CP.2.7.4, Sauer3

Apply Newton's Method to find both solutions of the system of three equations.

$$\begin{cases} 2u^2 - 4u + v^2 + 3w^2 + 6w + 2 &= 0 \\ u^2 + v^2 - 2v + 2w^2 - 5 &= 0 \\ 3u^2 - 12u + v^2 + 3w^2 + 8 &= 0 \end{cases}$$

Special Instructions:

a. Please first compute the two solutions using either `scipy.optimize.fsolve` or `scipy.optimize.root`. This will be useful to compute the forward error.

b. Start Newton's method from a relative distance (as measured by the infinity norm) of at least 0.1 from the solution.

c. At each iteration of Newton's method, you must print:

   (a) k, the iteration number

   (b) the absolute backward error at iteration number k defined by

   $$\|F(x_k)\|_\infty$$

   where x_k is the current iterate.

   (c) the relative forward error at iteration number k defined by

   $$\|x_k - x\|_\infty / \|x\|_\infty$$

   where x_k is the current iterate and x is the solution as computed by `scipy.optimize.fsolve` or `scipy.optimize.root`.

You can also print the current iterate x_k if you want.

Hint: Here are the two roots:

$$\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1.0960178410014330 \\ -1.1592471842154839 \\ -0.2611479367011116 \end{pmatrix}.$$

(Approximation for the second one.)

Hint:
Here are some function values and Jacobian function values that you can check before starting implementing Newton's method

$$F(\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}) = \begin{pmatrix} 25 \\ 7 \\ 12 \end{pmatrix}, \quad F(\begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix}) = \begin{pmatrix} 57 \\ 51 \\ 72 \end{pmatrix},$$

$$DF(\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}) = \begin{pmatrix} 0 & -2 & 18 \\ 2 & -4 & 8 \\ -6 & -2 & 12 \end{pmatrix}, \quad DF(\begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix}) = \begin{pmatrix} 8 & 10 & -18 \\ 6 & 8 & -16 \\ 6 & 10 & -24 \end{pmatrix}$$