

# Divergence Frontiers for Generative Models: Sample Complexity, Quantization Level, and Frontier Integral

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# Team



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# Image and Text Generation

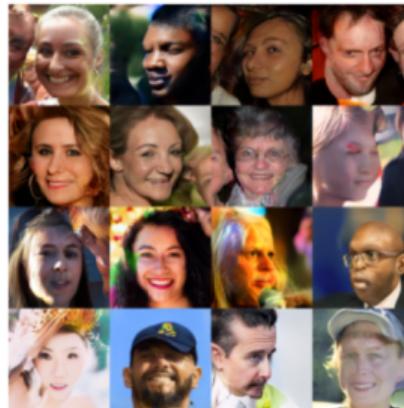
High quality but low variety



Kynkäanniemi et al. (2019)

Several people have asked about the techniques we used when cleaning out my mom's fabric stash last week.....  
Next, you need to get a **small, sharp knife**. I like to use a **small, sharp knife**. I like to use a **small, sharp knife**.

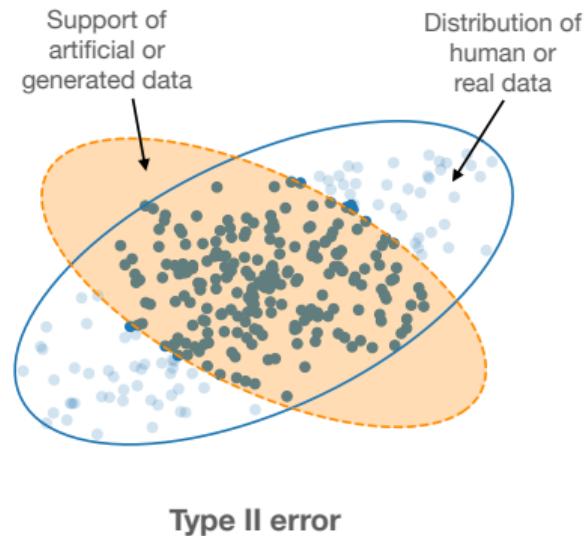
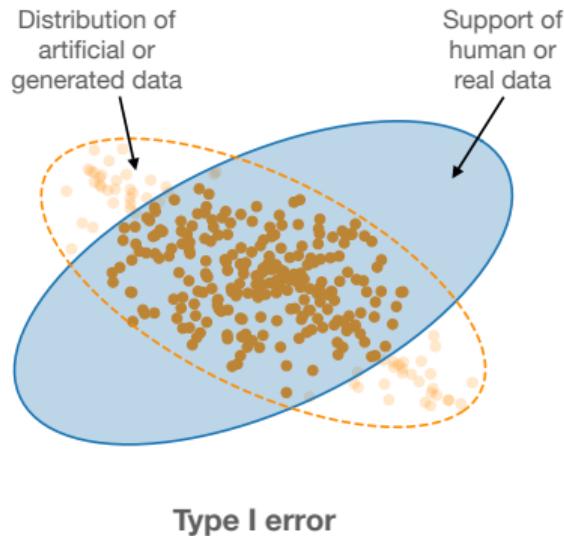
Low quality but high variety



Pillutla et al. (2021)

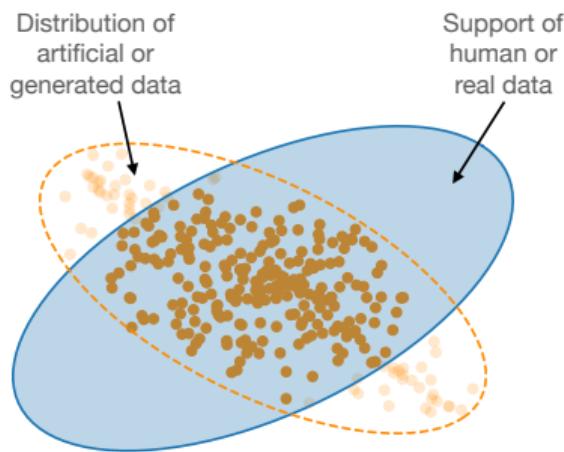
Several people have asked about the techniques we used when cleaning out my mom's fabric stash last week.....  
I had a great deal of **décor management** and was able to **stash the excess items away for safekeeping**.

# Type I and Type II Errors in Generative Modeling



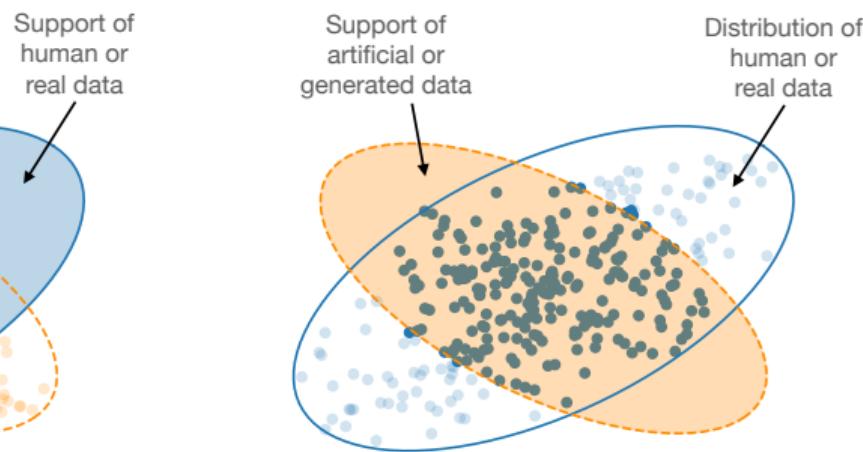
How to quantify them?

# Type I and Type II Errors in Generative Modeling



Type I error

$$\text{KL}(Q\|P)$$



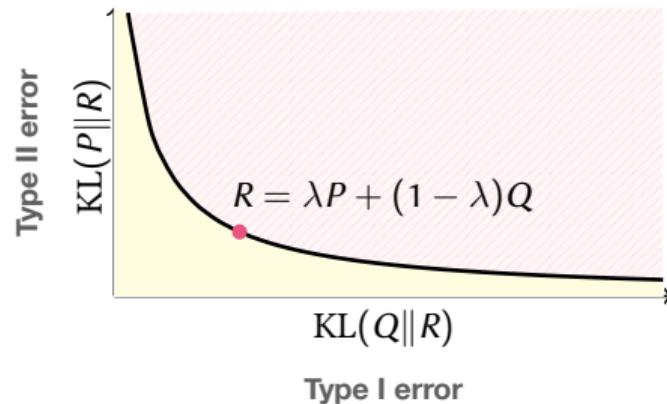
Type II error

$$\begin{aligned} P: & \text{real data distribution} \\ Q: & \text{generated data distribution} \end{aligned}$$

$$\text{KL}(P\|Q)$$

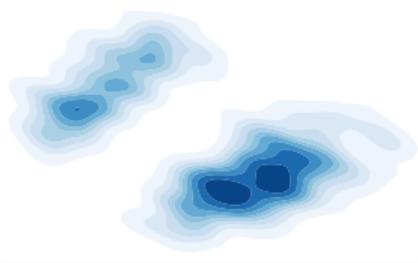
# Divergence Frontiers for Generative Models

- ▶ Divergence frontiers for data distribution  $P$  and model distribution  $Q$ .
- ▶ Applications in vision (Sajjadi et al. '18, Kynkäanniemi et al. '19, Djolonga et al. '20).
- ▶ Applications in NLP (Pillutla et al. '21; this NeurIPS).

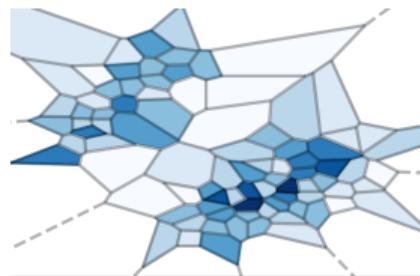


# Estimation Procedure of Divergence Frontiers

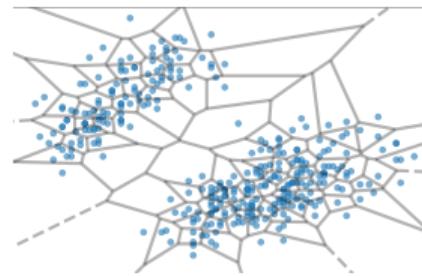
Continuous Distribution



Quantized Distribution

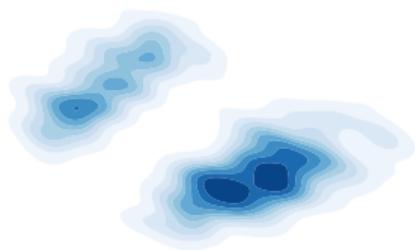


Empirical Estimator



# Estimation Procedure of Divergence Frontiers

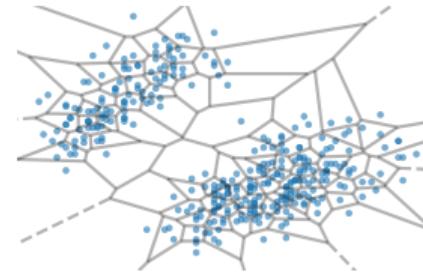
Continuous Distribution



Quantized Distribution



Empirical Estimator



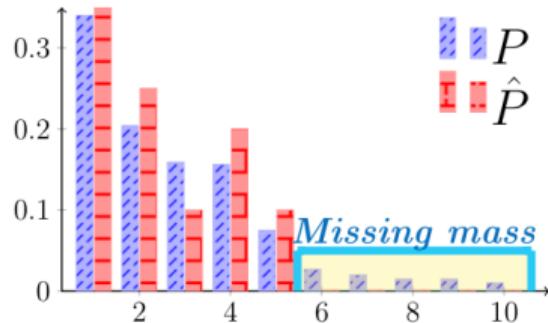
1. How to select the quantization level  $k$ ?

2. Can we do better than the naïve empirical estimator?

3. How many data points are needed to achieve a good accuracy?

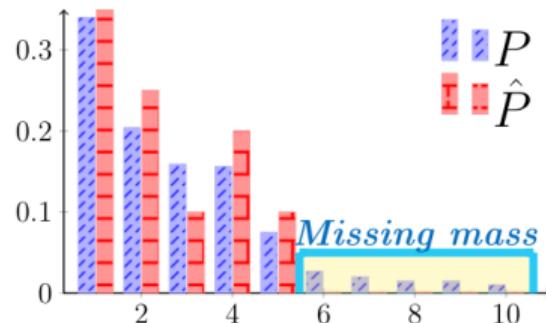
# Main Results

- ▶ Finite-sample bounds.
  - ▷ **Quantization level**  $k \propto O(n^{1/3})$ .
  - ▷ **Missing-mass adaptive smoothing** improves the estimation accuracy (e.g., add-constant and Good-Turing).
  - ▷ **Sample complexity**  $O(n^{-1/2} \log n)$ .
- ▶
- ▶



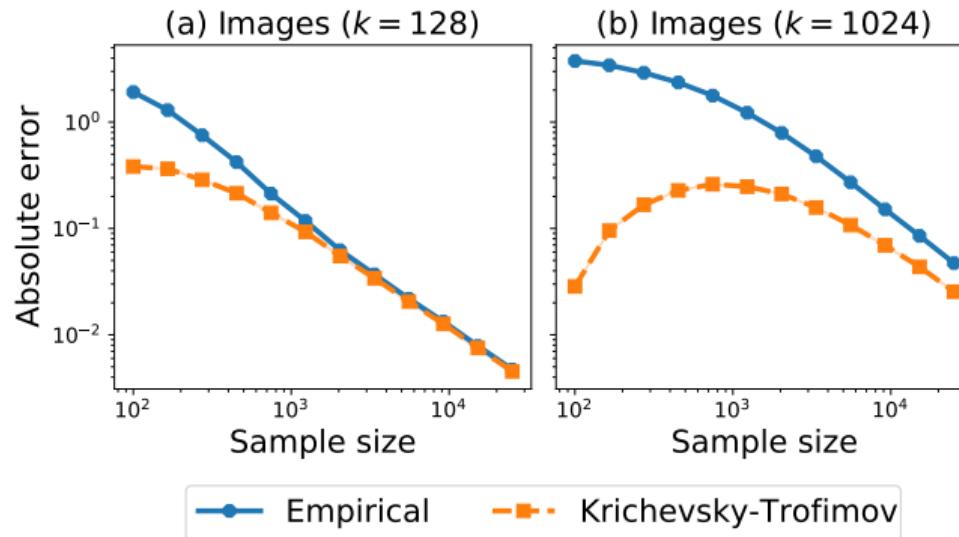
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- ▶ Statistical summary—**frontier integral**.
- ▶ Generalization to **f-divergences**.



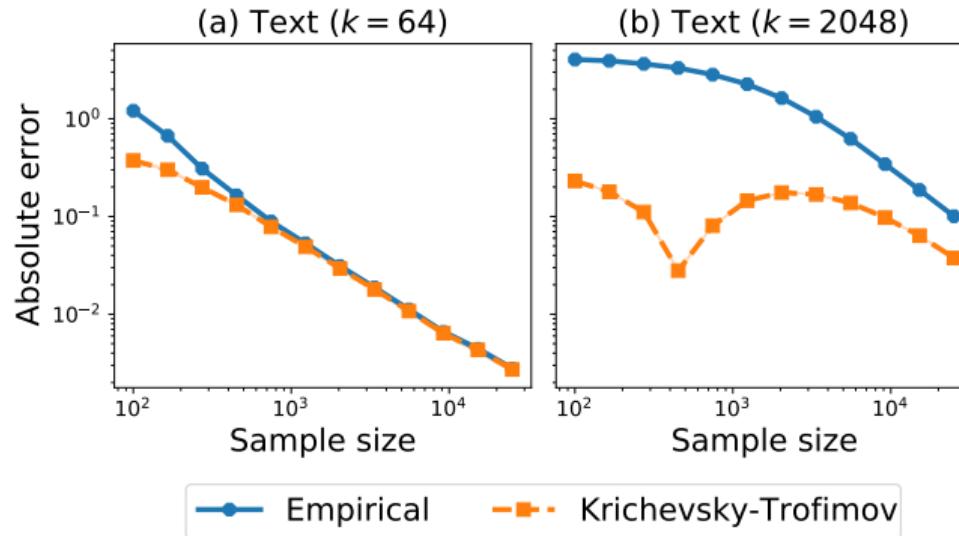
# Experimental Results

**Missing-mass adaptive smoothing improves the estimation accuracy.**



# Experimental Results

**Missing-mass adaptive smoothing improves the estimation accuracy.**



Thank You

Paper: [arxiv.org/abs/2106.07898](https://arxiv.org/abs/2106.07898)

Thank you!