EX.2.4.3, Sauer3

Solve the system by finding the $PA = LU$ factorization and then carrying out the two-step back substitution.

a. $\begin{pmatrix} 3 & 7 \\ 6 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -11 \end{pmatrix}$ b. $\begin{pmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$

Hint: The two $PA = LU$ factorization (with partial pivoting) are:
P = [[0. 1.] [1. 0.]] L = [[1. 0.] [0.5 1.]] U = [[6. 1.] [0. 6.5]]

a.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 7 \\ 6 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} 6 & 1 \\ 0 & \frac{13}{2} \end{pmatrix}$$

b.

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & 1 \end{pmatrix}\begin{pmatrix} 6 & 3 & 4 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

EX.2.4.3, Sauer3, solution, Langou

Colab: `https://colab.research.google.com/drive/1xmC3zRLV6RhRFHzMGZTYlLKtCH7ElGDj`

We want to solve: $Ax = b$. We know that $PA = LU$. So we want to apply P to $Ax = b$ to get $PAx = Pb$ and then replace PA by LU to get

$$LUx = Pb.$$

So we first apply P to b to get Pb, then solve $Ly = Pb$ to get y, then solve $Ux = y$ to get x.
(a)

$$Pb = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -11 \end{pmatrix} = \begin{pmatrix} -11 \\ 1 \end{pmatrix}$$

$$Ly = Pb \Rightarrow \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -11 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -11 \\ \frac{13}{2} \end{pmatrix}$$

$$Ux = y \Rightarrow \begin{pmatrix} 6 & 1 \\ 0 & \frac{13}{2} \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -11 \\ \frac{13}{2} \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$x = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

(b)

$$Pb = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

$$Ly = Pb \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 3 \end{pmatrix}$$

$$Ux = y \Rightarrow \begin{pmatrix} 6 & 3 & 4 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$x = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$