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EX.1.4.7, Sauer3

Let $f(x) = x^4 - 7x^3 + 18x^2 - 20x + 8$. Does Newton's Method converge quadratically to the root r = 2? Find $\lim_{i \to \infty} e_{i+1}/e_i$, where e_i denotes the error at step i.

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EX.1.4.7, Sauer3, solution, Langou

To determine the multiplicity of the root, we take derivatives and evaluate these at the root.

$$f'(x) = 4x^{3} - 21x^{2} + 36x - 20$$

$$f''(x) = 12x^{2} - 42x + 36$$

$$f'''(x) = 24x^{2} - 42$$

$$f'''(x) = 6$$

Thus we have that x = 2 is a root of multiplicity three. The convergence is not quadratic. Using Theorem 1.12, Newton's Method is locally convergent to r = 2 and the convergence rate is

$$\lim_{i \to \infty} \frac{e_{i+1}}{e_i} = \frac{(m-1)}{m} = \frac{2}{3}.$$