Name:____________________________________

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1. **(10 points)** Give the solution set in parametric form.

$$\begin{cases} x_1 - 2x_2 + 2x_5 = -1 \\ x_3 - 8x_5 = 9 \\ x_4 - 8x_5 = 9 \end{cases}$$

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2. **(10 points)** Give the solution set in parametric form.

$$\left\{\begin{array}{rcrcrcrcr} & & 2x_2 & + & 4x_3 & - & 4x_4 & = & -2 \\ -x_1 & + & 3x_2 & + & 5x_3 & & & = & 1 \\ x_1 & - & 2x_2 & - & 3x_3 & - & 2x_4 & = & -3 \end{array}\right.$$

3. **(10 points)** Give the solution set in parametric form.

$$\begin{cases} x_1 + 2x_2 - 3x_3 = -3 \\ -x_1 - 3x_2 + x_3 = 5 \\ x_1 + 4x_2 + x_3 = -5 \end{cases}$$

4. **(10 points)** Give the solution set in parametric form. (Note: solutions are in $\mathbb{R}^5$, so there are "x_1" and "x_3" in the equations although you do not see them.)

$$\left\{ \begin{array}{rrrrrrl} - & x_2 & + & 8x_4 & - & 6x_5 & = 9, \end{array} \right.$$

5. **(10 points)** For all values of k, give the solution set in parametric form.

$$\begin{cases} 2x & - & 4y & = & 8 \\ -x & + & 2y & = & k \end{cases}$$

Note: *The question ask for whether "there is no solution, a unique solution or infinitely many solutions". But the question asks for more. The question asks for the solution set in parametric form. So if "no solution", write "no solution". If "unique solution", write what "the unique solution" is. If "infinitely many solutions", write what "the infinitely many solutions" in parametric form are. Your answer depends on the value of k. So you need to say "if k is ... then", etc. You need to cover all possible k values in* $\mathbb{R}$.

6. **(10 points)** For all values of k, give the solution set in parametric form.

$$\begin{cases} x - 3y = 1 \\ -x + ky = 2 \end{cases}$$

Note: The question ask for whether "there is no solution, a unique solution or infinitely many solutions". But the question asks for more. The question asks for the solution set in parametric form. So if "no solution", write "no solution". If "unique solution", write what "the unique solution" is. If "infinitely many solutions", write what "the infinitely many solutions" in parametric form are. Your answer depends on the value of k. So you need to say "if k is ... then", etc. You need to cover all possible k values in $\mathbb{R}$.

7. **(10 points)** Justification required.
Let

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \\ 0 \\ 3 \end{pmatrix}, v_3 = \begin{pmatrix} 2 \\ 1 \\ -3 \\ 1 \\ 0 \\ 1 \end{pmatrix}.$$

Determine whether the vectors $\{v_1, v_2, v_3\}$ are linearly dependent or linearly independent.

8. **(10 points)** Justification required.
Let
$$v_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}.$$
Determine whether the vectors $\{v_1, v_2, v_3\}$ are linearly dependent or linearly independent.

9. **(10 points) No justification required.** Short questions. If not possible, write "not possible". If possible, give an example. **No justification required.**

   (a) Give an example of 3 vectors v_1, v_2, and v_3 in $\mathbb{R}^4$ such that (i) v_1, v_2, and v_3 are linearly dependent and (ii) v_3 is not a linear combination of v_1 and v_2.

   (b) Give an example of 3 vectors v_1, v_2, and v_3 in $\mathbb{R}^4$ such that (i) v_1, v_2, and v_3 are linearly independent and (ii) v_3 is a linear combination of v_1 and v_2.

   (c) Give an example of 3 vectors v_1, v_2, and v_3 in $\mathbb{R}^4$ such that (i) v_1, v_2, and v_3 are linearly dependent; and (ii) v_1 and v_2 are linearly independent; and (iii) v_1 is not a linear combination of v_2, and v_3; and (iv) v_2 is not a linear combination of v_1, and v_3.
   So, you need to give three vectors that satisfies **all of these four** *conditions. If you think this is not possible, write "not possible".*

10. **(10 points) No justification required.** True or False or Makes no sense? Determine whether the following statements are (i) true or (ii) false or (iii) the first "Let" statement makes no sense.

A question might "make no sense". "makes no sense" means that there does not exist any matrix A such that the first statement "Let A be m-by-n matrix with x free/leading variables." is true.

A question might seem to be "cannot answer". "true" means "always true". "false" means "false for at least one case". In other words, if you think "cannot answer" (becasue you can find examples where this is "true" and examples where this is "false"), then this means that the answer is "false".

To help answer, you can start the second statement by "it is **always** true that"

For example, the first question reads:

(a) Let A be 9-by-3 matrix with 6 free variables. The columns of A are linearly dependent.

You can think of it like that

(a) Let A be 9-by-3 matrix with 6 free variables. **It is always true, for any such matrix** A**, that** the columns of A are **always** linearly dependent.

(a) Let A be 9-by-3 matrix with 6 free variables. The columns of A are linearly dependent.

true false makes no sense

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no justification required, this space is left blank in case you need scracth space.

(b) Let A be 9-by-3 matrix with no free variable. For all b in $\mathbb{R}^9$ there always exists a solution x in $\mathbb{R}^3$ such that $Ax = b$.

true false makes no sense

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no justification required, this space is left blank in case you need scracth space.

(c) Let A be 4-by-7 matrix with 4 leading variables. For all b in $\mathbb{R}^4$ there always exists a solution x in $\mathbb{R}^7$ such that $Ax = b$.

true false makes no sense

no justification required, this space is left blank in case you need scracth space.

(d) Let A be 4-by-7 matrix with 3 free variables. For all b in $\mathbb{R}^4$ there always exists a solution x in $\mathbb{R}^7$ such that $Ax = b$.

true false makes no sense

no justification required, this space is left blank in case you need scracth space.

(e) Let A be 7-by-5 matrix. The columns of A are linearly independent.

true false makes no sense

no justification required, this space is left blank in case you need scracth space.

(f) Let A be 7-by-5 matrix. The columns of A are linearly dependent.

true false makes no sense

no justification required, this space is left blank in case you need scracth space.

(g) Let A be 5-by-7 matrix. The columns of A are linearly independent.

true false makes no sense

no justification required, this space is left blank in case you need scracth space.

(h) Let A be 5-by-7 matrix. The columns of A are linearly dependent.

true false makes no sense

no justification required, this space is left blank in case you need scracth space.

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1	2	3	4	5

6	7	8	9	10