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EX.0.1.1, Sauer

Rewrite the following polynomials in nested form. Evaluate with and without nested form at x = 1/3.

a.
$$p(x) = 6x^4 + x^3 + 5x^2 + x + 1$$

b.
$$p(x) = -3x^4 + 4x^3 + 5x^2 - 5x + 1$$

c.
$$p(x) = 2x^4 + x^3 - x^2 + 1$$

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EX.0.1.1, Sauer, solution, Langou

See as well the Colaboratory Jupyter Notebook.

In nested form,

a.
$$p(x) = 6x^4 + x^3 + 5x^2 + x + 1 = ((6x+1)x+5)x+1)x+1$$

b.
$$p(x) = -3x^4 + 4x^3 + 5x^2 - 5x + 1 = ((-3x + 4)x + 5)x - 5)x + 1$$

c.
$$p(x) = 2x^4 + x^3 - x^2 + 1 = ((2x+1)x - 1)x^2 + 1$$

And we can check that, using either standard form or nested form to evaluate the polynomial at $x = \frac{1}{3}$, we get

```
a. p(\frac{1}{3}) = 2
```

```
x = 1./3.
print( 6. * x**4 + x**3 + 5. * x**2 + x + 1. )
print( ( ( 6. * x + 1. ) * x + 5. ) * x + 1. ) * x + 1. )
2.0
2.0
2.0
```

b.
$$p(\frac{1}{3}) = 0$$

```
x = 1./3.

print( -3. * x**4 + 4. * x**3 + 5. * x**2 - 5. * x + 1. )

print( ( ( ( -3. * x + 4. ) * x + 5. ) * x + -5. ) * x + 1. )

1.1102230246251565e-16

0.0
```

c.
$$p(\frac{1}{3}) = \frac{77}{81}$$

```
x = 1./3.
print( 2. * x**4 + x**3 - x**2 + 1. )
print( ( ( ( 2. * x + 1. ) * x - 1. ) * x ) * x + 1. )
print( 77./81. )

0.9506172839506173
0.9506172839506173
0.9506172839506173
```