EX.5.1.4, Sauer3 – Problem rewritten by Julien.

Carry out the steps of EX.5.1.3 using the three-point centered-difference formula.
We want to study the three-point centered-difference formula to approximate $f'(x)$ where $f(x) = \sin x$ and $x = \frac{\pi}{3}$.

a. (handwritten) For this problem, what is $f'(x)$?

b. (handwritten) What is the **three-point centered-difference formula** in its general form?

c. (python) Use the **three-point centered-difference formula** to approximate $f'(x)$ for (a) $h = 0.1$, (b) $h = 0.01$, and (c) $h = 0.001$? Observe that, as h gets smaller, the approximation gets better. (However if we were to take h too small, we would suffer from cancellation error and start losing accuracy.)

d. (handwritten) What is the **three-point centered-difference** approximation error in its general form? (Hint: There is a "c" in the formula. Specify an interval where c is.)

e. (handwritten) Since we work with small positive h's, let us assume that h is in $(-0.2, 0.2)$, so that $x-h$ and $x+h$ are in $(0, \frac{\pi}{2})$, find lower and upper bounds for the **three-point centered-difference** approximation error (independent of c). We call this an "interval where the approximation error is". We want this interval to be "relevant".

f. (python) Check that the "true approximation error" is within your interval of part (e).

g. (handwritten) In general, we are more interested in guaranteeing that "the error is not larger than". We mean the absolute value of the error. (As opposed to find an interval where the error is.) Based on part (e), find an upper bound for the absolute value of the "true approximation error". We call this an "upper bound on the approximation error".

h. (python) Check that an "upper bound on the approximation error" is an upper bound on the approximation error. We want this upper bound to be "relevant".