EX.5.3.2.a, Sauer3
Apply Romberg Integration to find R_{33} for the integral

$$\int_0^1 xe^x \, dx.$$

EX.5.3.2.a, Sauer3, solution, Langou

Colab: `https://colab.research.google.com/drive/1YVuXc4AH7CFZRS5QVmQMlOVvdMnQRtcA`

- This is the same integral as in EX.5.2.4.a, we recall that the exact value of the integral we are trying to compute is 1.
- ```
  f = @(x) x*exp(x);
  h1 = 1; h2 = 1/2; h3 = 1/4;
  R11 = 1/2 * h1 * ( f(0) + f(1) );
  R21 = 1/2 * R11 + h2 * f( 1/2 );
  R31 = 1/2 * R21 + h3 * ( f( 1/4 ) + f( 3/4 ) );
  R22 =  ( 4 * R21 - R11 ) / 3;
  R32 =  ( 4 * R31 - R21 ) / 3;
  R33 =  ( 16 * R32 - R22 ) / 15;
  ```
- The final answer of the integration process is R_{33}, we get

  ```
  1.0000056017291137
  ```
- We can report the value obtained in a table:

  ```
  R11 = 1.3591409142295225
  R21 = 1.0917507747897934   R22 = 1.0026207283098838
  R31 = 1.0230644790527572   R32 = 1.0001690471404119   R33 = 1.0000056017291137
  ```

  Associated errors are:

  ```
  3.6e-01
  9.2e-02   2.6e-03
  2.3e-02   1.7e-04   5.6e-06
  ```

  Computing R_{33} only required 5 function evaluations.
- We note that R_{22} is exactly Simpson with $m = 1$ as computed in EX.5.2.4.a, R_{32} is exactly Simpson with $m = 2$ as computed in EX.5.2.4.a. This was expected.
  ```