EX.5.2.6.a, Sauer3

Apply the composite Midpoint Rule with $m = 1$, 2 and 4 panels to approximate the following integral.

$$\text{(a)} \qquad \int_0^{\frac{\pi}{2}} \frac{1-\cos x}{x^2}\, dx$$

Some comments:

- We note that in zero the function
$$f(x) = \frac{1-\cos x}{x^2}$$
is not defined. This is why we need an open rule (called in the book: "open Newton-Cotes method" or "midpoint rule"). A closed rule would use $f(0)$ and that would be a problem.

- To repeat, for this integral, Simpson's rule would utterly fail becasue $f(0)$ is not defined, so we need to use a midpoint rule. (To avoid 0.)

- We also note that this is not that bad since
$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}.$$
So we could very easily (and legitimally) extend f by continuity at 0, by defining $f(0) = 1/2$, and then we could use a closed rule. (Be careful when x gets close to 0 though. But this issue will be for "open" and "closed" rules.)

- We note that there is no closed-form formula for this integral and so the only way to obtain a value for this integral is by numerical integration.