EX.0.2.8, Sauer

Convert the following binary numbers to base 10. (a) 11011, (b) 110111.001, (c) $111.\overline{001}$, (d) $1010.\overline{01}$, (e) $10111.1\overline{0101}$ (f) $1111.010\overline{0001}$

a. $(11011)_2 = (2^4+2^3+2^1+2^0)_{10}$

$= (16+8+2+1)_{10} = (27)_{10}$

b. $(110111.001)_2 = (2^5+2^4+2^2+2^1+2^0+(\frac{1}{2})^3)_{10}$

$= (32+16+4+2+1+\frac{1}{8})_{10}$

$= (55\frac{1}{8})_{10} = (55.125)_{10}$

c. $(111.\overline{001})_2 = (2^2+2^1+2^0)_{10} + (0.\overline{001})_2$

set $x = (0.\overline{001})_2$ $\quad 2^3 \cdot x = (1.\overline{001})_2$

then $2^3 \cdot x - x = (1)_2 = 1$

$(2^3-1)x = 1$

$x = \frac{1}{7}$

$\therefore (111.\overline{001})_2 = (7\frac{1}{7})_{10}$

d. $(1010.\overline{01})_2 = (2^3+2^1)_{10} + (0.\overline{01})_2$

set $x = (0.\overline{01})_2$ $\quad 2^2 \cdot x = (1.\overline{01})_2$

then $(2^2-1)x = (1)_2 = 1$

$x = \frac{1}{3}$

$\therefore (1010.\overline{01})_2 = (10\frac{1}{3})_{10}$

e. $(10111.1\overline{0101})_2 = (2^4+2^2+2^1+2^0+(\frac{1}{2})^1)_{10} + (0.0\overline{0101})_2$

set $x=(0.00\overline{0101})_2$ $\quad 2^2\cdot x=(0.\overline{0101})_2 = y$

$$(1.\overline{0101})_2 = 2^2\cdot y$$

$$\therefore y=\frac{1}{3},\quad x=\frac{1}{6}$$

$\therefore (10111.1\overline{0101})_2=(16+4+2+1+\frac{1}{2}+\frac{1}{6})_{10}=(23\frac{2}{3})_{10}$

f. $(1111.010\overline{001})_2=(2^3+2^2+2^1+2^0+(\frac{1}{2})^2)_{10}+(0.000\overline{001})_2$

set $x=(0.000\overline{001})_2$ $\quad 2^6\cdot x=(1.001\overline{001})_2$

then $(2^6-1)x=(1.001)_2=(2^0+(\frac{1}{2})^3)_{10}=(\frac{9}{8})_{10}$

$$x=\frac{1}{63}\cdot\frac{9}{8}=(\frac{1}{56})_{10}$$

$\therefore (1111.010\overline{001})_2=(15+\frac{1}{4}+\frac{1}{56})_{10}=(15\frac{15}{56})_{10}$