EX.0.4.1, Sauer **Langou**

Identify for which values of x there is subtraction of nearly equal numbers, and find an alternate form that avoids the problem.

$$\text{(a)}\quad \frac{1-\sec x}{\tan^2 x} \qquad \text{(b)}\quad \frac{1-(1-x)^3}{x} \qquad \text{(c)}\quad \frac{1}{1+x}-\frac{1}{1-x}$$

EX.0.4.1, Sauer, solution, Langou

a. We see that there is subtraction of nearly equal numbers whenever $\sec x = 1$, so whenever $\cos x = 1$, so whenever x is near $2k\pi$ for $k \in \mathbb{Z}$.

   We remember our trig formula:
$$\sec^2 = 1 + \tan^2 x.$$

   This is coming from
$$1+\tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

   So in other words, we have that
$$\sec^2 - 1 = \tan^2 x.$$

   Once we see this and our function at hand, it makes sense to multiply and divide by $(1-\sec x)$, we get
$$\frac{1-\sec x}{\tan^2 x} = \left(\frac{1-\sec x}{\tan^2 x}\right)\left(\frac{1+\sec x}{1+\sec x}\right) = \left(\frac{1-\sec^2 x}{\tan^2 x}\right)\left(\frac{-1}{1+\sec x}\right) = \frac{-1}{1+\sec x}.$$

   The $\frac{-1}{1+\sec x}$ alternate form avoids the cancellation problem for x near $2k\pi$ for $k \in \mathbb{Z}$.

b. We see that there is subtraction of nearly equal numbers whenever $(1-x)^3 = 1$, so whenever x is near 0.

   Some algebra:
$$\frac{1-(1-x)^3}{x} = \frac{x^3-3x^2+3x}{x} = x^2-3x+3.$$

   So an alternate form that avoids the cancellation problem for x near 0 is x^2-3x+3.

c. We see that there is subtraction of nearly equal numbers whenever x is near 0.

   Some algebra:
$$\frac{1}{1+x} - \frac{1}{1-x} = \frac{-2x}{(1+x)(1-x)} = \frac{2x}{(1+x)(x-1)}$$

   So an alternate form that avoids the cancellation problem for x near 0 is $\frac{2x}{(1+x)(x-1)}$.