EX.0.3.7, Langou

Write each of the given numbers in Matlab's *format hex*. Show your work. Then check your answers with Matlab. (a) 16 (b) 130 (c) 1/4 (d) fl(1/7) (e) fl(4/7) (f) fl(0.01) (g) fl(−0.01)

a. $16 \div 2 = 8 \text{ R } 0$

$8 \div 2 = 4 \text{ R } 0$

$4 \div 2 = 2 \text{ R } 0$

$2 \div 2 = 1 \text{ R } 0$

$1 \div 2 = 0 \text{ R } 1$

$(16)_{10} = (10000)_2$

$= (1.0) \times 2^4$

Biased exponent is $4 + 1023 = 1027 = 2^{10} + 2^1 + 2^0$

The sign is 0 (positive)

Machine representation is the 64 bits

$0100\ 0000\ 0011\ \underbrace{0000 \cdots 0000}_{\#52,\,0}$

hex format is 4030000000000000

b. $130 \div 2 = 65 \text{ R } 0$

$65 \div 2 = 32 \text{ R } 1$

$32 \div 2 = 16 \text{ R } 0$

$16 \div 2 = 8 \text{ R } 0$

$8 \div 2 = 4 \text{ R } 0$

$4 \div 2 = 2 \text{ R } 0$

$2 \div 2 = 1 \text{ R } 0$

$1 \div 2 = 0 \text{ R } 1$

$(130)_{10} = (10000010)_2$

$= (1.000001) \times 2^7$

Biased exponent is $7 + 1023 = 1030 = 2^{10} + 2^2 + 2^1$

The sign is 0 (positive)

Machine representation is the 64 bits

$0100\ 0000\ 0110\ 0000\ 0100\ \underbrace{0000 \cdots 0000}_{\#44,\,0}$

hex format is 4060400000000000

c. $\frac{1}{4} \times 2 = \frac{1}{2} + 0$

$\frac{1}{2} \times 2 = 0 + 1$

$(\frac{1}{4})_{10} = (0.01)_2$

$= (1.0) \times 2^{-2}$

Biased exponent is $-2 + 1023 = 1021$

$= 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0$

The sign is 0 (positive)

Machine representation is the 64 bits

$0011\ 1111\ 1101\ \underbrace{0000 \cdots 0000}_{\#52,\,0}$

hex format is 3FD0000000000000

d. $\frac{1}{7} \times 2 = \frac{2}{7} + 0$

$\frac{2}{7} \times 2 = \frac{4}{7} + 0$

$\frac{4}{7} \times 2 = \frac{1}{7} + 1$

$\frac{1}{7} \times 2 = \frac{2}{7} + 0$

$$\left(\frac{1}{7}\right)_{10} = (0.\overline{001})_2$$
$$= (1.\overline{001})_2 \times 2^{-3}$$

$$fl\left(\frac{1}{7}\right) = 1.\boxed{001\cdots0010} \times 2^{-3}$$

Biased exponent is $-3 + 1023 = 1020$
$= 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2$

The sign is 0 (positive)

Machine representation is the 64 bits

0011 1111 1100 0010 0100 1001 0010 0100 1001
0010 0100 1001 0010

hex format is 3FC2492492492

e. $\frac{4}{7} \times 2 = \frac{1}{7} + 1$

$\frac{1}{7} \times 2 = \frac{2}{7} + 0$

$\frac{2}{7} \times 2 = \frac{4}{7} + 0$

$\frac{4}{7} \times 2 = \frac{1}{7} + 1$

$$\left(\frac{4}{7}\right)_{10} = (0.\overline{100})_2$$
$$= (1.\overline{001})_2 \times 2^{-1}$$

$$fl\left(\frac{4}{7}\right) = 1.\boxed{001\cdots0010} \times 2^{-1}$$

Biased exponent is $-1 + 1023 = 1022$
$= 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1$

The sign is 0 (positive)

Machine representation is the 64 bits

0011 1111 1110 0010 0100 1001 0010 0100 1001
0010 0100 1001 0010

hex format is 3FE2492492492

Approach 2: $\frac{4}{7} = 2^2 \times \frac{1}{7}$, exponent +2.

f. $0.01\times2=0.02+0$
$0.02\times2=0.04+0$
$0.04\times2=0.08+0$
$0.08\times2=0.16+0$
$0.16\times2=0.32+0$
$0.32\times2=0.64+0$
$0.64\times2=0.28+1$
$0.28\times2=0.56+0$
$0.56\times2=0.12+1$
$0.12\times2=0.24+0$
$0.24\times2=0.48+0$
$0.48\times2=0.96+0$
$0.96\times2=0.92+1$
$0.92\times2=0.84+1$
$0.84\times2=0.68+1$
$0.68\times2=0.36+1$
$0.36\times2=0.72+0$
$0.72\times2=0.44+1$
$0.44\times2=0.88+0$
$0.88\times2=0.76+1$
$0.76\times2=0.52+1$
$0.52\times2=0.04+1$
$0.04\times2=0.08+0$

$$(0.01)_{10}=(0.00\overline{00001010001111010111})_2$$
$$=(1.010001111010111\overline{00001010001111010111})_2\times2^{-7}$$

$fl(0.01)=$

$$1.\boxed{01000111101011100001010001111010111}$$
$$\boxed{00001010001111011}\times2^{-7}$$

Biased exponent is $-7+1023=1016$
$=2^9+2^8+2^7+2^6+2^5+2^4+2^3$

The sign is 0 (positive)

Machine representation is the 64 bits

0011 1111 1000 0100 0111 1010 1110 0001
0100 0111 1010 1110 0001 0100 0111 1011

hex format is 3F847AE147AE147B

g. $fl(-0.01)$

According to $fl(0.01)$ we only need to change the sign.

Machine representation is the 64 bits

1011 1111 1000 0100 0111 1010 1110 0001
0100 0111 1010 1110 0001 0100 0111 1011

hex format is BF847AE147AE147B