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EX.0.4.2, Sauer

Find the roots of the equation $x^2 + 3x - 8^{-14} = 0$ with three-digit accuracy.

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EX.0.4.2, Sauer, solution, Langou

$$x_1 = \frac{1}{2}(-3 + \sqrt{9 + 4 \cdot 8^{-14}})$$
$$x_2 = \frac{1}{2}(-3 - \sqrt{9 + 4 \cdot 8^{-14}}).$$

Using a calculator, we find that $8^{-14} \approx 2.27374 \cdot 10^{-13}$, and can compute directly

$$x_2 \approx -0.5 * (3 + \sqrt{9 + 4(2.27374 \cdot 10^{-13})}) \approx -3.000.$$

This will not work for x_1, however, since $\sqrt{9 + 4(2.27374 \cdot 10^{-13})} = 3.000$ to three decimal places, which would result in $x_1 = 0$. If we instead use

$$\begin{aligned} x_1 &= \frac{1}{2}(-3 + \sqrt{9 + 4 \cdot 8^{-14}}) = \frac{-9 + 9 + 4 \cdot 8^{-14}}{2(3 + \sqrt{9 + 4 \cdot 8^{-14}})} = \frac{4 \cdot 8^{-14}}{2(3 + \sqrt{9 + 4 \cdot 8^{-14}})} = \frac{2 \cdot 8^{-14}}{3 + \sqrt{9 + 4 \cdot 8^{-14}}} \\ &\approx \frac{2 \cdot 2.27374 \cdot 10^{-13}}{6.000} \approx 0.758 \cdot 10^{-14} \end{aligned}$$