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EX.2.4.3, Sauer3

Solve the system by finding the PA = LU factorization and then carrying out the two-step back substitution.

a.
$$\begin{pmatrix} 3 & 7 \\ 6 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -11 \end{pmatrix}$$
 b. $\begin{pmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$

Hint: The two PA = LU factorization (with partial pivoting) are: P = [[0, 1, 1], [1, 0, 1], L = [[1, 0, 1], [0.5, 1, 1], L = [[6, 1, 1], [0, 6.5]]

a.

$$\left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right) \left(\begin{array}{cc} 3 & 7 \\ 6 & 1 \end{array}\right) = \left(\begin{array}{cc} 1 & 0 \\ \frac{1}{2} & 1 \end{array}\right) \left(\begin{array}{cc} 6 & 1 \\ 0 & \frac{13}{2} \end{array}\right)$$

b.

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & 1 \end{pmatrix} \begin{pmatrix} 6 & 3 & 4 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

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EX.2.4.3, Sauer3, solution, Langou

Colab: https://colab.research.google.com/drive/1xmC3zRLV6RhRFHzMGZTY1LKtCH7ElGDj

We want to solve: Ax = b. We know that PA = LU. So we want to apply P to Ax = b to get PAx = Pb and then replace PA by LU to get

$$LUx = Pb$$
.

So we first apply P to b to get Pb, then solve Ly = Pb to get y, then solve Ux = y to get x.

(a)

$$Pb = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -11 \end{pmatrix} = \begin{pmatrix} -11 \\ 1 \end{pmatrix}$$

$$Ly = Pb \Rightarrow \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -11 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -11 \\ \frac{13}{2} \end{pmatrix}$$

$$Ux = y \Rightarrow \begin{pmatrix} 6 & 1 \\ 0 & \frac{13}{2} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -11 \\ \frac{13}{2} \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$x = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

(b)
$$Pb = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

$$Ly = Pb \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 3 \end{pmatrix}$$

$$Ux = y \Rightarrow \begin{pmatrix} 6 & 3 & 4 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$x = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$