

Summer 2025

MATH 4650-E01

CSCI 4650-E01

MATH 5660-E01

Homework 1

Due on Monday June 16th 2025

Exercises:

1. EX.0.1.2.Sauer [using Colab](#)
2. EX.0.1.6.Sauer [using Colab](#)
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10. EX.0.5.6.Sauer [handwritten](#)

### EX.0.1.2, Sauer

Rewrite the following polynomials in nested form and evaluate at  $x = -1/2$ .

a.  $p(x) = 6x^3 - 2x^2 - 3x + 7$

b.  $p(x) = 8x^5 - x^4 - 3x^3 + x^2 - 3x + 1$

c.  $p(x) = 4x^6 - 2x^4 - 2x + 4$

You can use python to perform the computation but you cannot use a `for` loop or a function. Only multiplications, additions and parenthesis.

**EX.0.1.6, Sauer**

Explain how to evaluate the polynomial below for a given input  $x$ , using as few operations as possible. You are not allowed to use the power operator. So no  $x**5$  for example. You can only multiplications and additions. How many multiplications and how many additions are required?

a.  $p(x) = a_0 + a_5x^5 + a_{10}x^{10} + a_{15}x^{15}$

b.  $p(x) = a_7x^7 + a_{12}x^{12} + a_{17}x^{17} + a_{22}x^{22} + a_{27}x^{27}$

### CP.0.1.2, Sauer

Use `nest.m` to evaluate  $p(x) = 1 - x + x^2 - x^3 + \dots + x^{98} - x^{99}$  at  $x = 1.00001$ . Find a simpler, equivalent expression and use it to estimate the error of the nested multiplication.

**EX.0.2.4, Sauer**

Convert the following base 10 numbers to binary. (a) 11.25 (b)  $\frac{2}{3}$  (c)  $\frac{3}{5}$  (d) 3.2 (e) 30.6 (f) 99.9.

**EX.0.2.8, Sauer**

Convert the following binary numbers to base 10. (a) 11011, (b) 110111.001, (c)  $111.\overline{001}$ , (d)  $1010.\overline{01}$ , (e)  $10111.\overline{10101}$  (f)  $1111.01\overline{0001}$

### **EX.0.3.6, Sauer**

Do the following sums by hand in IEEE double precision computer arithmetic, using the Rounding to Nearest Rule.

a.  $(1 + (2^{-51} + 2^{-52} + 2^{-54})) - 1$

b.  $(1 + (2^{-51} + 2^{-52} + 2^{-60})) - 1$

### EX.0.3.7, Langou

Write each of the given numbers in Matlab's *format hex*. Show your work. Then check your answers with Matlab. (a) 16 (b) 130 (c)  $1/4$  (d)  $\text{fl}(1/7)$  (e)  $\text{fl}(4/7)$  (f)  $\text{fl}(0.01)$  (g)  $\text{fl}(-0.01)$  (h)  $\text{fl}(-0.02)$



**EX.0.4.2, Sauer**

Find the roots of the equation  $x^2 + 3x - 8^{-14} = 0$  with three-digit accuracy.

**EX.0.5.2, Sauer**

Find  $c$  satisfying the Mean Value Theorem for  $f(x)$  on the interval  $[0, 1]$ .

(a)  $f(x) = e^x$ .

(b)  $f(x) = x^2$ .

(c)  $f(x) = 1/(x + 1)$ .

Note: In this exercise, we pretty much do not consider round-off errors. For example, we will assume that the values returned by matlab when evaluating  $f(x) = x^{-2}$  are exact. This is justified because our approximation errors (using Taylor polynomials) are much larger than the round-off errors. We will see some limitations (when round-off errors are not negligible any longer) at the end of the exercise.

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**EX.0.5.6, Sauer**

- a. Find the Taylor polynomial of degree 4 for  $f(x) = x^{-2}$  about the point  $x_0 = 1$ .
- b. Use the result of (a) to approximate  $f(0.9)$  and  $f(1.1)$ .
- c. Use the Taylor remainder to find an error formula for the Taylor polynomial. Give error bounds for each of the two approximations made in part (b). Which of the two approximations in part (b) do you expect to be closer to the correct value?
- d. Use a calculator to compare the actual error in each case with your error bound from part (c).