}{2}$?

$$\sum_{k=1}^{N}x_k^2\ \overset{H_1}{\gtrless}\ 2BN_0(1+\frac{N_0}{S_0})\left[\ln th+BT\ln(1+\frac{S_0}{N_0})\right]\triangleq th'$$

$$\xrightarrow{x(t)} 采样 \rightarrow \sum_{k=1}^{N}x_k^2 \rightarrow \text{Compare} \rightarrow \overset{H_1}{\gtrless}$$

$$\uparrow$$

th'

e.g.7 $s(t)=\begin{cases}A, & t\in[0,T]\\ 0, & else\end{cases}$

(1) $SNR_o=\frac{E}{\frac{N_0}{2}}=\frac{2}{N_0}\int_0^T s(t)^2dt=\frac{2A^2T}{N_0}$

(2) $h(t)=\begin{cases}e^{-\alpha t}, & t\in[0,T]\\ 0, & else\end{cases}$

$s(t)=s(t)*h(t)=\int s(t-\tau)h(\tau)d\tau$

$\tau\in[0,T]$

$t-\tau\in[0,T]\rightarrow\tau\in[t-T,t]$

① $t\in[0,T]$, $\therefore\tau\in[0,t]$

$S_0(t)=\frac{A}{\alpha}(1-e^{-\alpha t})$ $t\uparrow$

② $t\in[T,2T]$ $\therefore\tau\in(t-T,T]$

$S_0(t)=\frac{A}{\alpha}[e^{-\alpha(t-T)}-e^{-\alpha T}]$ $t\downarrow$

$\max S_0(t)=S_0(T)=\frac{A}{\alpha}(1-e^{-\alpha T})$

output noise

$E\,n_0(t)^2=R_{n_0}(0)=\frac{1}{2\pi}\int S_{n_0}(\omega)d\omega$

$=\frac{1}{2\pi}\int\frac{N_0}{2}|H(j\omega)|^2d\omega$

$=\frac{N_0}{2}\int h(t)^2dt$

$=\frac{N_0}{4\alpha}(1-e^{-2\alpha T})$

$\max OSNR=\frac{S_0^2(T)}{E\,n_0(t)^2}=\frac{4A^2(1-e^{-\alpha T})}{\alpha N_0(1+e^{-\alpha T})}$

由 $\frac{d\,OSNR}{d\alpha}=0$ 得: $\alpha=0$

$OSNR\big|_{\alpha=0}=\frac{2A^2}{N_0}\cdot\lim_{\alpha\to0}\frac{1-e^{-\alpha T}}{\alpha}$

$=\frac{2A^2}{N_0}\cdot T$ = Match Filter

(3) $S_0(t)=s(t)*h(t)$

$\tau\geq0$

$t-\tau\in[0,T]\rightarrow\tau\in[t-T,t]$

① $t\in[0,T]$ $\therefore\tau\in[0,t]$

$S_0(t)=\frac{A}{\alpha}[1-e^{-\alpha t}]$ $t\uparrow$

② $t\in(T,+\infty)$ $\therefore\tau\in[t-T,t]$

$S_0(t)=\frac{A}{\alpha}[e^{\alpha T}-1]e^{-\alpha t}$ $t\downarrow$

$E\,n_0(t)^2=\frac{N_0}{2}\int^{+\infty}h(t)^2dt$

$=\frac{N_0}{4\alpha}$

$OSNR_{max}=\frac{4A^2}{N_0\alpha}(1-e^{-\alpha T})^2$

$\leq(\cdot\cdot)\times\frac{1}{1-e^{-2\alpha T}}=OSNR_{max}$ in

$\because(1-e^{-2\alpha T})<1$

(4) $S_0(t)=\int_{\frac{t-T-\tau_0}{\alpha}}^{\frac{t-\tau_0}{\alpha}}A\cdot e^{-\frac{z^2}{2}}dz$

$E\,n_0(t)^2=\frac{N_0}{2}\int_{-\infty}^{+\infty}h(t)^2dt$

$=\frac{N_0\sqrt{\pi}}{2\cdot}$

$$OSNR=\frac{S_0(t)^2}{E\,n_0(t)^2}=\frac{4\pi A^2\alpha}{N_0}\left[\Phi\left(\frac{t-\tau_0}{\alpha}\right)-\Phi\left(\frac{t-T-\tau_0}{\alpha}\right)\right]^2$$

白噪声中已知signal检

假设检验

max OSNR

实为这种检

max OSNR 即 max $S_0(t)$

由: $\frac{d}{dt}S_0(t)=0=\frac{1}{\alpha}\cdot A e^{-\frac{1}{2}\left(\frac{t-\tau_0}{\alpha}\right)^2}-\frac{A}{\alpha}e^{-\frac{1}{2}\left(\frac{t-T-\tau_0}{\alpha}\right)^2}$

得: $t=\tau_0+\frac{T}{2}$

色噪声

LRT

预白

K-L

max OSNR :

$$I_0(x)=\int_0^{2\pi}e^{x\cos(\theta-\alpha)}\frac{d\theta}{2\pi}$$

	H_0
雷达	n
通信	S_0+n

通信系统 随

$\lambda(x(t))=$ 死

e.g. 胎儿 ECG signal process

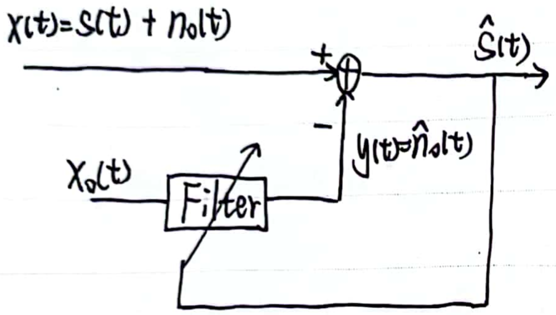
腹导信号：

$$x(t) = s(t) + \underbrace{\overset{\text{Mother ECG}}{m(t)} + \overset{\text{EMG}}{n(t)}}_{n_0(t)}$$

胸导信号：

$x_0(t)$ 用于估计 $n_0(t)$

$$\hat{s}(t) = x(t) - \hat{n}_0(t)$$



$$E(x(t)-y(t))^2 \approx E\,x(t)^2 + E(n_0(t)-y(t))^2$$

$$\therefore \min_{h(t)} E(n_0(t)-y(t))^2$$

☆ 正交条件 $E\{[x(t)-y(t)]\,x_0(t')\} = 0$

$$\because y(t) = h(t) * x_0(t) = \int_{-\infty}^{+\infty} h(\tau)\,x_0(t-\tau)\,d\tau$$

$$\therefore R_{xx_0}(\underbrace{t-t'}_{\Downarrow\ \eta}) = \int_{-\infty}^{+\infty} h(\tau)\,R_{x_0}(t-\tau-t')\,d\tau$$

$$R_{xx_0}(\eta) = h(\eta) * R_{x_0}(\eta)$$

$$H(j\omega) = \frac{S_{xx_0}(\omega)}{S_{x_0}(\omega)}$$ 物理不可实现

eg.8 GWN, GCN 混合检测 已知 $s(t)$

H_0: $x(t) = m(t) + n(t)$ $\quad t \in [0,T]$

H_1: $x(t) = s(t) + m(t) + n(t)$

$m(t)$ $R_m(\tau)$ 色 GN

$n(t)$ $R_n(\tau) = \frac{N_0}{2}\delta(\tau)$ 独立

NP

解: KL展开:

$$x(t) = \sum_k f_k(t) x_k \quad , x_k = \int_0^T x(t) f_k(t) dt$$

$$\int_0^T f_k(\tau) R_m(t-\tau) d\tau = \lambda_k f_k(t)$$

△ $\underline{x_k | H_1 \sim N(S_k, \frac{N_0}{2} + \lambda_k)}$

$$E(x_k|H_1) = E\int_0^T (s(t) + m(t) + n(t)) f_k(t) dt$$

$$= \int_0^T s(t) f_k(t) dt \triangleq S_k$$

$$Var(x_k|H_1) = E(x_k - \bar{x}_k)^2$$

$$= E\left[\int_0^T (m(t) + n(t)) f_k(t) dt\right]^2$$

$$= \iint R_m(t-\tau) f_k(t) f_k(\tau) dt d\tau \quad \text{由积分方程} \int_0^T \lambda_k f_k(t) f_k(t) dt$$

$$+ \frac{N_0}{2} \int_0^T f_k(t)^2 dt$$

$$= \left(\frac{N_0}{2} + \lambda_k\right) \int_0^T f_k(t)^2 dt$$

$$= \frac{N_0}{2} + \lambda_k$$

$$Cov(x_k, x_i | H_1) = E\{(x_k - \bar{x}_k)(x_i - \bar{x}_i) | H_1\}$$

$$= E\left\{\int_0^T [m(t) + n(t)] f_k(t) dt \cdot \int_0^T (m(\tau) + n(\tau)) f_i(\tau) d\tau\right\}$$

同上:

$$= \left(\frac{N_0}{2} + \lambda_i\right) \int_0^T f_k(t) f_i(t) dt$$

$$= \left(\frac{N_0}{2} + \lambda_i\right) \delta_{ik}$$

类似地:

△ $\underline{x_k | H_0 \sim N(0, \lambda_k + \frac{N_0}{2})}$

$$\lambda(x(t)) = \prod_{k=1}^{+\infty} \frac{f(x_k|H_1)}{f(x_k|H_0)} = \prod e^{-\frac{(x_k - S_k)^2}{2\sigma^2} + \frac{x_k^2}{2\sigma^2}}$$

$$= \exp\left\{\sum_k \frac{2x_k S_k - S_k^2}{2\lambda_k + N_0}\right\} \overset{H_1}{\gtrless} th$$

$$\triangleq e^G$$

$$G = \sum_k \frac{2S_k}{2\lambda_k + N_0} \int_0^T \left[x(t) - \frac{1}{2}s(t)\right] f_k(t) dt$$

$$\triangleq \int_0^T \left(x(t) - \frac{1}{2}s(t)\right) \eta(t) dt$$

☆ $$\int_0^T \eta(\tau) R_m(t-\tau) d\tau = \int_0^T \sum \frac{2S_k}{2\lambda_k + N_0} f_k(\tau) R_m(t-\tau)$$

$$= \sum \frac{2S_k \lambda_k}{2\lambda_k + N_0} f_k(t)$$

$$= \sum_k \left(1 - \frac{N_0}{2\lambda_k + N_0}\right) S_k f_k(t)$$

$$= s(t) - \frac{N_0}{2} \eta(t)$$

$$\therefore \int_0^T \eta(\tau) R_m(t-\tau) d\tau = s(t) - \frac{N_0}{2} \eta(t)$$

非齐次积分方程给出 $\eta(t)$ 形式.

规则: $G \overset{H_1}{\gtrless} \ln th$

e.g.9. 随相

H_0 : $x(t)=n(t)$

H_1 : $x(t)=Af(t)\sin(\omega_c t+\theta)+n(t)$

$\triangleq s(t)+n(t)$

$f(t)$ 慢变包络 $\qquad \theta\sim U[0,2\pi]$

$A,\ \omega_c$ const. $\qquad \omega_c \gg \frac{2\pi}{T}$

解

$$\lambda(x(t))=\frac{f(x(t)|H_1)}{f(x(t)|H_0)}$$

$$=\frac{1}{f(x(t)|H_0)}\cdot\int_0^{2\pi} f(x(t)|\theta,H_1)\cdot\frac{1}{2\pi}d\theta$$

$$=\frac{1}{2\pi}\cdot\int_0^{2\pi}\exp\left[-\frac{1}{N_0}\int_0^T[s(t)^2-2x(t)s(t)]dt\right]d\theta$$

$$\int_0^T s(t)^2dt=A^2\int_0^T f(t)^2\sin^2(\omega_c t+\theta)\,dt$$

$f(t)$ 慢变, $\omega_c\to+\infty$

$$\approx\frac{A^2}{2}\int_0^T f(t)^2dt$$

$$\triangleq\frac{A^2}{2}\cdot E$$

$$\int_0^T x(t)s(t)dt=A\cos\theta\int x(t)f(t)\sin\omega_c t\,dt+A\sin\theta\int x(t)f(t)\cos\omega_c t\,dt$$

$$=A(\cos\theta\cdot\underbrace{q\cos\theta_0}_{\triangleq b}+\sin\theta\cdot\underbrace{q\sin\theta_0}_{\triangleq a})$$

$$=Aq\cos(\theta-\theta_0)$$

$$\therefore\lambda(x(t))=\frac{1}{2\pi}\int_0^{2\pi}e^{-\frac{1}{N_0}\left[\frac{A^2}{2}E-2Aq\cos(\theta-\theta_0)\right]}d\theta$$

$$=e^{-\frac{A^2E}{2N_0}}\cdot I_0\left(\frac{2Aq}{N_0}\right)\ \overset{H_1}{\gtrless}\ th$$

$$\Leftrightarrow\quad q\ \overset{H_1}{\gtrless}\ th'$$

where. $q=\sqrt{(\int xf\sin)^2+(\int xf\cos)^2}$.

P159. q 3种计算方法 (3种结构)☆

$x(t)\to\boxed{f(T-t)\sin\omega_c t}\xrightarrow{y(t)}\boxed{\text{Envelope Detect}}\xrightarrow{l(t)}\ T\ \to$ compare $\to\ \overset{H_1}{\gtrless}$, th'

$h(t)=f(T-t)\sin\omega_c t$

$$y(t)=x(t)*h(t)=\int_0^T x(\tau)f(T-t+\tau)\sin\omega_c(t-\tau)d\tau$$

$$=\sin\omega_c t\cdot\underbrace{\int x(\tau)f(T-t+\tau)\cos\omega_c\tau\,d\tau}_{=q(t)\cos\theta_0(t)}$$

$$-\cos\omega_c t\cdot\underbrace{\int x(\tau)f(T-t+\tau)\sin\omega_c\tau\,d\tau}_{=q(t)\sin\theta_0(t)}$$

$$=q(t)\ \sin[\omega_c t-\theta_0(t)]$$

ED $\to q(t)$

取 $q(t)|_{t=T}=q$

☆ 思路: $f(a,b)\xrightarrow{\text{Jacobi}}f(q,\theta_0)\xrightarrow{\text{margin}}f(q)$ ☆ P156~161

随机性来自 $n(t),\theta$.

r.v.

$$\begin{cases}a=\int x(t)f(t)\cos\omega_c t\,dt\\ b=\int x(t)f(t)\sin\omega_c t\,dt\end{cases}$$

先求 $a|\theta,H_1$, $b|\theta,H_1$?

可得: $a|\theta,H_1\sim N(\frac{AE}{2}\sin\theta,\frac{N_0}{4}E)$

$b|\theta,H_1\sim N(\frac{AE}{2}\cos\theta,\frac{N_0}{4}E)$ P159 自己算

$Cov(a,b|\theta,H_1)\approx 0$ ∴条件独立

$$f(q,\theta_0|\theta,H_1)=f(a,b|\theta,H_1)\cdot\underbrace{q}_{|\text{Jacobi}|}.$$

$$f(q,\theta_0|H_1)=\int_0^{2\pi}f(q,\theta_0|\theta,H_1)\frac{1}{2\pi}d\theta$$

$$f(q|H_1)=\int_0^{2\pi}f(q,\theta_0|H_1)\,d\theta_0$$

$$=\iint f(q,\theta_0|\theta,H_1)\frac{d\theta}{2\pi}\cdot d\theta_0$$

$$=\frac{q}{\sigma^2}\exp\left[-\frac{q^2+(\frac{AE}{2})^2}{2\sigma^2}\right]I_0\left(\frac{AEq}{2\sigma^2}\right)$$

(Rice)

$$f(q|H_0)=\frac{q}{\sigma^2}e^{-\frac{q^2}{2\sigma^2}}\qquad\text{(Rayleigh)}$$

$$\therefore P(D_1|H_0)=\int_{th'}^{+\infty}f(q|H_0)dq$$

$=\alpha_0$ → 可得 th'

$$E\triangleq\int_0^T f(t)^2dt$$

$$J=\frac{\partial(a,b)}{\partial(q,\theta_0)}=\begin{vmatrix}\sin\theta_0 & q\cos\theta_0\\ \cos\theta_0 & -q\sin\theta_0\end{vmatrix}=-q$$

$$|J|=q$$