EX.2.4.4, Sauer3

Solve the system by finding the $PA = LU$ factorization and then carrying out the two-step back substitution.

a. $\begin{pmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$ b. $\begin{pmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ 17 \\ 3 \end{pmatrix}$

Hint: The two $PA = LU$ factorization (with partial pivoting) are:

a.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1/2 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{pmatrix}$$

b.

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 1/5 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 0 & 5/2 & 1/2 \\ 0 & 0 & 7/5 \end{pmatrix}$$