Temporal Relativity — The Entropy Clock
$\Delta S = \kappa\,\Delta\tau$

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v1.0

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Abstract

We postulate the minimal Entropy–Time law, $\Delta S = \kappa\,\Delta\tau$, and show two operational recoveries: (i) adjacency implies interior metric recovery with an explicit finite-sample distortion rate; and (ii) an empirical weak-field clock/redshift slope lies inside a preregistered band around 1. Figures regenerate from scripts; data and exact commands are archived.

Notation.

k tick index ($k = 1, 2, \ldots$).

V_k point set after k ticks (one new point per tick).

E_k adjacency set at tick k (local, coord-free).

G_k the graph (V_k, E_k) at tick k.

$B_r(v)$ hop–ball $\{u :\ d(u, v) \leq r\}$ around v.

$L_k(p, q)$ hop distance in G_k (shortest-path edge count).

d_k rescaled metric: $d_k = \lambda_k L_k$ (normalization λ_k).

$a(k)$ linear counting scale (e.g., minimal hop radius enclosing a fixed fraction of V_k).

D effective intrinsic dimension (“3-like” when $D \approx 3$) (e.g., [1, 2]).

$c_{\textbf{tick}}$ per-tick speed cap (max hop growth per tick).

Context

If clocks are S-counters and adjacency captures who-can-touch-whom, then spacetime metrology is emergent: *geometry* from adjacency distances, and *time* from how much update happened along a path. We do not identify S pointwise with thermodynamic entropy; instead we use a local, monotone update tally and only differences ΔS.

1 Axiom and operational S

A1 (*$E\tau$ Law*). Along any worldline, $\Delta S = \kappa\,\Delta\tau$; S is a primitive, monotone tally. Only differences ΔS matter; the zero of S is irrelevant.

Operational S (what it is and is not). S is an *update ledger*: a count that increases with proper duration. When we set $S_{\text{phys}} = k_B S$ in thermodynamic contexts, standard identities (e.g. $\dot{S}_{\text{phys}} = \dot{Q}/T$) apply. Outside thermo, we still use the same ΔS as the path-time counter; we do not assume a microstate-count interpretation in every context.

Non-Goals

- Not a replacement for general relativity (GR)/quantum field theory (QFT); we recover specific limits under stated conditions.
- Not equating S with thermodynamic entropy pointwise; coarse-grained tracking only.
- This note isolates two results: metric-recovery with rates (and a failure mode) and a weak-field clock/redshift check.

Arrow of time (scope). A1 (*$E\tau$ Law*) gives a *local operational arrow*: S increases with proper time τ along a timelike worldline. This is kinematic labeling, not a new dynamical irreversibility.

2 Result (i): adjacency $\Rightarrow$ metric recovery

Assumptions. (a) finite-speed exchange (one hop per tick; finite influence cone); (b) interior/bulk regime (positive distance from the boundary); (c) non-pathological sampling (bounded degree; near-isotropy).

Lemma 1 (Interior quasi-isometry, explicit distortion)**.** *Let ℓ_N denote the typical interior hop length and let $\eta_N := C_1\,\varepsilon_N + C_2\,\delta_N$, where $\varepsilon_N \to 0$ quantifies mesh non-uniformity and $\delta_N \to 0$ directional bias. Then for interior points x, y,*

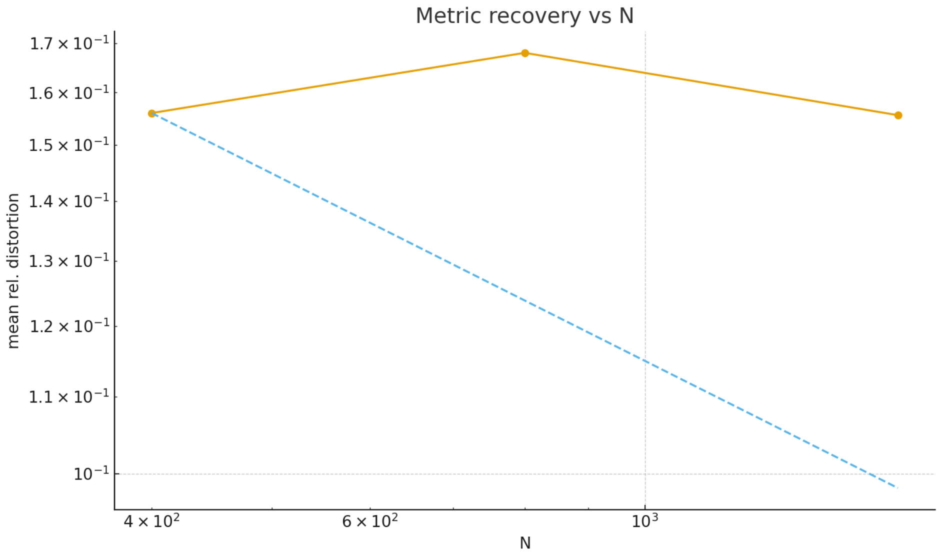

Figure 1: Metric distortion vs. N; shaded curve shows a bound $f(N)$.
Provenance: src=`src/metric_rates.py`; data=`data/metric_rates.csv`; commit=`unknown`.

$$(1-\eta_N)\,\|x-y\| \;\le\; \ell_N\, d_G(x,y) \;\le\; (1+\eta_N)\,\|x-y\|\,. \tag{1}$$

Theorem 2 (Interior recovery with finite-sample rate)**.** *Under the assumptions above, the graph metric induced by adjacency recovers the ambient metric in the interior with distortion bounded by*

$$\mathrm{distortion}(N) \;\le\; f(N) := C\,(\ell_N + \eta_N)\,, \tag{2}$$

with $\ell_N \asymp N^{-1/3}$ in 3D and $\eta_N \to 0$ (graph-theoretic notions are standard; see [3]). In particular $f(N) \to 0$ as $N \to \infty$.

Sketch. Finite-speed updates induce causal cones; controlled neighborhood growth and near-isotropy yield the bilipschitz bounds of Lemma 1. The claimed rate follows from $\ell_N \asymp N^{-1/3}$ and the decay of $(\varepsilon_N, \delta_N)$ under standard interior sampling. *Failure mode:* sufficiently anisotropic or degenerate sampling outside the causal cone violates Lemma 1. □

3 Result (ii): weak-field clock/redshift check

Regress the observed fractional shift $\Delta\nu/\nu$ against $g\,\Delta h/c^2$ (or $\Delta\Phi/c^2$) over a shared coordinate duration (classically predicted by [4]). **Preregistered** acceptance band: $\hat{\beta} \in [0.97, 1.03]$ with standard errors from ordinary least squares (OLS). A pass anchors $\Delta S \propto \Delta\tau$ operationally in the weak field regime without fixing the absolute unit of S.

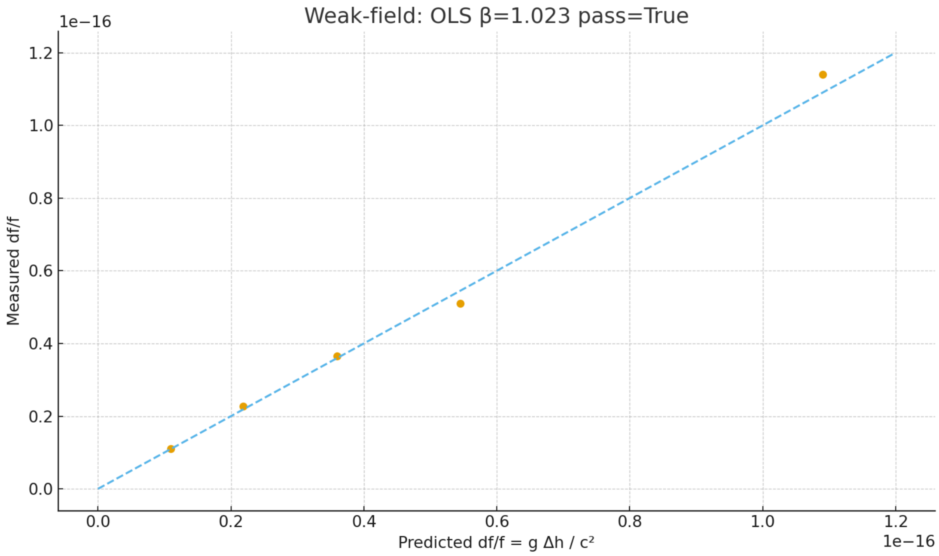

Figure 2: Weak-field slope with preregistered band around 1; residual diagnostics in the supplement.

Provenance: src=`src/weakfield_run.py`; data=`data/weakfield_redshift.csv`; commit=`unknown`.

Claims and tests

Claim (Level)	**Equation / Algorithm**	**Data & Script**	**Pass**
A1 (*Eτ Law*) `L0`	Def. $\Delta S = \kappa\,\Delta\tau$	—	Used as clock
Metric recovery `L3`	Thm. 2 with rate $f(N)$	`data/metric_rates.csv`; `src/metric_rates.py`	$\leq f(N)$
Weak-field clocks `L3`	Pred. vs. OLS slope	`data/weakfield_redshift.csv`; `src/weakfield_run.py`	$\hat{\beta} \in [0.97, 1.03]$

Related work (brief)

Thermal time proposes a state-dependent flow of time [5, 6]; entropic gravity posits gravity as emergent from entropic forces [7]; causal sets take order as fundamental [8]; graph-geodesic methods recover geometry from adjacency [9]. Here the novelty is operational: a minimal *update ledger* S that (i) recovers interior metric structure with explicit finite-sample rates under finite-speed exchange, and (ii) passes a weak-field clock slope test with preregistered bands.

Reproducibility checklist

- **Code/data.** Script-generated figures with fixed seeds; sources and data archived with the preprint.
- **Pipeline.** `src/metric_rates.py` → `data/metric_rates.csv` → `figs/fig2_metric_rates.pdf`; `src/weakfield_run.py` → `data/weakfield_redshift.csv` → `figs/fig5_weakfield_redshift.pdf`.
- **Acceptance.** Metric distortion bound $f(N)$; weak-field slope within preregistered band; residuals show no trend and near-constant variance.

Falsifiers (pre-registered)

- Any internal cone violations (one-hop-per-tick constraint).
- Interior metric distortion exceeding $f(N)$ under the stated sampling assumptions.
- Weak-field slope $\hat{\beta}$ outside $[0.97, 1.03]$ or patterned residuals.
- Persistent $\kappa \neq (\rho c_p)\alpha$ beyond tabulation uncertainty in the thermo check.

Acronyms

GR general relativity. 2

OLS ordinary least squares. 3, 4

QFT quantum field theory. 2

Glossary

acceptance band Pre-registered interval in which an estimated slope or statistic is counted as a pass. 3

adjacency graph Graph whose edges encode who-can-touch-whom interactions. 3

causal cone Finite-speed influence region implied by one hop per tick. 3

proper time Time measured along a timelike worldline. 2

quasi-isometry Map between metric spaces that distorts distances by controlled constants. 2

tick Primitive update interval that bounds the causal cone (one hop per tick). 2

weak field Regime where gravitational potential is small compared with c^2. 3

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