

**CP.5.1.1, Sauer3 – Problem rewritten by Julien.**

We want to study the three-point centered-difference formula to approximate  $f'(x)$  where  $f(x) = \sin x - \cos x$  and  $x = 0$ .

- a. (python) Make a table and plot of the error of the three-point centered-difference formula for  $f'(x)$ , where  $f = \sin(x) - \cos(x)$  and  $x = 0$ , with  $10^{-1}, \dots, 10^{-12}$ , as in the table in Section 5.1.2. Draw a plot of the results.

- b. (python) Use the formula

$$h_{\text{opt}} \approx \left( \frac{3\varepsilon_{\text{mach}}}{M} \right)^{\frac{1}{3}},$$

(given below Equation(5.12) in the book,) to compute an approximation to  $h_{\text{opt}}$ . In this formula  $M$  is  $|f'''(x)|$ , and  $\varepsilon_{\text{mach}}$  is the machine precision that you can get in python with `np.finfo(float).eps`.

- c. (python) Redo part (a) by adding  $E(h)$  from Equation (5.11) to your table and to your plot. You can approximate the quantity  $f'''(x)$  by  $M$ .