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EX.0.4.1, Sauer Langou

Identify for which values of x there is subtraction of nearly equal numbers, and find an alternate form that avoids the problem.

(a) 
$$\frac{1-\sec x}{\tan^2 x}$$
 (b)  $\frac{1-(1-x)^3}{x}$  (c)  $\frac{1}{1+x}-\frac{1}{1-x}$ 

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## EX.0.4.1, Sauer, solution, Langou

a. We see that there is subtraction of nearly equal numbers whenever  $\sec x = 1$ , so whenever  $\cos x = 1$ , so whenever x is near  $2k\pi$  for  $k \in \mathbb{Z}$ .

We remember our trig formula:

$$\sec^2 = 1 + \tan^2 x.$$

This is coming from

$$1 + \tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

So in other words, we have that

$$\sec^2 - 1 = \tan^2 x.$$

Once we see this and our function at hand, it makes sense to multiply and divide by  $(1 - \sec x)$ , we get

$$\frac{1-\sec x}{\tan^2 x} = \left(\frac{1-\sec x}{\tan^2 x}\right) \left(\frac{1+\sec x}{1+\sec x}\right) = \left(\frac{1-\sec^2 x}{\tan^2 x}\right) \left(\frac{-1}{1+\sec x}\right) = \frac{-1}{1+\sec x}.$$

The  $\frac{-1}{1+\sec x}$  alternate form avoids the cancellation problem for x near  $2k\pi$  for  $k\in\mathbb{Z}$ .

b. We see that there is subtraction of nearly equal numbers whenever  $(1-x)^3 = 1$ , so whenever x is near 0. Some algebra:

$$\frac{1 - (1 - x)^3}{x} = \frac{x^3 - 3x^2 + 3x}{x} = x^2 - 3x + 3.$$

So an alternate form that avoids the cancellation problem for x near 0 is  $x^2 - 3x + 3$ .

c. We see that there is subtraction of nearly equal numbers whenever x is near 0.

Some algebra:

$$\frac{1}{1+x} - \frac{1}{1-x} = \frac{-2x}{(1+x)(1-x)} = \frac{2x}{(1+x)(x-1)}$$

So an alternate form that avoids the cancellation problem for x near 0 is  $\frac{2x}{(1+x)(x-1)}$ .