EX.1.1.8, Langou

Let $(f : \mathbb{R} \to \mathbb{R})$ be continuous. Let a and b, and `tol`. Let k be the smallest number of iterations of the Bisection Method that guarantees a forward error bound on the approximate solution within `tol`. Let `numevals` be the smallest number of iterations of the Bisection Method that guarantees a forward error bound on the approximate solution within `tol`.

a. Derive a formula that relates a, b, k and `tol`.

b. Solve for k as a function of a, b, and `tol`.

c. Derive a formula that relates k and `numevals`.

Let a, b, and `tol` as given below. How many function evaluations of the Bisection Method are required to guarantee a forward error bound on the approximate solution within `tol`? Answer with an integer.

d. $a = -3$, $b = 16$, `tol` $=10^{-3}$

e. $a = -0.2$, $b = 2.7$, `tol` $=10^{-12}$