

EX.0.5.2, Sauer

Find c satisfying the Mean Value Theorem for $f(x)$ on the interval $[0, 1]$. (a) $f(x) = e^x$.

(b) $f(x) = x^2$.

(c) $f(x) = 1/(x+1)$.

$$a. f(x) = e^x, f'(x) = e^x$$

$$f'(c) = e^c = \frac{f(1) - f(0)}{1 - 0} = e - 1$$

$$\therefore c = \ln(e - 1)$$

$$b. f(x) = x^2, f'(x) = 2x$$

$$f'(c) = 2c = \frac{f(1) - f(0)}{1 - 0} = 1$$

$$\therefore c = \frac{1}{2}$$

$$c. f(x) = \frac{1}{1+x}, f'(x) = \frac{-1}{(1+x)^2}$$

$$f'(c) = \frac{-1}{(1+c)^2} = \frac{f(1) - f(0)}{1 - 0} = -\frac{1}{2}$$

$$\therefore c = \sqrt{2} - 1$$