EX.0.2.4, Sauer

Convert the following base 10 numbers to binary. (a) 11.25 (b) 2/3 (c) 3/5 (d) 3.2 (e) 30.6 (f) 99.9.

EX.0.2.4, Sauer, solution, Langou

a. Either we write:

$$11.25 = 8 + 2 + 1 + \frac{1}{4} = 2^3 + 2^1 + 2^0 + 2^{-2}.$$

Or we use our technique:

$$\begin{array}{rcccc} 11/2 & = & 5 & R & 1 \\ 5/2 & = & 2 & R & 1 \\ 2/2 & = & 1 & R & 0 \\ 1/2 & = & 0 & R & 1 \end{array} \qquad \begin{array}{ccccc} 0.25 & * & 2 & = & 0.50 & \to 0 \\ 0.50 & * & 2 & = & 1.00 & \to 1 \\ 0.00 & * & 2 & = & 0.00 & \to 0 \\ \hline 0.00 & * & 2 & = & 0.00 & \to 0 \end{array}$$

$$(11)_{10} = (1011)_2 \qquad (0.25)_{10} = (0.01\overline{0})_2$$

We find:

$$(11.25)_{10} = (1011.01)_2$$

b.

$$\begin{array}{ccccc} \frac{2}{3} & * & 2 & = & 1\frac{1}{3} & \to 1 \\ \frac{1}{3} & * & 2 & = & 0\frac{2}{3} & \to 0 \\ \hline \frac{2}{3} & * & 2 & = & 1\frac{1}{3} & \to 1 \end{array}$$

We find:

$$(\frac{2}{3})_{10} = (0.\overline{10})_2$$

<u>Check:</u>

$$\begin{aligned} (0.\overline{10})_2 &= \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \dots \\ &= \frac{1}{2}\left(1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \dots\right) \\ &= \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n = \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{1}{2}\frac{4}{3} = \frac{2}{3} \end{aligned}$$

c.

$$\begin{array}{ccccc} .6 & * & 2 & = & 1.2 & \to 1 \\ .2 & * & 2 & = & 0.4 & \to 0 \\ .4 & * & 2 & = & 0.8 & \to 0 \\ .8 & * & 2 & = & 1.6 & \to 1 \\ \hline .2 & * & 2 & = & 0.4 & \to 1 \end{array}$$

We find:

$$(\frac{3}{5})_{10} = (0.\overline{1001})_2$$

<u>Check:</u>

$$\begin{aligned}
(0.\overline{1001})_2 &= \left(\frac{1}{2}+\frac{1}{16}\right)+\frac{1}{16}\left(\frac{1}{2}+\frac{1}{16}\right)+\left(\frac{1}{16}\right)^2\left(\frac{1}{2}+\frac{1}{16}\right)+\left(\frac{1}{16}\right)^3\left(\frac{1}{2}+\frac{1}{16}\right)+\dots \\
&= \frac{9}{16}\left(1+\frac{1}{16}+\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3+\dots\right) \\
&= \frac{9}{16}\sum_{n=0}^{\infty}\left(\frac{1}{16}\right)^n = \frac{9}{32}\left(\frac{1}{1-\frac{1}{16}}\right) = \frac{9}{16}\frac{16}{15} = \frac{3}{5}
\end{aligned}$$

d.

$$\begin{array}{rcrcr}
3/2 &=& 1 & R & 1 \\
1/2 &=& 0 & R & 1
\end{array}$$

$$(3)_{10} = (11)_2$$

$$\begin{array}{ccccl}
0.2 & * & 2 & = & 0.4 \to 0 \\
0.4 & * & 2 & = & 0.8 \to 0 \\
0.8 & * & 2 & = & 1.6 \to 1 \\
0.6 & * & 2 & = & 1.2 \to 1 \\
\hline
0.2 & * & 2 & = & 0.4 \to 0
\end{array}$$

$$(0.2)_{10} = (0.\overline{0011})_2$$

We find:

$$(3.2)_{10} = (11.\overline{0011})_2$$

e.

$$\begin{array}{rcrcr}
30/2 &=& 15 & R & 0 \\
15/2 &=& 7 & R & 1 \\
7/2 &=& 3 & R & 1 \\
2/2 &=& 1 & R & 1 \\
1/2 &=& 0 & R & 1
\end{array}$$

$$(30)_{10} = (11110)_2$$

(Or we just eyeball that $30 = 16+8+4+2$.)

$$\begin{array}{ccccl}
0.6 & * & 2 & = & 1.2 \to 1 \\
0.2 & * & 2 & = & 0.4 \to 0 \\
0.4 & * & 2 & = & 0.8 \to 0 \\
0.8 & * & 2 & = & 1.6 \to 1 \\
\hline
0.6 & * & 2 & = & 1.2 \to 1
\end{array}$$

$$(0.6)_{10} = (0.\overline{1001})_2$$

We find:

$$(30.6)_{10} = (11110.\overline{1001})_2$$

f.

$$\begin{array}{rcrcr}
99/2 &=& 49 & R & 1 \\
49/2 &=& 24 & R & 1 \\
24/2 &=& 12 & R & 0 \\
12/2 &=& 6 & R & 0 \\
6/2 &=& 3 & R & 0 \\
3/2 &=& 1 & R & 1 \\
1/2 &=& 0 & R & 1
\end{array}$$

$$(99)_{10} = (1100011)_2$$

$$\begin{array}{ccccl}
0.9 & * & 2 & = & 1.8 \to 1 \\
0.8 & * & 2 & = & 1.6 \to 1 \\
0.6 & * & 2 & = & 1.2 \to 1 \\
0.2 & * & 2 & = & 0.4 \to 0 \\
0.4 & * & 2 & = & 0.8 \to 0 \\
\hline
0.8 & * & 2 & = & 1.6 \to 1
\end{array}$$

$$(0.9)_{10} = (0.1\overline{1100})_2$$

We find:

$$(99.9)_{10} = (1100011.1\overline{1100})_2$$