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EX.1.2.7, Sauer3

Use Theorem 1.6 to determine whether Fixed-Point Iteration of g(x) is locally convergent to the given fixed point r. (a) $g(x) = (2x-1)^{\frac{1}{3}}$, r = 1, (b) $g(x) = \frac{1}{2}(x^3+1)$, r = 1, (c) $g(x) = \sin x + x$, r = 0.

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EX.1.2.7, Sauer3, solution, Langou

- a. (a) The function $\left(g:\left(\frac{1}{2},\infty\right)\to\mathbb{R},\quad x\mapsto (2x-1)^{\frac{1}{3}}\right)$ is continuously differentiable in $\left(\frac{1}{2},\infty\right)$,
 - (b) by direct evaluation, we see that g(r) = r, (i.e., g(1) = 1,)
 - (c) we compute g'(x) and get $g'(x) = \frac{2}{3}(2x-1)^{-\frac{2}{3}}$, evaluate at r=1, and get $g'(r)=\frac{2}{3}$, and see that |g'(r)|<1.

We conclude (using Theorem 1.6) that Fixed Point Iteration is locally convergent to the fixed point r = 1.

- b. (a) The function $(g: \mathbb{R} \to \mathbb{R}, x \mapsto \frac{1}{2}(x^3+1))$ is continuously differentiable in \mathbb{R} ,
 - (b) by direct evaluation, we see that g(r) = r, (i.e., g(1) = 1,)
 - (c) we compute g'(x) and get $g'(x) = \frac{3}{2}x^2$, evaluate at r = 1, and get $g'(r) = \frac{3}{2}$, and see that $|g'(r)| \ge 1$.

The assumption of Theorem 1.6 are not satisfied. We cannot conclude anything.

- c. (a) The function $g: \mathbb{R} \to \mathbb{R}$, $x \mapsto \sin x + x$ is continuously differentiable in \mathbb{R} ,
 - (b) by direct evaluation, we see that g(r) = r, (i.e., g(0) = 0,)
 - (c) we compute g'(x) and get $g'(x) = \cos x + 1$, evaluate at r = 0, and get g'(r) = 2, and see that $|g'(r)| \ge 1$.

The assumption of Theorem 1.6 are not satisfied. We cannot conclude anything.