Note: In this exercise, we pretty much do not consider round-off errors. For example, we will assume that the values returned by matlab when evaluating $f(x) = x^{-2}$ are exact. This is justified because our approximation errors (using Taylor polynomials) are much larger than the round-off errors. We will see some limitations (when round-off errors are not negligible any longer) at the end of the exercise.

EX.0.5.6, Sauer

a. Find the Taylor polynomial of degree 4 for $f(x) = x^{-2}$ about the point $x_0 = 1$.

b. Use the result of (a) to approximate $f(0.9)$ and $f(1.1)$.

c. Use the Taylor remainder to find an error formula for the Taylor polynomial. Give error bounds for each of the two approximations made in part (b). Which of the two approximations in part (b) do you expect to be closer to the correct value?

d. Use a calculator to compare the actual error in each case with your error bound from part (c).