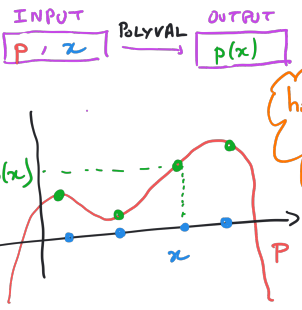
0.1 Evaluating a Polynomial

$$p(x) = -2x^4 + 9x^3 + 2x^2 - 7x + 4$$



BASIC POLYVAL

```
def polyval(c,x):
    px = c[0]
    for i in range(1, np.size(c)):
        px = px + c[i] * (x)**i
    return px
```

$$\begin{aligned} p(x) &= -2x^4 + 9x^3 + 2x^2 - 7x + 4 \\ &= \left(-2x^3 + 9x + 2x - 7\right) x + 4 \\ &= \left(\left(-2x^2 + 9x + 2\right)x - 7\right) x + 4 \\ &= \left(\left(\left(-2x + 9\right)x + 2\right)x - 7\right)x + 4 \end{aligned}$$

```
y = -2
y = y * x + 9
y = y * x + 2
y = y * x - 7
y = y * x + 4
```

```
def polyval_nested(c,x):
    d = np.size(c)
    px = c[d-1]
    for i in range(d-2,-1,-1):
        px = px * x + c[i]
    return px
```

NESTED POLYVAL

for a polynomial of degree d
NESTED POLYVAL performs
d multiplications
and d additions

for a polynomial of degree d
BASIC POLYVAL performs
d multiplications
and d additions
and d " power to the i ", i = 1 to d

Note: to compute 7^{100} → "cheating" (NOT good)
a good algorithm is NOT

NOT GOOD → 7 * 7 * 7 * 7 . . . .
↳ impossible to write

NOT GOOD →
```
x = 7
for i in range(1, 100):
    x = x * 7
```
→ COST
99 multiplications to compute x^{100}

$$100 = 64 + 32 + 4$$

$$\begin{aligned} x1 &= 7 \\ x2 &= x1 * x1 = 7^2 \\ x4 &= x2 * x2 = 7^4 \\ x8 &= x4 * x4 = 7^8 \\ x16 &= x8 * x8 = 7^{16} \\ x32 &= x16 * x16 = 7^{32} \\ x64 &= x32 * x32 = 7^{64} \end{aligned}$$

$$x100 = x64 * x32 * x4$$

8 multiplications to compute x^{100}