EX.2.3.5, Sauer3

Find the relative forward and backward errors and error magnification factor for the following approximate solutions of the system

$$\begin{cases} x_1 - 2\,x_2 = 3 \\ 3\,x_1 - 4\,x_2 = 7 \end{cases}$$

(a) $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$ (b) $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$

(e) What is the condition number of the coefficient matrix?

EX.2.3.5, Sauer3, solution, Langou

Colab link: https://colab.research.google.com/drive/1vNFzKXfm_Fd2-BttkXhUdT86M7dOTAVf

```
import numpy as np
```

```
A = np.array([
                  [  1., -2.  ],
                  [  3., -4.]])

b = np.array([[3.],[7.]])
```

```
# First off, we can find the solution right away.
# The solution is x1 = 1 and x2 = -1.
# Let us check this.

x = np.array([[1.],[-1.]])
print("|| b - Ax ||_oo / || b ||_oo = ",\
      f"{np.linalg.norm(b-A@x,np.infty)/np.linalg.norm(b,np.infty)}")
```

```
|| b - Ax ||_oo / || b ||_oo =  0.0
```

```
# (e) What is the condition number of the coefficient matrix?

K = np.linalg.norm(A,np.inf)*np.linalg.norm(np.linalg.inv(A),np.inf)
print("kappa(A) = ", f"{K:6.1f}")
```

```
kappa(A) =    21.0
```

```
# Find the relative forward and backward errors and error
# magnification factor for the following approximate solutions
# of the system
# (a) [ -2 ]  (b) [ -2 ]  (c) [  0 ]  (d) [ -1 ]
```

```
#     [ -4 ]      [ -3 ]      [ -2 ]      [ -1 ]
for xa in [ np.array([[-2.],[-4.]]),\
            np.array([[-2.],[-3.]]),\
            np.array([[ 0.],[-2.]]),\
            np.array([[-1.],[-1.]]) ]:
  relative_forward_error = np.linalg.norm( x - xa, np.inf )\
     / np.linalg.norm( x, np.inf )
  relative_backward_error = np.linalg.norm( b - A@xa, np.inf )\
    / np.linalg.norm( b, np.inf )
  error_magnification_factor = relative_forward_error\
    / relative_backward_error
  print( "xa = [ ",f"{xa[0,0]:.0f}",",",f"{xa[1,0]:.0f}","]" )
  print( "relative forward error     = ",\
    f"{relative_forward_error:8.2f}" )
  print( "relative backward error    = ",\
    f"{relative_backward_error:8.2f}" )
  print( "error magnification factor = ",\
   f"{error_magnification_factor:8.2f}" )
  print("\n")
```

```
xa = [  -2 , -4 ]
relative forward error     =      3.00
relative backward error    =      0.43
error magnification factor =      7.00


xa = [  -2 , -3 ]
relative forward error     =      3.00
relative backward error    =      0.14
error magnification factor =     21.00


xa = [  0 , -2 ]
relative forward error     =      1.00
relative backward error    =      0.14
error magnification factor =      7.00


xa = [  -1 , -1 ]
relative forward error     =      2.00
relative backward error    =      0.86
error magnification factor =      2.33
```