Name:____________________________________

MATH 3191 E01 :: Fall 2020 :: Exam 2

1. For each matrix equality, give the elementary coefficient matrix that enables to go from the right to the left. You can either multiply by the elementary matrix on the left or on the right but not on both sides.

   (a) $(L_2 \leftrightarrow L_5)$

$$\begin{pmatrix} -5 & -8 & 8 \\ 4 & 4 & -6 \\ 4 & 0 & -3 \\ -7 & 6 & 4 \\ -3 & 3 & 8 \end{pmatrix} = \begin{pmatrix} -5 & -8 & 8 \\ -3 & 3 & 8 \\ 4 & 0 & -3 \\ -7 & 6 & 4 \\ 4 & 4 & -6 \end{pmatrix}$$

   (b) $(C_3 \leftarrow C_3 + C_2)$

$$\begin{pmatrix} -5 & -8 & 0 \\ -3 & 3 & 11 \\ 4 & 0 & -3 \\ -7 & 6 & 10 \\ 4 & 4 & -2 \end{pmatrix} = \begin{pmatrix} -5 & -8 & 8 \\ -3 & 3 & 8 \\ 4 & 0 & -3 \\ -7 & 6 & 4 \\ 4 & 4 & -6 \end{pmatrix}$$

   (c) $(L_4 \leftarrow 2L_4)$

$$\begin{pmatrix} -5 & -8 & 8 \\ -3 & 3 & 8 \\ 4 & 0 & -3 \\ -14 & 12 & 8 \\ 4 & 4 & -6 \end{pmatrix} = \begin{pmatrix} -5 & -8 & 8 \\ -3 & 3 & 8 \\ 4 & 0 & -3 \\ -7 & 6 & 4 \\ 4 & 4 & -6 \end{pmatrix}$$

   (d) $(C_1 \leftrightarrow C_5)$

$$\begin{pmatrix} 4 & 6 & -5 & 0 & 2 \\ 0 & 2 & 8 & -7 & 2 \\ -1 & -6 & -9 & 10 & 7 \end{pmatrix} = \begin{pmatrix} 2 & 6 & -5 & 0 & 4 \\ 2 & 2 & 8 & -7 & 0 \\ 7 & -6 & -9 & 10 & -1 \end{pmatrix}$$

   (e) $(L_1 \leftarrow L_1 - L_2)$

$$\begin{pmatrix} 0 & 4 & -13 & 7 & 4 \\ 2 & 2 & 8 & -7 & 0 \\ 7 & -6 & -9 & 10 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 6 & -5 & 0 & 4 \\ 2 & 2 & 8 & -7 & 0 \\ 7 & -6 & -9 & 10 & -1 \end{pmatrix}$$

2. Give the inverse of the following matrices. If you think the matrix is not invertible, that might be possible.

   (a)
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} =$$

   (b)
$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}^{-1} =$$

   (c)
$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} =$$

   (d)
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} =$$

   (e)
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}^{-1} =$$

3. Circle: "makes no sense" or "cannot answer" or answer the question.
   - "makes no sense" means the problems makes no sense. More specifically it means that the first sentence that starts with "Let A ..." and ends at its period is not possible. No such matrix A exists.
   - "cannot answer" means "Such matrices A exists, but we do not have enough information to answer the question".
   - No justification needed.

   (a) Let A be a 10-by-8 matrix such that the dim(Null (A))=2. What is the rank of A?

   makes no sense cannot answer the rank of A is ___________

   (b) Let A be a 8-by-7 matrix such that the rank(A) = 7. Are the columns of A linearly dependent?

   makes no sense cannot answer yes no

   (c) Let A be a 5-by-8 matrix such that the columns are linearly independent. What is the dimension of the row space of A?

   makes no sense cannot answer the dimension of the row space of A is ________

   (d) Let A be a 9-by-7 matrix such that the rank(A) = 5. What is the dimension of the null space of A?

   makes no sense cannot answer the dimension of the null space of A is ________

   (e) Let A be a 4-by-7 matrix. Are the rows of A linearly independent?

   makes no sense cannot answer yes no

4. Either indicate whether that the matrix-matrix multiplication is not possible, xor, if it is possible, indicate the dimension of the output.

(a)

$$\begin{pmatrix} -1 & -8 & -2 & -9 & -2 \\ -1 & -7 & 7 & -2 & 3 \\ 7 & -7 & 6 & 1 & 3 \end{pmatrix} \begin{pmatrix} -4 & -8 \\ -1 & -3 \\ -10 & -6 \\ 10 & 0 \\ -7 & -3 \end{pmatrix}$$

makes no sense the output is ________-by-________

(b)

$$\begin{pmatrix} -4 & -8 \\ -1 & -3 \\ -10 & -6 \\ 10 & 0 \\ -7 & -3 \end{pmatrix} \begin{pmatrix} -1 & -8 & -2 & -9 & -2 \\ -1 & -7 & 7 & -2 & 3 \\ 7 & -7 & 6 & 1 & 3 \end{pmatrix}$$

makes no sense the output is ________-by-________

(c)

$$\begin{pmatrix} 9 \\ 8 \\ -9 \\ 5 \end{pmatrix} \begin{pmatrix} -5 & -2 & 1 & 9 \end{pmatrix}$$

makes no sense the output is ________-by-________

(d)

$$\begin{pmatrix} -5 & -2 & 1 & 9 \end{pmatrix} \begin{pmatrix} 9 \\ 8 \\ -9 \\ 5 \end{pmatrix}$$

makes no sense the output is ________-by-________

5. Compute the entry (2,5) of this matrix-matrix multiplication.

$$C = \begin{pmatrix} 4 & 1 & -7 \\ 3 & 8 & 7 \\ -6 & 3 & 3 \\ -7 & -6 & -2 \\ 10 & -3 & -6 \\ -7 & -1 & -1 \\ -9 & 10 & 0 \end{pmatrix} \begin{pmatrix} 0 & -2 & -4 & 6 & -2 \\ 2 & 0 & 2 & 0 & 1 \\ -5 & -5 & 0 & 5 & 0 \end{pmatrix}$$

The (2,5) entry of C is $c_{2,5} =$ _________.

6. Give the solution set for the following system using “inverse of a matrix”. So you must compute the inverse of the 2-by-2 matrix and then compute x with the formula $x = A^{-1}b$.

$$\left\{ \begin{array}{rcrcr} 2x_1 & - & x_2 & = & 1 \\ 3x_1 & - & 5x_2 & = & 0 \end{array} \right.$$

7. We consider the following system of equations

$$\begin{cases} 2x_1 & - & 3x_2 & = & 1 \\ 4x_1 & - & 6x_2 & = & 2 \end{cases}$$

Student claims

"the matrix $\begin{pmatrix} 2 & -3 \\ 4 & -6 \end{pmatrix}$ does not have an inverse, so there are no solutions (x_1, x_2) to the linear system of equations."

(a) Does the matrix $\begin{pmatrix} 2 & -3 \\ 4 & -6 \end{pmatrix}$ have an inverse? If yes, give the inverse? If no, explain why the inverse does not exist.

(b) Are there solutions $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ to the linear system of equations? If yes, give the solution set? If no, explain why there are no solutions.

8. Compute A^{-1}, the inverse of A.
(If the inverse of A exists. If the inverse of A does not exist, please explain.)

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

9. Compute the determinant of the matrix below

$$A = \begin{pmatrix} 1 & 3 & 1 & 1 \\ 1 & 2 & 2 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

10. Matrix A and the reduced echelon form of A are given below.

$$A = \begin{pmatrix} 8 & 16 & -1 & 5 & -6 & 33 & -8 & -5 & 8 \\ -10 & -20 & -9 & -37 & 5 & -54 & 2 & -25 & -41 \\ -1 & -2 & -6 & -19 & -5 & -16 & 4 & -11 & -16 \\ -2 & -4 & 4 & 10 & 8 & 2 & 1 & 16 & 11 \\ -1 & -2 & -1 & -4 & -5 & -11 & -1 & -16 & -11 \\ 5 & 10 & -7 & -16 & 5 & 23 & 3 & 24 & 9 \\ -4 & -8 & -3 & -13 & -6 & -29 & 3 & -21 & -21 \\ 6 & 12 & 2 & 12 & -4 & 28 & 4 & 20 & 22 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 5 & 0 & 3 & 3 \\ 0 & 0 & 1 & 3 & 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) Clearly indicate with arrows the columns / rows below that makes a basis for $\mathrm{Row}(A)$.

A basis for $\mathrm{Row}(A)$ is

$$\begin{pmatrix} 8 & 16 & -1 & 5 & -6 & 33 & -8 & -5 & 8 \\ -10 & -20 & -9 & -37 & 5 & -54 & 2 & -25 & -41 \\ -1 & -2 & -6 & -19 & -5 & -16 & 4 & -11 & -16 \\ -2 & -4 & 4 & 10 & 8 & 2 & 1 & 16 & 11 \\ -1 & -2 & -1 & -4 & -5 & -11 & -1 & -16 & -11 \\ 5 & 10 & -7 & -16 & 5 & 23 & 3 & 24 & 9 \\ -4 & -8 & -3 & -13 & -6 & -29 & 3 & -21 & -21 \\ 6 & 12 & 2 & 12 & -4 & 28 & 4 & 20 & 22 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 5 & 0 & 3 & 3 \\ 0 & 0 & 1 & 3 & 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(b) Clearly indicate with arrows the columns / rows below that makes a basis for $\mathrm{Col}(A)$.

A basis for $\mathrm{Col}(A)$ is

$$\begin{pmatrix} 8 & 16 & -1 & 5 & -6 & 33 & -8 & -5 & 8 \\ -10 & -20 & -9 & -37 & 5 & -54 & 2 & -25 & -41 \\ -1 & -2 & -6 & -19 & -5 & -16 & 4 & -11 & -16 \\ -2 & -4 & 4 & 10 & 8 & 2 & 1 & 16 & 11 \\ -1 & -2 & -1 & -4 & -5 & -11 & -1 & -16 & -11 \\ 5 & 10 & -7 & -16 & 5 & 23 & 3 & 24 & 9 \\ -4 & -8 & -3 & -13 & -6 & -29 & 3 & -21 & -21 \\ 6 & 12 & 2 & 12 & -4 & 28 & 4 & 20 & 22 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 5 & 0 & 3 & 3 \\ 0 & 0 & 1 & 3 & 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

11. Let A such that

$$A = \begin{pmatrix} 4 & 8 & 8 & 4 \\ -8 & -16 & -9 & -15 \\ 1 & 2 & -3 & -9 \\ 1 & 3 & -7 & 5 \end{pmatrix}$$

All the computation below are correct. The question is: what is the determinant of A?

It is OK to give the answer as a sign (+ or −) followed by a product of positive numbers. So for example: answers like $(-3 \times 9 \times 7 \times 5 \times 9)$ or $(+2 \times 8 \times 7 \times 7 \times 5)$ would be acceptable.

The determinant of A is __________________.

$$\begin{pmatrix} 4 & 8 & 8 & 4 \\ -8 & -16 & -9 & -15 \\ 1 & 2 & -3 & -9 \\ 1 & 3 & -7 & 5 \end{pmatrix} \quad \begin{matrix} L_1 \leftarrow L_1/4 \\ \rightsquigarrow \end{matrix} \quad \begin{pmatrix} 1 & 2 & 2 & 1 \\ -8 & -16 & -9 & -15 \\ 1 & 2 & -3 & -9 \\ 1 & 3 & -7 & 5 \end{pmatrix}$$

$$\begin{matrix} L_2 \leftarrow L_2 + 8L_1 \\ L_3 \leftarrow L_3 - L_1 \\ L_4 \leftarrow L_4 - L_1 \\ \rightsquigarrow \end{matrix} \quad \begin{pmatrix} 1 & 2 & 2 & 1 \\ 0 & 0 & 7 & -7 \\ 0 & 0 & -5 & -10 \\ 0 & 1 & -9 & 4 \end{pmatrix}$$

$$\begin{matrix} L_4 \leftrightarrow L_2 \\ \rightsquigarrow \end{matrix} \quad \begin{pmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & -9 & 4 \\ 0 & 0 & -5 & -10 \\ 0 & 0 & 7 & -7 \end{pmatrix}$$

$$\begin{matrix} L_3 \leftarrow L_3/5 \\ L_4 \leftarrow L_4/7 \\ \rightsquigarrow \end{matrix} \quad \begin{pmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & -9 & 4 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\begin{matrix} L_4 \leftarrow L_4 + L_3 \\ \rightsquigarrow \end{matrix} \quad \begin{pmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & -9 & 4 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

12. We want to solve the following linear system for x_4 using Cramer's rule. Please provide the formula for x_4 using Cramer's rule. Please give the formula with two four-by-four determinants. Do not attempt to compute anything, just write the formula. Assume a computer can compute the formula for you if the computer sees your formula. (Which, would computer be able to see, be pretty much true.)

$$\left\{\begin{array}{rcrcrrcrcr} 4x_1 & + & x_2 & + & 9x_3 & +6x_4 & - & 4x_5 & = & 0 \\ 3x_1 & + & 3x_2 & + & x_3 & -2x_4 & - & x_5 & = & 9 \\ -9x_1 & - & 2x_2 & - & 3x_3 & -8x_4 & + & x_5 & = & 3 \\ -9x_1 & + & 6x_2 & - & 8x_3 & -5x_4 & - & x_5 & = & 9 \\ -4x_1 & + & 4x_2 & + & 2x_3 & -7x_4 & + & 8x_5 & = & -5 \end{array}\right.$$

$$x_4 = \frac{\left|\phantom{\begin{matrix} \\ \\ \\ \end{matrix}}\right|}{\left|\phantom{\begin{matrix} \\ \\ \\ \end{matrix}}\right|}$$

Irrelevant facts: It turns out that

$$x_4 = -\frac{1549}{1019}.$$

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1	2	3	4	5	6

7	8	9	10	11	12