Summer 2024

MATH 4650-E01
CSCI 4650-E01
MATH 5660-E01 (additional exercises will be assigned by email for 5660 students)

Reality Check #5: Motion Control in Computer-Aided Modeling

Due on Friday July 26th 2024

Computer-aided modeling and manufacturing requires precise control of spatial position along a prescribed motion path. We will illustrate the use of Adaptive Quadrature to solve a fundamental piece of the problem: equipartition, or the division of an arbitrary path into equal-length subpaths.

In numerical machining problems, it is preferable to maintain constant speed along the path. During each second, progress should be made along an equal length of the machine–material interface. In other motion planning applications, including computer animation, more complicated progress curves may be required: A hand reaching for a doorknob might begin and end with low velocity and have higher velocity in between. Robotics and virtual reality applications require the construction of parametrized curves and surfaces to be navigated. Building a table of small equal increments in path distance is often a necessary first step.

Assume that a parametric path $P = \{x(t), y(t) | 0 \leq t \leq 1\}$ is given. Figure 5.6 shows the example path

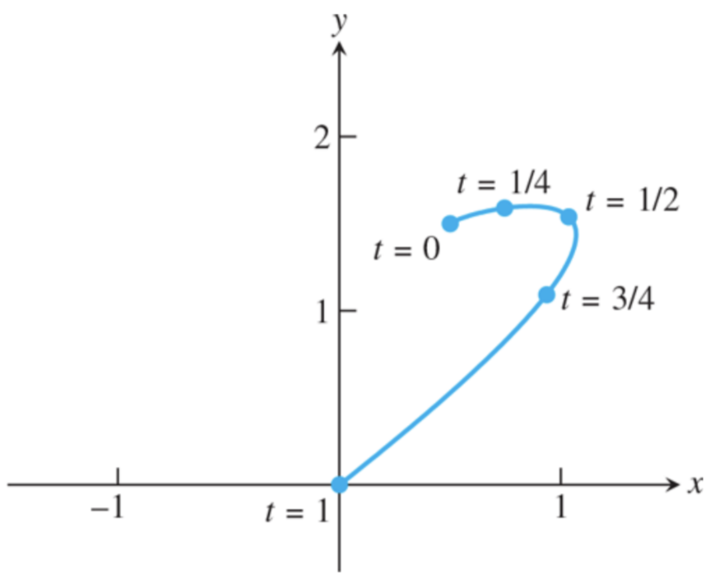

Figure 5.6. Parametrized curve given by Bézier spline.

Typically, equal intervals of the parameter t do not divide the path into segments of equal length.

$$P = \begin{cases} x(t) = 0.5 + 0.3t + 3.9t^2 - 4.7t^3 \\ y(t) = 1.5 + 0.3t + 0.9t^2 - 2.7t^3 \end{cases}$$

which is the Bézier curve defined by the four points (0.5,1.5), (0.6,1.6), (2,2), (0,0). (See Section 3.5.) Points defined by evenly spaced parameter values $t = 0, 1/4, 1/2, 3/4, 1$ are shown. Note that even spacing in parameter does not imply even spacing in arc length. Your goal is to apply quadrature methods to divide this path into n equal lengths.

Recall from calculus that the arc length of the path from t_1 to t_2 is

$$\int_{t_1}^{t_2} \sqrt{x'(t)^2 + y'(t)^2}\,dt.$$