EX.0.2.3, Sauer

Convert the following base 10 numbers to binary. (a) 10.5, (b) 1/3, (c) 5/7, (d) 12.8, (e) 55.4, (f) 0.1.

EX.0.2.3, Sauer, solution, Langou

- Only turning the Python code is not a good answer.
- The copy-paste from this PDF to python code does not work great. It is better to copy-paste from colab.
- The Colab Jupyter Notebook is available at: `https://colab.research.google.com/drive/15eH4ZJPSJ1K9fCefVra7IKw5Ws36WB6R`.
- The Python code and its ouput is at the end of this document.

a. Convert the base 10 number $(10.5)_{10}$ to binary.

Either we write:

$$10.5 = 8 + 2 + \frac{1}{2} = 2^3 + 2^1 + 2^{-1}.$$

Or we use our technique:

$$\begin{array}{rcccc} 10/2 & = & 5 & R & 0 \\ 5/2 & = & 2 & R & 1 \\ 2/2 & = & 1 & R & 0 \\ 1/2 & = & 0 & R & 1 \end{array}$$

$$(10)_{10} = (1010)_2$$

$$\begin{array}{ccccc} 0.50 & * & 2 & = & 1.00 & \to 1 \\ 0.00 & * & 2 & = & 0.00 & \to 0 \\ \hline 0.00 & * & 2 & = & 0.00 & \to 0 \end{array}$$

$$(0.5)_{10} = (0.1\overline{0})_2$$

We find:

$$(10.5)_{10} = (1010.1)_2$$

b. Convert the base 10 number 1/3 to binary.

$$\begin{array}{ccccc} \frac{1}{3} & * & 2 & = & 0\frac{2}{3} & \to 0 \\ \frac{2}{3} & * & 2 & = & 1\frac{1}{3} & \to 1 \\ \hline \frac{1}{3} & * & 2 & = & 0\frac{2}{3} & \to 0 \end{array}$$

We find:

$$(\frac{1}{3})_{10} = (0.\overline{01})_2$$

Check:

$$
\begin{aligned}
(0.\overline{01})_2 &= \frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\dots \\
&= \frac{1}{4}\left(1+\frac{1}{4}+\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^3+\dots\right) \\
&= \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n = \frac{1}{4}\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{1}{4}\frac{4}{3} = \frac{1}{3}
\end{aligned}
$$

c. Convert the base 10 number 5/7 to binary.

$$
\begin{array}{ccccc}
\frac{5}{7} & * & 2 & = & 1\frac{3}{7} & \to 1 \\
\frac{3}{7} & * & 2 & = & 0\frac{6}{7} & \to 0 \\
\frac{6}{7} & * & 2 & = & 1\frac{5}{7} & \to 1 \\
\hline
\frac{5}{7} & * & 2 & = & 1\frac{5}{7} & \to 1
\end{array}
$$

We find:

$$(\frac{5}{7})_{10} = (0.\overline{101})_2$$

Check:

$$
\begin{aligned}
(0.\overline{101})_2 &= \frac{5}{8}+\frac{5}{64}+\frac{5}{512}+\frac{5}{4168}+\dots \\
&= \frac{5}{8}\left(1+\frac{1}{8}+\left(\frac{1}{8}\right)^2+\left(\frac{1}{8}\right)^3+\dots\right) \\
&= \frac{5}{8}\sum_{n=0}^{\infty}\left(\frac{1}{8}\right)^n = \frac{5}{8}\left(\frac{1}{1-\frac{1}{8}}\right) = \frac{5}{8}\frac{8}{7} = \frac{5}{7}
\end{aligned}
$$

d. Convert the base 10 number 12.8 to binary.

For the integer part, we can see that $12 = 8+4 = 2^3+2^2$, so that $(12)_{10} = (1100)_2$.

$$
\begin{array}{rcccc}
12/2 & = & 6 & R & 0 \\
6/2 & = & 3 & R & 0 \\
3/2 & = & 1 & R & 1 \\
1/2 & = & 0 & R & 1
\end{array}
$$

$$(12)_{10} = (1100)_2$$

Now, for the decimal part,

$$
\begin{array}{ccccc}
0.8 & * & 2 & = & 1.6 & \to 1 \\
0.6 & * & 2 & = & 1.2 & \to 1 \\
0.2 & * & 2 & = & 0.4 & \to 0 \\
0.4 & * & 2 & = & 0.8 & \to 0 \\
\hline
0.8 & * & 2 & = & 1.6 & \to 1
\end{array}
$$

$$(0.8)_{10} = (0.\overline{1100})_2$$

We find:

$$(12.8)_{10} = (1100.\overline{1100})_2$$

e. Convert the base 10 number 55.4 to binary.

For the integer part, we can see that $55 = 32+16+4+2+1 = 2^5+2^4+2^2+2^1+2^0$, so that $(55)_{10} = (110111)_2$.

$$\begin{array}{rcrcc} 55/2 & = & 27 & R & 1 \\ 27/2 & = & 13 & R & 1 \\ 13/2 & = & 6 & R & 1 \\ 6/2 & = & 3 & R & 0 \\ 3/2 & = & 1 & R & 1 \\ 1/2 & = & 0 & R & 1 \end{array}$$

$$(55)_{10} = (110111)_2$$

Now, for the decimal part,

$$\begin{array}{ccccl} 0.4 & * & 2 & = & 0.8 \;\rightarrow 0 \\ 0.8 & * & 2 & = & 1.6 \;\rightarrow 1 \\ 0.6 & * & 2 & = & 1.2 \;\rightarrow 1 \\ 0.2 & * & 2 & = & 0.4 \;\rightarrow 0 \\ \hline 0.4 & * & 2 & = & 0.8 \;\rightarrow 0 \end{array}$$

$$(0.4)_{10} = (0.\overline{0110})_2$$

We find:

$$(55.4)_{10} = (110111.\overline{0110})_2$$

f. Convert the base 10 number 0.1 to binary.

$$\begin{array}{ccccl} 0.1 & * & 2 & = & 0.2 \;\rightarrow 0 \\ 0.2 & * & 2 & = & 0.4 \;\rightarrow 0 \\ 0.4 & * & 2 & = & 0.8 \;\rightarrow 0 \\ 0.8 & * & 2 & = & 1.6 \;\rightarrow 1 \\ 0.6 & * & 2 & = & 1.2 \;\rightarrow 1 \\ \hline 0.2 & * & 2 & = & 0.4 \;\rightarrow 0 \end{array}$$

$$(0.1)_{10} = (0.0\overline{0011})_2$$

We find:

$$(0.1)_{10} = (0.0\overline{0011})_2$$

Please note that we can obtain 0.1 by dividing 0.4 by 2^2 and so indeed 0.1 in base 2 is "close" to 0.4 in base 2.

```
# (a) 10.5
x = 0b1010
y = 0b1
z = x + y * 2**(-1)
print( z )
print( 10.5 )
```

```
10.5
10.5
```

```python
# (b) 1/3
x = 0
y = 0b01
for i in range(0,30):
  y = y * ( 2 ** (-2) )
  x = x + y
print( x )
print( 1./3. )
```

```
0.3333333333333333
0.3333333333333333
```

```python
# (c) 5/7
x = 0
y = 0b101
for i in range(0,30):
  y = y * ( 2 ** (-3) )
  x = x + y
print( x )
print( 5./7. )
```

```
0.7142857142857142
0.7142857142857143
```

```python
# (d) 12.8
x = 0b1100
y = 0b1100
for i in range(0,30):
  y = y * ( 2 ** (-4) )
  x = x + y
print( x )
print( 12.8 )
```

```
12.8
12.8
```

```python
# (e) 55.4
x = 0b110111
y = 0b0011
for i in range(0,30):
  y = y * ( 2 ** (-4) )
  x = x + y
print( x )
print( 55.4 )
```

```
55.4
55.4
```

```python
# (f) 0.1
x = 0
y = 0b0011 * ( 2 ** (-1) )
```

```python
for i in range(0,20):
  y = y * ( 2 ** (-4) )
  x = x + y
print( x )
print( 0.1 )
```

```
0.1
0.1
```