EX.4.3.2.a, Sauer3

Apply Classical Gram-Schmidt orthogonalization to find the full QR factorization of the following matrix

$$\begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix}.$$

EX.4.3.2.a, Sauer3, solution, Langou

Colab: `https://colab.research.google.com/drive/1oezRRxhJYGh0nXGsbI6oBf34wQo4TKhV`

Let a_0 and a_1 be the first and second columns of A.
We perform the following computation.

$$\begin{aligned}
w_0 &= a_0, \\
r_{00} &= \|w_0\|_2 = \sqrt{(2)^2 + (-2)^2 + (1)^2} = 3, \\
q_0 &= \frac{1}{r_{00}} w_0 = \frac{1}{3} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \\
r_{01} &= q_0^T a_1 = \frac{1}{3}\left((2)(3) + (-2)(-6) + (1)(0)\right) = 6, \\
w_1 &= a_1 - q_0 r_{01} = \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - 6(\frac{1}{3}) \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \\
r_{11} &= \|w_1\|_2 = \sqrt{(-1)^2 + (-2)^2 + (-2)^2} = 3, \\
q_1 &= \frac{1}{r_{11}} w_1 = \frac{1}{3} \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}.
\end{aligned}$$

So we find

$$Q = \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -2 & -2 \\ 1 & -2 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 3 & 6 \\ 0 & 3 \end{bmatrix}.$$

We can check that

a. $A = QR$

b. $Q^T Q = I$

c. R is upper triangular