Temporal Relativity — The Entropy Clock

$\Delta S = \kappa\,\Delta\tau$

Christopher Michael Langstaff

v1.2

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Abstract

We postulate the minimal Entropy–Time law, $\Delta S = \kappa\,\Delta\tau$, and (i) *recall* the standard interior metric-recovery result for adjacency graphs with a finite-sample rate $f(N)$ under common assumptions (e.g., [1, 2]), and (ii) present an empirical weak-field clock/redshift check with a preregistered acceptance band around 1. Figures regenerate from scripts; data and exact commands are archived.

Notation.

k tick index ($k = 1, 2, \dots$).

V_k point set after k ticks (one new point per tick).

E_k adjacency set at tick k (local, coord-free).

G_k the graph (V_k, E_k) at tick k.

$B_r(v)$ hop–ball $\{u : d(u, v) \le r\}$ around v.

$L_k(p, q)$ hop distance in G_k (shortest-path edge count).

d_k rescaled metric: $d_k = \lambda_k L_k$ (normalization λ_k).

$a(k)$ linear counting scale (e.g., minimal hop radius enclosing a fixed fraction of V_k).

D effective intrinsic dimension (“3-like” when $D \approx 3$) (e.g., [3, 4]).

$c_{\mathbf{tick}}$ per-tick speed cap (max hop growth per tick).

Context

If clocks are S-counters and adjacency captures who-can-touch-whom, then spacetime metrology is emergent: *geometry* from adjacency distances, and *time* from how much update happened along a path. We do not identify S pointwise with thermodynamic entropy; instead we use a local, monotone update tally and only differences ΔS.

1 Axiom and operational S

A1 (*$E\tau$ Law*). Along any worldline, $\Delta S = \kappa\,\Delta\tau$; S is a primitive, monotone tally. Only differences ΔS matter; the zero of S is irrelevant.

Operational S (what it is and is not). S is an *update ledger*: a count that increases with proper duration. When we set $S_{\text{phys}} = k_B S$ in thermodynamic contexts, standard identities (e.g. $\dot{S}_{\text{phys}} = \dot{Q}/T$) apply. Outside thermo, we still use the same ΔS as the path-time counter; we do not assume a microstate-count interpretation in every context.

Operational mapping and calibration (modality invariance). Each clock modality m carries its own ledger S_m with calibration constant $\kappa_m > 0$ so that $\Delta S_m = \kappa_m\,\Delta\tau$. The device readout (cycles, phase, ticks) maps monotonically to S_m; any positive rescaling $S_m \mapsto a_m S_m + b_m$ simply rescales κ_m and leaves A1 intact, so only ΔS_m is operational. For two colocated clocks m, n along the same worldline segment, $\Delta S_m/\Delta S_n = \kappa_m/\kappa_n$ is constant (fixed calibrations).

Non-Goals

- Not a replacement for general relativity (GR)/quantum field theory (QFT); we recover specific limits under stated conditions.
- Not equating S with thermodynamic entropy pointwise; coarse-grained tracking only.
- This note isolates two results: metric-recovery with rates (and a failure mode) and a weak-field clock/redshift check.

Arrow of time (scope). A1 (*$E\tau$ Law*) gives a *local operational arrow*: S increases with proper time τ along a timelike worldline. This is kinematic labeling, not a new dynamical irreversibility.

2 Background (i): adjacency $\Rightarrow$ metric recovery

Assumptions. (a) finite-speed exchange (one hop per tick; finite influence cone); (b) interior/bulk regime (positive distance from the boundary); (c) non-pathological sampling (bounded degree; near-isotropy).

Lemma 1 (Interior quasi-isometry, explicit distortion)**.** *Let ℓ_N denote the typical interior hop length and let $\eta_N := C_1\,\varepsilon_N + C_2\,\delta_N$, where $\varepsilon_N \to 0$ quantifies mesh non-uniformity and $\delta_N \to 0$ directional bias. Then for interior points x, y,*

$$(1-\eta_N)\,\|x-y\| \;\le\; \ell_N\, d_G(x,y) \;\le\; (1+\eta_N)\,\|x-y\|\,. \tag{1}$$

Theorem 2 (Interior recovery with finite-sample rate)**.** *Under the assumptions above, the graph metric induced by adjacency recovers the ambient metric in the interior with distortion bounded by*

$$\mathrm{distortion}(N) \;\le\; f(N) := C\,(\ell_N + \eta_N)\,, \tag{2}$$

with $\ell_N \asymp N^{-1/3}$ in 3D and $\eta_N \to 0$ (graph-theoretic notions are standard; see [2]). In particular $f(N) \to 0$ as $N \to \infty$.

Remark (References and intuition)*.* As in the standard adjacency→geometry literature (e.g., [1, 2]), shortest-path (hop) distances approximate ambient/-geodesic distances under interior sampling. Intuitively, finite-speed updates induce causal cones and near-isotropy controls bilipschitz distortion; the rate follows from the typical interior hop length $\ell_N \asymp N^{-1/3}$ in 3D together with decay of nonuniformity and directional bias. *Failure mode:* sufficiently anisotropic or degenerate sampling outside the causal cone violates Lemma 1.

3 Result (ii): weak-field clock/redshift check

Regress the observed fractional shift $\Delta\nu/\nu$ against $g\,\Delta h/c^2$ (or $\Delta\Phi/c^2$) over a shared coordinate duration (classically predicted by [5]). **Preregistered** acceptance band: $\hat{\beta} \in [0.97,\, 1.03]$ with standard errors from ordinary least squares (OLS). A pass anchors $\Delta S \propto \Delta\tau$ operationally in the weak field regime without fixing the absolute unit of S.

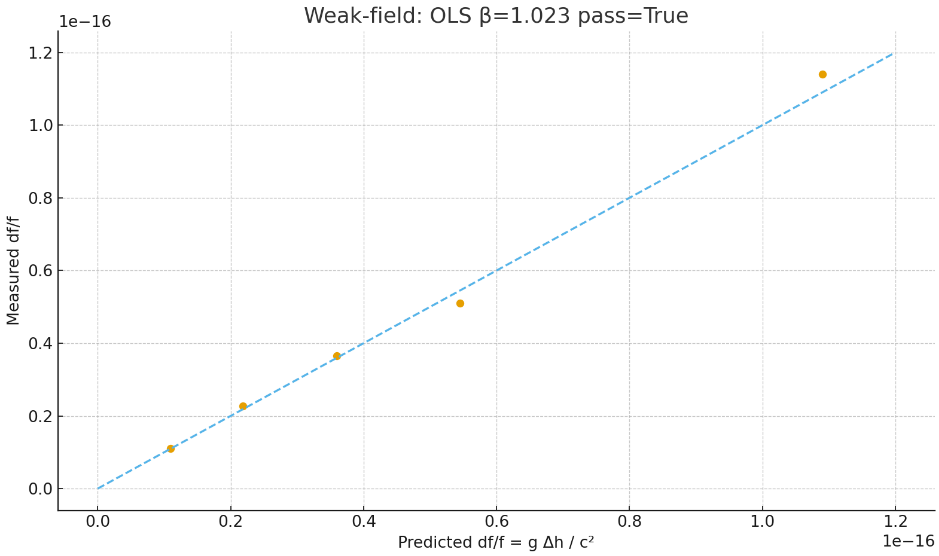

Figure 1: Weak-field slope with preregistered band around 1; residual diagnostics in the supplement.

Provenance: src=`src/weakfield_run.py`; data=`data/weakfield_redshift.csv`; commit=`v0.1.3`.

Calibration invariance. Because the analysis uses the *fractional* rate change $\Delta\nu/\nu$, the unknown modality calibration κ_m cancels; the slope probes $\mathrm{d}\tau$ directly.

Claims and tests

- **A1 (*Eτ Law*) — L0**. *Equation:* $\Delta S = \kappa\,\Delta\tau$. *Data & Script:* —. *Pass:* Used as clock.
- **Metric recovery — L3**. *Equation:* Thm. 2 with rate $f(N)$ (e.g., [1, 2]). *Data & Script:* —. *Pass:* Bound $f(N)$ (theoretical).
- **Weak-field clocks — L3**. *Equation:* Pred. vs. OLS slope. *Data & Script:* `src/weakfield_run.py`; `data/weakfield_redshift.csv`; `figs/fig_weakfield_redshift.png`. *Pass:* $\hat{\beta} \in [0.97, 1.03]$.

Related work (brief)

Thermal time proposes a state-dependent flow of time [6, 7]; entropic gravity posits gravity as emergent from entropic forces [8]; causal sets take order as fundamental [9]; graph-geodesic methods recover geometry from adjacency [1]. Here the novelty is operational: a minimal *update ledger* S that (i) recovers interior metric structure with explicit finite-sample rates under finite-speed exchange, and (ii) passes a weak-field clock slope test with preregistered bands.

Reproducibility checklist

- **Code/data.** Script-generated figures with fixed seeds; sources and data archived with the preprint.
- **Pipeline.** `src/weakfield_run.py` → `data/weakfield_redshift.csv` → `figs/fig_weakfield_redshift.png`.
- **Acceptance.** Weak-field slope within preregistered band; residuals show no trend and near-constant variance.

Falsifiers (pre-registered)

- Any internal cone violations (one-hop-per-tick constraint).
- Interior metric distortion exceeding $f(N)$ under the stated sampling assumptions.
- Weak-field slope $\hat{\beta}$ outside $[0.97, 1.03]$ or patterned residuals.
- Persistent $\kappa \neq (\rho c_p)\alpha$ beyond tabulation uncertainty in the thermo check.

Data and code availability

All materials to reproduce the analysis and figure are openly available: GitHub () and Zenodo v0.1.2 (https://doi.org/10.5281/zenodo.17127328). Key files: `src/weakfield_run.py` (OLS slope + figure regeneration), `data/weakfield_redshift.csv` (input data), and the output figure `figs/fig_weakfield_redshift.png` regenerated from the script.

AI use disclosure

ChatGPT was used as a tool—under the author's direction—for wording/LaTeX polish and to draft a short reproducibility script (`src/weakfield_run.py`). The author reviewed, edited, and executed all code; all concepts, analyses, and conclusions are the author's. No confidential data were provided to the tool. AI is not an author.

Acronyms

GR general relativity. 2

OLS ordinary least squares. 3, 4

QFT quantum field theory. 2

Glossary

acceptance band Pre-registered interval in which an estimated slope or statistic is counted as a pass. 3

adjacency graph Graph whose edges encode who-can-touch-whom interactions. 3

causal cone Finite-speed influence region implied by one hop per tick. 3

proper time Time measured along a timelike worldline. 2

quasi-isometry Map between metric spaces that distorts distances by controlled constants. 3

tick Primitive update interval that bounds the causal cone (one hop per tick). 3

weak field Regime where gravitational potential is small compared with c^2. 3

References

[1] Joshua Tenenbaum, Vin de Silva, and John C. Langford. "A global geometric framework for nonlinear dimensionality reduction". In: *Science* 290.5500 (2000), pp. 2319–2323.

[2] László Lovász. "Random Walks on Graphs: A Survey". In: *Combinatorics, Paul Erdős is Eighty*. Ed. by Miklós Deza, Imre Z. Ruzsa, and Tamás Sós. Vol. 2. Budapest: János Bolyai Mathematical Society, 1996, pp. 1–46.

[3] Kenneth Falconer. *Fractal Geometry: Mathematical Foundations and Applications*. 3rd ed. Chichester: John Wiley & Sons, 2014.

[4] Peter Grassberger and Itamar Procaccia. "Measuring the Strangeness of Strange Attractors". In: *Physica D: Nonlinear Phenomena* 9.1-2 (1983), pp. 189–208.

[5] A. Einstein. “On the Influence of Gravitation on the Propagation of Light”. In: *Annalen der Physik* 35 (1911), pp. 898–908.

[6] Carlo Rovelli. “Statistical mechanics of gravity and the thermodynamical origin of time”. In: *Classical and Quantum Gravity* 10.8 (1993), pp. 1549–1566.

[7] Carlo Rovelli. *Quantum Gravity*. Cambridge University Press, 2004.

[8] Erik Verlinde. “On the Origin of Gravity and the Laws of Newton”. In: *Journal of High Energy Physics* 2011.4 (2011), p. 29.

[9] Luca Bombelli, Joohan Lee, David Meyer, and Rafael D. Sorkin. “Space-Time as a Causal Set”. In: *Physical Review Letters* 59.5 (1987), pp. 521–524.