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EX.2.1.3, Sauer3

Solve by back substitution:

a.

$$3x - 4y + 5z = 2$$
$$3y - 4z = -1$$
$$5z = 5$$

b.

$$x - 2y + z = 2$$
$$4y - 3z = 1$$
$$-3z = 3$$

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EX.2.1.3, Sauer3, solution, Langou

 $See~Colab~\texttt{https://colab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHotY2mhI3c49Doorder{\texttt{polab.research.google.com/drive/1FHXErhF1TkEg-_iF9EHo$

- a. We have (x, y, z) = (1/3, 1, 1).
- b. We have (x, y, z) = (2, -1/2, -1).

```
# (a)

# 3x - 4y + 5z = 2

# 3y - 4z = -1

# 5z = 5

A = np.array([

[ 3., -4., 5.],

[ 0., 3., -4.],

[ 0., 0., 5.]])

b = np.array([

[ 2.],

[ -1.],

[ 5.]])
```

```
# solve Ax=b with np.linalg.solve
# and check that Ax is b

x = np.linalg.solve(A, b)
print("With np.linalg.solve, we find x = \n", x)

print("\nWe check that Ax is b. Indeed [ Ax, b ] =\n",\
```

```
np.concatenate((A@x, b), axis=1) )
With np.linalg.solve, we find x =
 [[0.33333333]]
 [1.
             ]
 [1.
            ]]
We check that Ax is b. Indeed [ Ax, b ] =
 [[2. 2.]
 [-1. -1.]
 Γ5.
       5.]]
\# use our bucket algorithm
\# b a c k w a r d \_ s u b s t i t u t i o n
# for the solve
x = backward_substitution( A, b )
print("\nx=\n", x)
\mathbf{x} =
 [[0.33333333]]
 [1.
            ]
            11
 [1.
\# (b)
#
                       x - 2y + z = 2
#
                           4y - 3z = 1
#
                               -3z = 3
A = np.array([
                   1., -2., 1.],
                   0., 4., -3. ],
                     0., 0., -3.
b = np.array([
                     2.],
                  1.],
                     3.]])
\# solve Ax=b with np.linalg.solve
# and check that Ax is b
x = np.linalg.solve(A, b)
print("\nWith np.linalg.solve, we find x = \n", x)
print("\nWe check that Ax is b. Indeed [ Ax, b ] =\n",\
 np.concatenate((A@x, b), axis=1) )
With np.linalg.solve, we find x =
 [[ 2. ]
 [-0.5]
 [-1.]
```

```
We check that Ax is b. Indeed [ Ax, b ] =
    [[2. 2.]
    [1. 1.]
    [3. 3.]]

# use our bucket algorithm
# backward_substitution
# for the solve
x = backward_substitution( A, b )
print("x=\n", x )

x=
    [[ 2. ]
    [-0.5]
    [-1. ]]
```