Tutorial on Language Generation

Organizers: *Moses Charikar, Anay Mehrotra, Charlotte Peale, Chirag Pabbaraju, Grigoris Velegkas*

Validity–Breadth Trade-Off (Part I)

This Talk:

- Why Breadth?
- Definitions of Breadth + Abstractions
- Results
- Step Back
- Future Directions

Proofs in Part II by Grigoris

Recap of Language Generation in the Limit

Language Generation

Kleinberg, Mullainathan, 2024

string of K →

← guess new string from K

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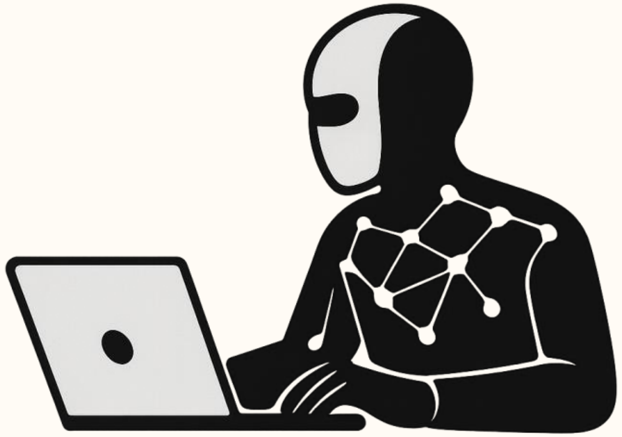
…



Success: guess correct (i.e., valid and unseen) for every $t > t^*$
(We say that algorithm has generated K in the limit)

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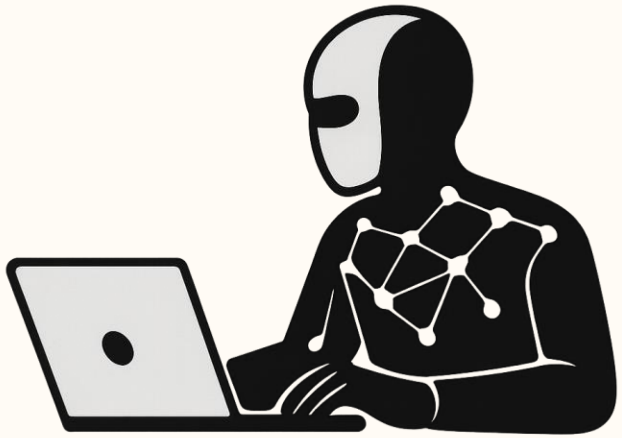

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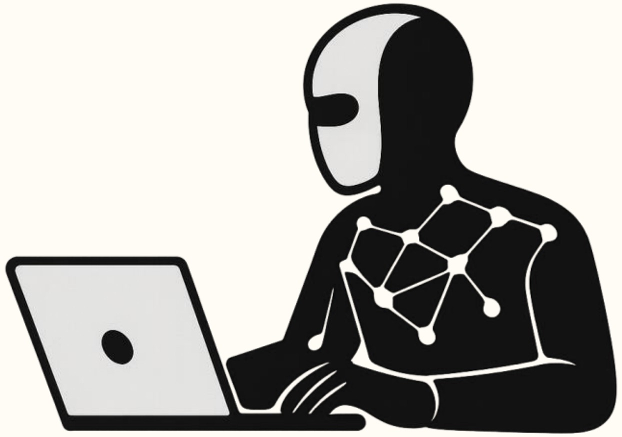

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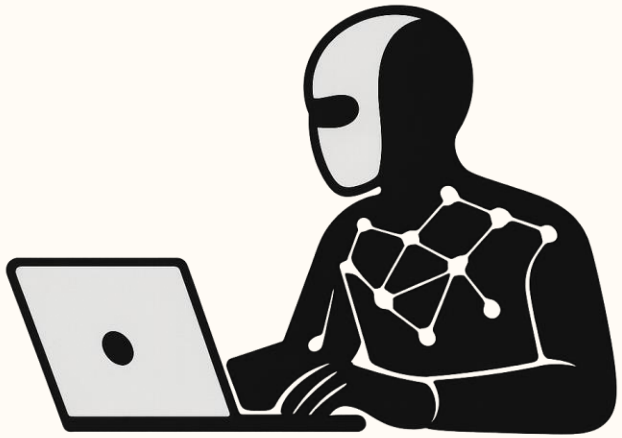

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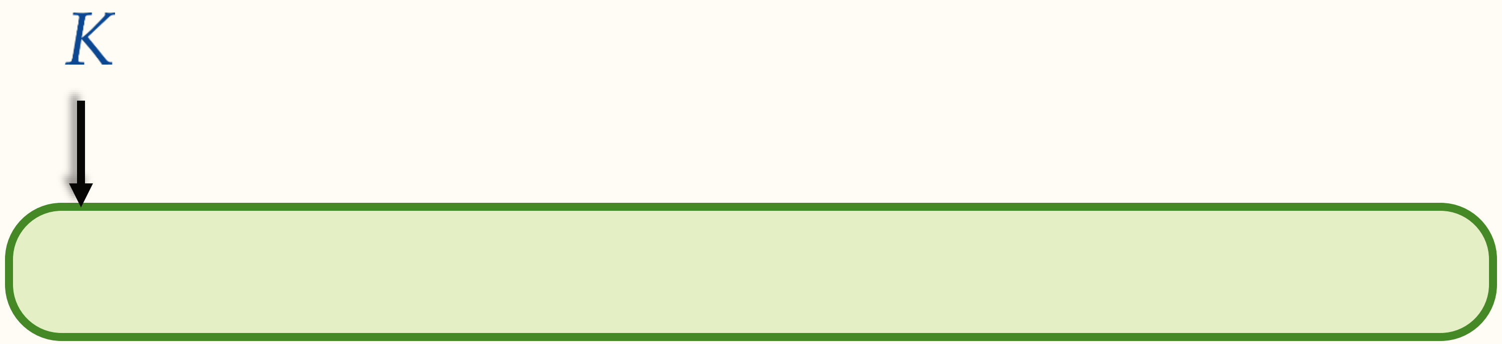
Assume all languages infinite

Language Generation in the Limit

Theorem [Kleinberg, Mullainathan 2024]
Language generation in the limit is possible for any countable collection of languages

[KM’24]’s Generator Loses Breadth



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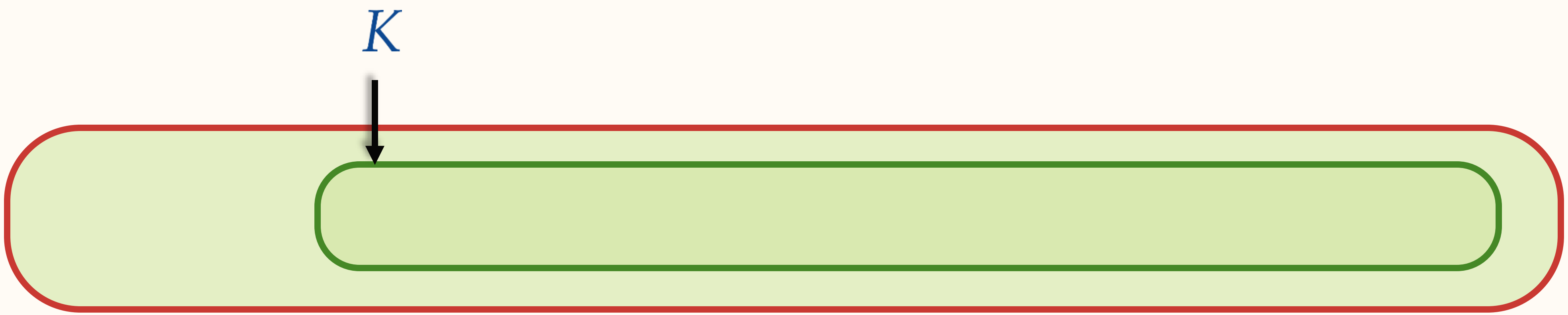

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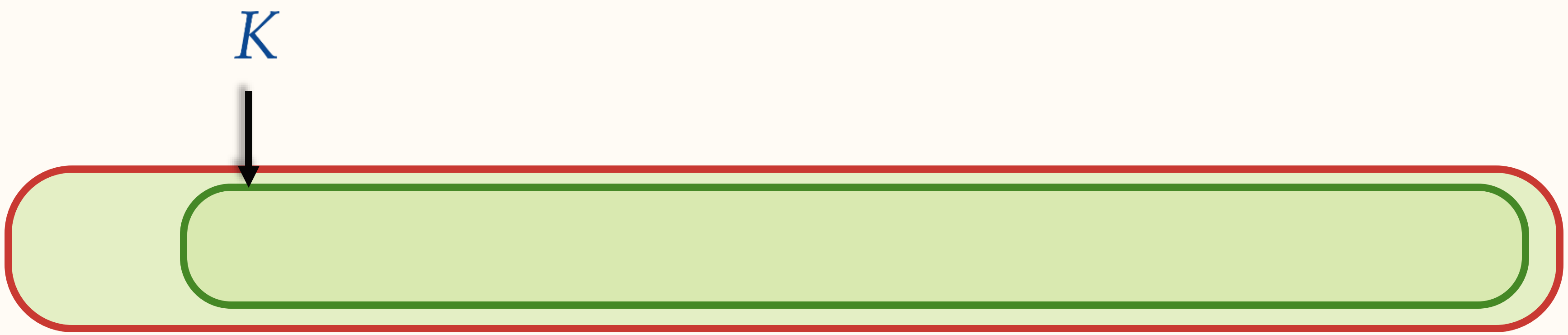

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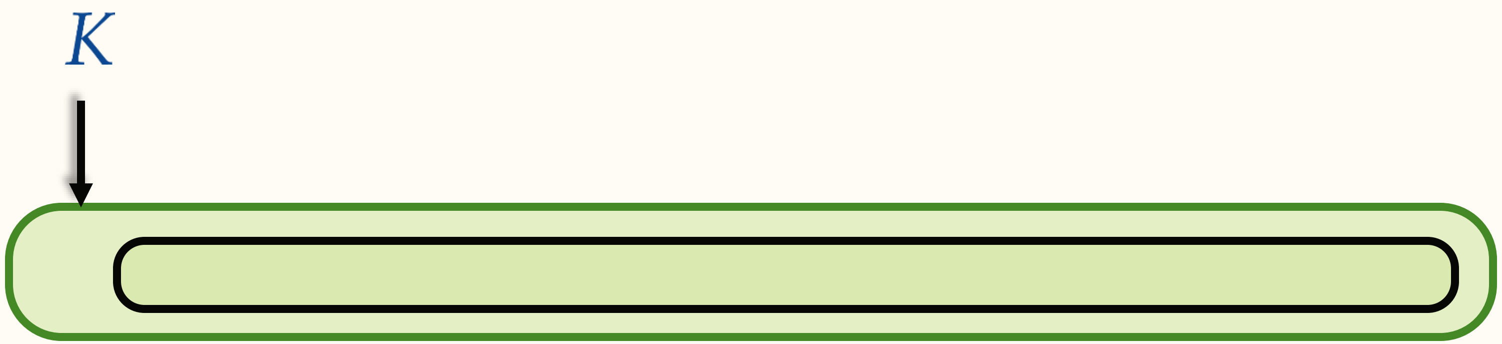

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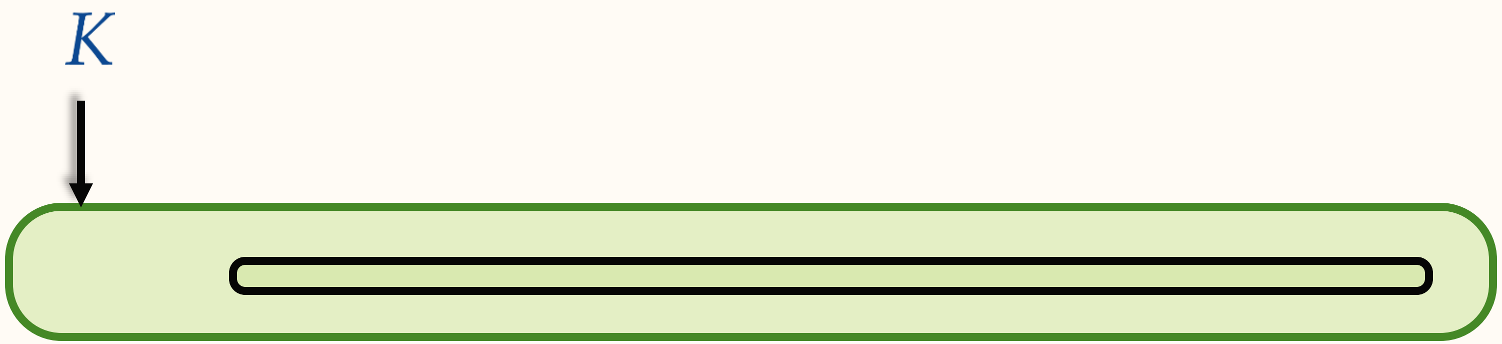

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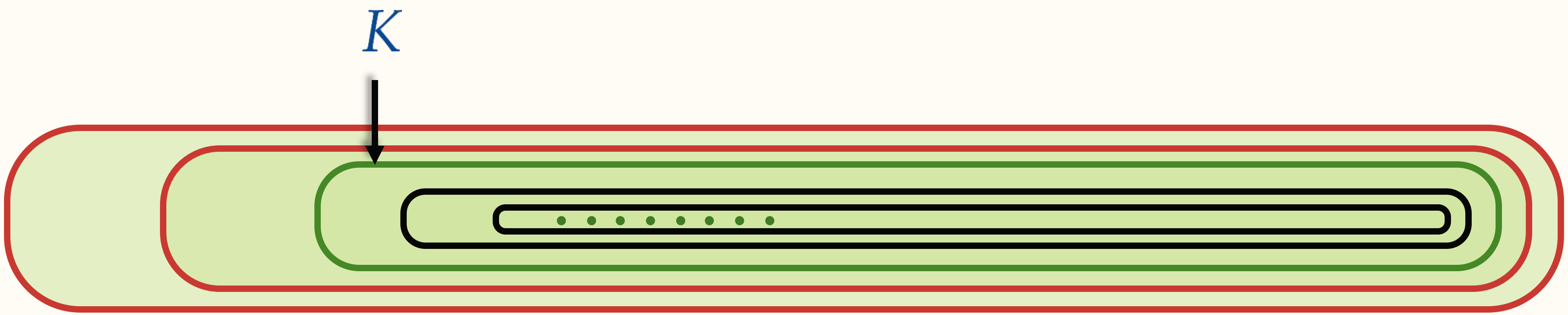

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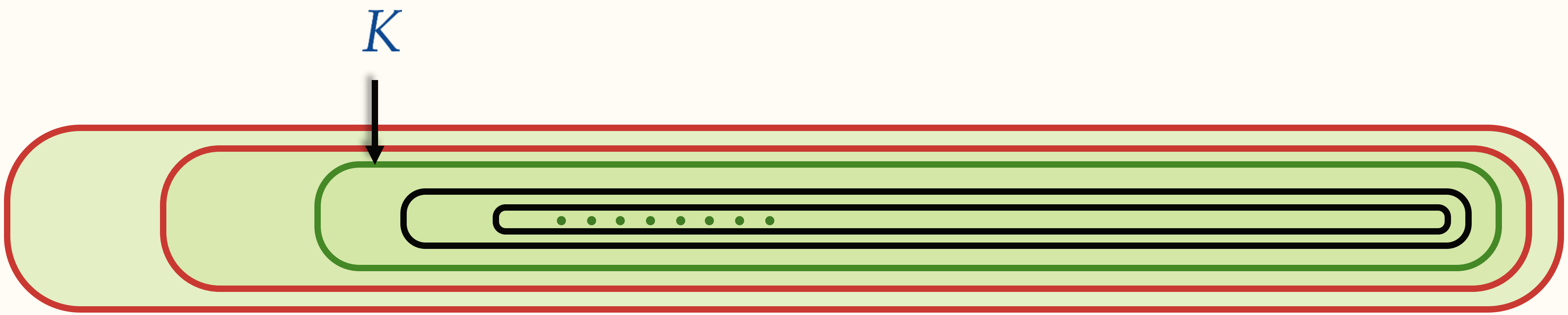

One question asked by [KM24]: Can a generator avoid hallucinations while maintaining some notion of "breadth"

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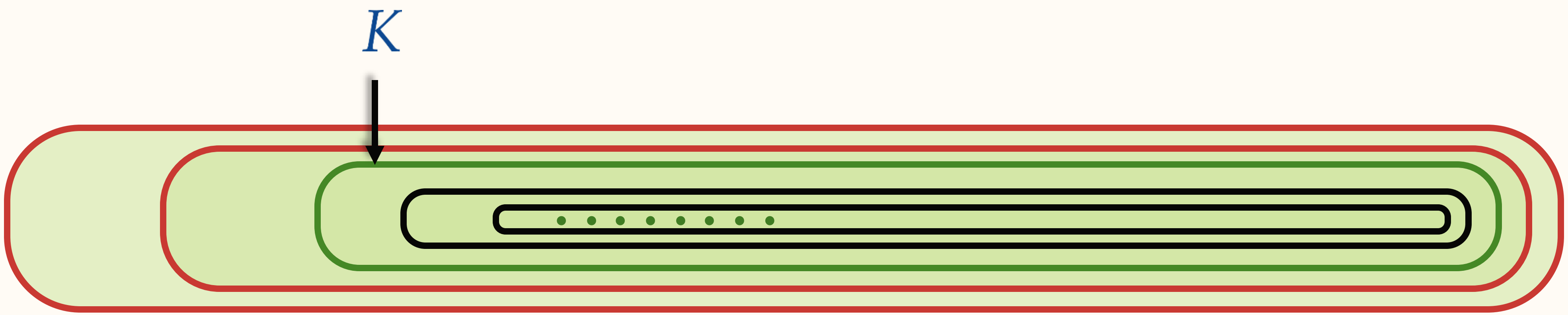
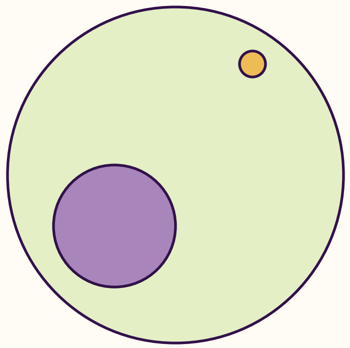


One question asked by [KM24]: Can a generator avoid hallucinations while maintaining some notion of "breadth"

Or what is *the limit in generation in the limit*

What is Breadth?



Generator's support G

Training data S

Target language K

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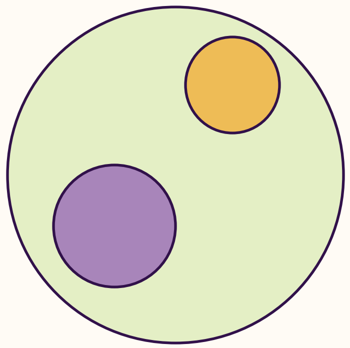

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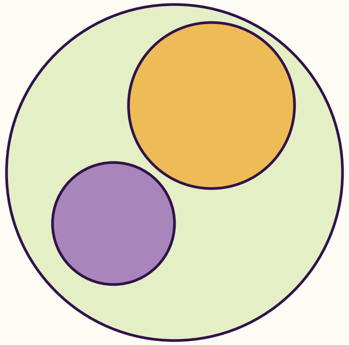

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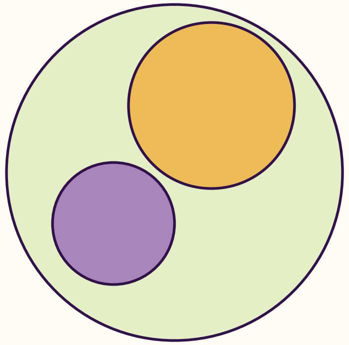

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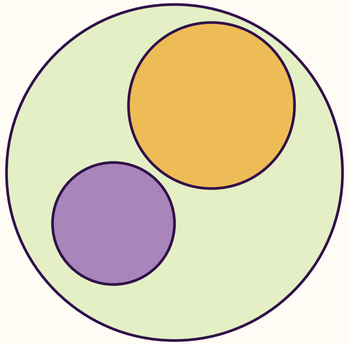

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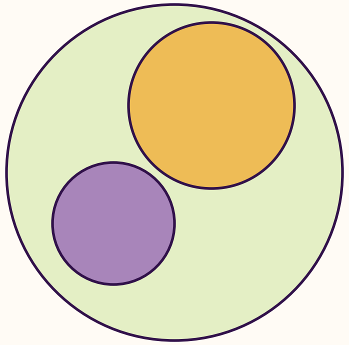

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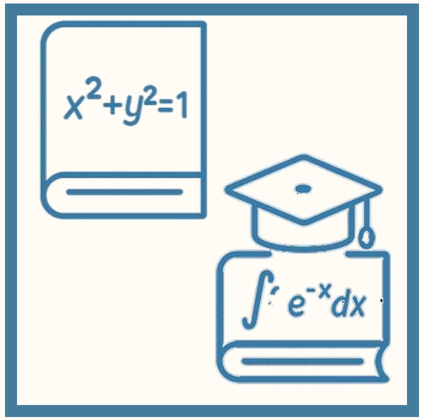
“panda surfing in a watermelon juice with sunglasses while singing.”



Solve new math problems

Some Definitions of Breadth

For generator G, let $G(S) \subseteq \mathcal{X}$ be the output-set of G trained on S

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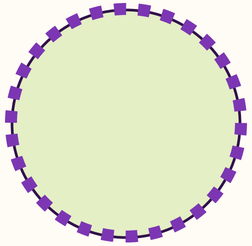
Set Theoretic Definitions

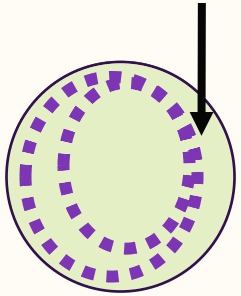
Exact Breadth

$G(S) = K \setminus S$



Approximate Breadth

$|(K \setminus S) \setminus G(S)| < \infty$



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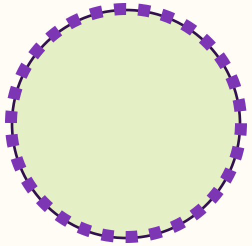

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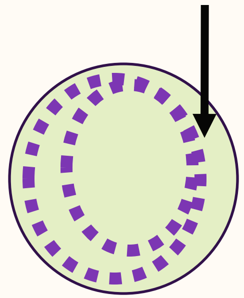

Consider $K = \mathbb{N}$, $\quad G(S) = \{i, i+1, \dots\} \quad$ and $\quad G(S) = \{2, 4, 6, \dots\}$

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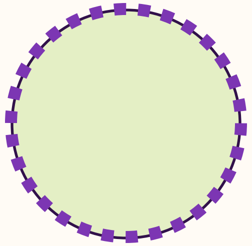

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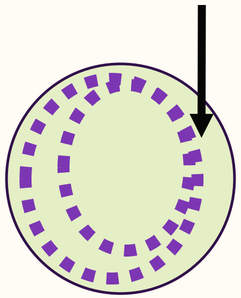

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Can also require/relax additional properties [CP'25] [KMV'24]

Abstractions of Breadth

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Core Question (in diagonalization-based proofs): How many languages can G achieve the property for *simultaneously*?

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Definition (Finite Non-Uniqueness). Property P has finite non-uniqueness, if G can achieve P for L_1 and L_2 only if $|L_1 \Delta L_2| < \infty$

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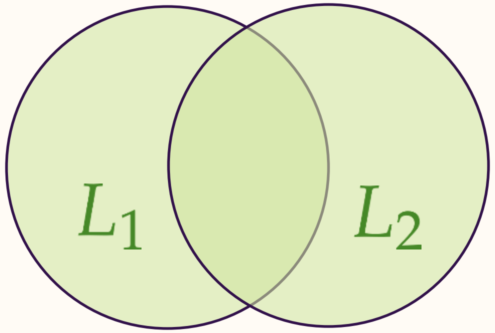

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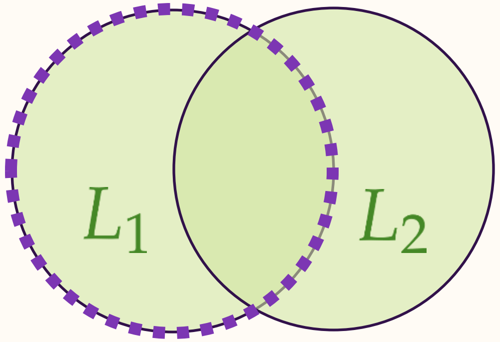

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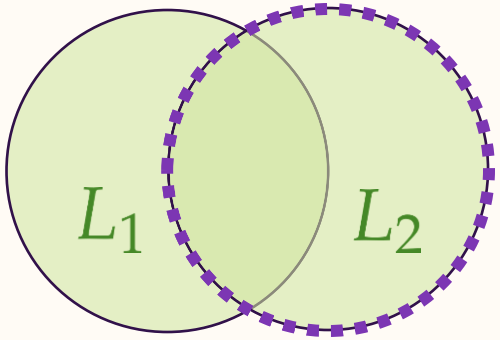

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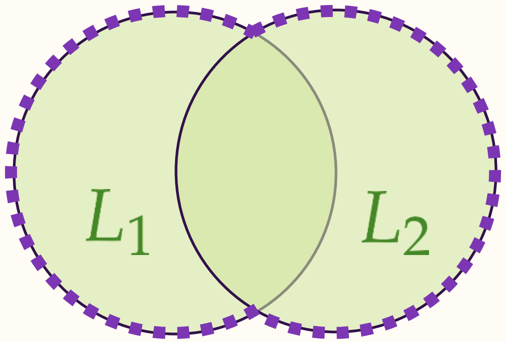

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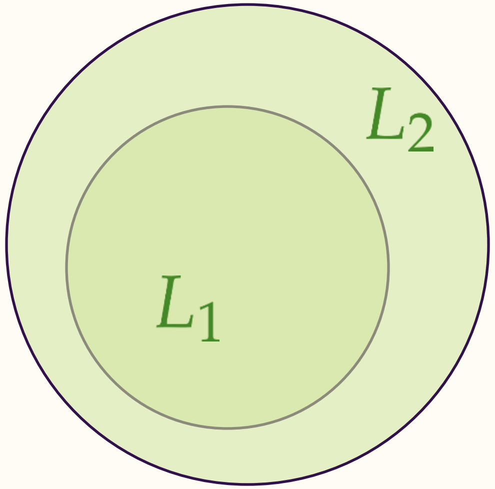

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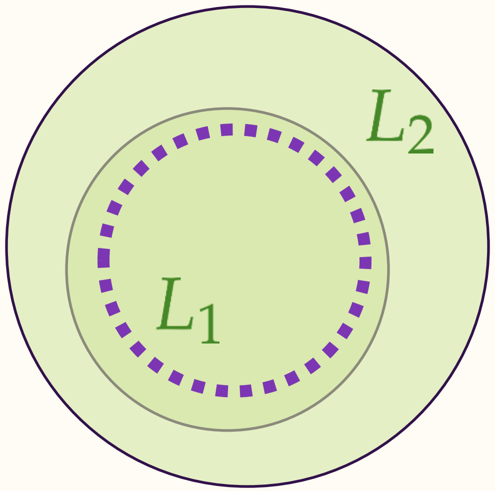

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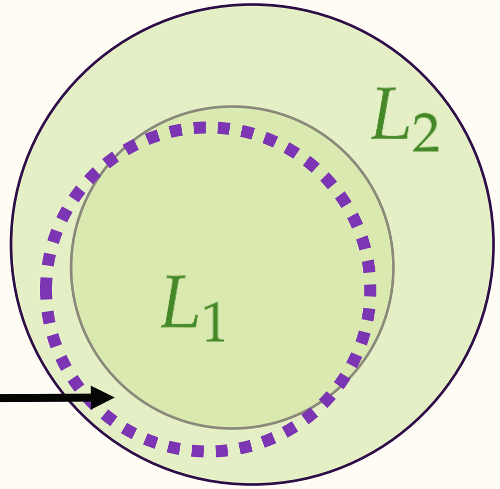
Can generate things not in L_1!



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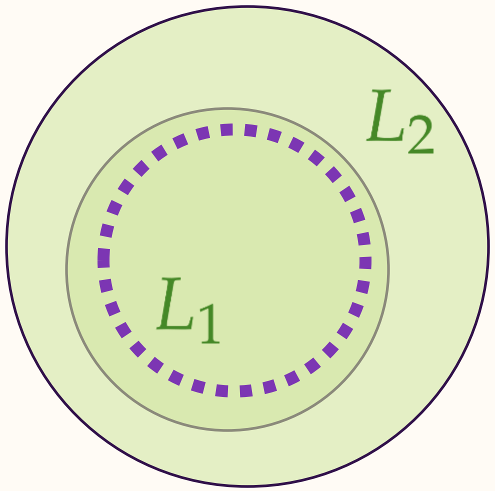

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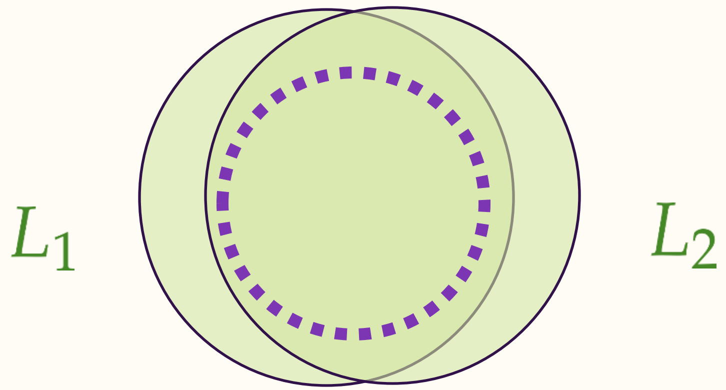

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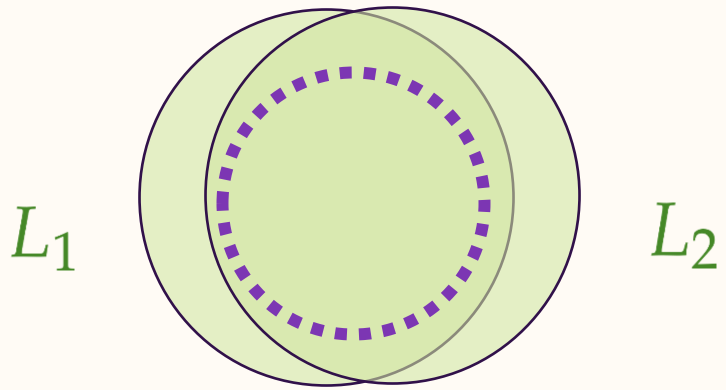

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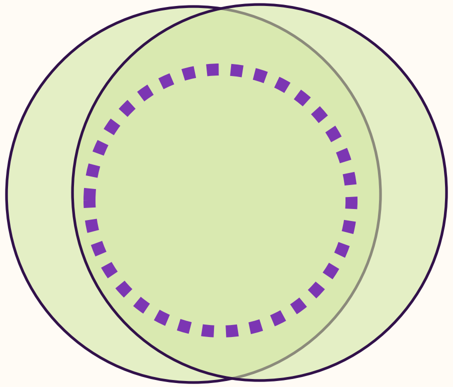
L_1  L_2

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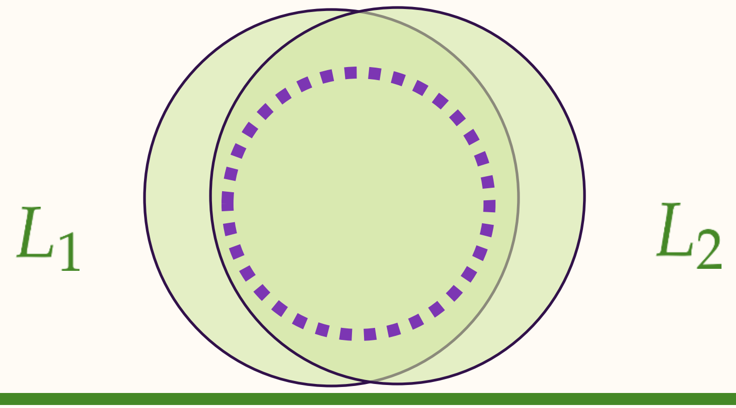

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Other examples: *Allow generators to have finite hallucinations*

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Will be formalized in the next talk!

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Will be formalized in the next talk!

Proof strategy? *Diagonalization via characterization of identification*

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[KMV'25]: (a) extends to statistical model, (b) extends conditionally

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Implication: For non-identifiable collections exact breadth is violated infinitely many times

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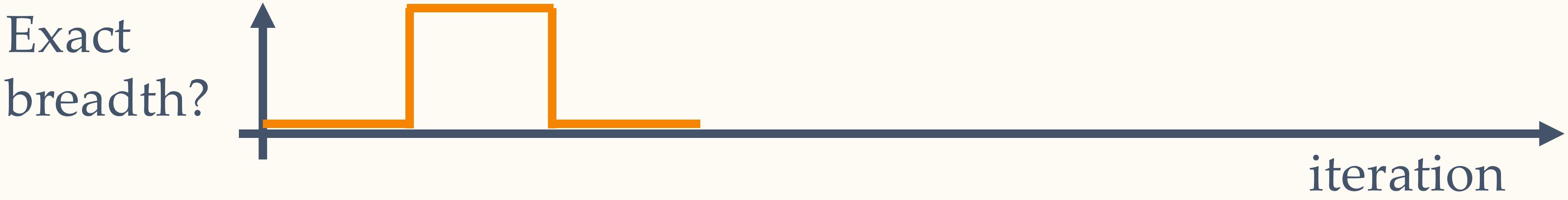

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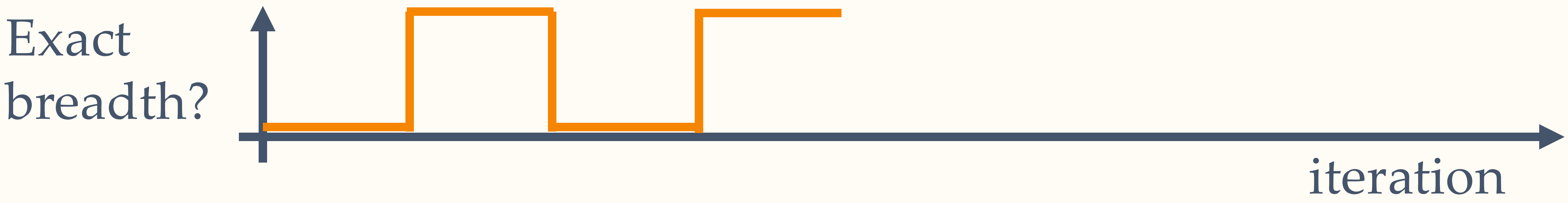
Implication: For non-identifiable collections exact breadth is violated infinitely many times



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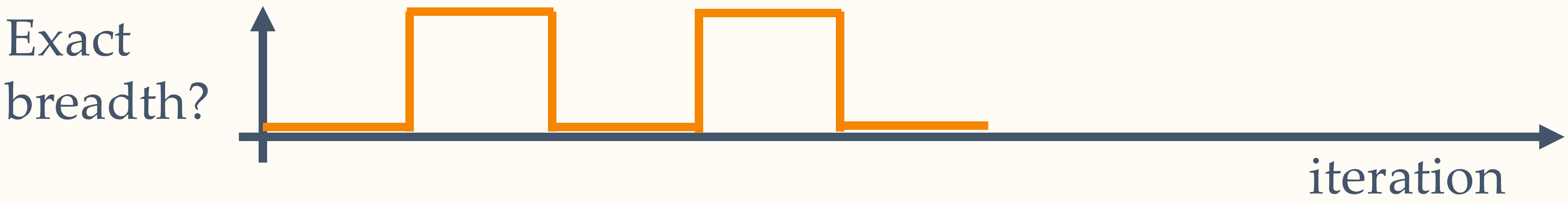

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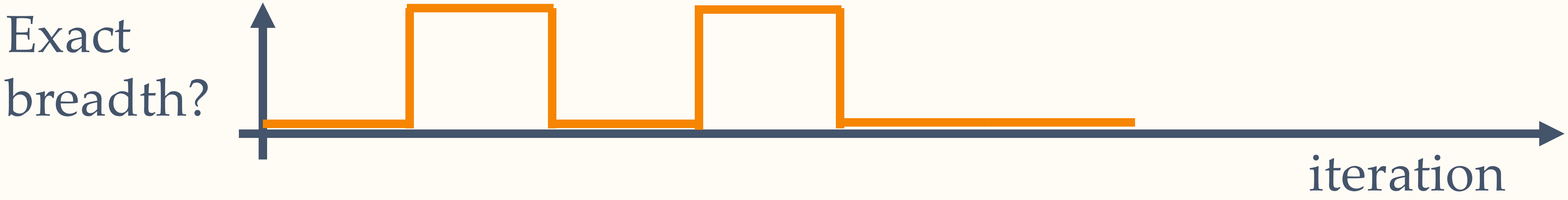

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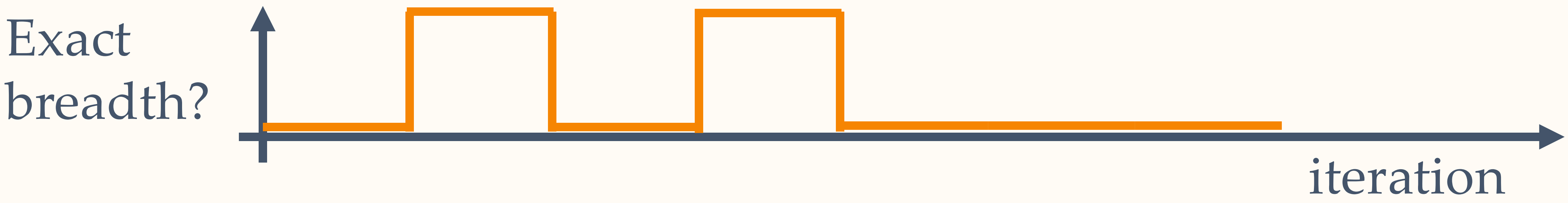

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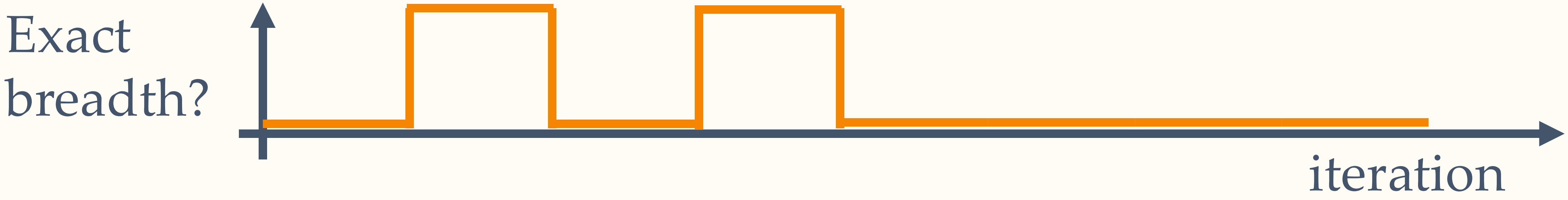

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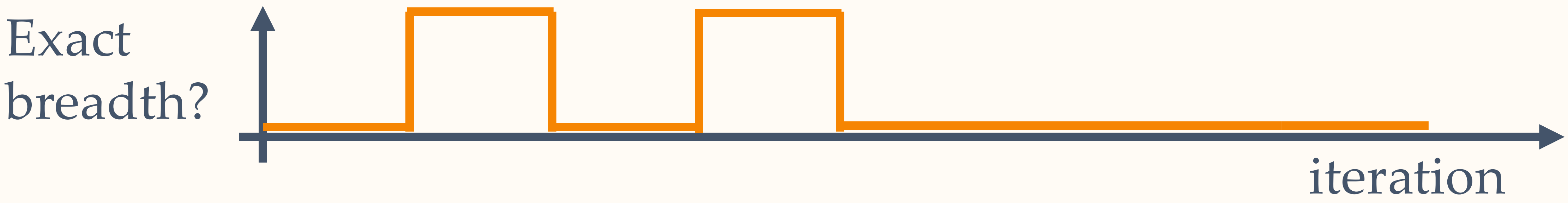

Question: Is it possible to achieve exact breadth infinitely many times?

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Theorem. [KW’25] There is a generator $\mathcal{G}$, that achieves exact breadth *infinitely* many times for *any* countable collection $\mathcal{L}$

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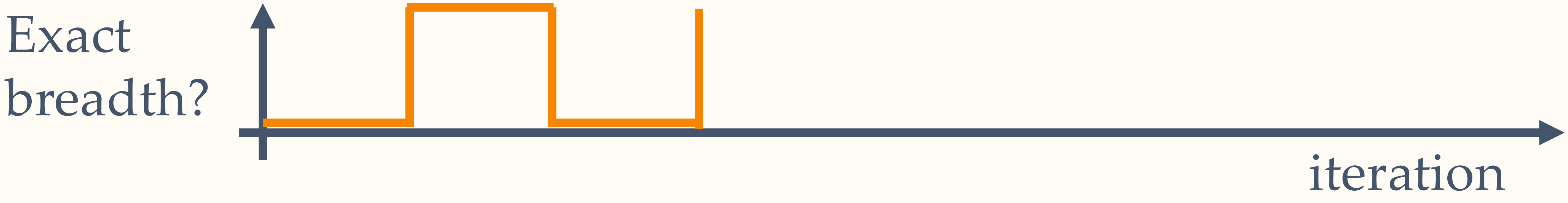

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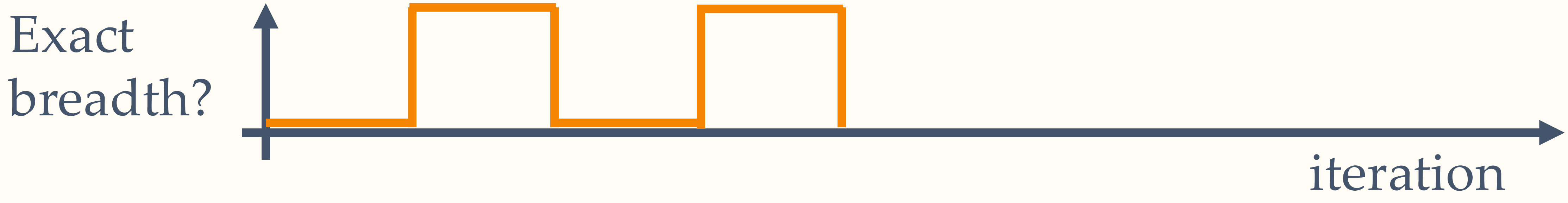

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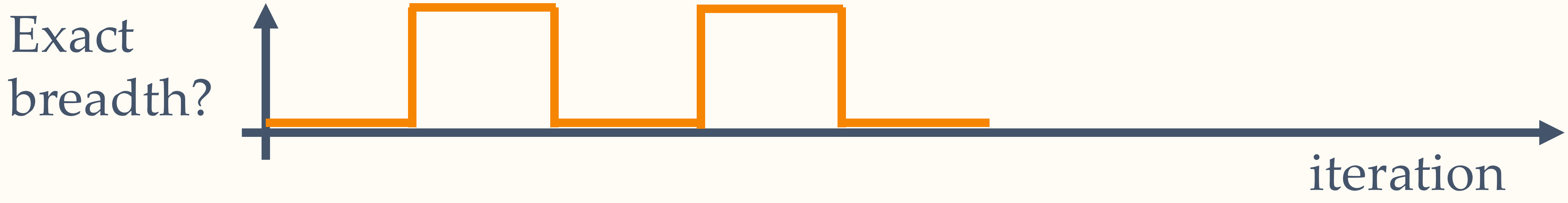

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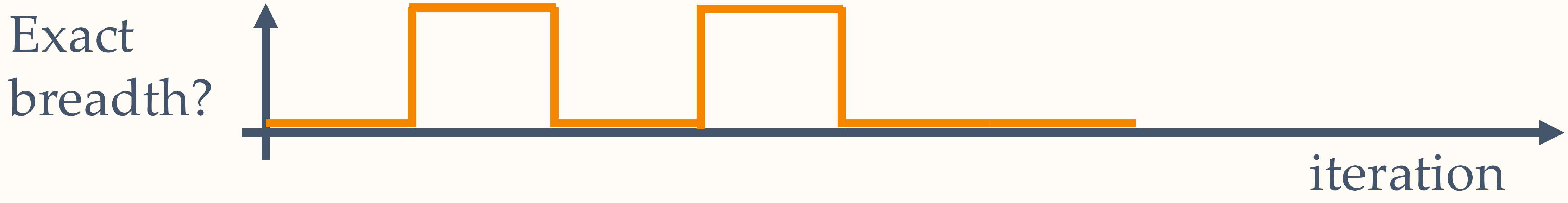

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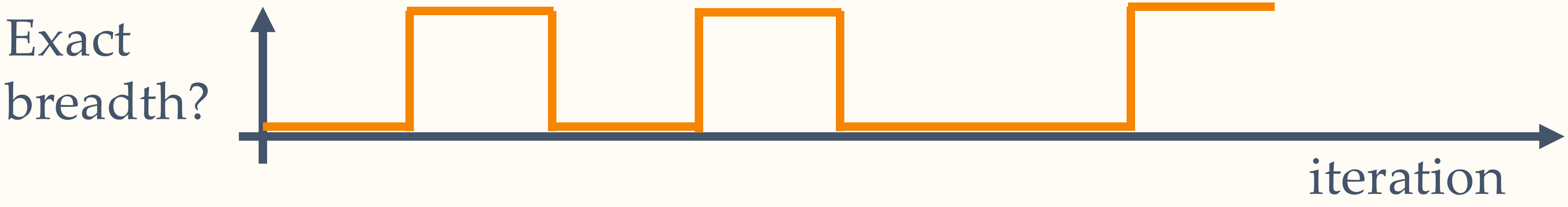

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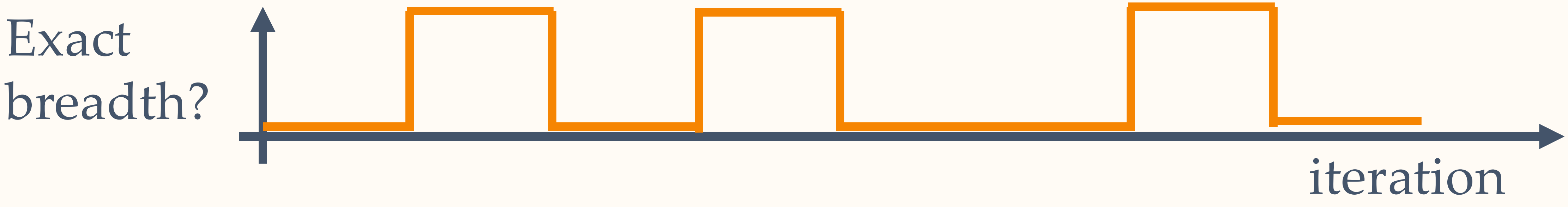

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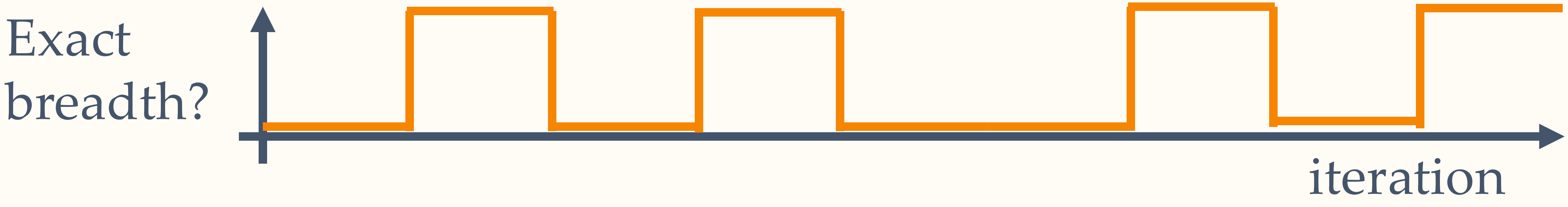

Immediate Open Questions

1. Fine-grained trade-offs between hallucinations and breadth

   Partial results [CP'25], [KMV'24], [KW'25]

2. Allow multiple responses (could bypass impossiblity results)
3. What other type of feedback is useful?

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COLT 2025
On Thursday in the
Language Model Session

Step Back – How Strong is Exact Breadth?

Example

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Does not contradict Godel (does contradict Turing's decidability)

References

[CP'25] Exploring Facets of Language Generation in the Limit, COLT'25
[HKMV'25] On Union-Closedness of Language Generation, arXiv'25
[KM'24] Language Generation in the Limit, NeurIPS'24
[KW'24] Density Measures for Language Generation, arXiv'25
[KMV'25] On the Limits of Language Generation: … STOC'25
[KMV'24] Characterizations of Language Generation With Breadth, arXiv'24
[PRR'25] Representative Language Generation, ICML'25

Thank you!