EX.2.1.2.b2c, Sauer3

Use Gaussian elimination to solve the systems:

b.
$$\begin{aligned} x + 2y - z &= 2 \\ 3y + z &= 4 \\ 2x - y + z &= 2 \end{aligned}$$

c.
$$\begin{aligned} 2x + y - 4z &= -7 \\ x - y + z &= -2 \\ -x + 3y - 2z &= 6 \end{aligned}$$

Colab: `https://colab.research.google.com/drive/14daK08RnGf2uTi-6pTWZLGPNFjy7LZER`

Ex. 2.1.2

b) $x+2y-z=2$
$3y+z=4$
$2x-y+z=2$
$$\Rightarrow \left|\begin{array}{cccc} 1 & 2 & -1 & 2 \\ 0 & 3 & 1 & 4 \\ 2 & -1 & 1 & 2 \end{array}\right|$$

$$\xrightarrow{L_2=L_2-2L_0} \left|\begin{array}{cccc} 1 & 2 & -1 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & -5 & 3 & -2 \end{array}\right|$$

$$\xrightarrow{L_2=L_2+\frac{5}{3}L_1} \left|\begin{array}{cccc} 1 & 2 & -1 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & \frac{14}{3} & \frac{14}{3} \end{array}\right| \Rightarrow \begin{array}{l} x+2y-z=2 \\ 3y+z=4 \\ \frac{14}{3}z=\frac{14}{3} \end{array}$$

$$\Rightarrow \begin{cases} x=1 \\ y=1 \\ z=1 \end{cases} \quad \text{solution } \vec{x}=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

c) $2x+y-4z=-7$
$x-y+z=-2$
$-x+3y-2z=6$
$$\Rightarrow \left|\begin{array}{cccc} 2 & 1 & -4 & -7 \\ 1 & -1 & 1 & -2 \\ -1 & 3 & -2 & 6 \end{array}\right|$$

$$\xrightarrow{L_2=L_2+L_1} \left|\begin{array}{cccc} 2 & 1 & -4 & -7 \\ 1 & -1 & 1 & -2 \\ 0 & 2 & -1 & 4 \end{array}\right|$$

$$\xrightarrow{L_1=L_1-\frac{1}{2}L_0} \left|\begin{array}{cccc} 2 & 1 & -4 & -7 \\ 0 & -\frac{3}{2} & 3 & \frac{3}{2} \\ 0 & 2 & -1 & 4 \end{array}\right|$$

$$\xrightarrow{L_2=L_2+\frac{4}{3}L_1} \left|\begin{array}{cccc} 2 & 1 & -4 & -7 \\ 0 & -\frac{3}{2} & 3 & \frac{3}{2} \\ 0 & 0 & 3 & 6 \end{array}\right| \Rightarrow \begin{array}{l} 2x+y-4z=-7 \\ -\frac{3}{2}y+3z=\frac{3}{2} \\ 3z=6 \end{array}$$

$$\Rightarrow \begin{cases} x=-1 \\ y=3 \\ z=2 \end{cases} \quad \text{solution } \vec{x}=\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

EX.2.1.2.b2c, Sauer3, solution, Langou

b.

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 2 \\ 0 & 3 & 1 & 4 \\ 2 & -1 & 1 & 2 \end{array}\right] \quad \Rightarrow \quad \left[\begin{array}{rrr|r} 1 & 2 & -1 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & \frac{14}{3} & \frac{14}{3} \end{array}\right]$$

then using back substitution we have $(x, y, z) = (1, 1, 1)$.

c.

$$\left[\begin{array}{rrr|r} 2 & 1 & -4 & -7 \\ 1 & -1 & 1 & -2 \\ -1 & 3 & -2 & 6 \end{array}\right] \quad \Rightarrow \quad \left[\begin{array}{rrr|r} 2 & 1 & -4 & -7 \\ 0 & -\frac{3}{2} & 3 & \frac{3}{2} \\ 0 & 0 & 3 & 6 \end{array}\right]$$

then using back substitution we have $(x, y, z) = (-1, 3, 2)$.