## EX.0.2.7, Sauer

Convert the following binary numbers to base 10. (a) 1010101 (b) 1011.101 (c) $10111.\overline{01}$ (d) $110.\overline{10}$ (e) $10.\overline{110}$ (f) $110.1\overline{101}$ (g) $10.0101\overline{1101}$ (h) $111.\overline{1}$.



## EX.0.2.7, Sauer, solution, Langou

- Only turning the Python code is not a good answer.
- The copy-paste from this PDF to python code does not work great. It is better to copy-paste from colab.
- The Colab Jupyter Notebook is available at: `https://colab.research.google.com/drive/1prkH1Saji2f2Th8Lom5 JcOhuuBJQSoY`.

a.

$$(1010101)_2 = 64 + 16 + 4 + 1 = (85)_{10}$$

$$(1010101)_2 = (85)_{10}$$

```
print( 0b1010101 )
print( int( '1010101', 2 ) )
```

```
85
85
```

b.

$$(1011.101)_2 = 8 + 2 + 1 + 0.5 + 0.125 = (11.625)_{10}.$$

$$(1011.101)_2 = (11.625)_{10}.$$

```
print( ( 0b1011101 ) * ( 2 ** -3 ) )
```

```
11.625
```

c. $(10111)_2 = 16 + 4 + 2 + 1 = 23$

$$\begin{aligned}
(0.\overline{01})_2 &= \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots \\
&= \frac{1}{4}\left(1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \dots\right) \\
&= \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n = \frac{1}{4}\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{1}{4}\frac{4}{3} = \frac{1}{3}
\end{aligned}$$

$$(10111.\overline{01})_2 = (23\frac{1}{3})_{10}.$$

```python
# appending 6 times the repetend 01
x = ( 0b10111010101010101 ) * ( 2 ** (-12) )
print( x )
print( 23. + 1./3. )
```

```
23.333251953125
23.333333333333332
```

```python
# appending 30 times the repetend 01
x = 0b10111
y = 0b01
for i in range(0,30):
  y = y * ( 2 ** (-2) )
  x = x + y
print( x )
print( 23. + 1./3. )
```

```
23.333333333333332
23.333333333333332
```

d. $(110)_2 = 4 + 2 = 6$

$$\begin{aligned}(0.\overline{10})_2 &= \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \dots \\ &= \frac{1}{2}\left(1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \dots\right) \\ &= \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n = \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{1}{2}\frac{4}{3} = \frac{2}{3}\end{aligned}$$

$$(110.\overline{10})_2 = (6\frac{2}{3})_{10}.$$

```python
# appending 6 times the repetend 10
x = ( 0b110101010101010 ) * ( 2 ** (-12) )
print( x )
print( 6. + 2./3. )
```

```
6.66650390625
6.666666666666667
```

```python
# appending 30 times the repetend 10
x = 0b110
y = 0b10
for i in range(0,30):
  y = y * ( 2 ** (-2) )
  x = x + y
print( x )
print( 6. + 2./3. )
```

```
6.666666666666666
6.666666666666667
```

e. $(10)_2 = 2$

$$
\begin{aligned}
(0.\overline{110})_2 &= \left(\frac{1}{2}+\frac{1}{4}\right)+\frac{1}{8}\left(\frac{1}{2}+\frac{1}{4}\right)+\left(\frac{1}{8}\right)^2\left(\frac{1}{2}+\frac{1}{4}\right)+\left(\frac{1}{8}\right)^3\left(\frac{1}{2}+\frac{1}{4}\right)+\dots \\
&= \frac{3}{4}\left(1+\frac{1}{8}+\left(\frac{1}{8}\right)^2+\left(\frac{1}{8}\right)^3+\dots\right) \\
&= \frac{3}{4}\sum_{n=0}^{\infty}\left(\frac{1}{8}\right)^n = \frac{3}{4}\left(\frac{1}{1-\frac{1}{8}}\right) = \frac{3}{4}\frac{8}{7} = \frac{6}{7}
\end{aligned}
$$

$$(10.\overline{110})_2 = (2\frac{6}{7})_{10}.$$

```
# appending 5 times the repetend 110
x = ( 0b10110110110110110 ) * ( 2 ** (-15) )
print( x )
print( 2. + 6./7. )
```

```
2.85711669921875
2.857142857142857
```

```
# appending 20 times the repetend 110
x = 0b10
y = 0b110
for i in range(0,20):
  y = y * ( 2 ** (-3) )
  x = x + y
print( x )
print( 2. + 6./7. )
```

```
2.857142857142857
2.857142857142857
```

f. $(110)_2 = 6$

$$
\begin{aligned}
(0.1\overline{101})_2 &= \frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{8}\right)+\frac{1}{2}\frac{1}{8}\left(\frac{1}{2}+\frac{1}{8}\right)+\frac{1}{2}\left(\frac{1}{8}\right)^2\left(\frac{1}{2}+\frac{1}{8}\right)+\frac{1}{2}\left(\frac{1}{8}\right)^3\left(\frac{1}{2}+\frac{1}{8}\right)+\dots \\
&= \frac{1}{2}+\frac{5}{16}\left(1+\frac{1}{8}+\left(\frac{1}{8}\right)^2+\left(\frac{1}{8}\right)^3+\dots\right) \\
&= \frac{1}{2}+\frac{5}{16}\sum_{n=0}^{\infty}\left(\frac{1}{8}\right)^n = \frac{1}{2}+\frac{5}{16}\left(\frac{1}{1-\frac{1}{8}}\right) = \frac{1}{2}+\frac{5}{16}\frac{8}{7} = \frac{1}{2}+\frac{5}{14} = \frac{6}{7}
\end{aligned}
$$

$$(110.1\overline{101})_2 = (6\frac{6}{7})_{10}.$$

```
# appending 4 times the repetend 101
x = ( 0b1101101101101101 ) * ( 2 ** (-13) )
print( x )
print( 6. + 6./7. )
```

```
6.8570556640625
6.857142857142857
```

```
# appending 20 times the repetend 101
x = 0b1101 * ( 2 ** (-1) )
y = 0b101 * ( 2 ** (-1) )
for i in range(0,20):
  y = y * ( 2 ** (-3) )
  x = x + y
print( x )
print( 6. + 6./7. )
```

```
2.857142857142857
2.857142857142857
```

g. $(10)_2 = 2$

$$\begin{aligned}
(0.010\overline{1101})_2 &= \frac{1}{4} + \frac{1}{8}\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{16}\right) + \frac{1}{8}\frac{1}{16}\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{16}\right) + \frac{1}{8}\left(\frac{1}{16}\right)^2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{16}\right) + \ldots \\
&= \frac{1}{4} + \frac{13}{128}\left(1 + \frac{1}{16} + \left(\frac{1}{16}\right)^2 + \ldots\right) \\
&= \frac{1}{4} + \frac{13}{128}\sum_{n=0}^{\infty}\left(\frac{1}{16}\right)^n = \frac{1}{4} + \frac{13}{128}\left(\frac{1}{1-\frac{1}{16}}\right) = \frac{1}{4} + \frac{13}{128}\frac{16}{15} = \frac{1}{4} + \frac{13}{120} = \frac{43}{120}
\end{aligned}$$

$$(10.010\overline{1101})_2 = (2\frac{43}{120})_{10}.$$

```
# appending 4 times the repetend 1101
x = ( 0b1001011011101 ) * ( 2 ** (-11) )
print( x )
print( 2. + 43./120. )
```

```
2.35791015625
2.3583333333333334
```

```
# appending 20 times the repetend 1101
x = 0b10010 * ( 2 ** (-3) )
y = 0b1101 * ( 2 ** (-3) )
for i in range(0,20):
  y = y * ( 2 ** (-4) )
  x = x + y
print( x )
print( 2. + 43./120. )
```

```
2.3583333333333334
2.3583333333333334
```

h. $(111.\bar{1})_2$ is same as $(1000)_2$ so it is $(8)_{10}$. This is like saying that in base 10 the number $3.9999999\ldots$ is same a 4.

A more rigorous explanation $(111)_2 = 4 + 2 + 1 = 7$

$$
\begin{aligned}
(0.\bar{1})_2 &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \\
&= \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} \left(\frac{1}{1 - \frac{1}{2}}\right) = \frac{1}{2}(2) = 1
\end{aligned}
$$

So $(111.\bar{1})_2 = 7 + 1 = 8.$

$$(111.\bar{1})_2 = (8)_{10}.$$

```
# appending 17 times the repetend 1
x = ( 0b11111111111111111111 ) * ( 2 ** (-17) )
print( x )
```

```
7.999992370605469
```

```
# appending 60 times the repetend 1
x = 0b111
y = 0b1
for i in range(0,60):
  y = y * ( 2 ** (-1) )
  x = x + y
print( x )
```

```
8.0
```