CP.2.7.1.a, Sauer3

Implement Newton's Method with appropriate starting points to find all solutions. Check with EX.2.7.3 to make sure your answers are correct.

$$\text{(a)} \begin{cases} u^2 + v^2 &= 1 \\ (u-1)^2 + v^2 &= 1 \end{cases}$$

Hint from Sauer from his solution as in EX.2.7.3:

a. The curves are circles with radius 1 centered at $(u, v) = (0, 0)$ and $(1, 0)$, respectively. Solving the first equation for v^2 and substituing into the second yields $(u-1)^2 + 1 - u^2 = 1$ or $-2u + 1 = 0$, so $u = \frac{1}{2}$. The two solutions are

$$\begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} u_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix}.$$

Special Instructions:

a. Please first compute the two solutions using either Sauer's exact solutions (see Hint above) or `scipy.optimize.fsolve` or `scipy.optimize.root`. This will be useful to compute the forward error.

b. Start Newton's method from a relative distance (as measured by the infinity norm) of at least 0.1 from the solution.

c. At each iteration of Newton's method, you must print:

   (a) k, the iteration number

   (b) the absolute backward error at iteration number k defined by

   $$\|F(x_k)\|_\infty$$

   where x_k is the current iterate.

   (c) the relative forward error at iteration number k defined by

   $$\|x_k - x\|_\infty / \|x\|_\infty$$

   where x_k is the current iterate and x is the solution as computed by `scipy.optimize.fsolve` or `scipy.optimize.root`.

   You can also print the current iterate x_k if you want.