CP.5.1.2, Sauer3 – Problem rewritten by Julien.

We want to study the three-point centered-difference formula to approximate $f'(x)$ where $f(x) = (x+1)^{-1}$ and $x = 1$.

a. (python) Make a table and plot of the error of the three-point centered-difference formula for $f'(x)$, where $f = (x+1)^{-1}$ and $x = 1$, with 10^{-1},..., 10^{-12}, as in the table in Section 5.1.2. Draw a plot of the results.

b. (python) Use the formula

$$h_{\text{opt}} \approx \left(\frac{3\varepsilon_{\text{mach}}}{M}\right)^{\frac{1}{3}},$$

(given below Equation(5.12) in the book,) to compute an approximation to h_{opt}. In this formula M is $|f'''(x)|$, and $\varepsilon_{\text{mach}}$ is the machine precision that you can get in python with `np.finfo(float).eps`.

c. (python) Redo part (a) by adding $E(h)$ from Equation (5.11) to your table and to your plot. You can approximate the quantity $f'''(x)$ by M.