Summer 2022

MATH 4650-E02
CSCI 4650-E02

Homework 2

Due on Saturday June 18th 2022

Exercises:

1. EX.1.1.2 handwritten
2. EX.1.1.4 handwritten
3. EX.1.1.8 handwritten, not in book, I wrote it
4. CP.1.1.2 using Colab
5. EX.1.2.8 handwritten
6. EX.1.2.14 handwritten
7. CP.1.2.4 using Colab
8. EX.1.3.8 handwritten
9. CP.1.3.2 using Colab
10. EX.1.4.2 handwritten
11. EX.1.4.4 handwritten
12. CP.1.4.2 using Colab
13. CP.1.4.8 using Colab
14. CP.1.5.1 using Colab
15. CP.1.5.2 using Colab
16. CP.1.5.3 using Colab

EX.1.1.2, Sauer3

Use the Intermediate Value Theorem to find an interval of length one that contains a root of the equation (a) $x^5 + x = 1$, (b) $\sin(x) = 6x + 5$, (c) $\ln(x) + x^2 = 3$.

EX.1.1.4, Sauer3

Consider the equations (a) $x^5 + x = 1$, (b) $\sin(x) = 6x + 5$, (c) $\ln(x) + x^2 = 3$.
Apply two steps of the Bisection Method to find an approximate root within 1/8 of the true root.

EX.1.1.8, Langou

Let $(f : \mathbb{R} \to \mathbb{R})$ be continuous. Let a and b, and `tol`. Let k be the smallest number of iterations of the Bisection Method that guarantees a forward error bound on the approximate solution within `tol`. Let `numevals` be the smallest number of iterations of the Bisection Method that guarantees a forward error bound on the approximate solution within `tol`.

a. Derive a formula that relates a, b, k and `tol`.

b. Solve for k as a function of a, b, and `tol`.

c. Derive a formula that relates k and `numevals`.

Let a, b, and `tol` as given below. How many function evaluations of the Bisection Method are required to guarantee a forward error bound on the approximate solution within `tol`? Answer with an integer.

d. $a = -3$, $b = 16$, `tol` $=10^{-3}$

e. $a = -0.2$, $b = 2.7$, `tol` $=10^{-12}$

CP.1.1.2, Sauer3

Use the Bisection Method to find the root to eight correct decimal places for
(a) $x^5 + x = 1$, (b) $\sin(x) = 6x + 5$, (c) $\ln(x) + x^2 = 3$.

EX.1.2.8, Sauer3

Use Theorem 1.6 to determine whether Fixed-Point Iteration of $g(x)$ is locally convergent to the given fixed point r. (a) $g(x) = (2x-1)/x^2, \quad r = 1$, (b) $g(x) = \cos x + \pi + 1, \quad r = \pi$, (c) $g(x) = e^{2x} - 1, \quad r = 0$.

EX.1.2.14, Sauer3

Which of the following three Fixed-Point Iteration converge to $\sqrt{2}$? Rank the ones that converge from fastest to slowest.

$$(a)\quad x \longrightarrow \frac{1}{2}x + \frac{1}{x}; \quad (b)\quad x \longrightarrow \frac{2}{3}x + \frac{2}{3x}; \quad (c)\quad x \longrightarrow \frac{3}{4}x + \frac{1}{2x}.$$

CP.1.2.4, Sauer3

Calculate the cube roots of the following numbers to eight correct decimal places, by using Fixed-Point Iteration with $g(x) = (2x + A/x^2)/3$, where A is (a) 2, (b) 3, (c) 5. State your initial guess and the number of steps needed.

EX.1.3.8, Sauer3

Let $f(x) = x^n - ax^{n-1}$, and set $g(x) = x^n$. (a) Use the Sensitivity Formula to give a prediction for the nonzero root of $f_\varepsilon(x) = (1+\varepsilon)x^n - ax^{n-1}$ for small ε. (b) Find the nonzero root and compare with the prediction.

CP.1.3.2, Sauer3

Let $f(x) = \sin(x^3) - x^3$. (a) Find the multiplicity of the root $r = 0$. (b) Use `scipy.optimize.fsolve` command with initial guess $x = 0.1$ to locate a root. What are the forward and backward errors of `scipy.optimize.fsolve`'s response?

EX.1.4.2, Sauer3

Apply two steps of Newton's Method with initial guess $x_0 = 1$.

a. $x^3 + x^2 - 1 = 0$;

b. $x^2 + \frac{1}{x+1} - 3x = 0$;

c. $5x - 10 = 0$.

EX.1.4.4, Sauer3

Use Theorem 1.11 or 1.12 to estimate the error e_{i+1} in terms of the previous error e_i as Newton's Method converges to the given roots. Is the convergence linear or quadratic?

a. $32x^3 - 32x^2 - 6x + 9 = 0, \qquad r = -\frac{1}{2},\ r = \frac{3}{4};$

b. $x^3 - x^2 - 5x - 3 = 0, \qquad r = -1,\ r = 3.$

CP.1.4.2, Sauer3

Each equation has one real root. Use Newton's Method to approximate the root to eight correct decimal places.

a. $x^5 + x = 1$;

b. $\sin x = 6x + 5$;

c. $\ln x + x^2 = 3$.

CP.1.4.8, Sauer3

Consider the function

$$f(x) = 94\cos^3 x - 24\cos x + 177\sin^2 x - 108\sin^4 x - 72\cos^3 x\sin^2 x - 65$$

on the interval $[0, 3]$. Plot the function on the interval, and find all three roots to six correct decimal places. Determine which roots converge quadratically, and find the multiplicity of the roots that converge linearly.

CP.1.5.1, Sauer3 **Langou**

Use the Secant Method to find the (single) solution of each of the following equations. Use initial guesses $x_0 = 1$ and $x_1 = 2$.

a. $x^3 = 2x + 2$

b. $e^x + x = 7$

c. $e^x + \sin x = 4$

CP.1.5.2, Sauer3 **Langou**

Use the Method of False Position to find the (single) solution of each of the following equations. Start from initial bracket $[1, 2]$

a. $x^3 = 2x + 2$

b. $e^x + x = 7$

c. $e^x + \sin x = 4$

CP.1.5.3, Sauer3 **Langou**

Use Inverse Quadratic Interpolation to find the (single) solution of each of the following equations. Use initial guesses $x_0 = 1$, $x_1 = 2$, $x_2 = 0$, and update by retaining the three most recent iterates.

a. $x^3 = 2x + 2$

b. $e^x + x = 7$

c. $e^x + \sin x = 4$