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EX.2.3.2.a, Sauer3

Find the infinity norm condition number of

$$\text{(a)} \quad A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

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EX.2.3.2.a, Sauer3, solution, Langou

Colab: `https://colab.research.google.com/drive/14vOZOyhkkon5EBVUPjRILlRaC-P8Ktfi`

$$\|A\|_\infty = \max\left(|1|+|2|, |3|+|4|\right) = \max\left(3, 7\right) = 7.$$

$$A^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} = \frac{1}{(1)(4)-(2)(3)} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}.$$

$$\|A^{-1}\|_\infty = \frac{1}{2}\max\left(|-4|+|2|, |3|+|-1|\right) = \frac{1}{2}\max\left(6, 4\right) = 3.$$

$$\kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty = 7 * 3 = 21.$$

$$\boxed{\kappa_\infty(A) = 21}$$

```python
import numpy as np

A = np.array([
                 [  1., 2.],
                 [  3., 4.]])

print( "|| A ||_oo =",\
  f"{np.linalg.norm(A,np.infty):6.1f}")

print( "\n|| A^{-1} ||_oo =",\
  f"{np.linalg.norm(np.linalg.inv(A),np.infty):6.1f}")

print( "\nkappa( A ) = || A ||_oo * || A^{-1} ||_oo =",\
  f"{np.linalg.norm(A,np.infty)\
   * np.linalg.norm(np.linalg.inv(A),np.infty):6.1f}")
```

```
|| A ||_oo =    7.0

|| A^{-1} ||_oo =    3.0

kappa( A ) = || A ||_oo * || A^{-1} ||_oo =   21.0
```