Summer 2024

MATH 4650-E01
CSCI 4650-E01
MATH 5660-E01

Homework 1

Due on Monday June 10th 2024

Exercises:

1. EX.0.1.2.Sauer using Colab
2. EX.0.1.6.Sauer using Colab
3. CP.0.1.2.Sauer using Colab
4. EX.0.2.4.Sauer handwritten
5. EX.0.2.8.Sauer handwritten
6. EX.0.3.6.Sauer handwritten
7. EX.0.3.7.Langou handwritten
8. EX.0.4.2.Sauer handwritten
9. EX.0.5.2.Sauer handwritten
10. EX.0.5.6.Sauer handwritten

EX.0.1.2, Sauer

Rewrite the following polynomials in nested form and evaluate at $x = -1/2$.

a. $p(x) = 6x^3 - 2x^2 - 3x + 7$

b. $p(x) = 8x^5 - x^4 - 3x^3 + x^2 - 3x + 1$

c. $p(x) = 4x^6 - 2x^4 - 2x + 4$

EX.0.1.6, Sauer

Explain how to evaluate the polynomial below for a given input x, using as few operations as possible. You are not allowed to use the power operator. So no `x**5` for example. You can only multiplications and additions. How many multiplications and how many additions are required?

a. $p(x) = a_0 + a_5 x^5 + a_{10} x^{10} + a_{15} x^{15}$

b. $p(x) = a_7 x^7 + a_{12} x^{12} + a_{17} x^{17} + a_{22} x^{22} + a_{27} x^{27}$

CP.0.1.2, Sauer

Use nest.m to evaluate $p(x) = 1 - x + x^2 - x^3 + \ldots + x^{98} - x^{99}$ at $x = 1.00001$. Find a simpler, equivalent expression and use it to estimate the error of the nested multiplication.

EX.0.2.4, Sauer

Convert the following base 10 numbers to binary. (a) 11.25 (b) 2/3 (c) 3/5 (d) 3.2 (e) 30.6 (f) 99.9.

EX.0.2.8, Sauer

Convert the following binary numbers to base 10. (a) 11011, (b) 110111.001, (c) $111.\overline{001}$, (d) $1010.\overline{01}$, (e) $10111.1\overline{0101}$ (f) $1111.010\overline{001}$

EX.0.3.6, Sauer

Do the following sums by hand in IEEE double precision computer arithmetic, using the Rounding to Nearest Rule.

a. $(1 + (2^{-51} + 2^{-52} + 2^{-54})) - 1$

b. $(1 + (2^{-51} + 2^{-52} + 2^{-60})) - 1$

EX.0.3.7, Langou

Write each of the given numbers in Matlab's *format hex*. Show your work. Then check your answers with Matlab. (a) 16 (b) 130 (c) 1/4 (d) $\mathrm{fl}(1/7)$ (e) $\mathrm{fl}(4/7)$ (f) $\mathrm{fl}(0.01)$ (g) $\mathrm{fl}(-0.01)$ (h) $\mathrm{fl}(-0.02)$

EX.0.4.2, Sauer

Find the roots of the equation $x^2 + 3x - 8^{-14} = 0$ with three-digit accuracy.

EX.0.5.2, Sauer

Find c satisfying the Mean Value Theorem for $f(x)$ on the interval $[0, 1]$.
(a) $f(x) = e^x$.
(b) $f(x) = x^2$.
(c) $f(x) = 1/(x + 1)$.

Note: In this exercise, we pretty much do not consider round-off errors. For example, we will assume that the values returned by matlab when evaluating $f(x) = x^{-2}$ are exact. This is justified because our approximation errors (using Taylor polynomials) are much larger than the round-off errors. We will see some limitations (when round-off errors are not negligible any longer) at the end of the exercise.

EX.0.5.6, Sauer

a. Find the Taylor polynomial of degree 4 for $f(x) = x^{-2}$ about the point $x_0 = 1$.

b. Use the result of (a) to approximate $f(0.9)$ and $f(1.1)$.

c. Use the Taylor remainder to find an error formula for the Taylor polynomial. Give error bounds for each of the two approximations made in part (b). Which of the two approximations in part (b) do you expect to be closer to the correct value?

d. Use a calculator to compare the actual error in each case with your error bound from part (c).