Tutorial on Language Generation

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Stronger Notions of Generation and Comparison to Prediction

Chirag Pabbaraju

Tutorial on Language Generation in the Limit

COLT 2025

Visit: LanguageGeneration.github.io

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$t^\star$ can depend on target L_z as well as enumeration order! 😓

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Kleinberg-Mullainathan's algorithm also faces this issue 😓

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Li, Raman, Tewari '24

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z_1' z_2' z_3' z_4' ....

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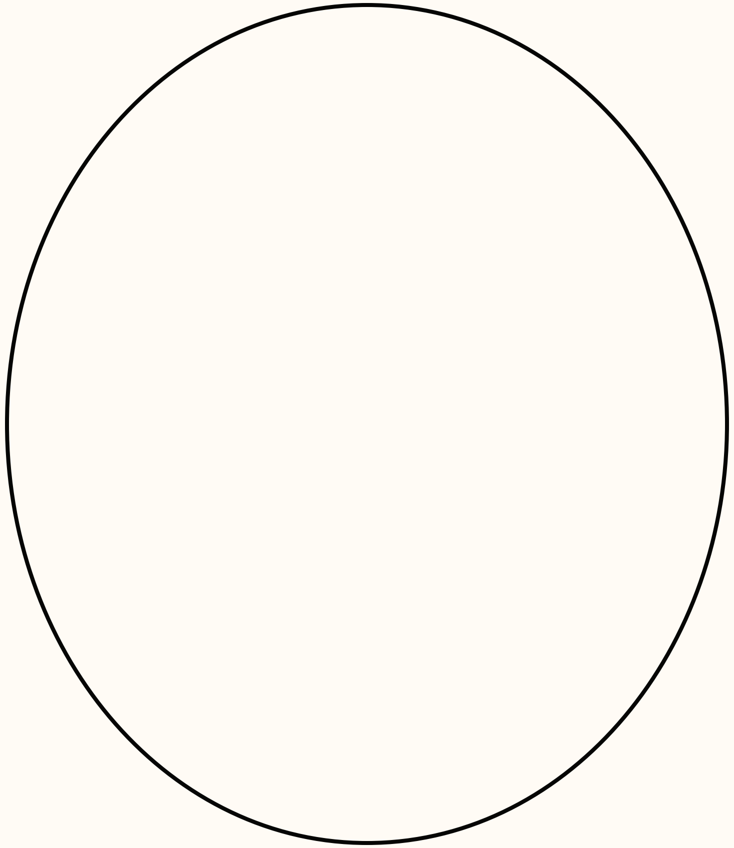

Countable language collections

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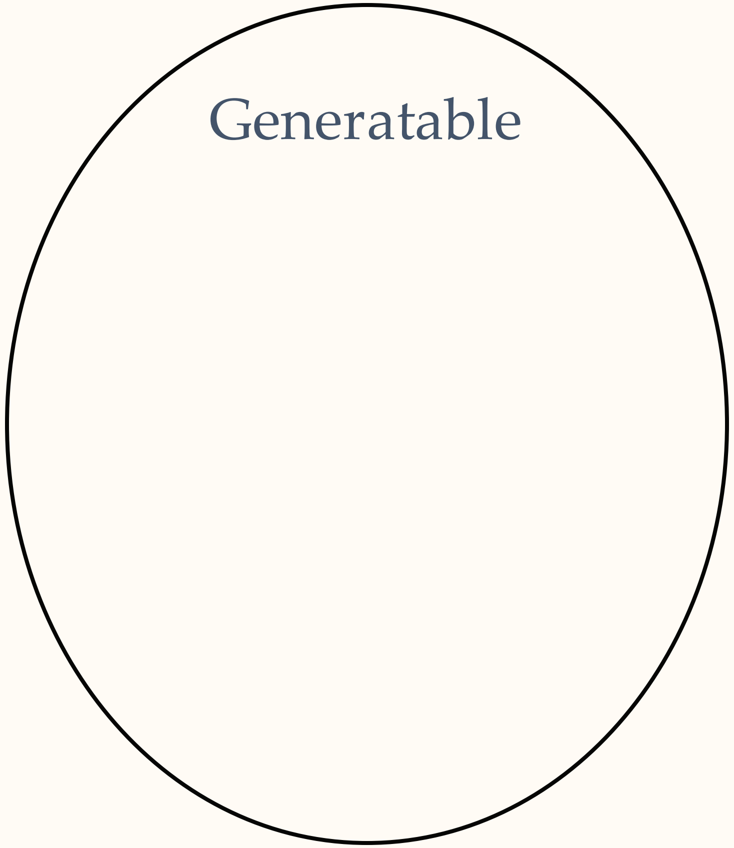

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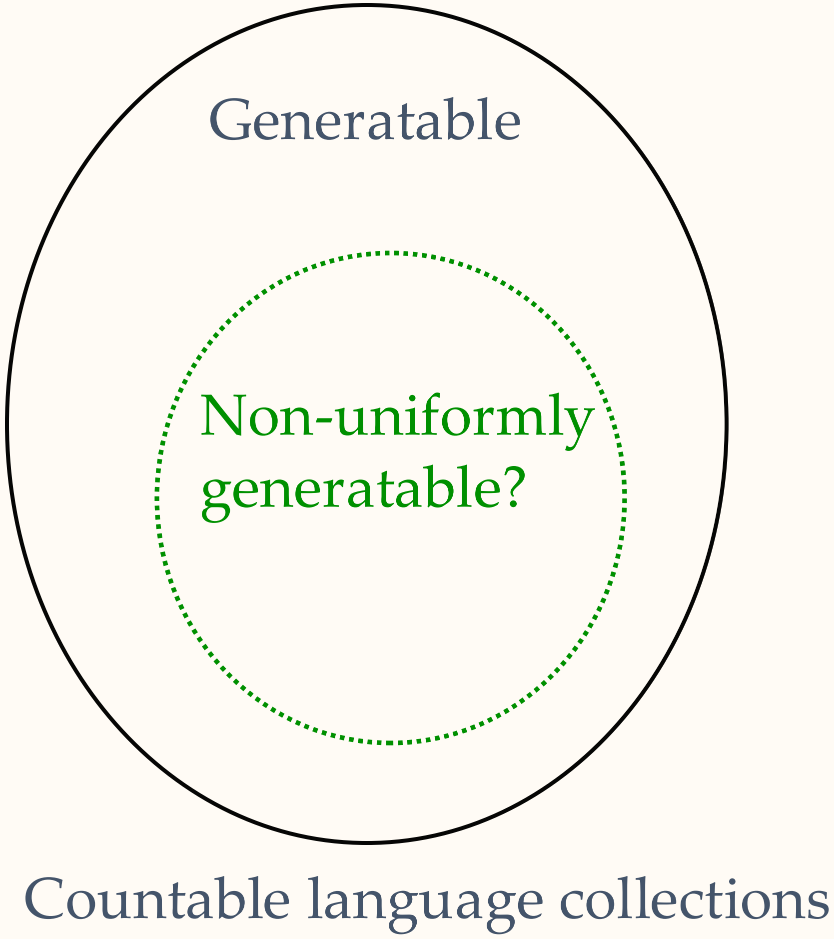

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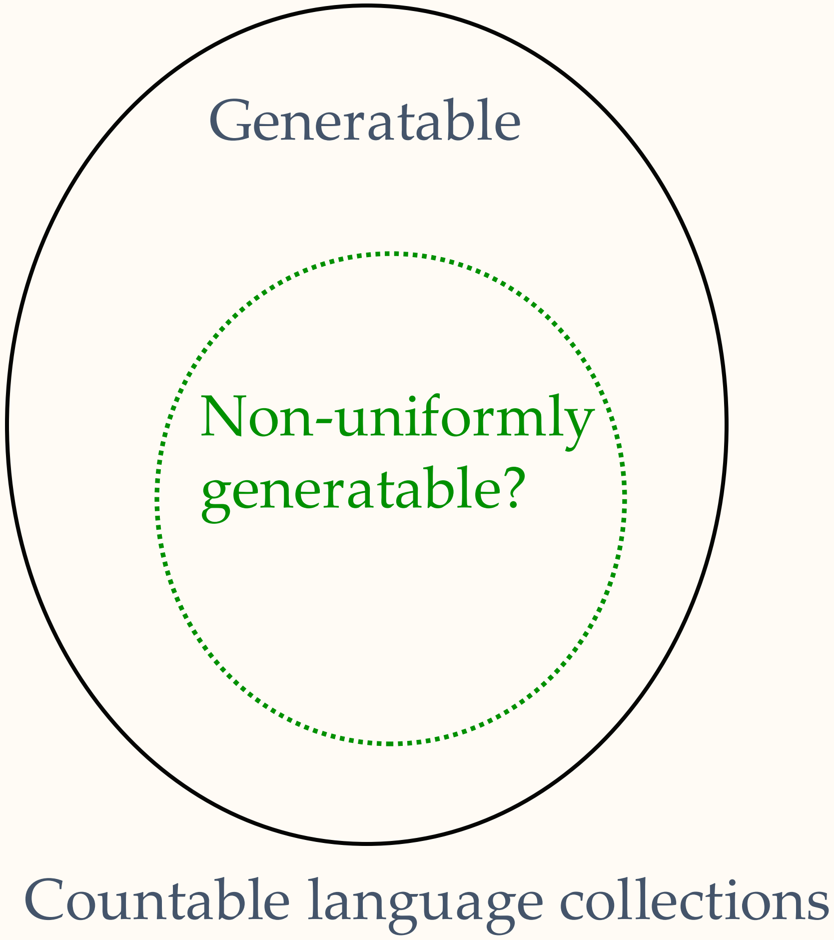

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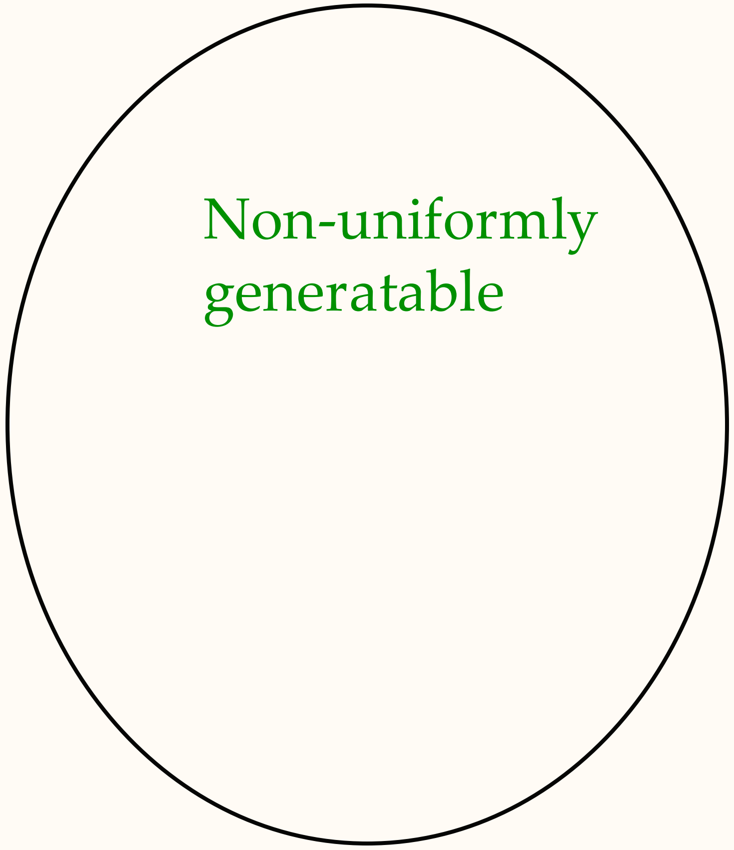
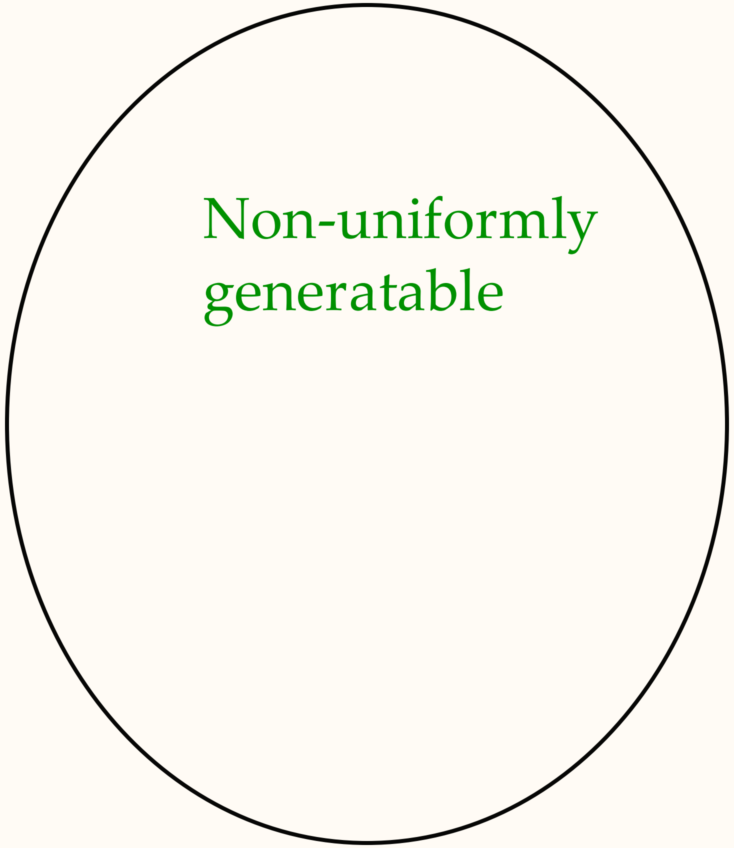


Countable language collections

Non-uniform Generation in the Limit

- Question (Li, Raman, Tewari '24): Is every countable collection of languages non-uniformly generatable in the limit?
- Theorem (Charikar, P '24, Li, Raman, Tewari '24): Yes! 🙃
- However, provably requires stronger oracles



Countable language collections

Non-uniform Generation in the Limit

- $\mathcal{C} = \{L_1, L_2, L_3, \dots\}$, each L_i countably infinite subset of universe $\mathcal{X}$
- Adversary chooses some target language L_z, starts enumerating it in an order of their choosing

$$x_1, x_2, x_3, x_4, x_5, \dots\dots$$

- At each time step t, algorithm generates a string z_t
- Non-uniformly generates in the limit if the moment the algorithm sees $t^\star = t^\star(\mathcal{C}, L_z)$ distinct strings , all strings generated thereafter are new and in L_z

chooses L_{70}

😈	x'_1	x'_2	x'_3	x'_4	....	$x'_{t^\star}$	$x'_{t^\star+1}$	....
😇	z'_1	z'_2	z'_3	z'_4	....	$z'_{t^\star}$ new, $\in L_{70}$	$z'_{t^\star+1}$ new, $\in L_{70}$	....

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Can we further get $t^\star$ to depend only on $\mathcal{C}$? 🤔

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new, $\in L_{70}$ new, $\in L_{70}$

Uniform Generation in the Limit

Li, Raman, Tewari ‘24

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x_1' x_2' x_3' x_4' ....

z_1' z_2' z_3' z_4' ....

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Uniform Generation in the Limit

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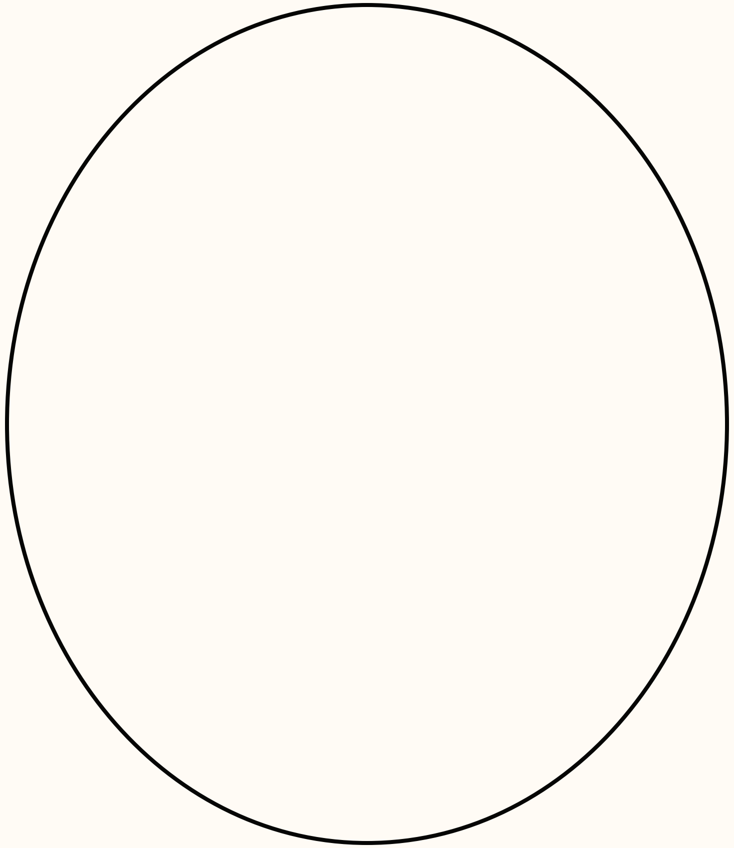

Countable language collections

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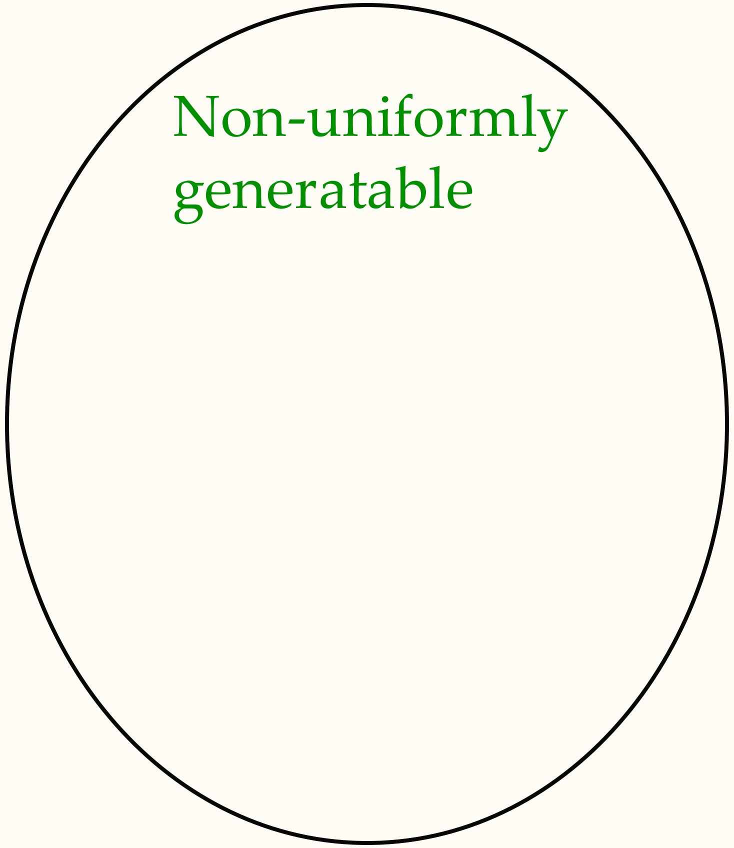

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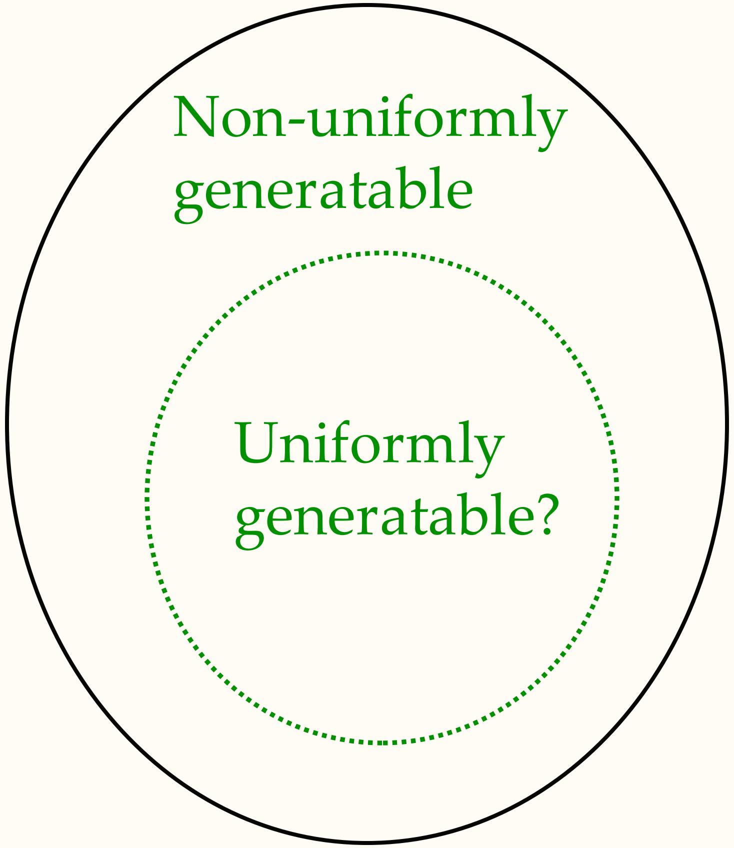
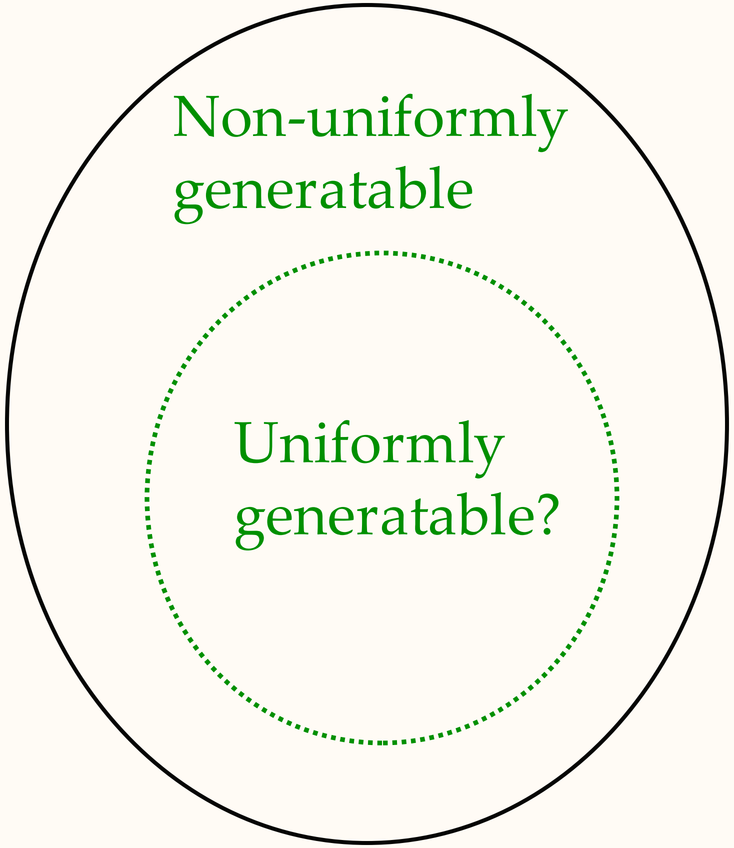


Countable language collections

Uniform Generation in the Limit

- $O = \{-1, -3, -5, \ldots\}$



Countable language collections

Uniform Generation in the Limit

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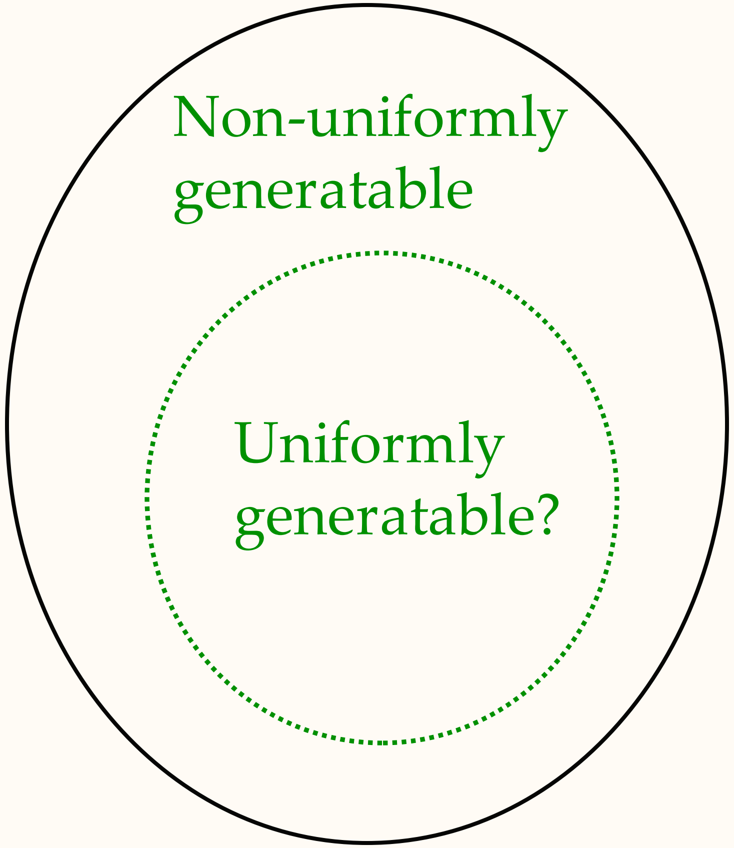

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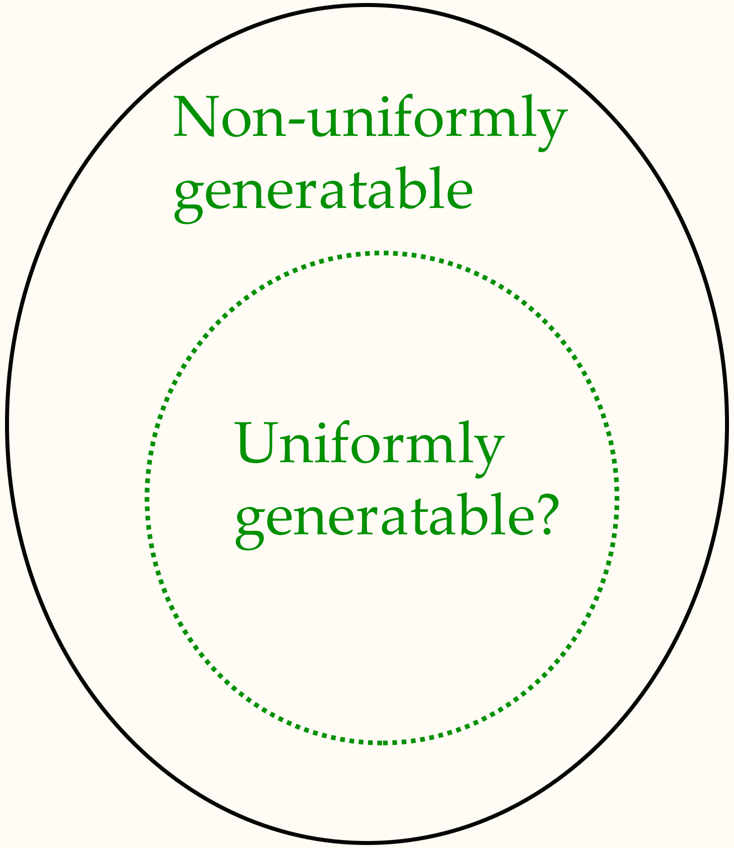

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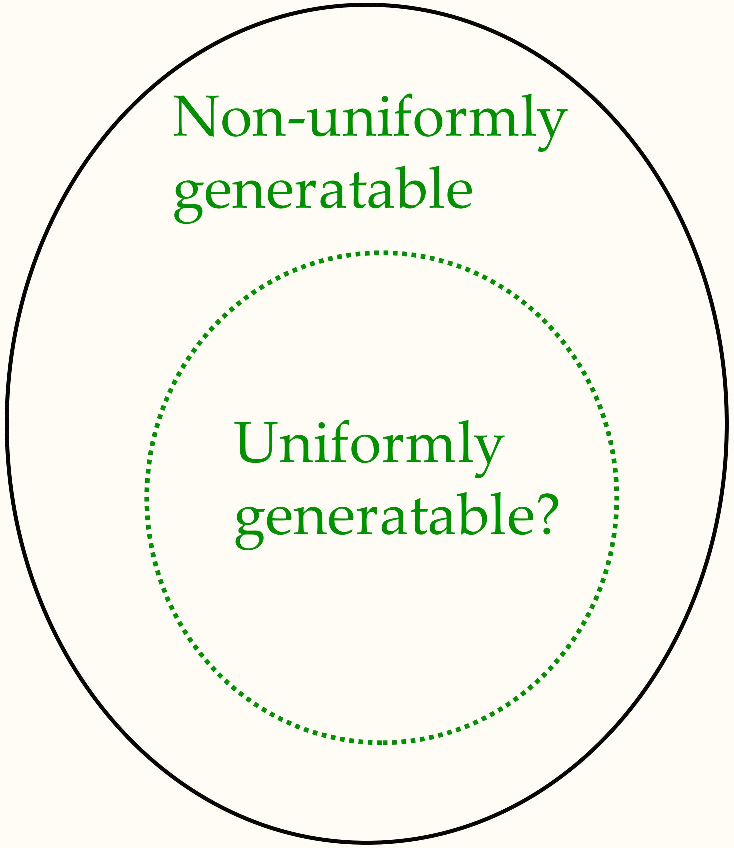

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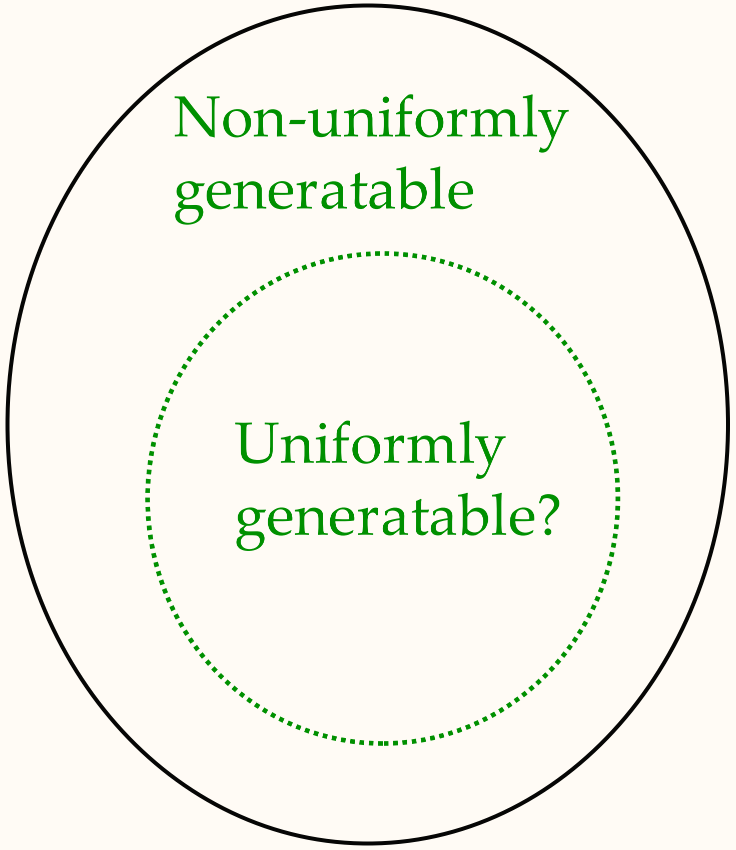

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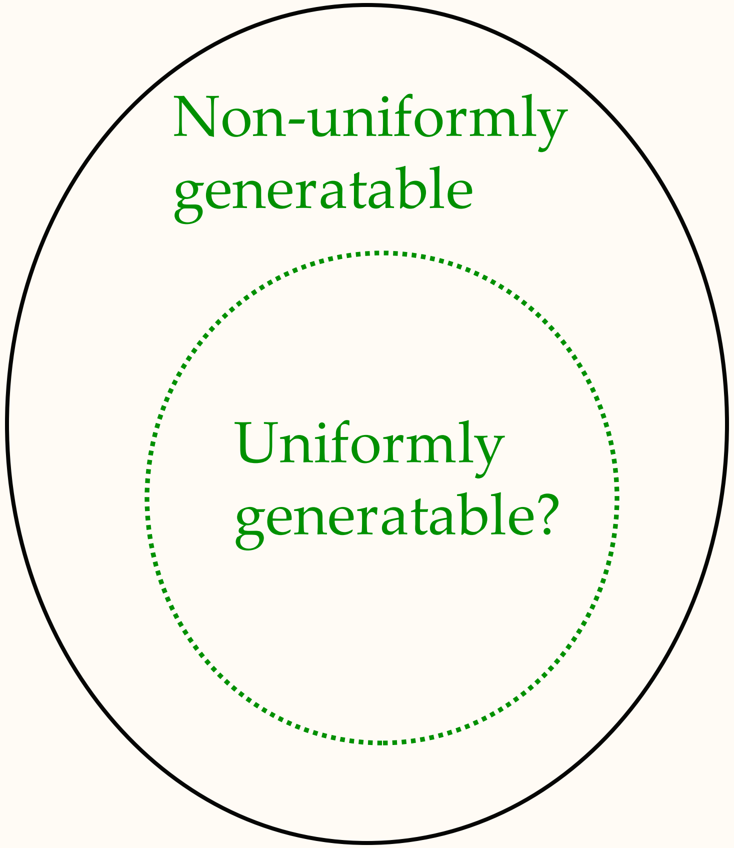
$$\vdots$$



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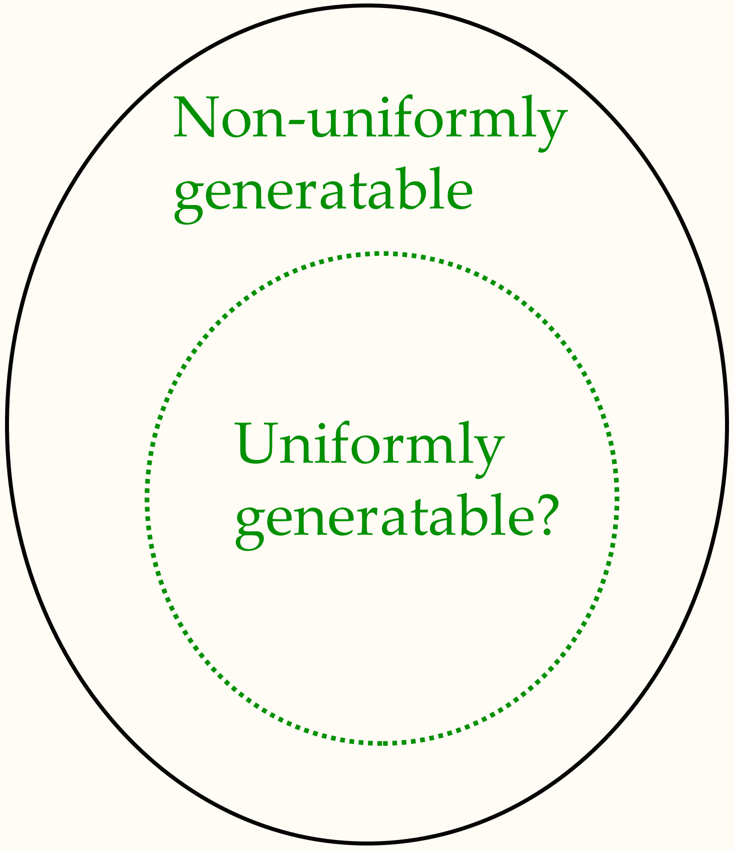

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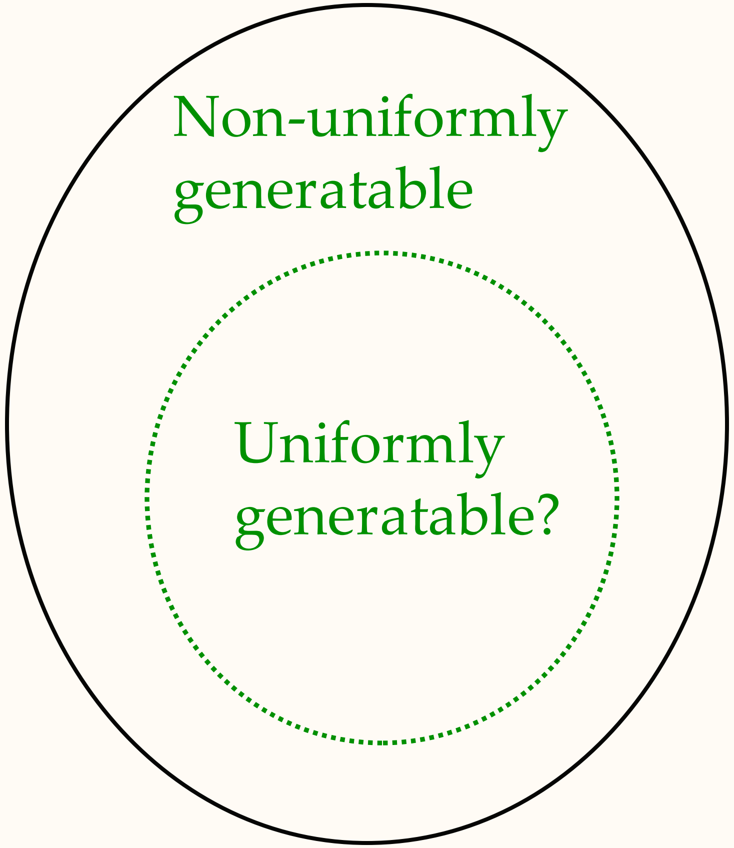


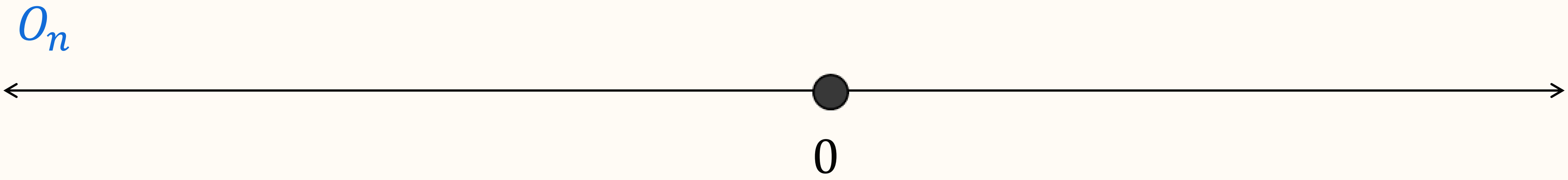
Countable language collections

Uniform Generation in the Limit

Uniform Generation in the Limit

o_n

Uniform Generation in the Limit



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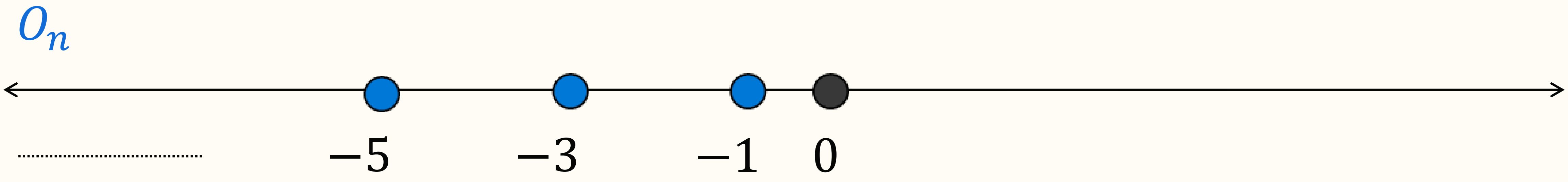

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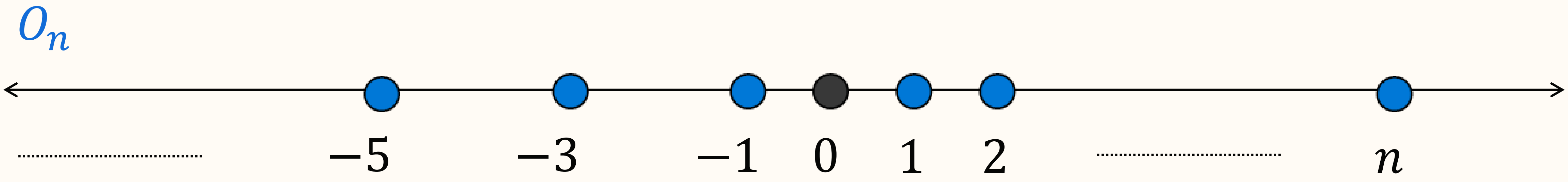
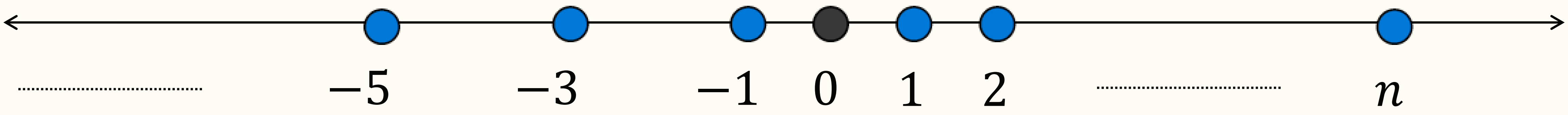


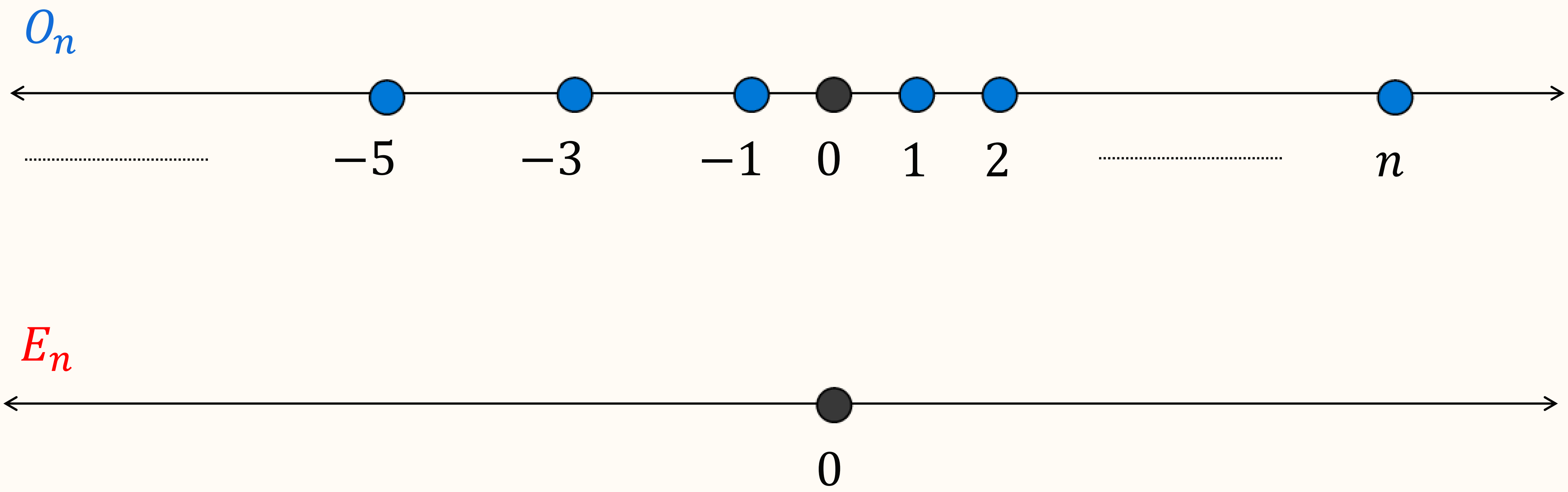
Uniform Generation in the Limit

O_n



E_n

Uniform Generation in the Limit



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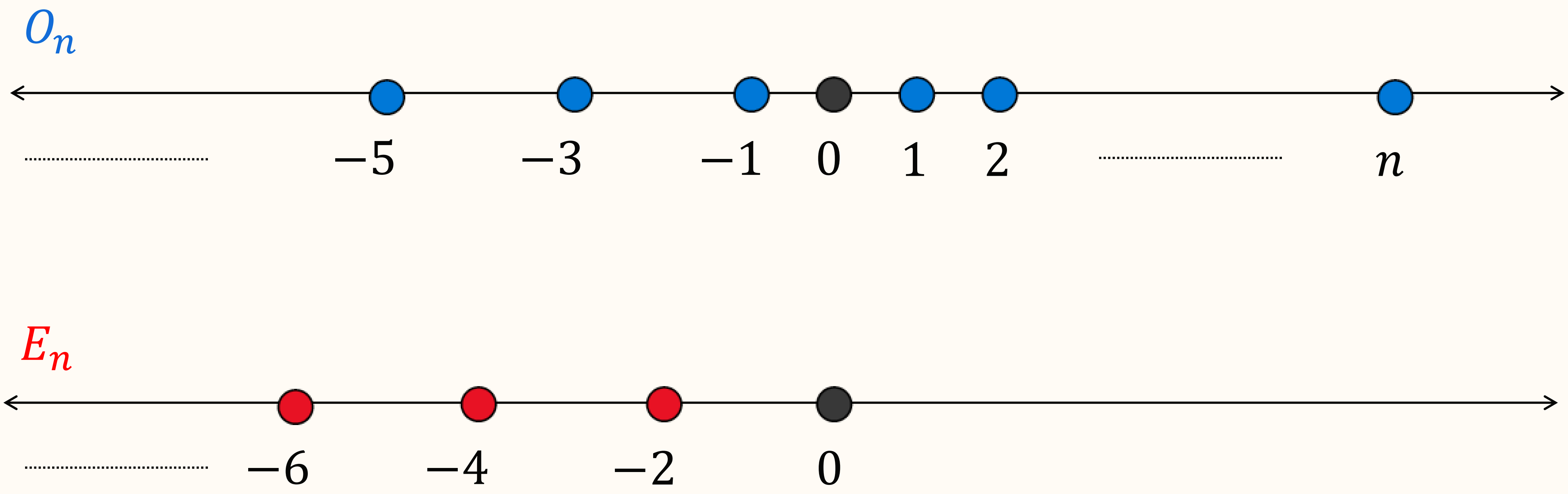

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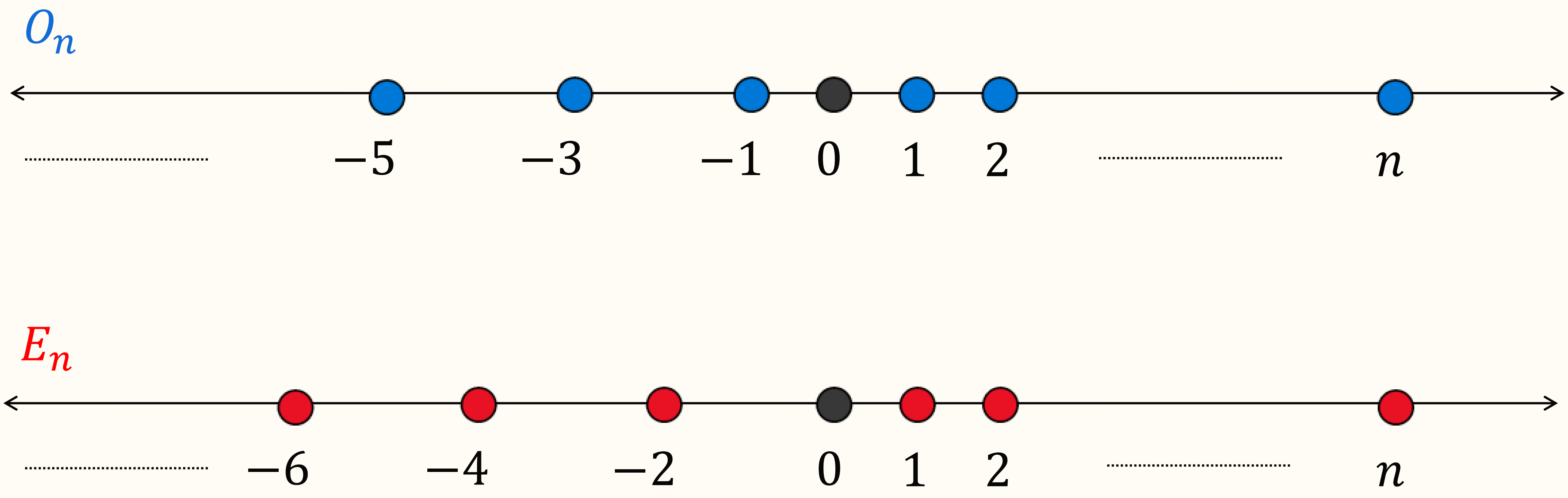

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- Suppose algorithm claims a uniform time bound of $t^\star = t^\star(\mathcal{C})$

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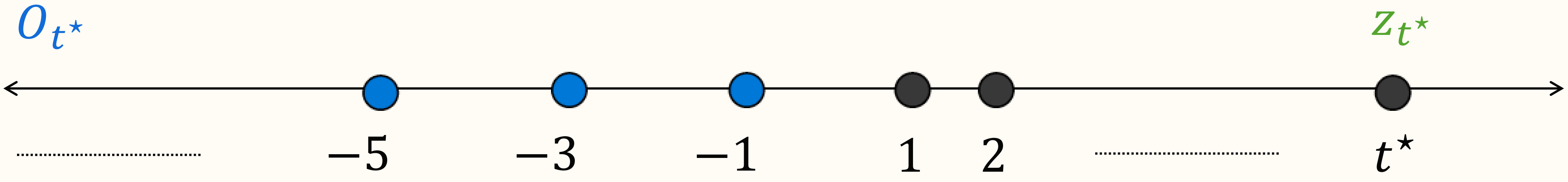

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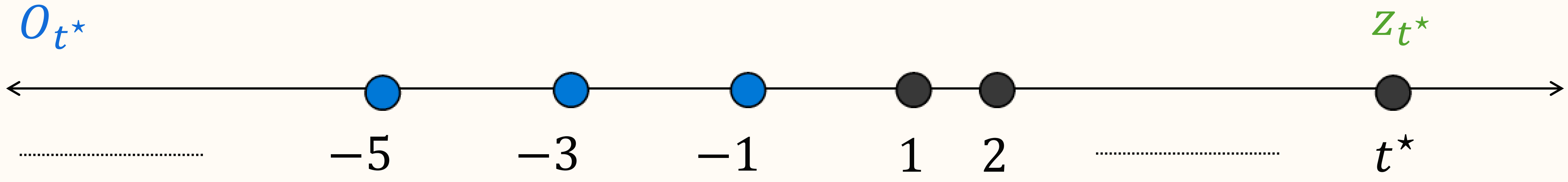

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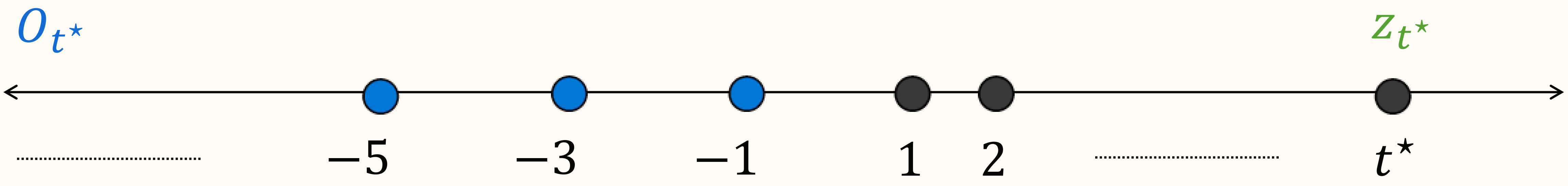

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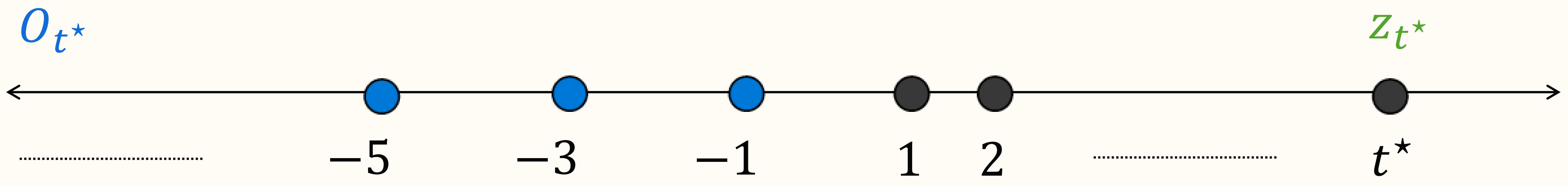

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Uniform Generation in the Limit

- Suppose algorithm claims a uniform time bound of $t^\star = t^\star(\mathcal{C})$
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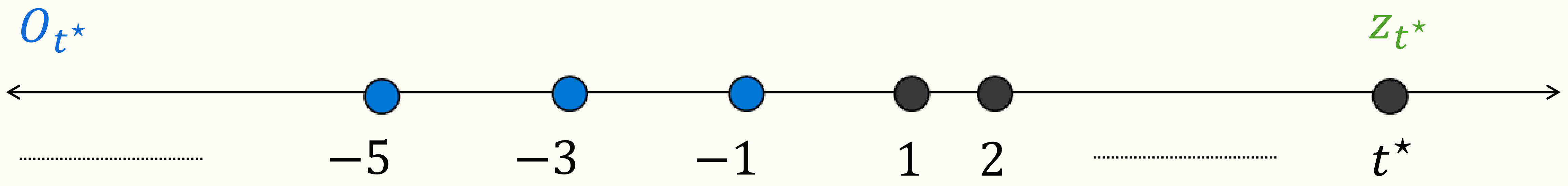

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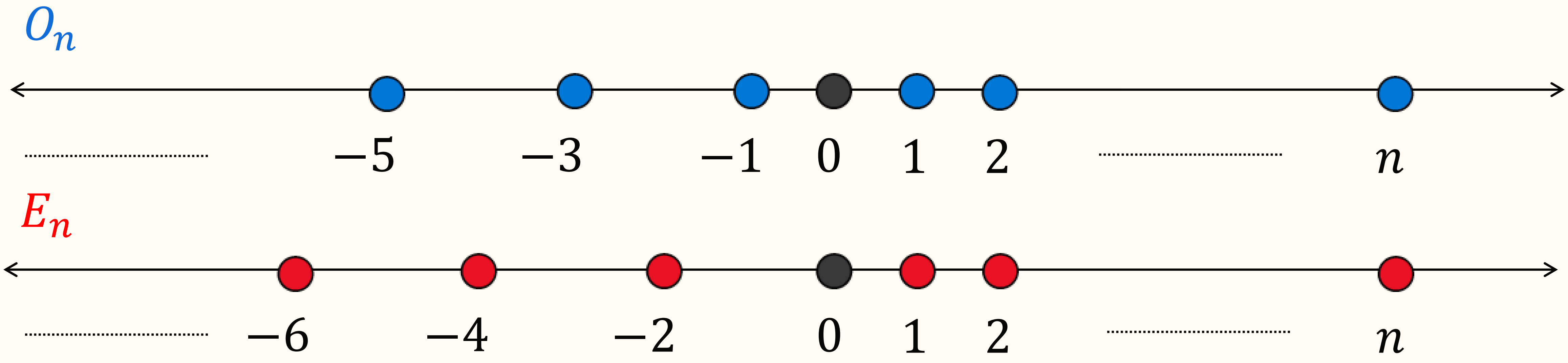

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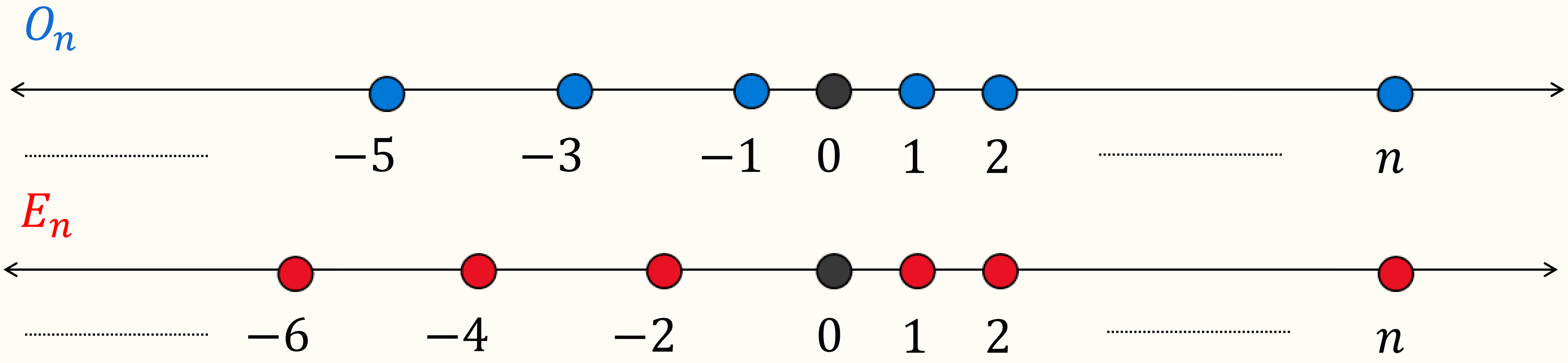

- Issue: Arbitrarily large **finite intersections**

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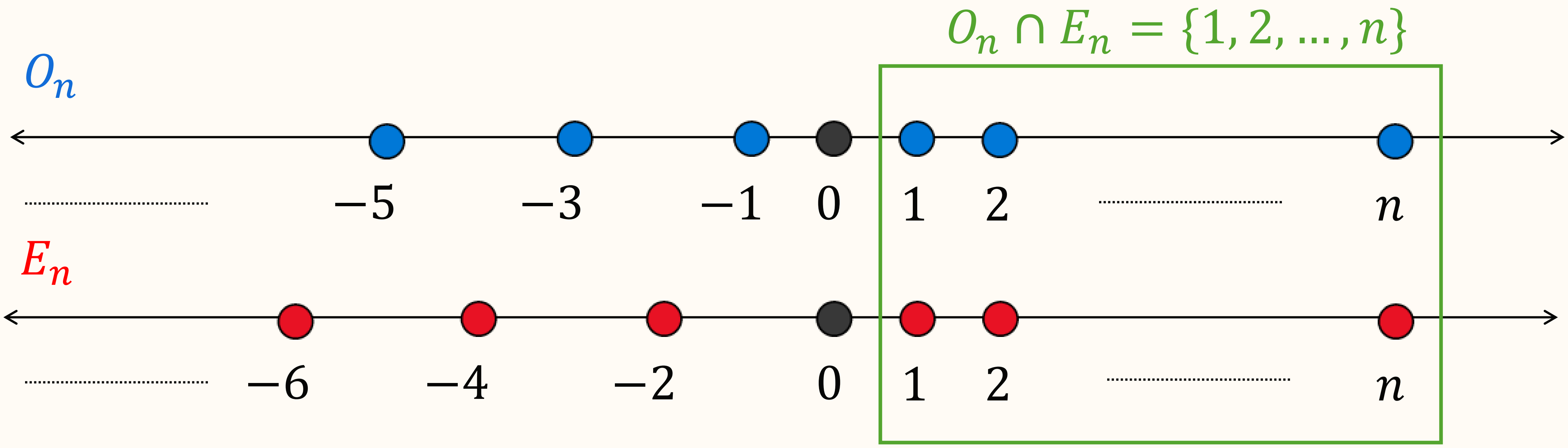

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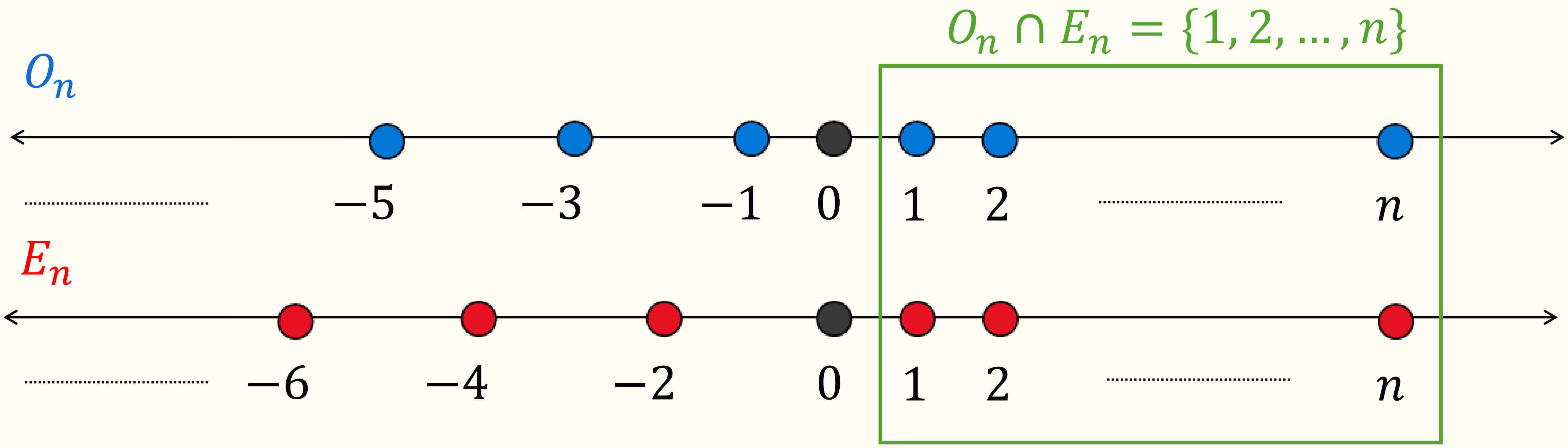

- Issue: Arbitrarily large **finite intersections**
- <u>Definition:</u> Largest finite intersection $:=$ "closure dimension"

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- Theorem (Kleinberg, Mullainathan ‘24, Li, Raman, Tewari ‘24): Uniformly generatable if and only closure dimension is bounded

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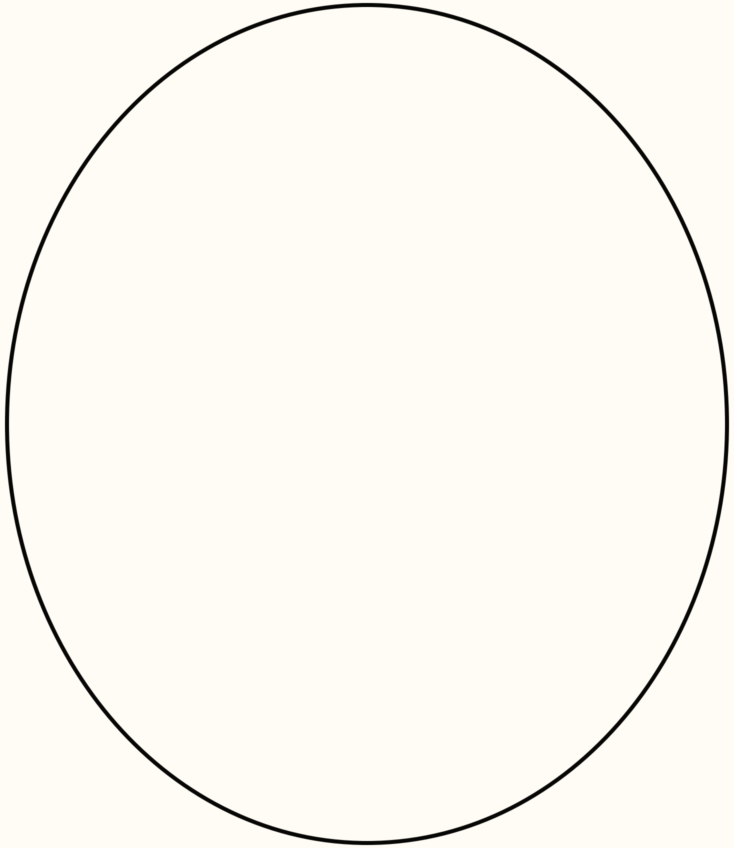

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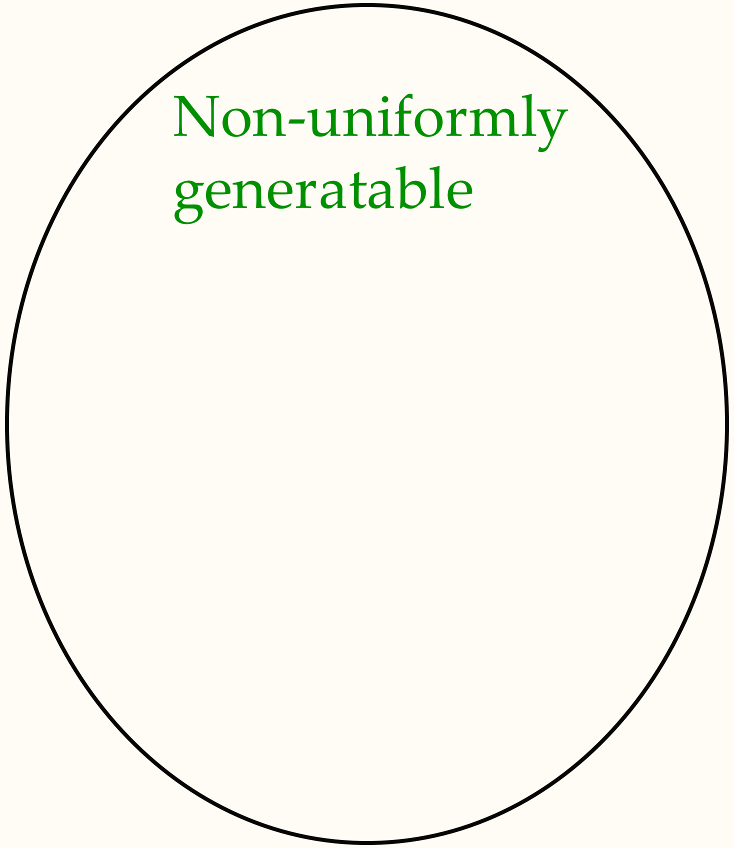

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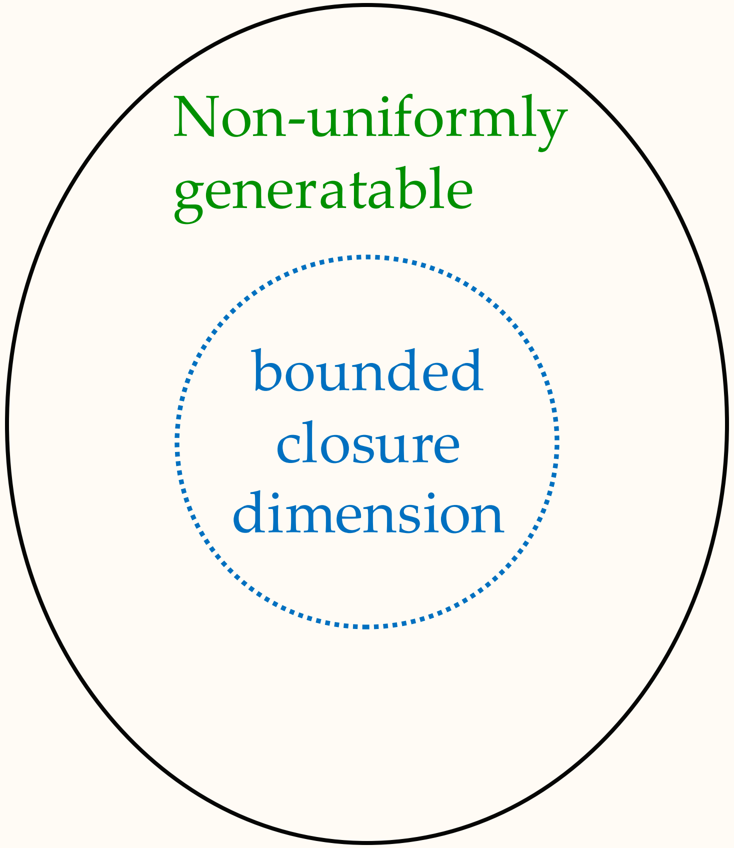

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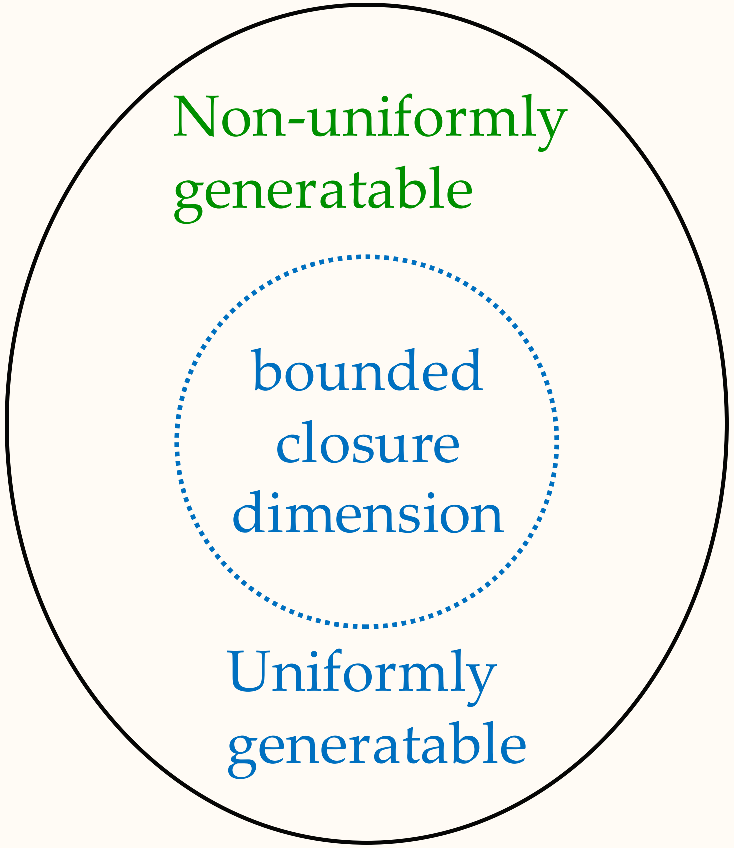

Countable language collections

Languages as binary concepts

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Associate every language with its indicator function

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$\mathcal{H}$ \ $\mathcal{X}$	x_1	x_2	x_3	x_4	x_5	…
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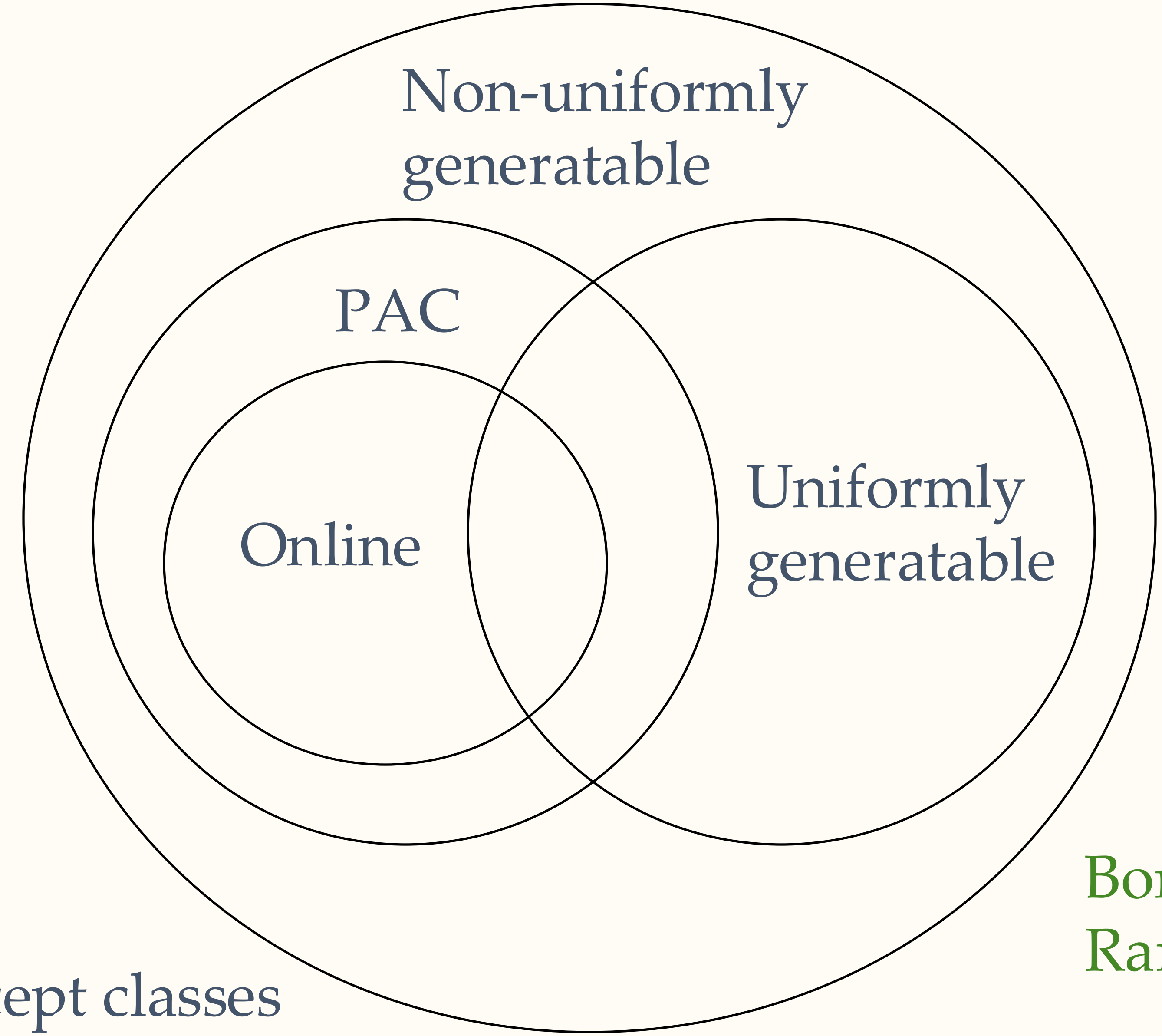
Can then ask standard learning theory questions for this concept class

Generation versus Prediction



Borrowed from Li, Raman, Tewari '24

Countable concept classes

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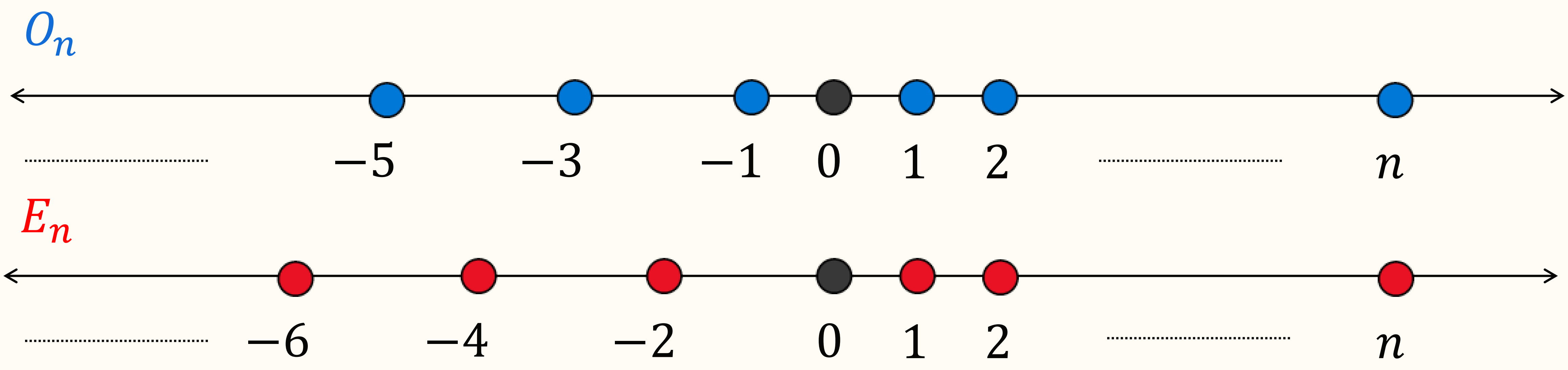

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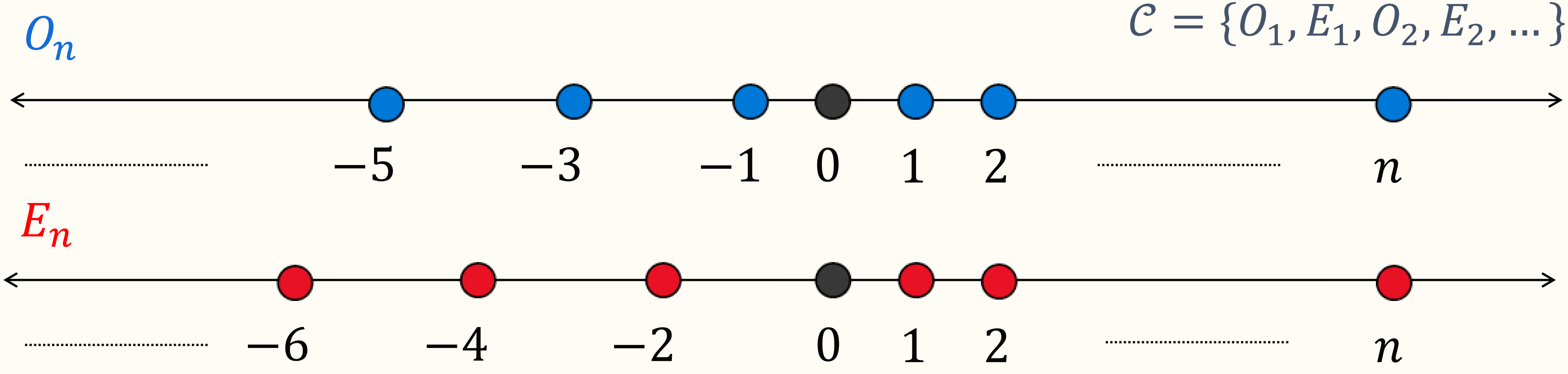

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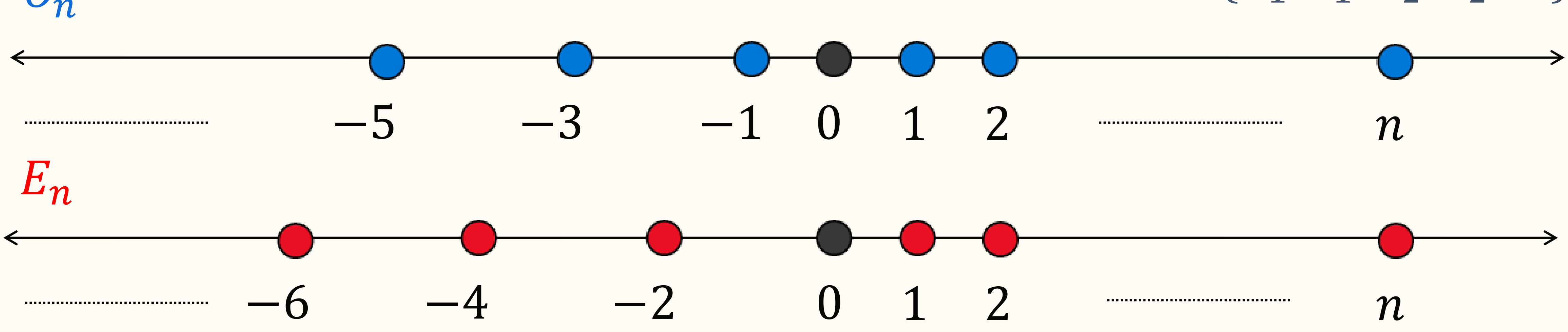

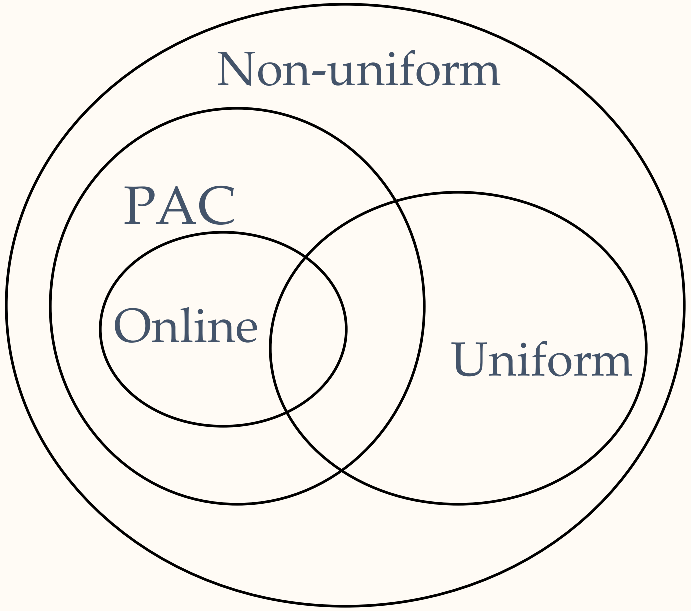

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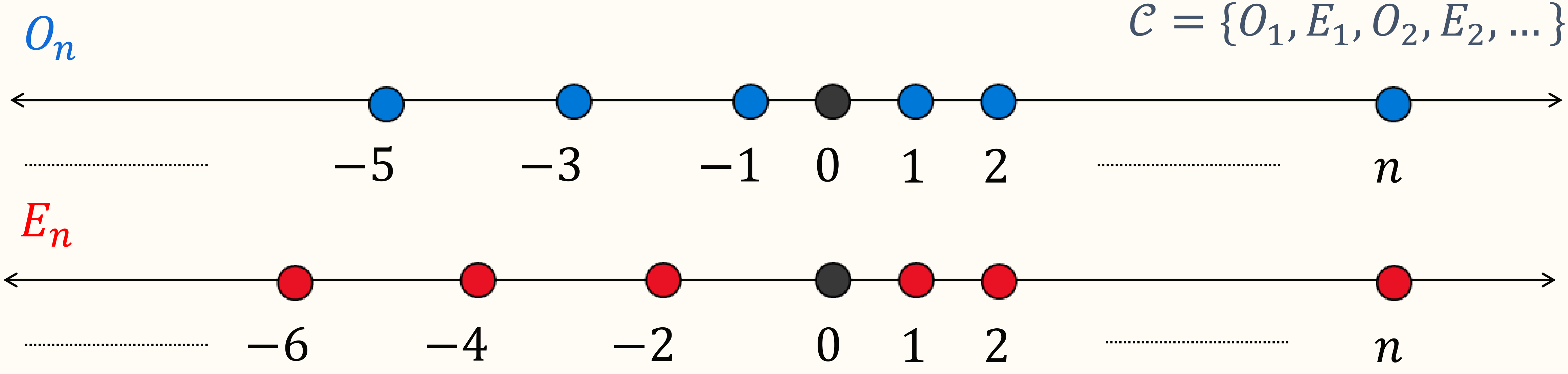

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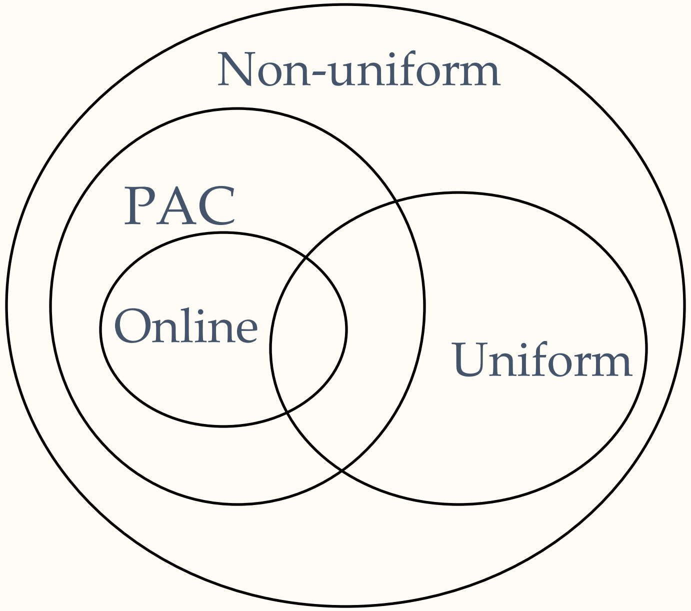

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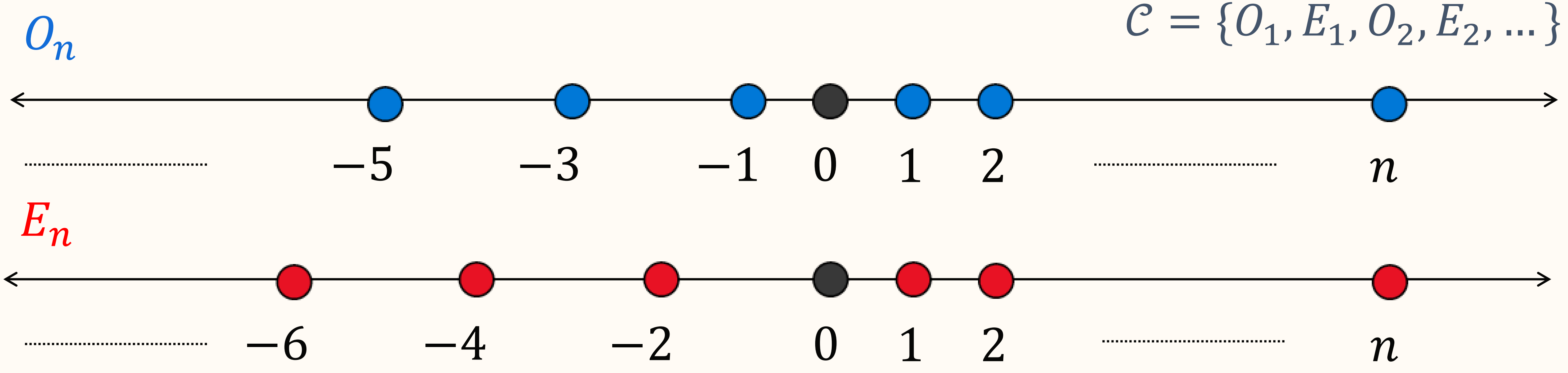

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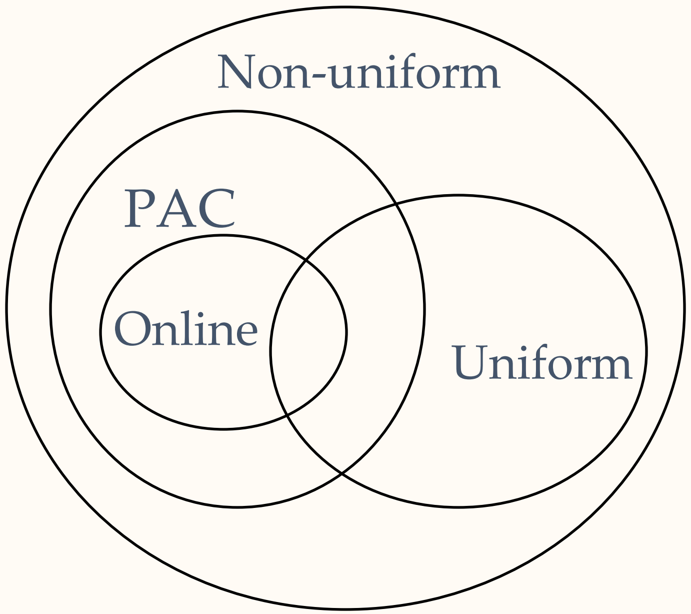

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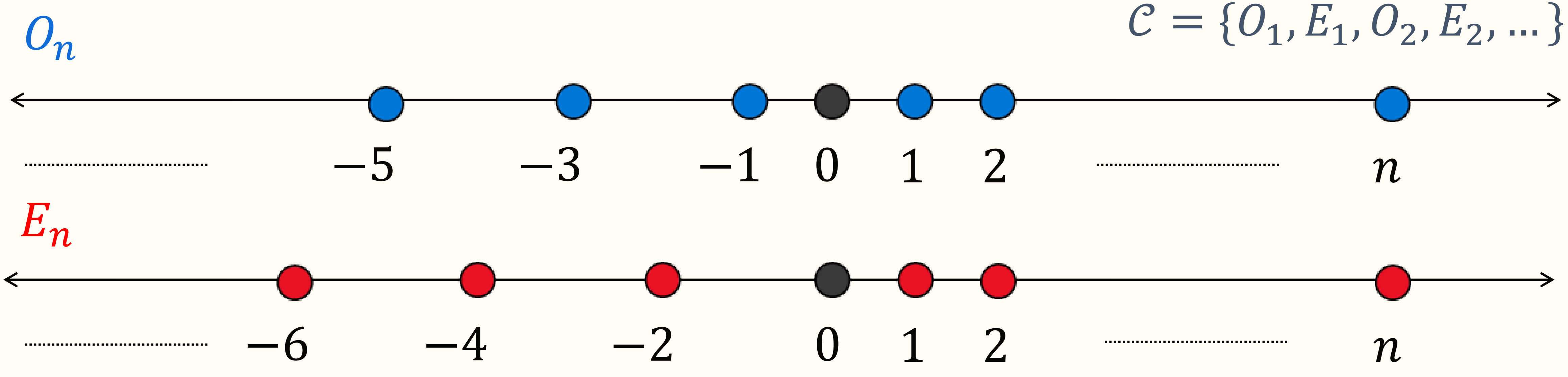

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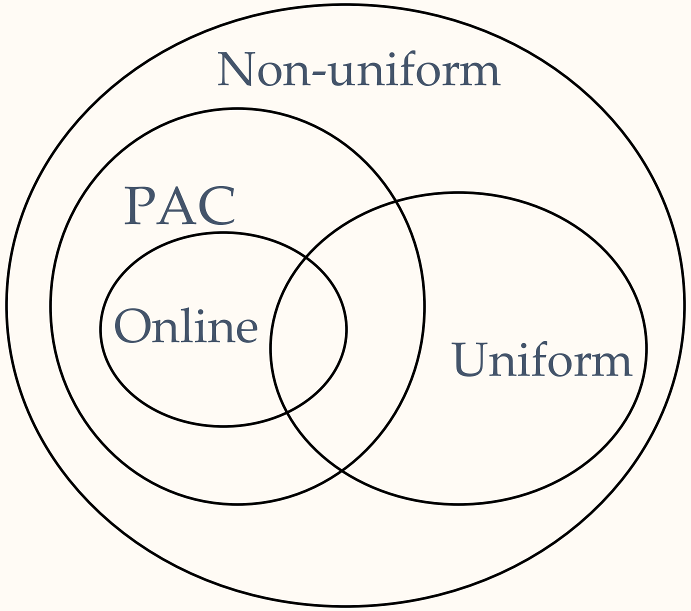

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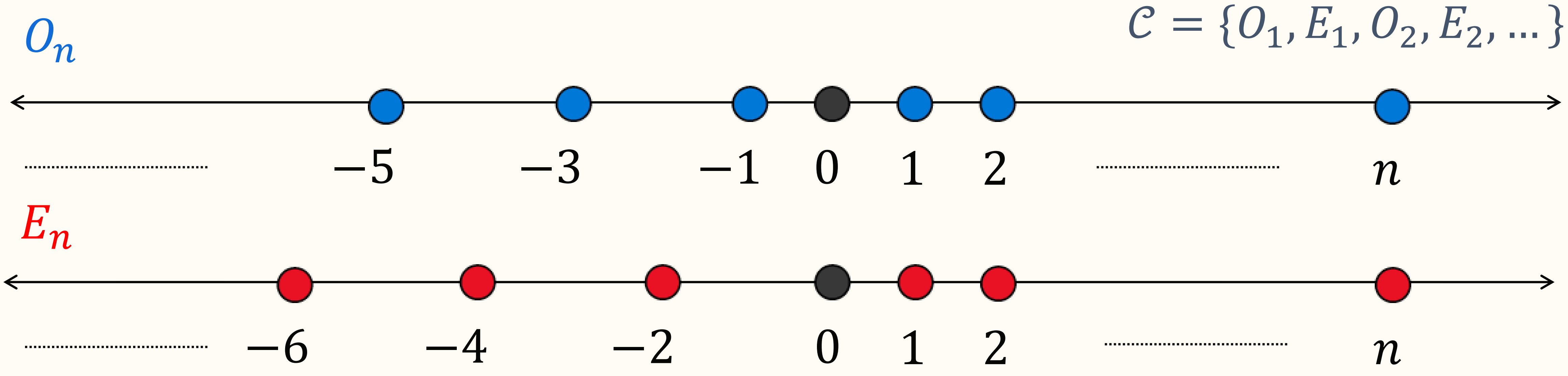

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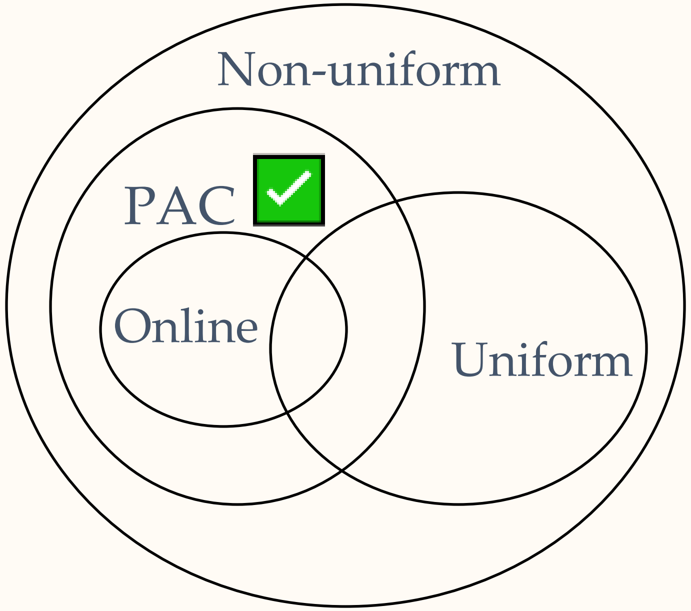

Generation versus Prediction

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O_n

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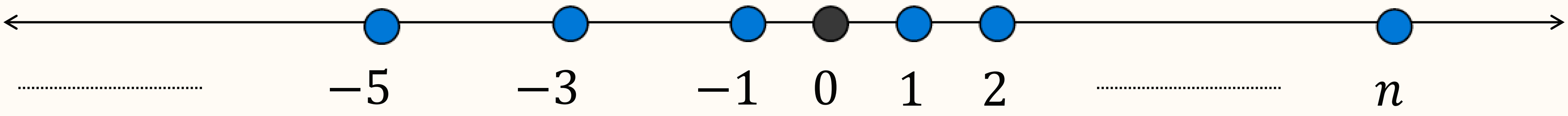

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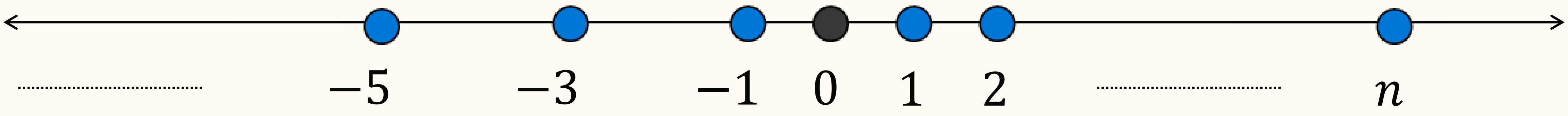

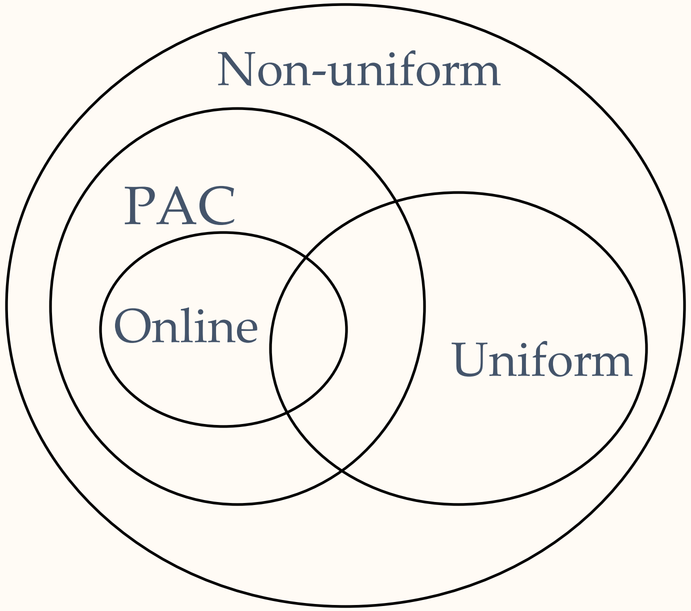

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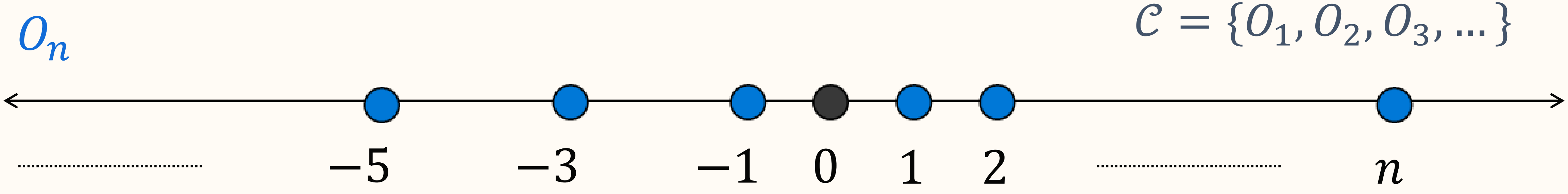

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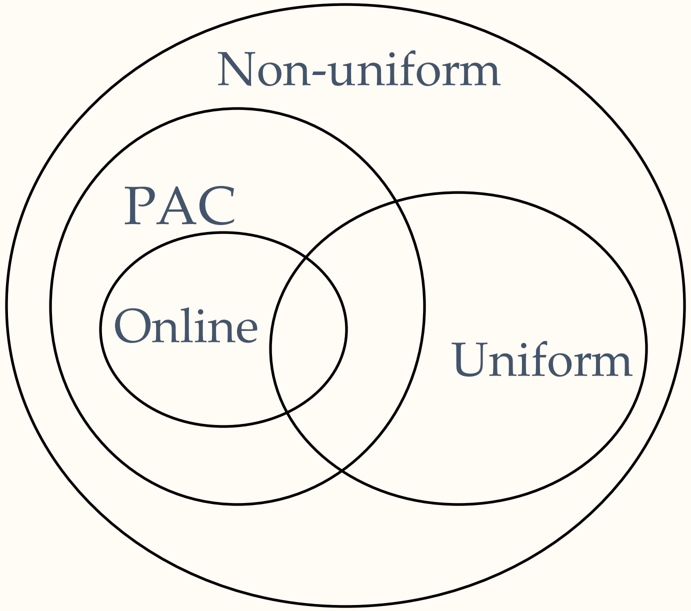

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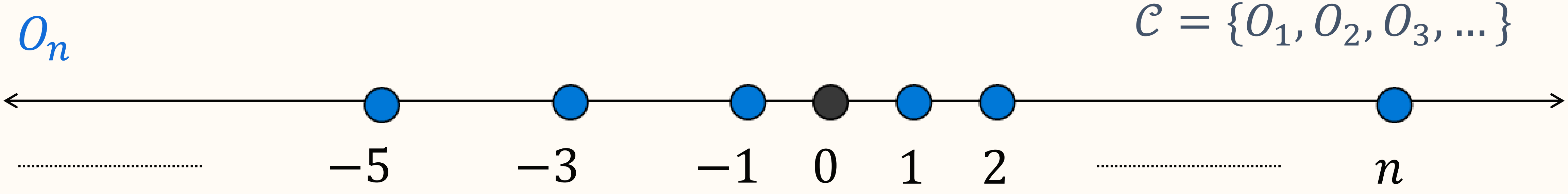

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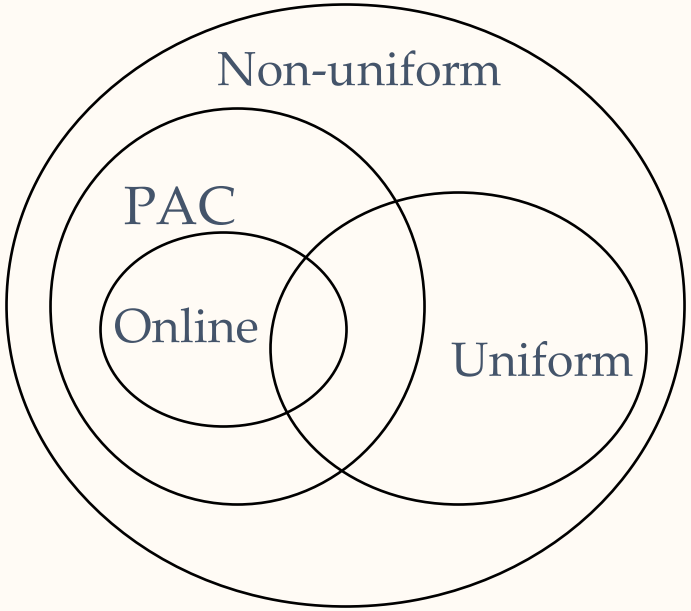

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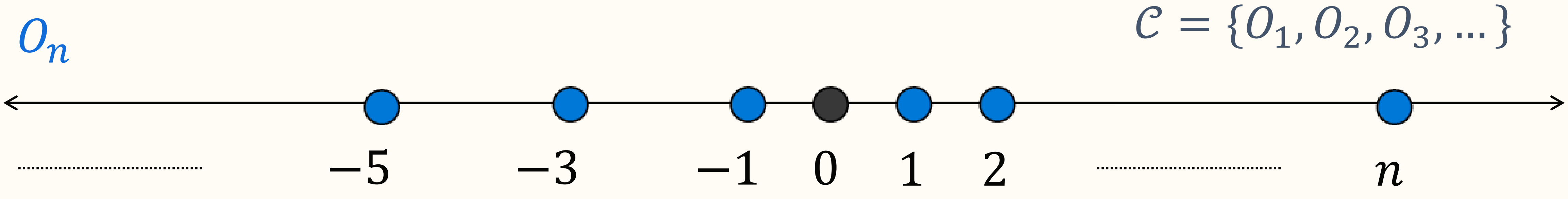

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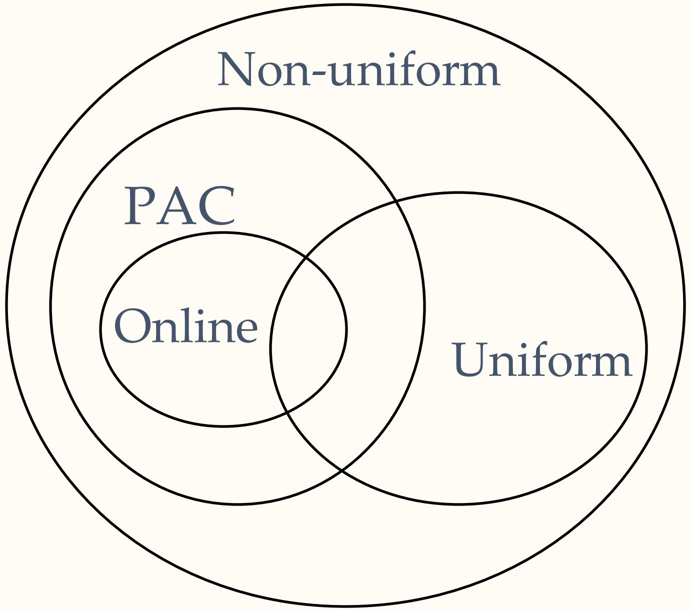

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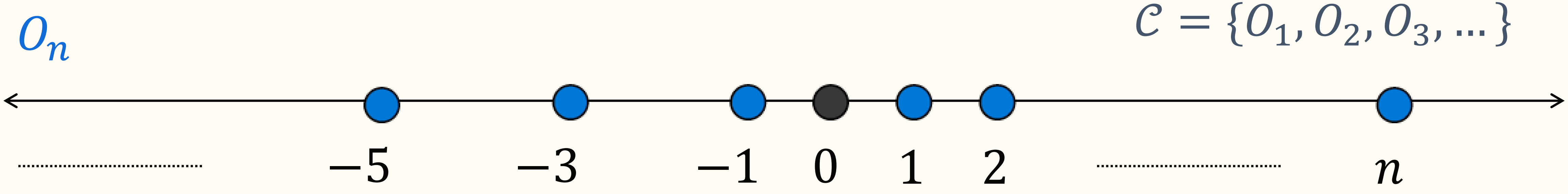

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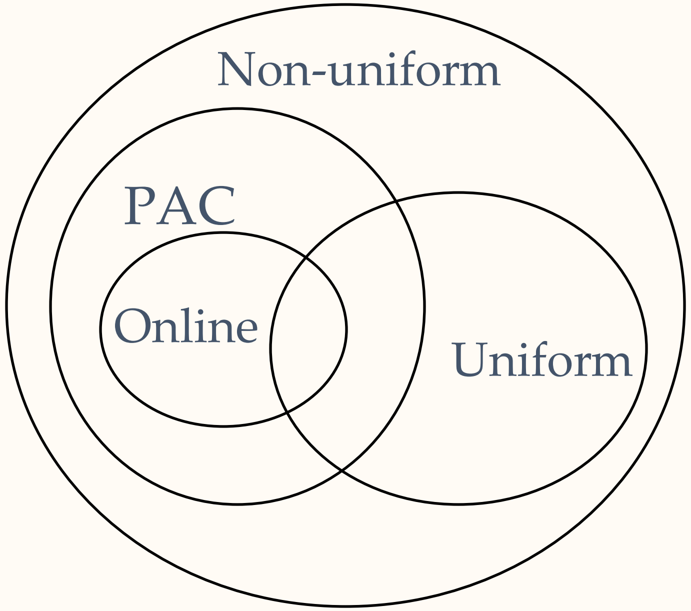

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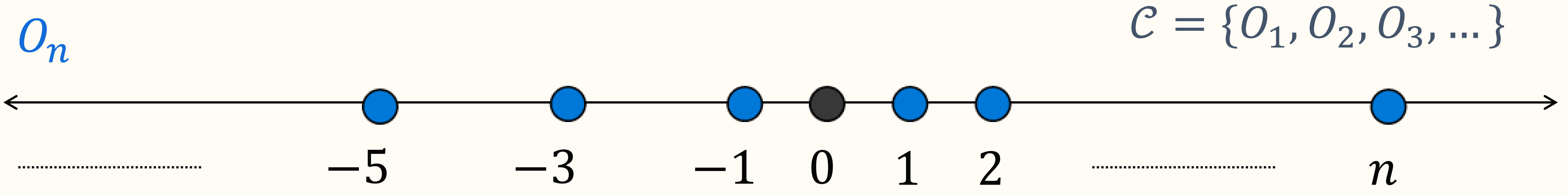

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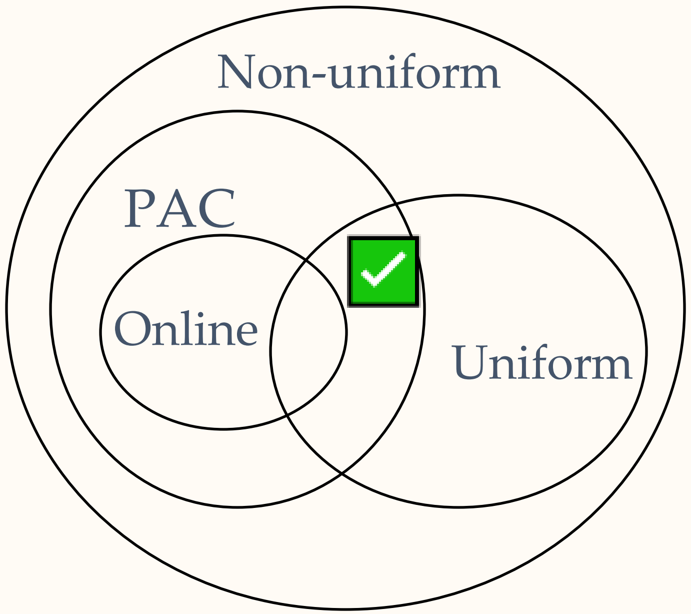

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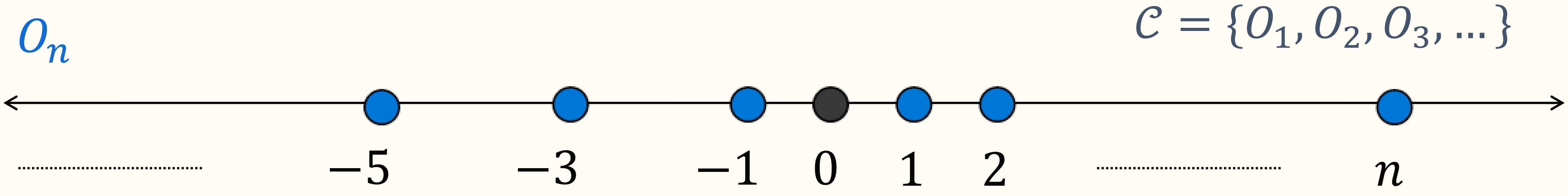

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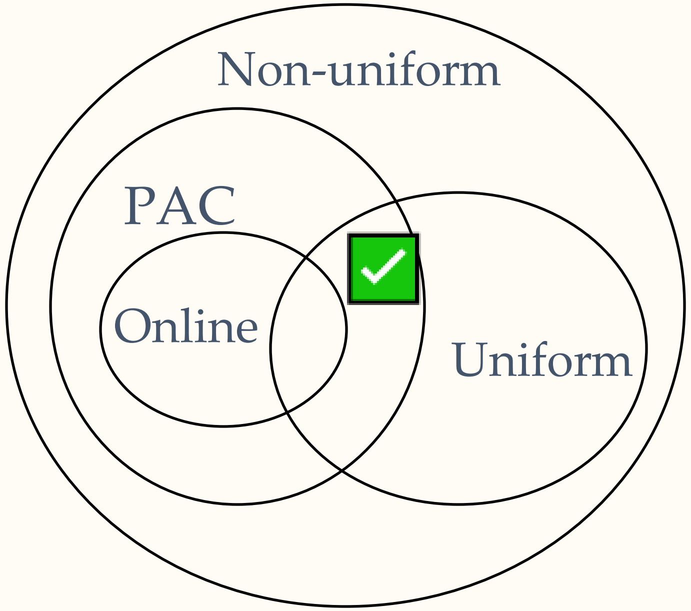

recent work by Hanneke, Karbasi, Mehrotra, Velegkas '25 shows that generatability not even closed under finite unions! 😓

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- Kalavasis, Mehrotra, Velegkas '24 show that for the Kleinberg, Mullainathan '24 algorithm, this is bounded as $C \cdot \exp(-c \cdot t)$ for all countable collections!

Summary

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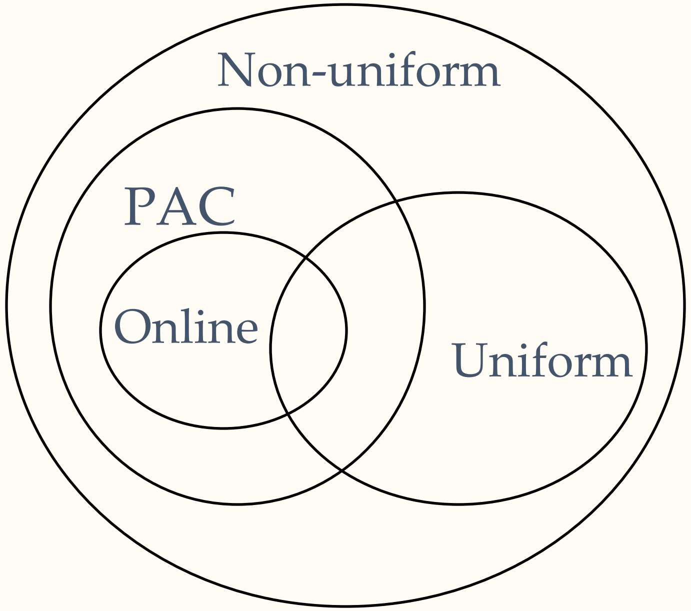

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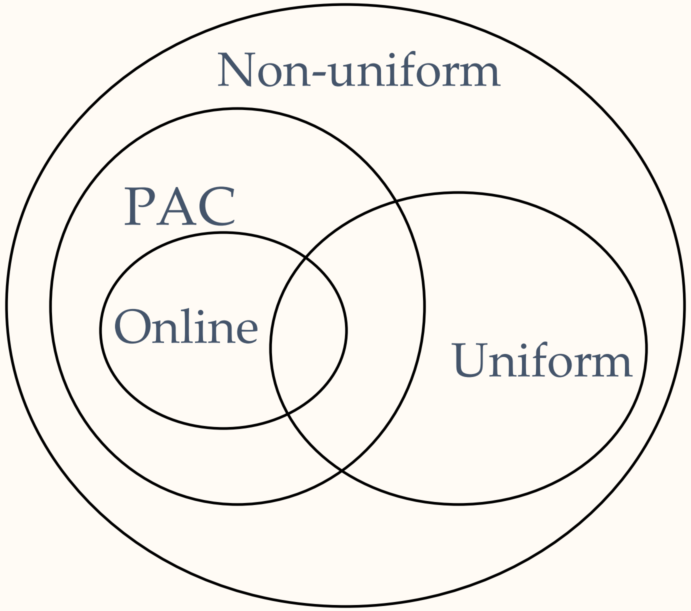

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- <u>Open:</u> black-box transforming algorithm that generates in the limit $\Longrightarrow$ exponential rate