CP.4.1.5, Sauer3

A company test-markets a new soft drink in 22 cities of approximately equal size. The selling price (in dollars) and the number sold per week in the cities are listed as follows:

city	price	sales/week
1	0.59	3980
2	0.80	2200
3	0.95	1850
4	0.45	6100
5	0.79	2100
6	0.99	1700
7	0.90	2000
8	0.65	4200
9	0.79	2440
10	0.69	3300
11	0.79	2300
12	0.49	6000
13	1.09	1190
14	0.95	1960
15	0.79	2760
16	0.65	4330
17	0.45	6960
18	0.60	4160
19	0.89	1990
20	0.79	2860
21	0.99	1920
22	0.85	2160

a. First, the company wants to find the "demand curve": how many it will sell at each potential price. Let P denote price and S denote sales per week. Find the line $S = c_1 + c_2 P$ that best fits the data from the table in the sense of least squares. Find the normal equations and the coefficients c_1 and c_2 of the least squares line. Plot the least squares line along with the data, and calculate the root mean square error.

b. After studying the results of the test marketing, the company will set a single selling price P throughout the country. Given a manufacturing cost of \$0.23 per unit, the total profit (per city, per week) is $S(P - 0.23)$ dollars. Use the results of the preceding least squares approximation to find the selling price for which the company's profit will be maximized.

Getting started: https://colab.research.google.com/drive/13gw6Vwiz6cQjx3lElAJtqnxC6vhabWkv