EX.1.4.7, Sauer3 **Langou**

Let $f(x) = x^4 - 7x^3 + 18x^2 - 20x + 8$. Does Newton's Method converge quadratically to the root $r = 2$? Find $\lim_{i\to\infty} e_{i+1}/e_i$, where e_i denotes the error at step i.

EX.1.4.7, Sauer3, solution, Langou

To determine the multiplicity of the root, we take derivatives and evaluate these at the root.

$$\begin{aligned} f'(x) &= 4x^3 - 21x^2 + 36x - 20 & f'(2) &= 0 \\ f''(x) &= 12x^2 - 42x + 36 & f''(2) &= 0 \\ f'''(x) &= 24x^2 - 42 & f'''(2) &= 6 \end{aligned}$$

Thus we have that $x = 2$ is a root of multiplicity three. The convergence is not quadratic. Using Theorem 1.12, Newton's Method is locally convergent to $r = 2$ and the convergence rate is

$$\lim_{i\to\infty} \frac{e_{i+1}}{e_i} = \frac{(m-1)}{m} = \frac{2}{3}.$$