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EX.4.3.2.a, Sauer3

Apply Classical Gram-Schmidt orthogonalization to find the full QR factorization of the following matrix

$$\left[\begin{array}{cc} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{array}\right].$$

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EX.4.3.2.a, Sauer3, solution, Langou

Colab: https://colab.research.google.com/drive/10ezRRxhJYGhOnXGsbI6oBf34wQo4TKhV

Let a_0 and a_1 be the first and second columns of A.

We perform the following computation.

$$w_{0} = a_{0},$$

$$r_{00} = \|w_{0}\|_{2} = \sqrt{(2)^{2} + (-2)^{2} + (1)^{2}} = 3,$$

$$q_{0} = \frac{1}{r_{00}}w_{0} = \frac{1}{3}\begin{bmatrix} 2\\ -2\\ 1 \end{bmatrix},$$

$$r_{01} = q_{0}^{T}a_{1} = \frac{1}{3}\left((2)(3) + (-2)(-6) + (1)(0)\right) = 6,$$

$$w_{1} = a_{1} - q_{0}r_{01} = \begin{bmatrix} 3\\ -6\\ 0 \end{bmatrix} - 6(\frac{1}{3})\begin{bmatrix} 2\\ -2\\ 1 \end{bmatrix},$$

$$r_{11} = \|w_{1}\|_{2} = \sqrt{(-1)^{2} + (-2)^{2} + (-2)^{2}} = 3,$$

$$q_{1} = \frac{1}{r_{11}}w_{1} = \frac{1}{3}\begin{bmatrix} -1\\ -2\\ -2 \end{bmatrix}.$$

So we find

$$Q = \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -2 & -2 \\ 1 & -2 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 3 & 6 \\ 0 & 3 \end{bmatrix}.$$

We can check that

a.
$$A = QR$$

b.
$$Q^TQ = I$$

c. R is upper triangular