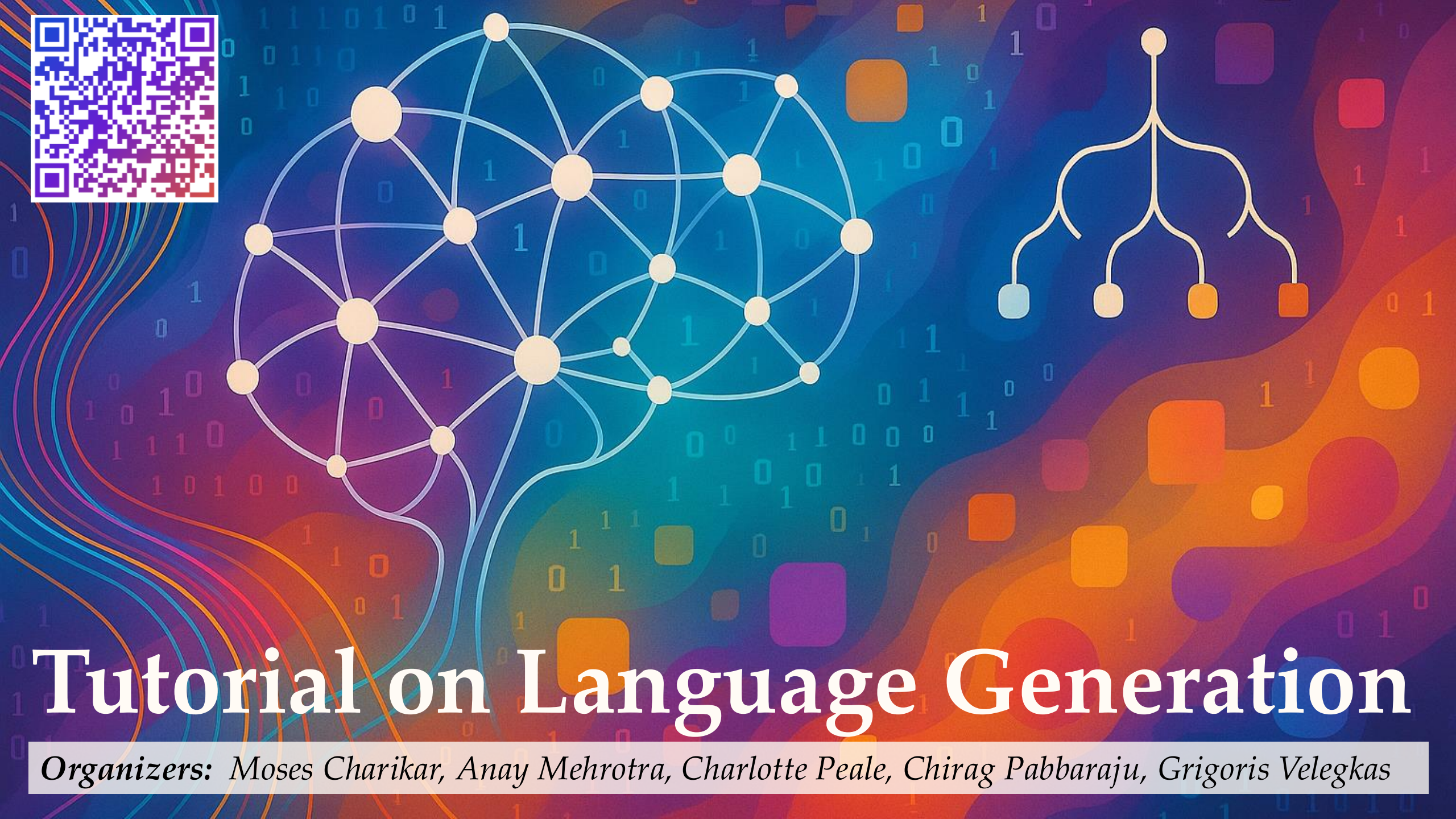


Tutorial on Language Generation

Organizers: *Moses Charikar, Anay Mehrotra, Charlotte Peale, Chirag Pabbaraju, Grigoris Velegkas*

Schedule

Session 1 (40/20 mins)

- *Language Generation and Learning Theory* — *Moses*
- *Stronger Notions of Generation and Comparison to Prediction* — *Chirag*

Session 2 (20/20/20 mins)

- *Validity–Breadth Trade-Off (Part I)* — *Anay*
- *Validity–Breadth Trade-Off (Part II)* — *Grigoris*
- *Diverse and Robust Generation* — *Charlotte*

Tutorial on Language Generation

Visit: LanguageGeneration.github.io

Organizers:

Moses Charikar
Stanford

Anay Mehrotra
Yale University

Chirag Pabbaraju
Stanford

Charlotte Peale
Stanford

Grigoris Velegkas
Yale → Google Research

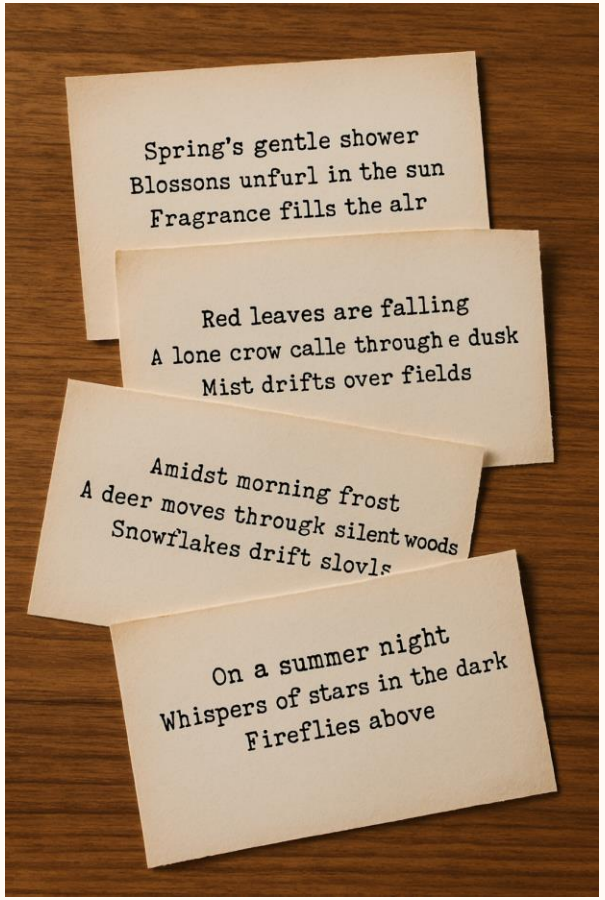
COLT 2025

LLMs and Language Generation 

Computer scientists have been fascinated by language acquisition by humans and machines for decades

LLMs: what is the problem?

Kleinberg, Mullainathan, 2024



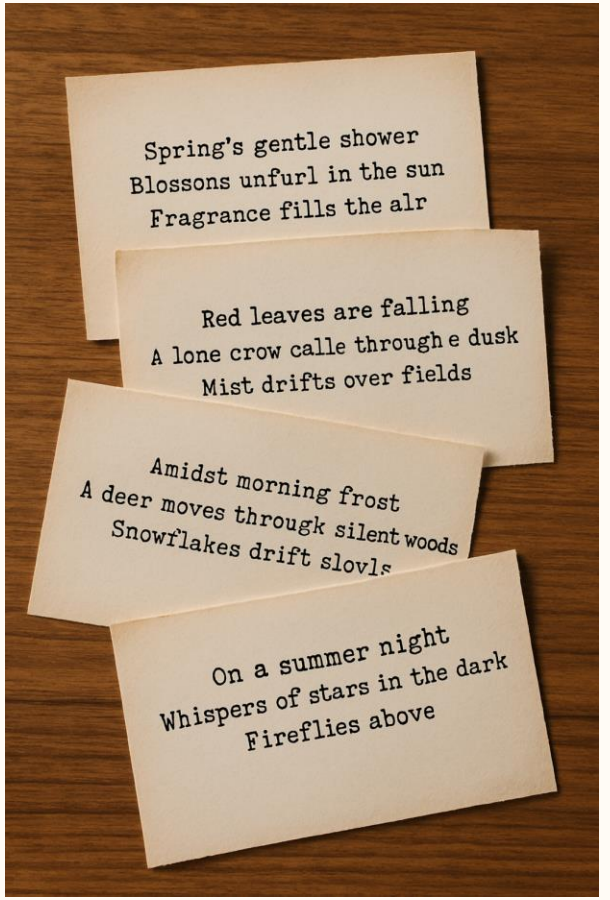
LLMs: what is the problem?

Kleinberg, Mullainathan, 2024

From a large collection of text:

1. assign probabilities to every sequence of words

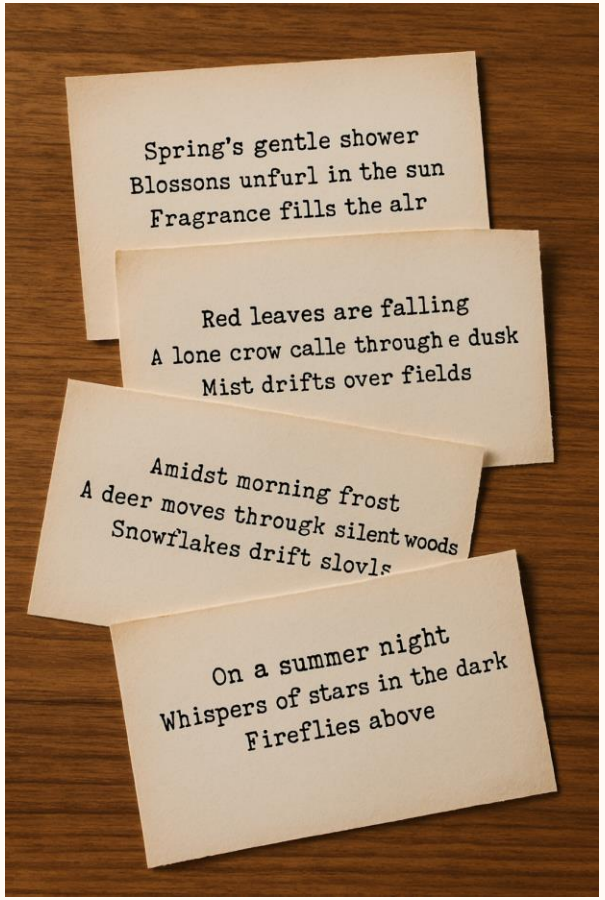
2. produce strings with high assigned probability



LLMs: simplest problem statement?

Kleinberg, Mullainathan, 2024

Given strings from an unknown language,
produce valid unseen strings from the language.



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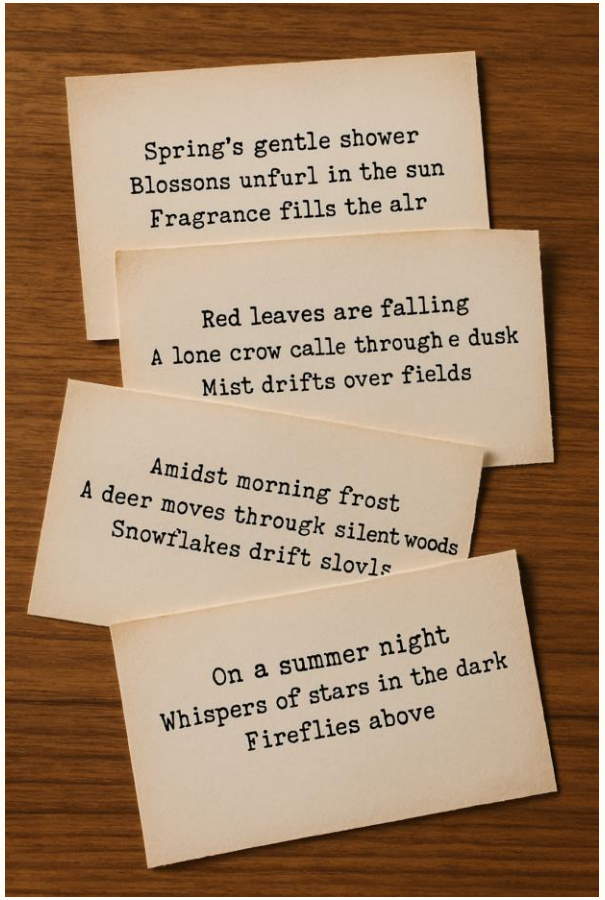
string: sequence of symbols

language: set of strings
e.g. C programs



Language Identification in the Limit

E Mark Gold*

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I wish to construct a precise model for "able to speak English"...
to investigate theoretically how it can be achieved artificially

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I wish to construct a precise model for "able to speak English"…
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Since we cannot explicitly write down the rules of English…
artificial intelligence… will have to learn… from implicit data…

Language Identification

Gold 1967, Angluin 1979

Game between adversary and algorithm.

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Adversary thinks of target language K from countable list
(e.g. all context-free grammars)

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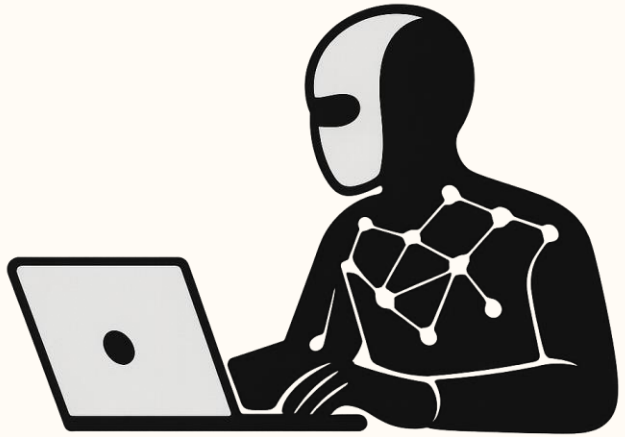

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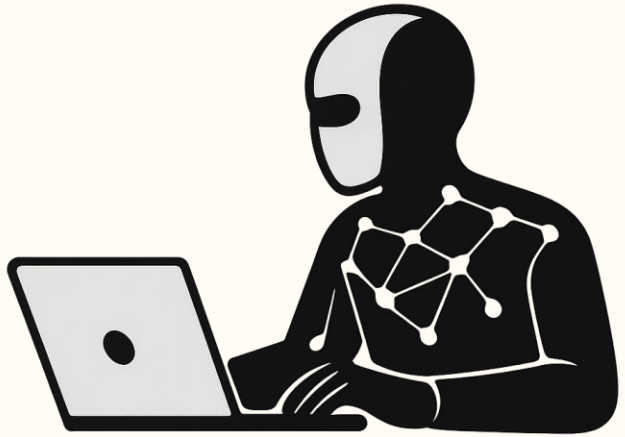

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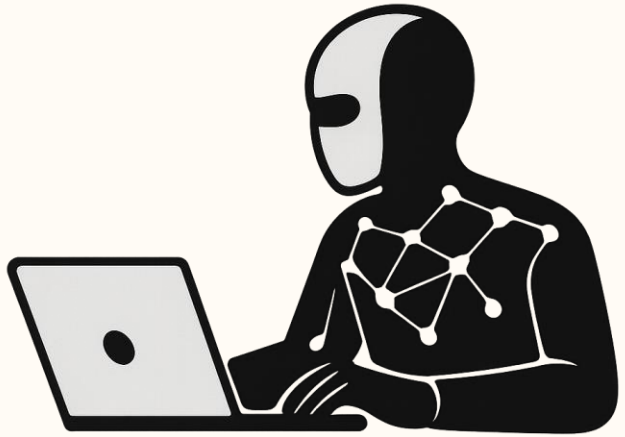

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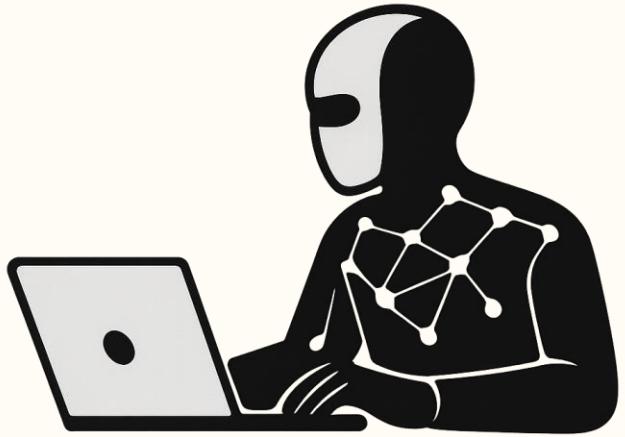

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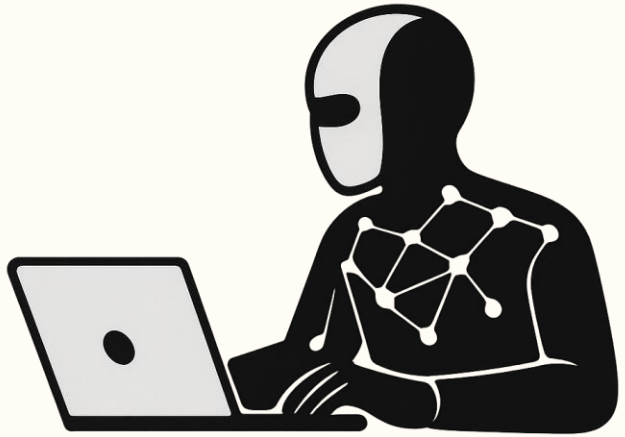

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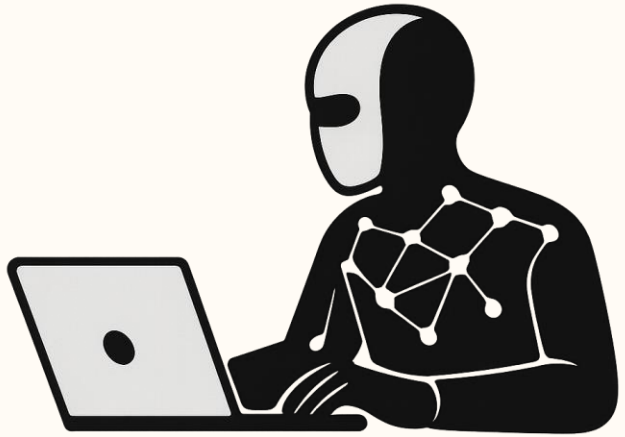

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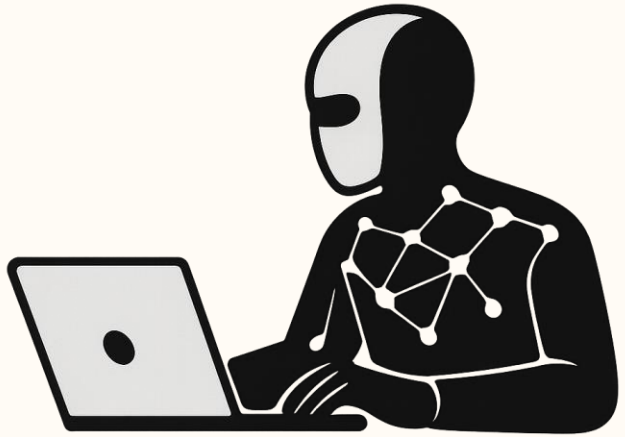

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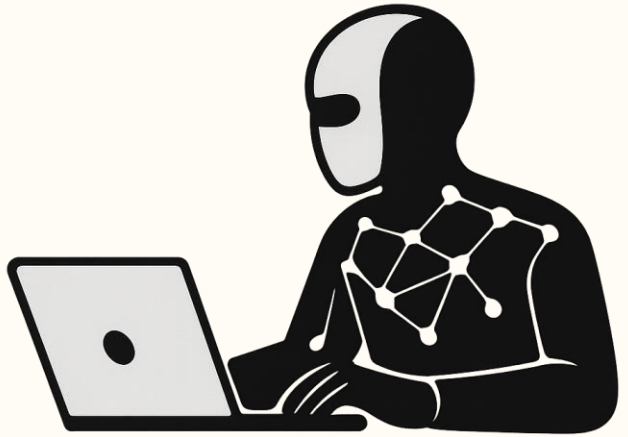

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Assume all languages infinite

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Theorem [Kleinberg, Mullainathan 2024]
Language generation in the limit is possible
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Generation vs Identification

After seeing many C programs

(generation) output valid C programs

(identification) output valid grammar for C

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validity (only generate valid strings) vs
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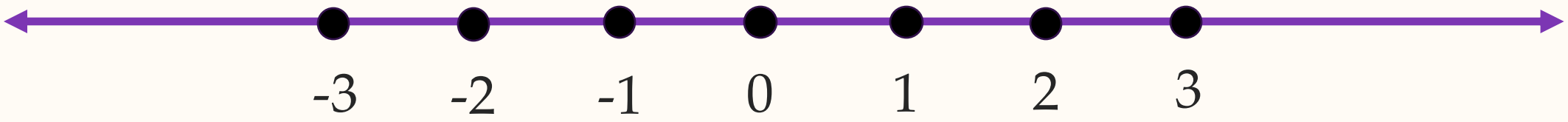
hallucination vs mode-collapse

Language Identification Lower Bound

$$L_i = \{-i, -i+1, -i+2, \ldots\}$$

$$L_\infty = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$



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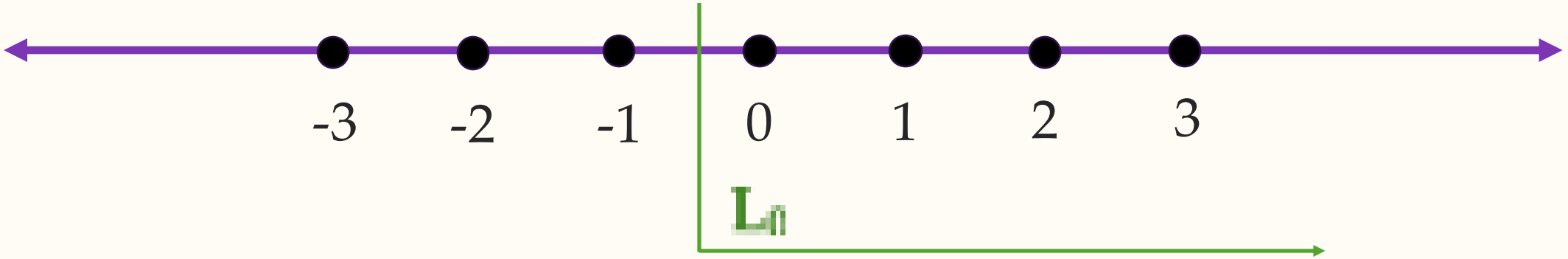

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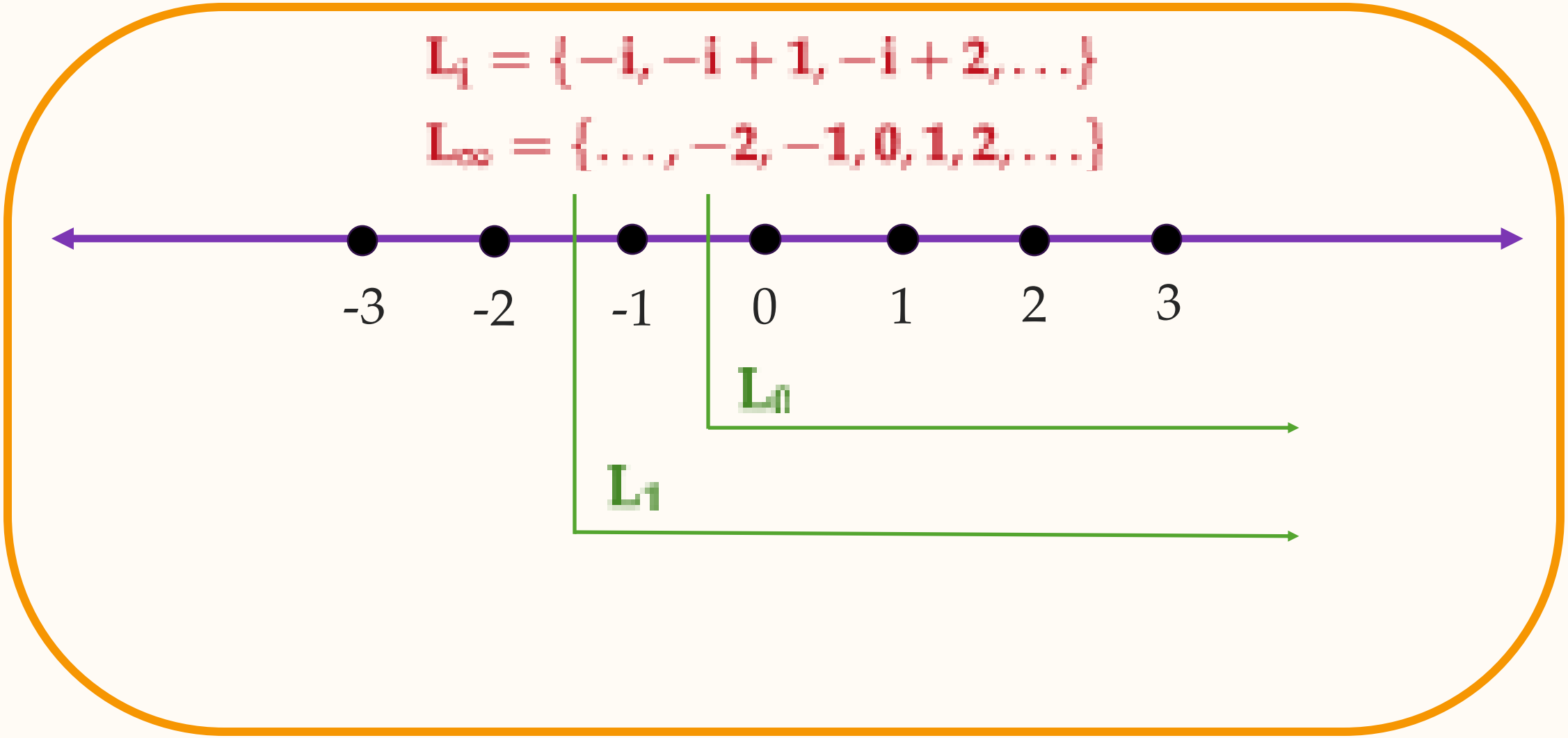

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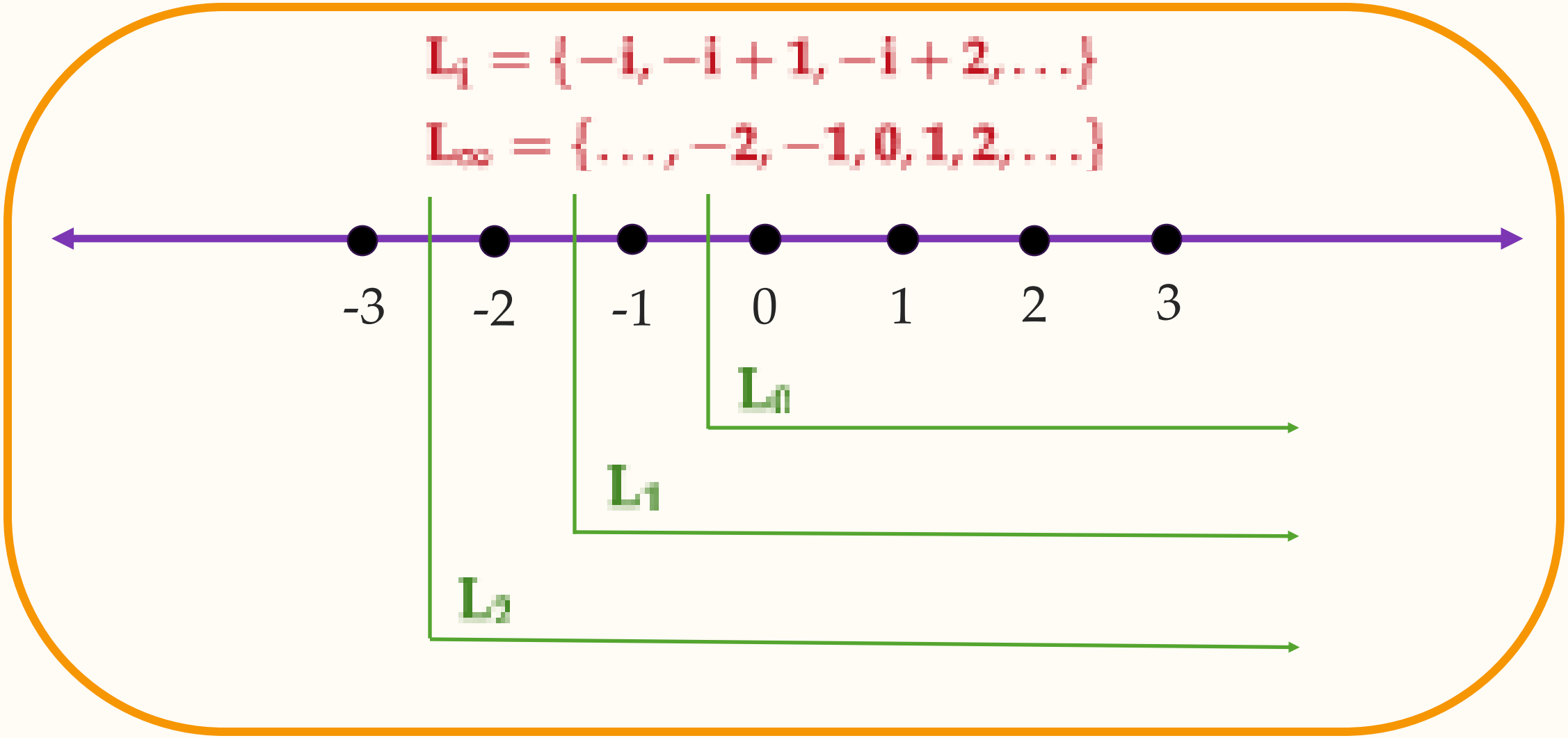

Closure operation

$\langle S \rangle$: intersection of all languages containing S

Output an element of $\langle S_t \rangle \setminus S_t$?

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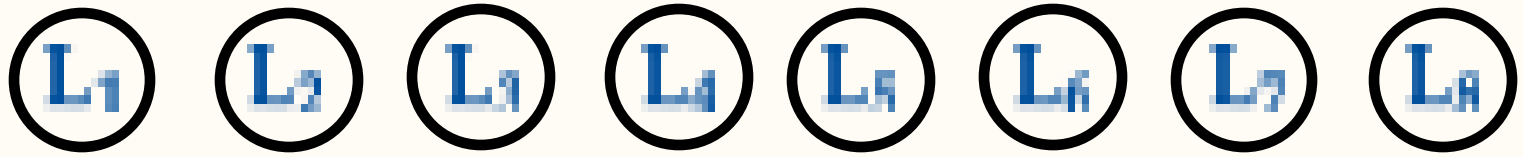
e.g. modify previous example

$$L_{t,V} = V \cup \{-t, -t+1, -t+2, \ldots\}$$

where V ranges over all finite sets

Natural language elimination strategy



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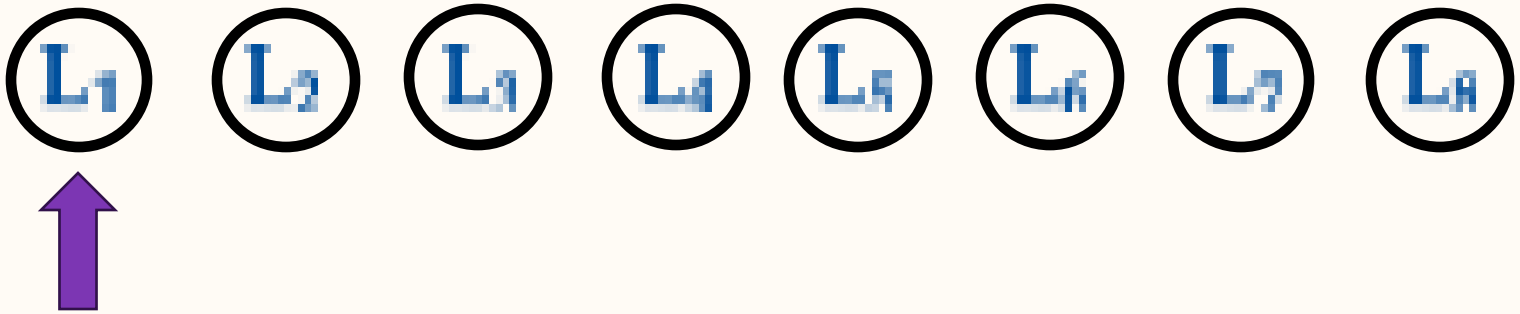

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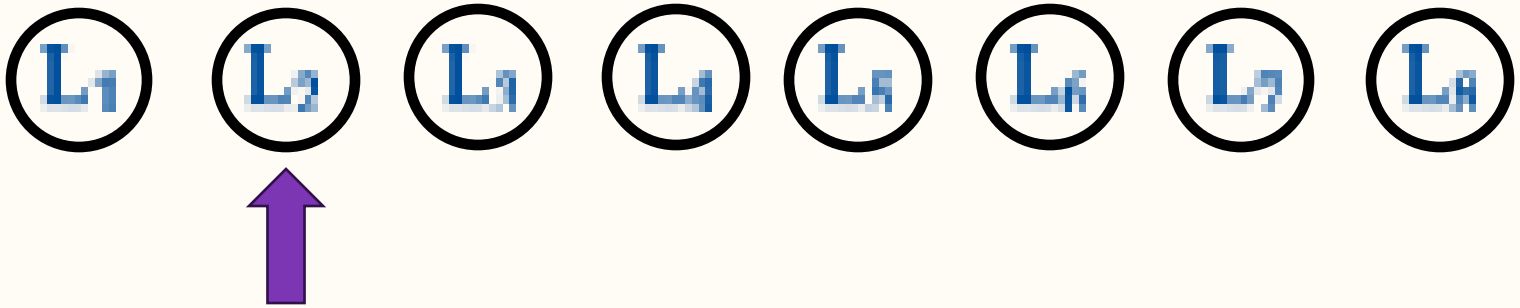

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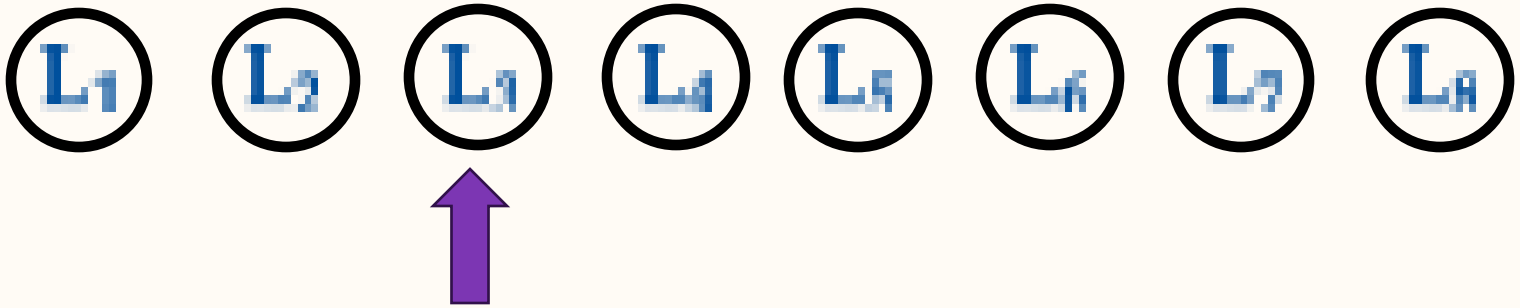

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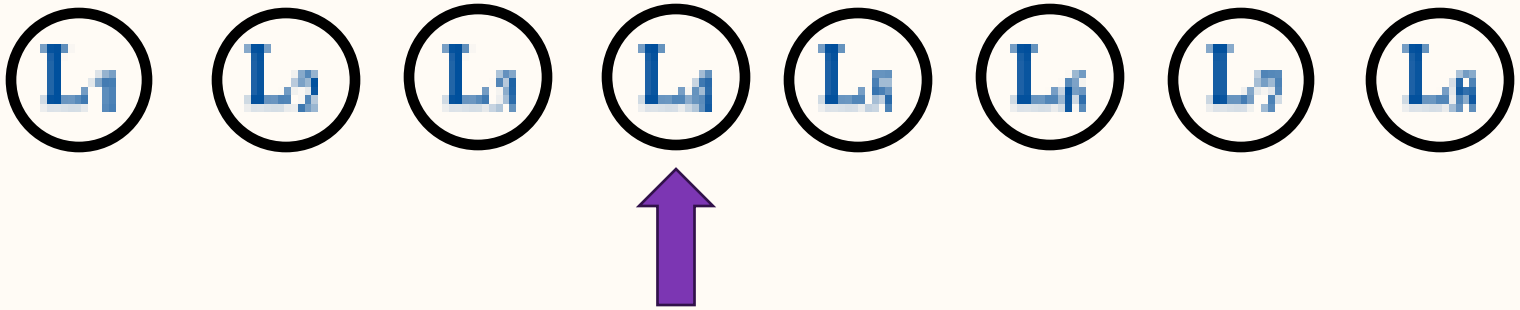

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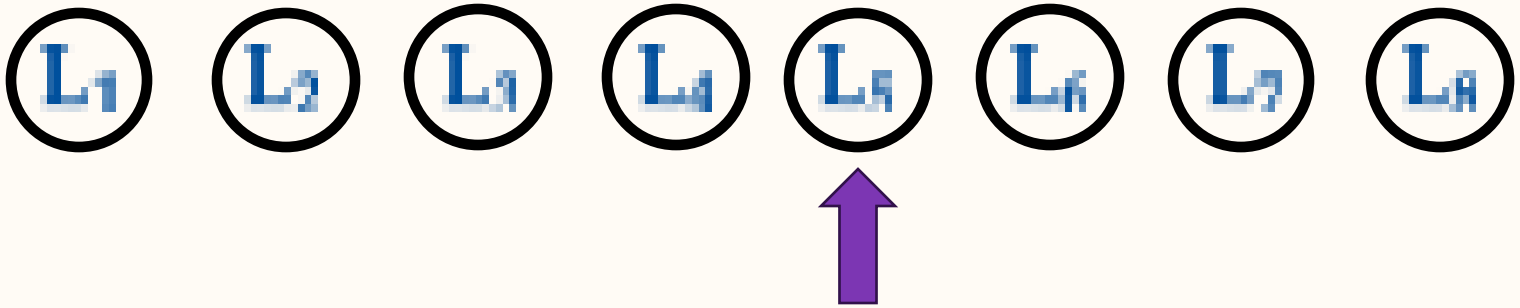

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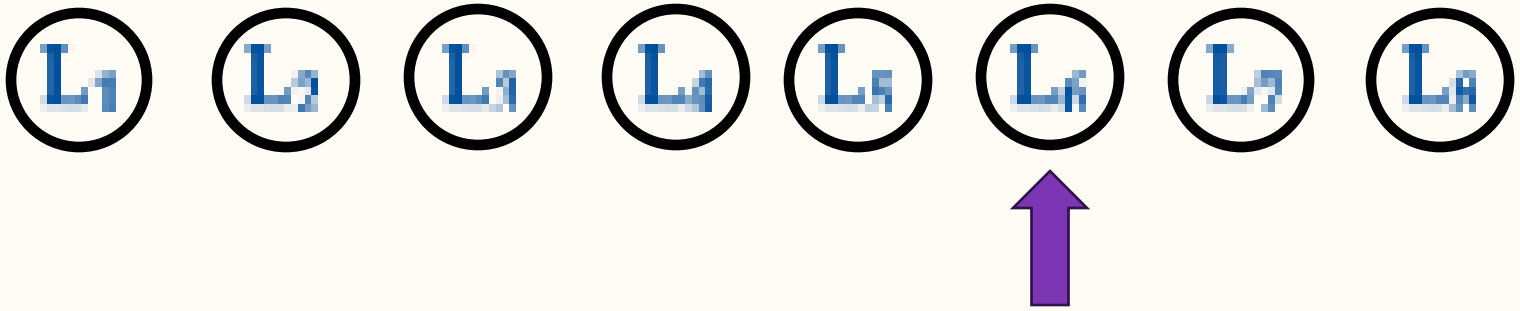

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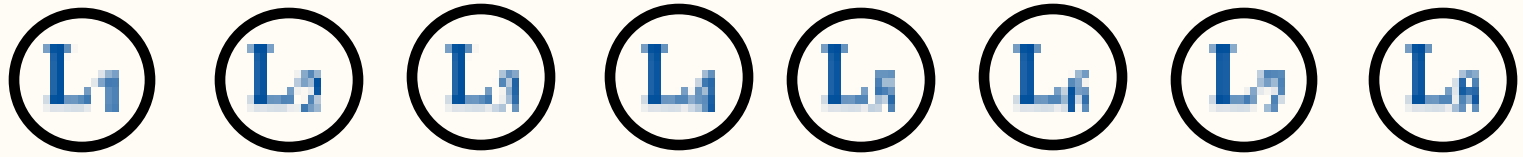

(wrong) Proof :

If $L_i \neq K$ enumeration of K will eventually reveal this

Since $K = L_z$ for some z, we will eventually get to $K = L_z$

We will never move beyond K

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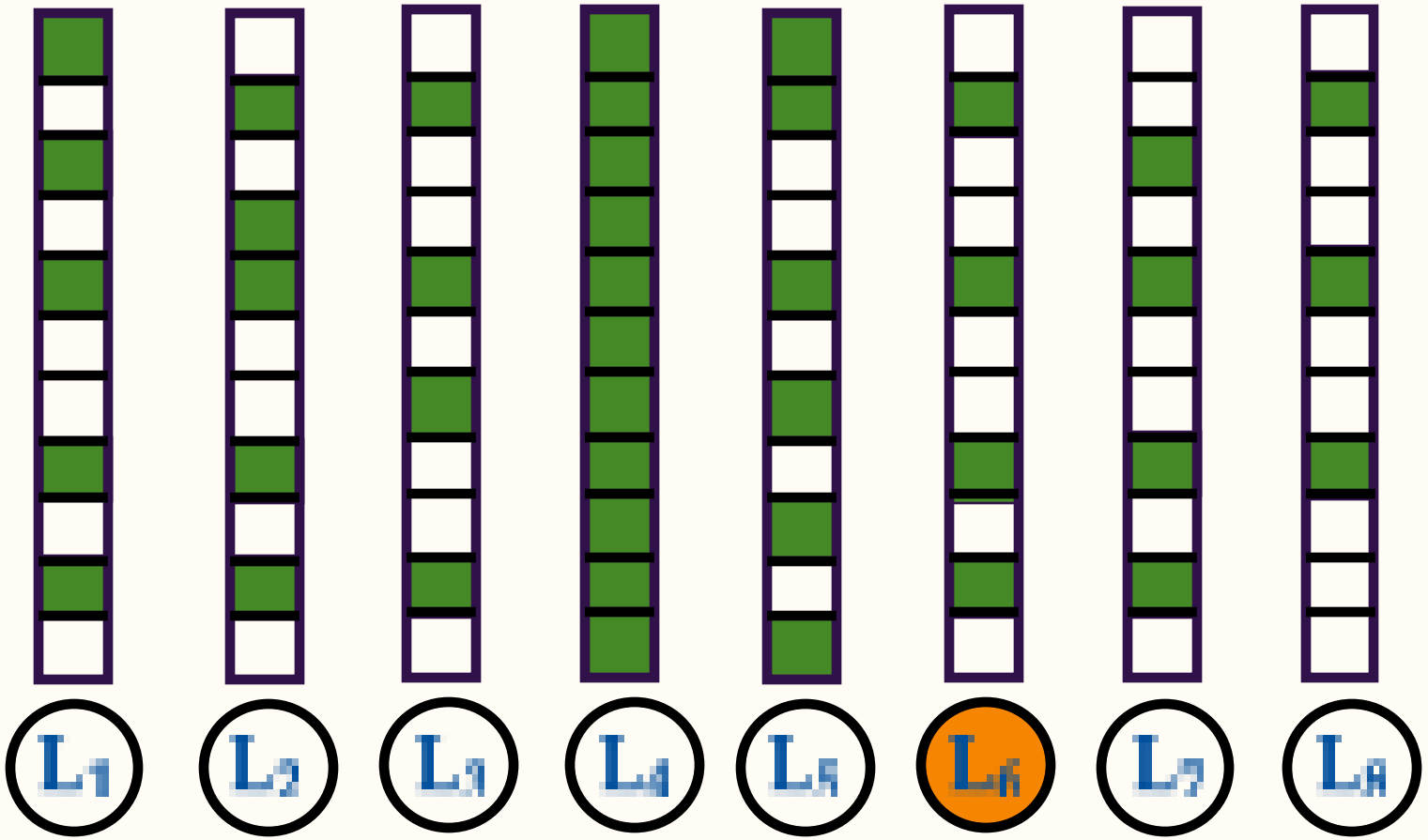

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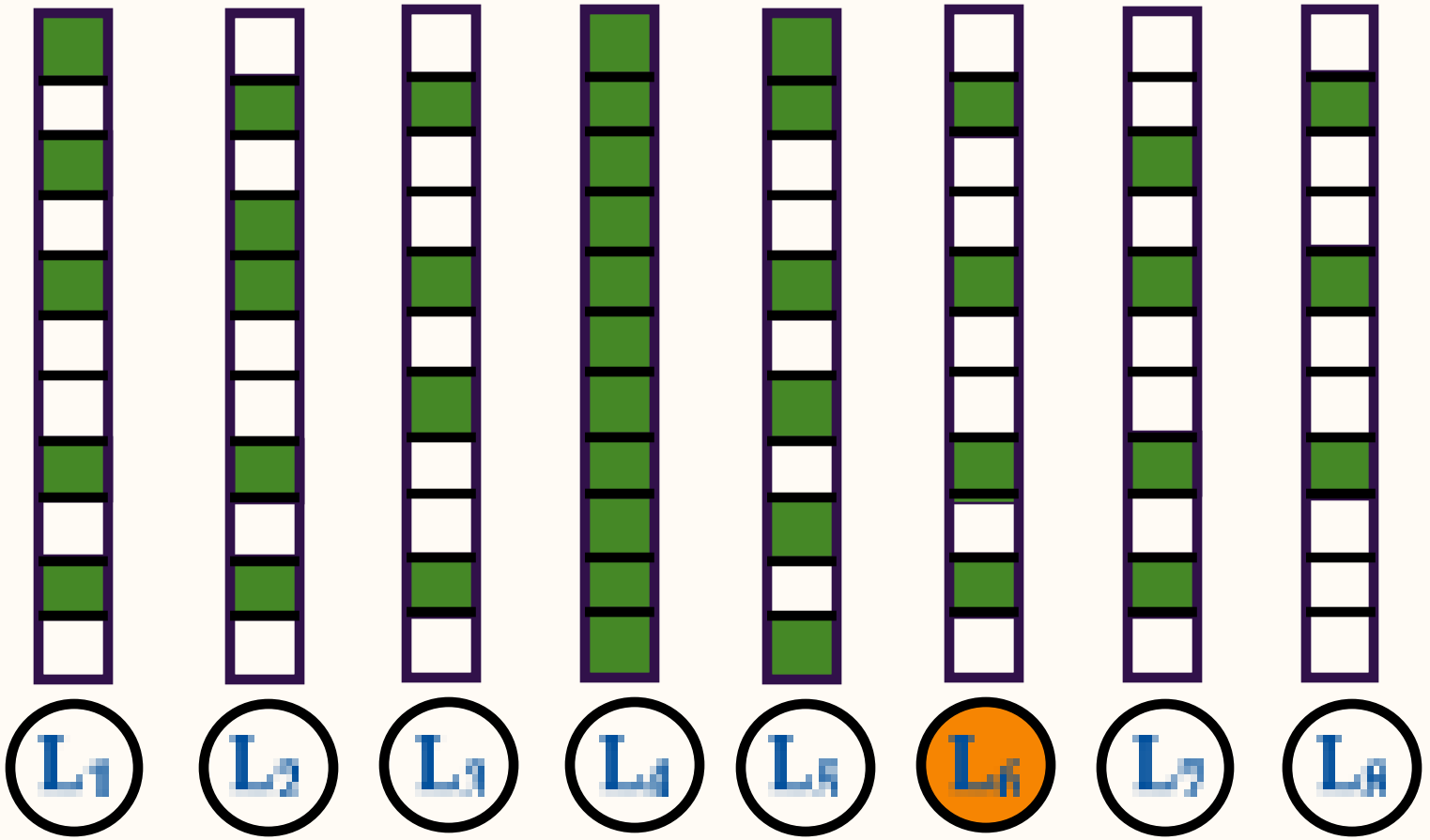
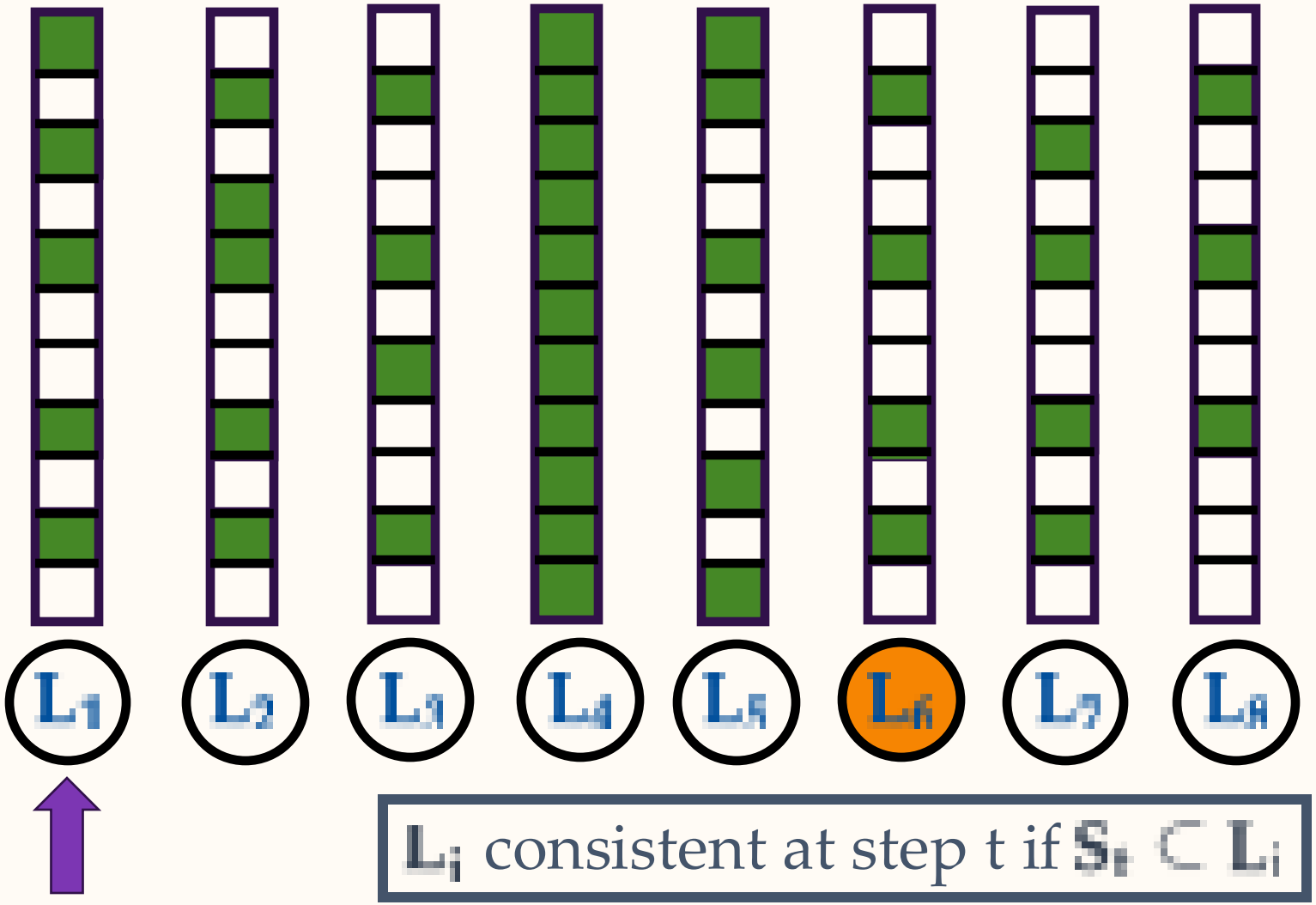


L_i consistent at step t if $S_t \subset L_i$

Natural language elimination strategy



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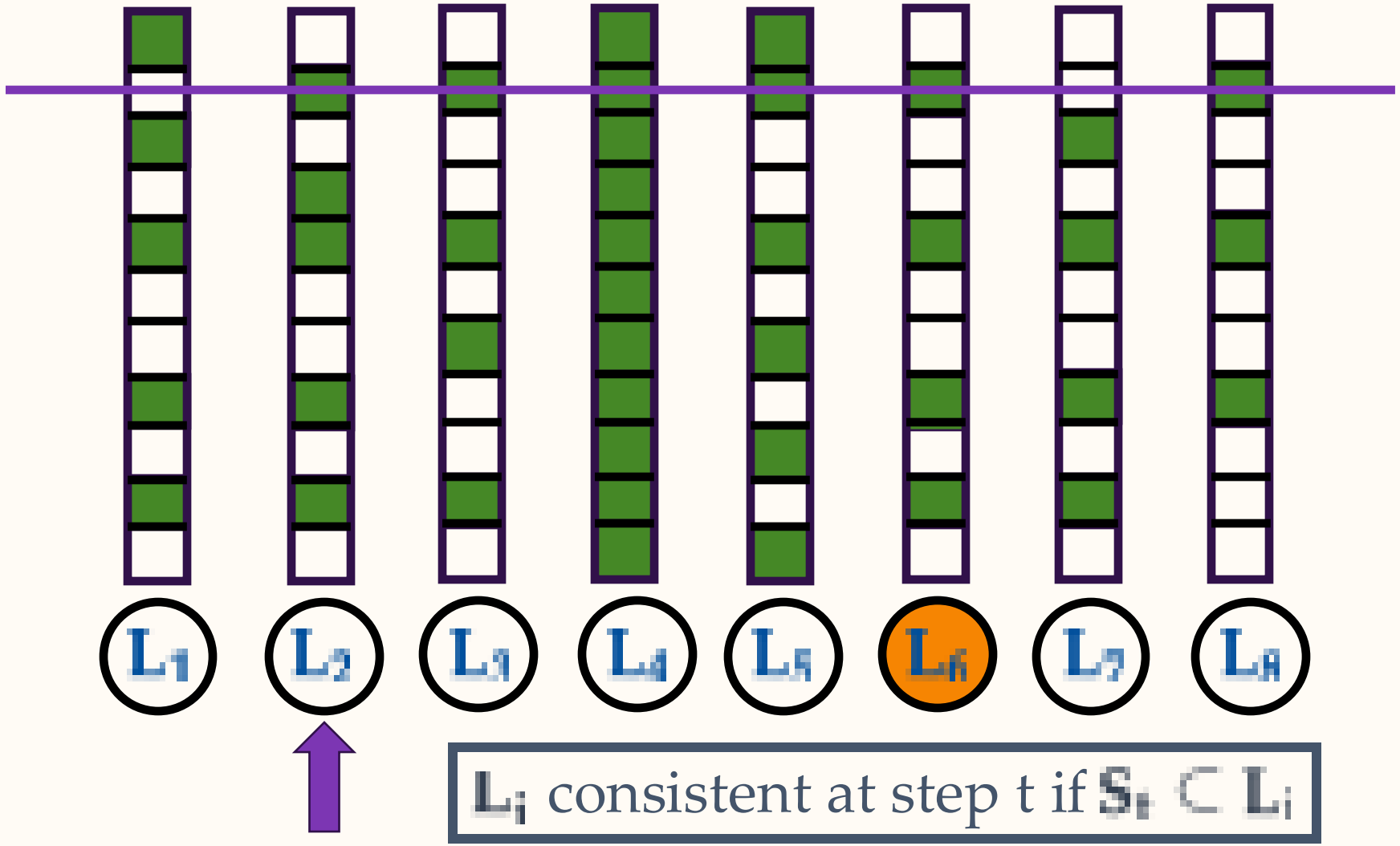

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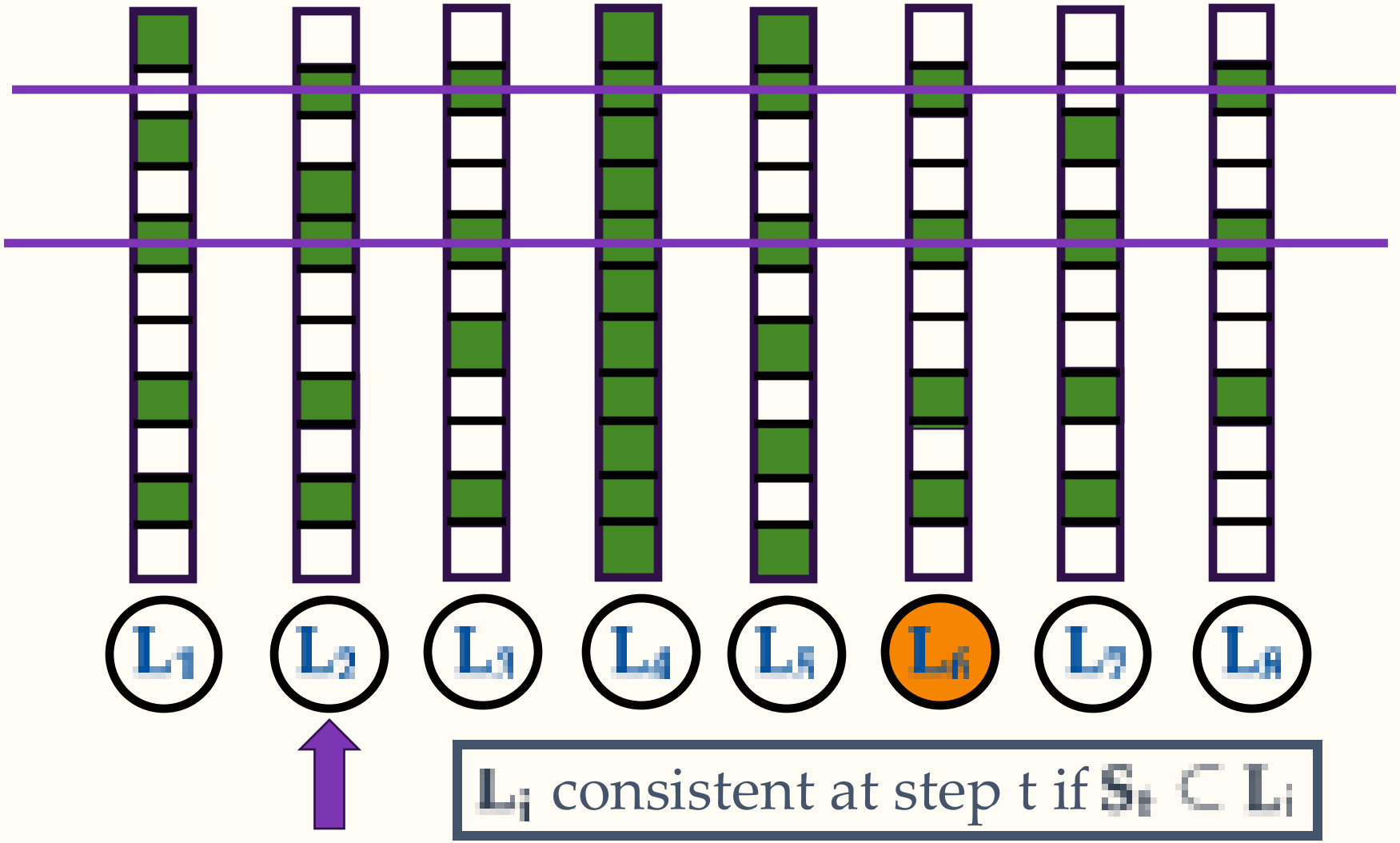

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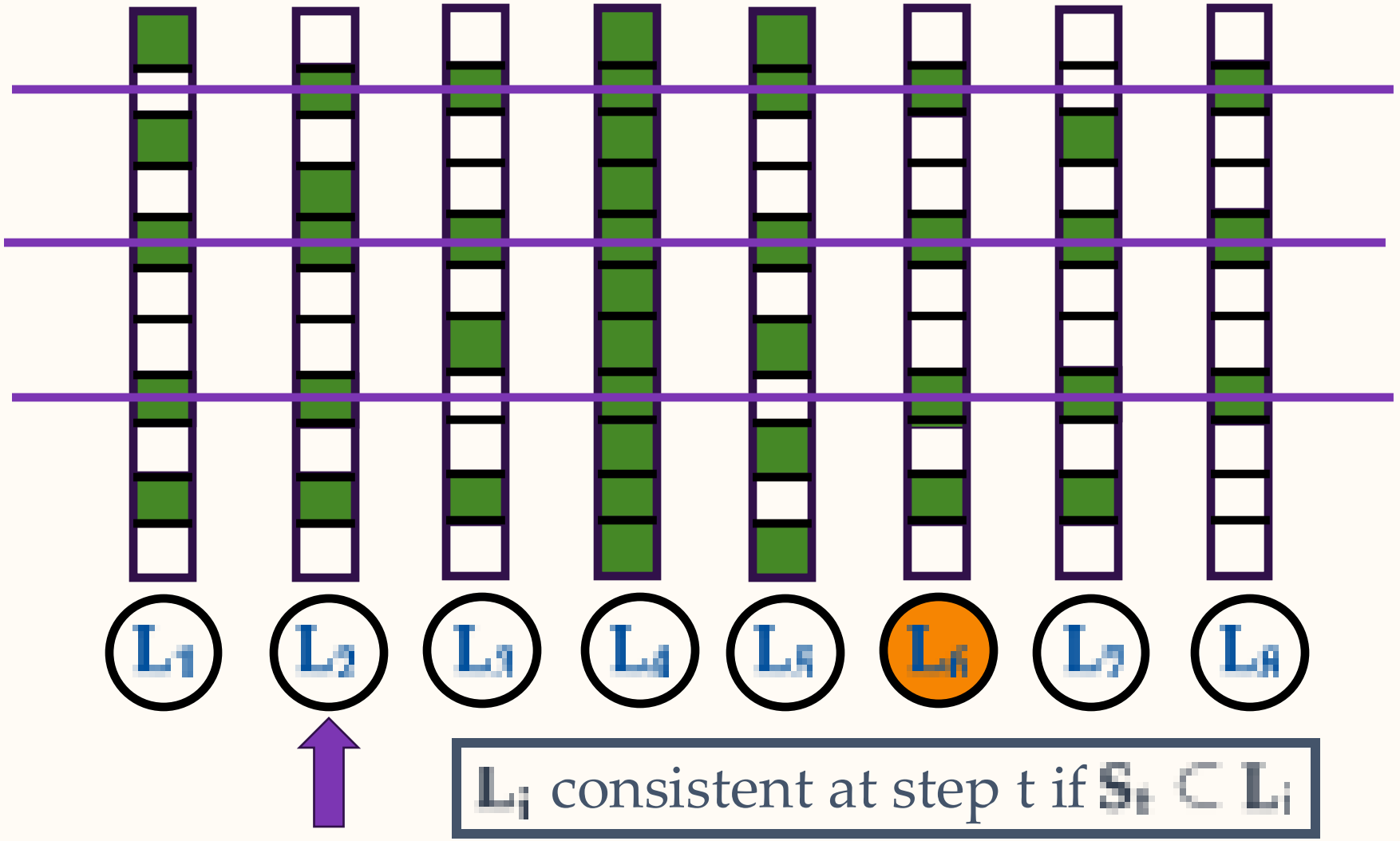

Minimal Consistent Languages?

Consistent language L_i such that there is no consistent L_j with $L_j \subsetneq L_i$

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Consistent language L_i such that there is no consistent L_j with $L_j \subsetneq L_i$

Problem: There may not be a minimal consistent language!

Critical Languages

Definition: L_n is critical at step t if
1. L_n is consistent at step t
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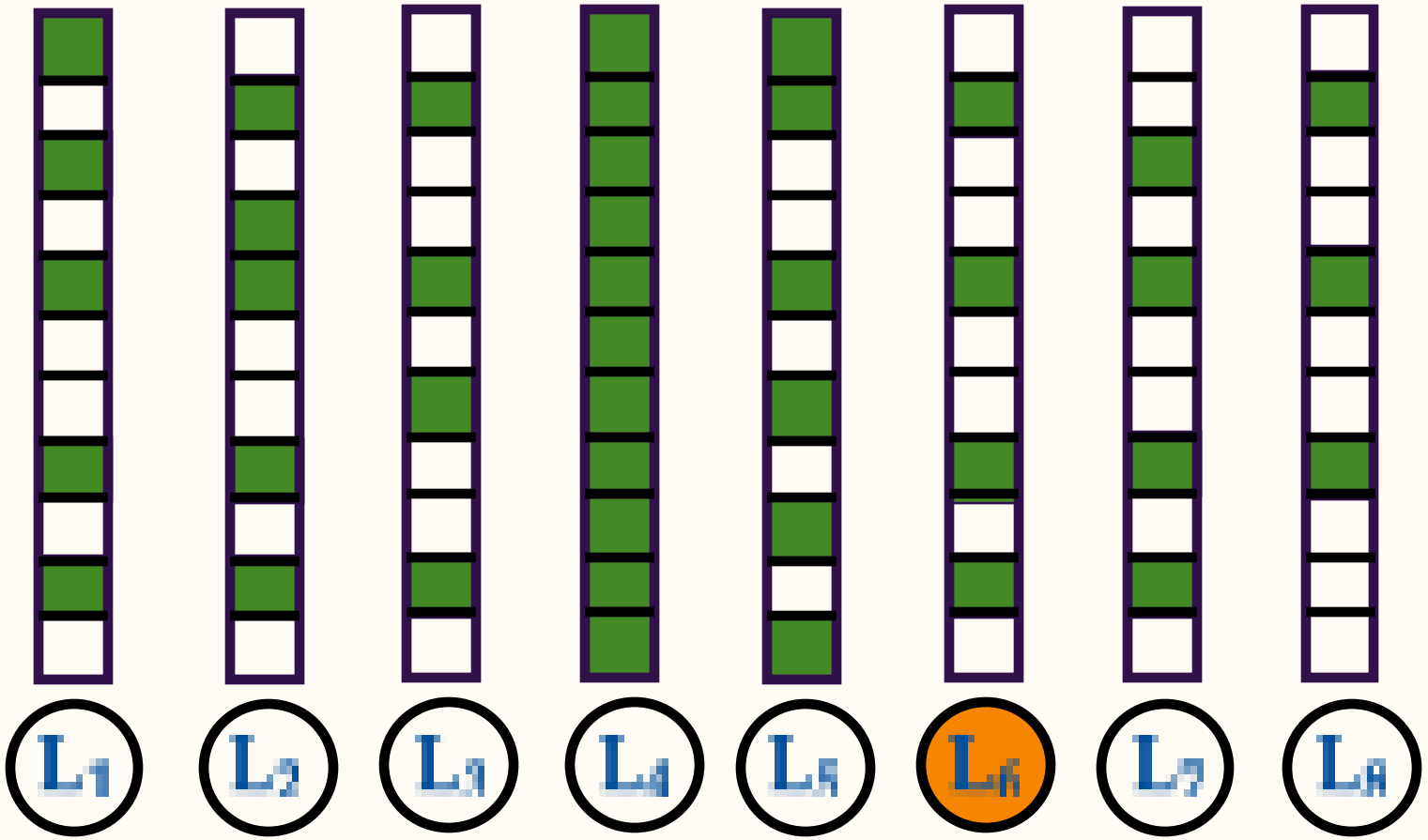
2. Target language will eventually become critical

Target language will become critical



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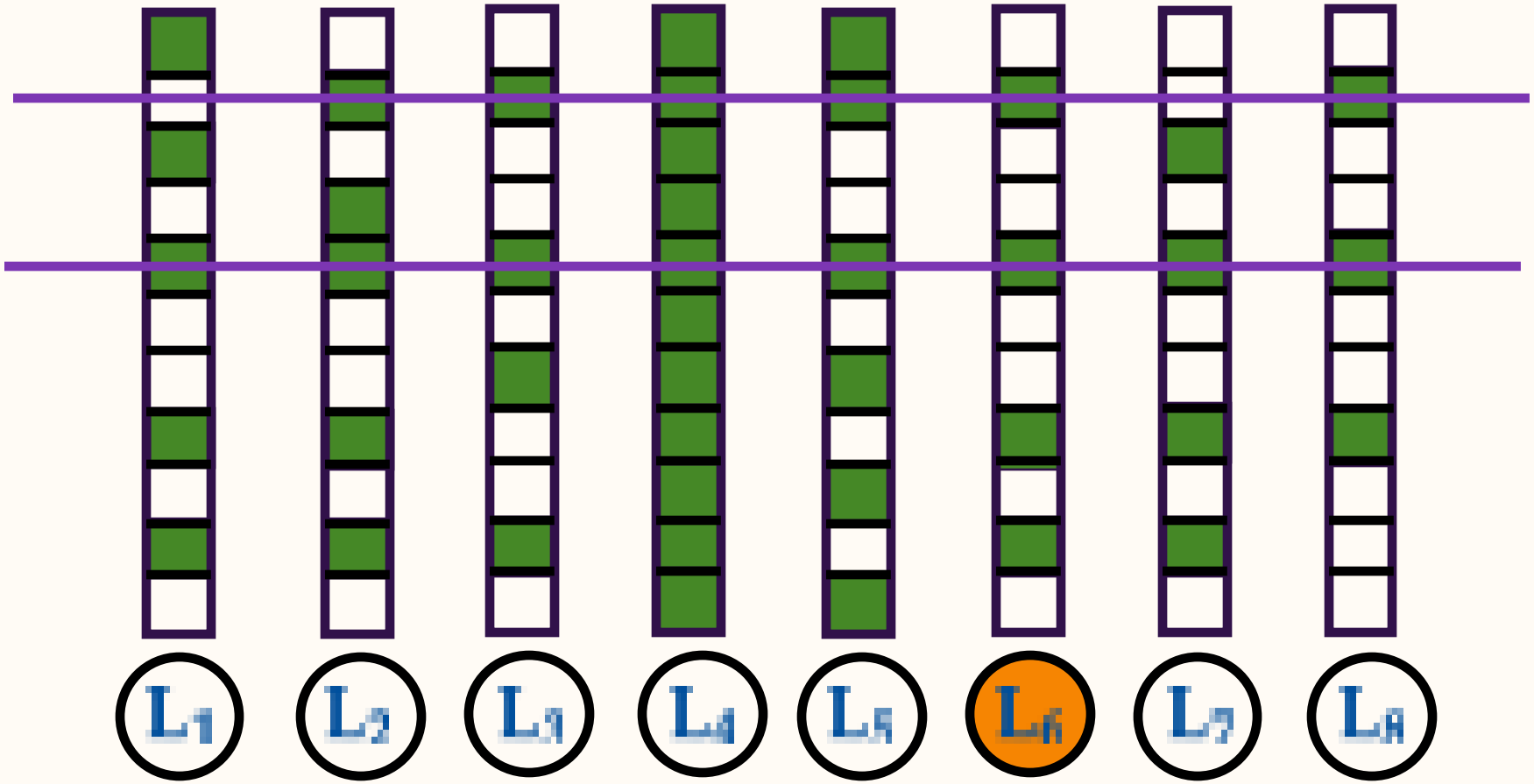

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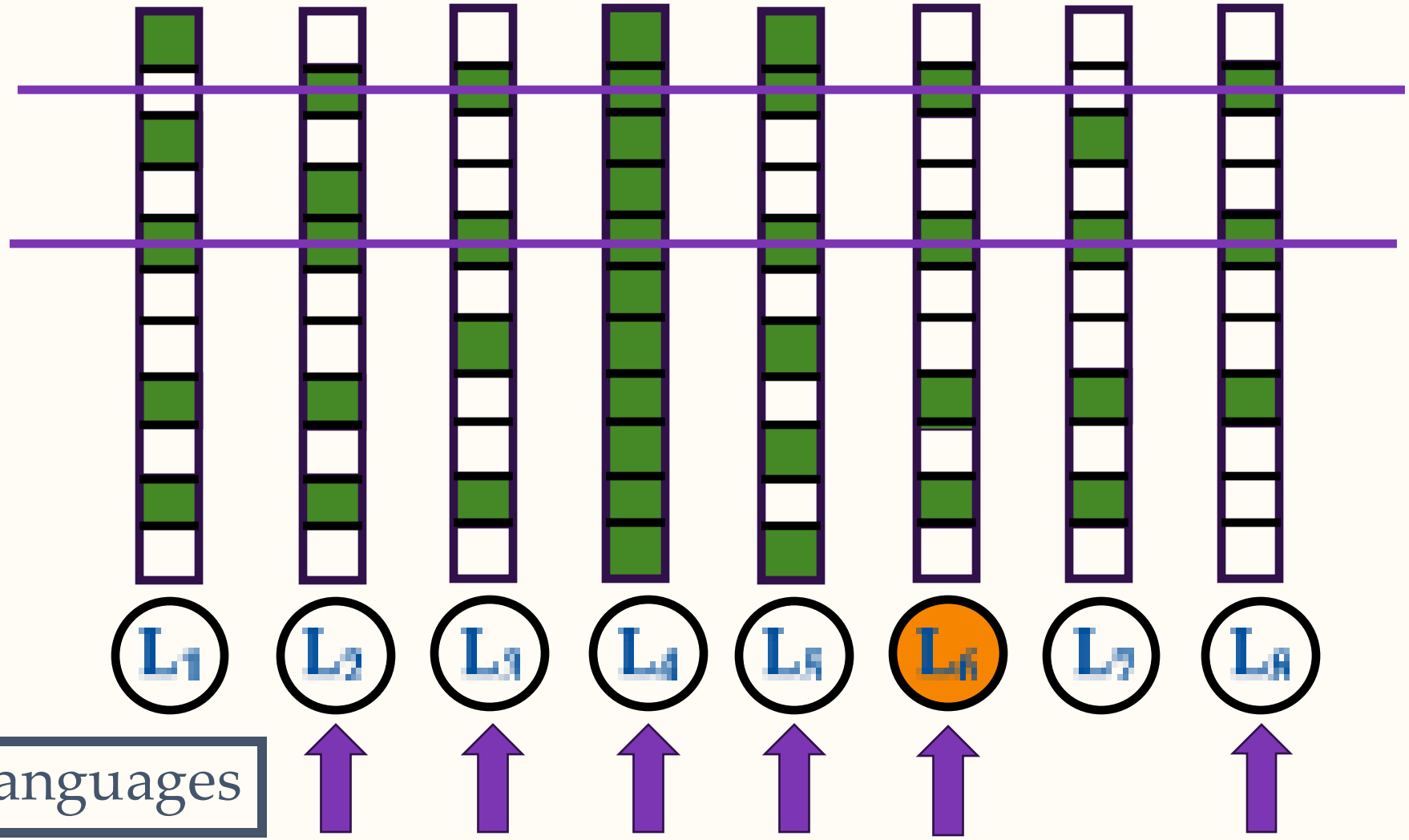

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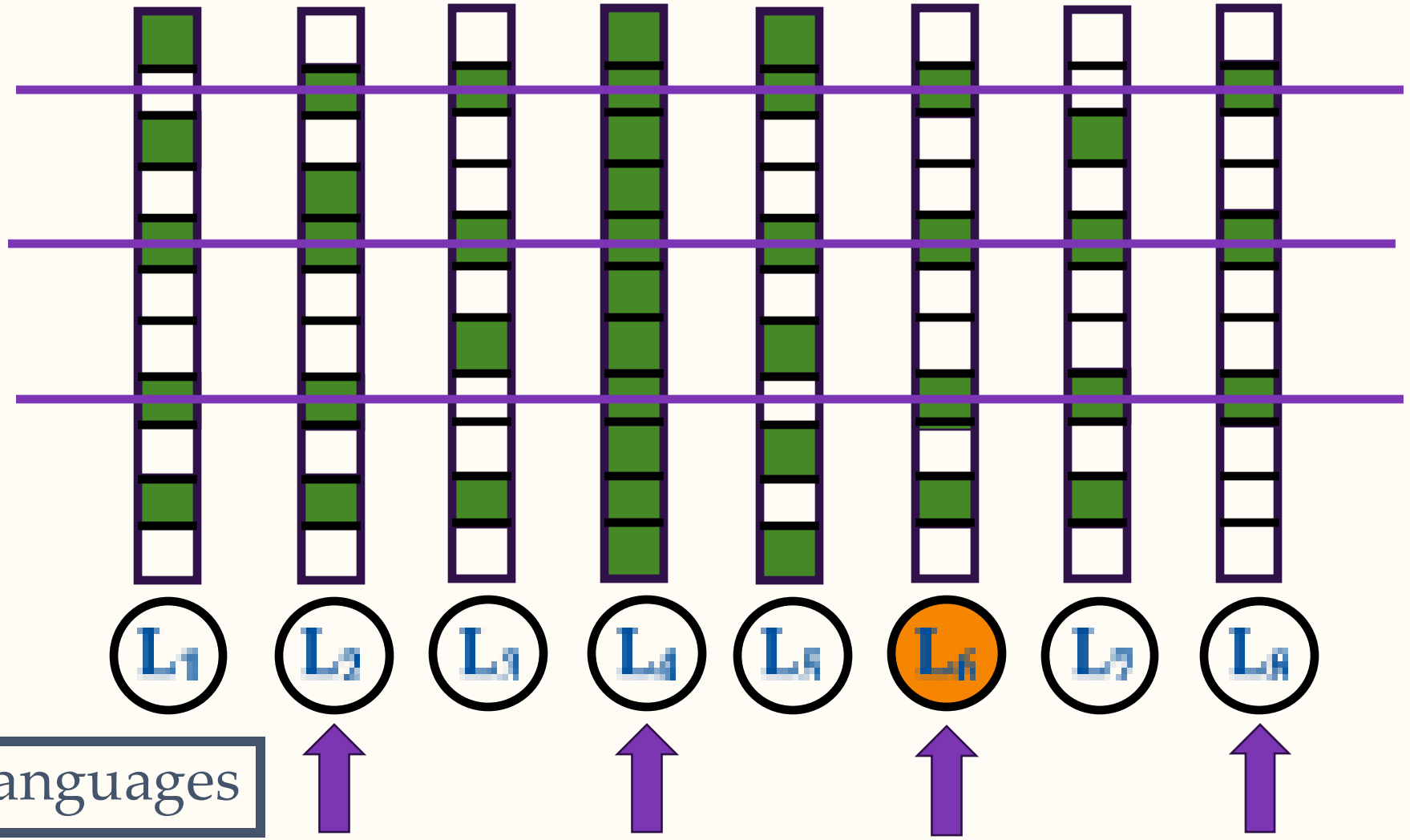

Algorithm for Generation in the Limit

At step t, only consider $L_1, \ldots, L_t$

L_{n_t} : critical language with highest index $n_t < t$

Generate a string from $L_{n_t} \setminus S_t$

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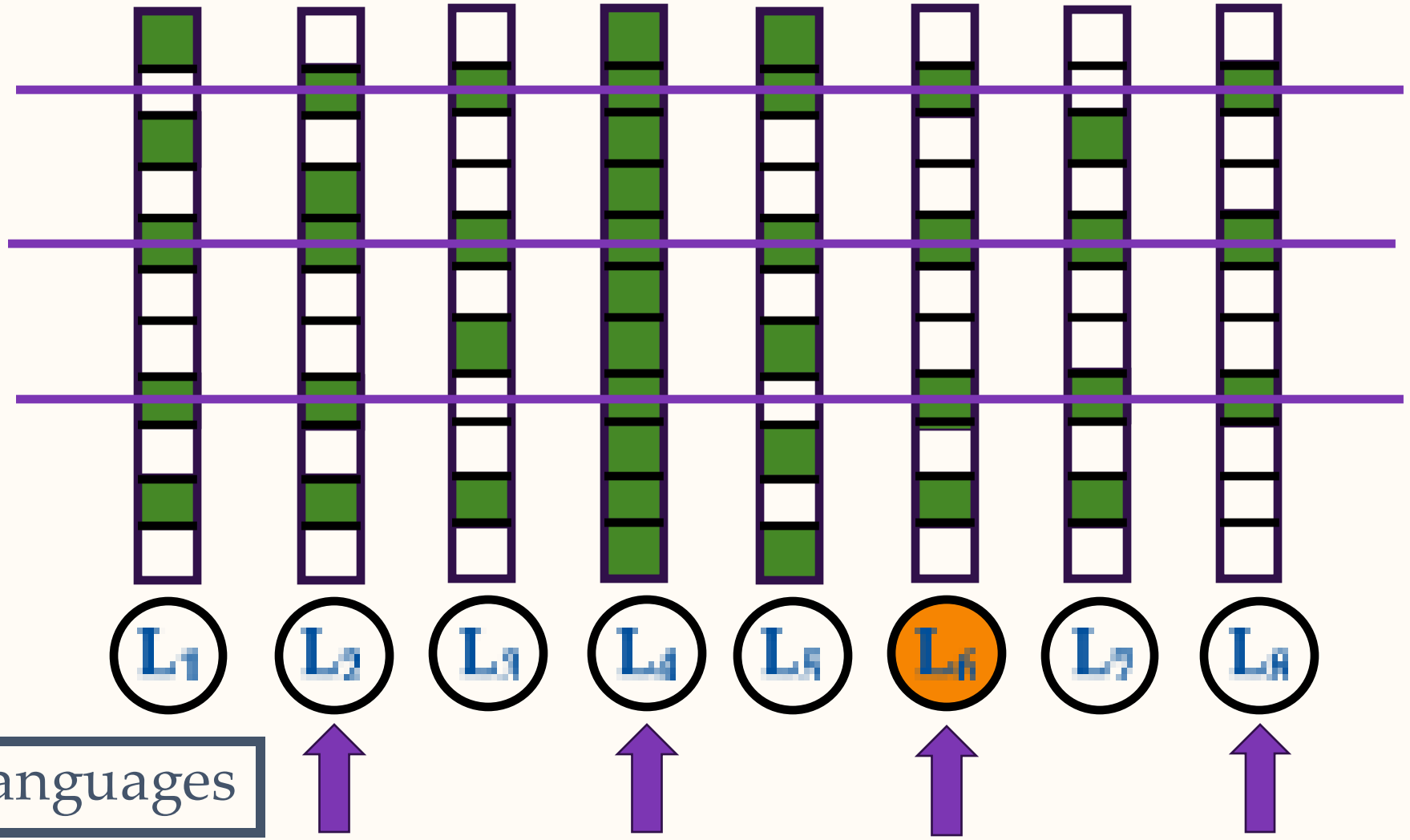
$L_{n_t} \subseteq L_z$, so any string from $L_{n_t} \setminus S_t$ also belongs to $L_z \setminus S_t$

Generation in the Limit correctness



Language Generation: Tradeoffs & Extensions

Mode-collapse in [Kleinberg, Mullainathan 2024] algorithm.
validity vs breadth tradeoff: Anay and Grigoris

Stronger requirements (uniform/non-uniform generation): Chirag

Diversity constraints and noisy data: Charlotte