Quality of Wine

#Panel Data For this assignment, we examined the relationship between the variables "quality" and "fixed acidity", "volatile acidity", "citric acid", "residual sugar", "chlorides", "free sulfur dioxide", "total sulfur dioxide", "density", "pH", "sulphates", and "alcohol". The variable we're trying to predict, is the quality of wine based on our 11 predictors.

```
Wine_Quality<- read.csv("~/Downloads/WineQT.csv")
View(Wine_Quality)
```

Question 1

Histogram

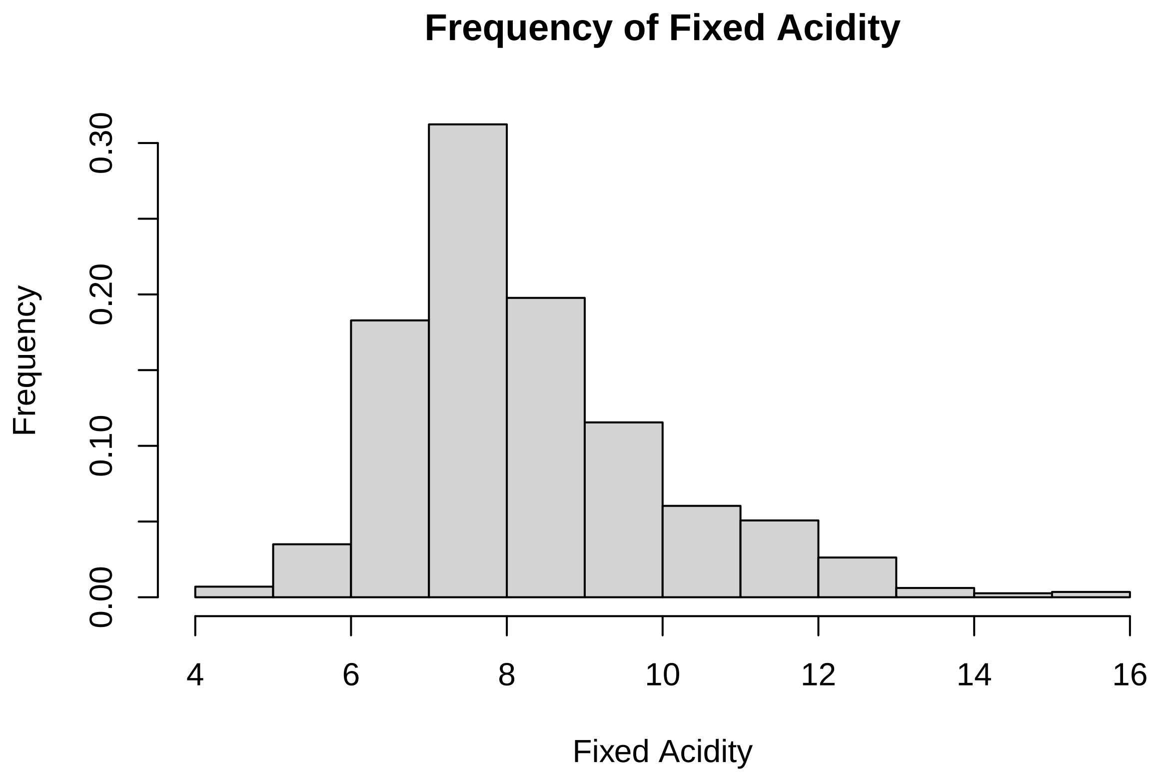
In histograms 1 (fixed acidity), 2 (volatile acidity), 4 (residual sugar), 5 (chlorides), 6 (free sulfur dioxide), 7 (total sulfur dioxide), 10 (sulphates), and 11 (alcohol) the models are right-skewed, meaning there is a positive distribution. Histograms 8 (pH) and 9 (density) have a normal distribution. By fitting a distribution to the histogram, we can predict as well as understand the observations in a data set better. The red line, inputted as curve(), shows the distribution that best fits the data.

```
library(fitdistrplus)
```

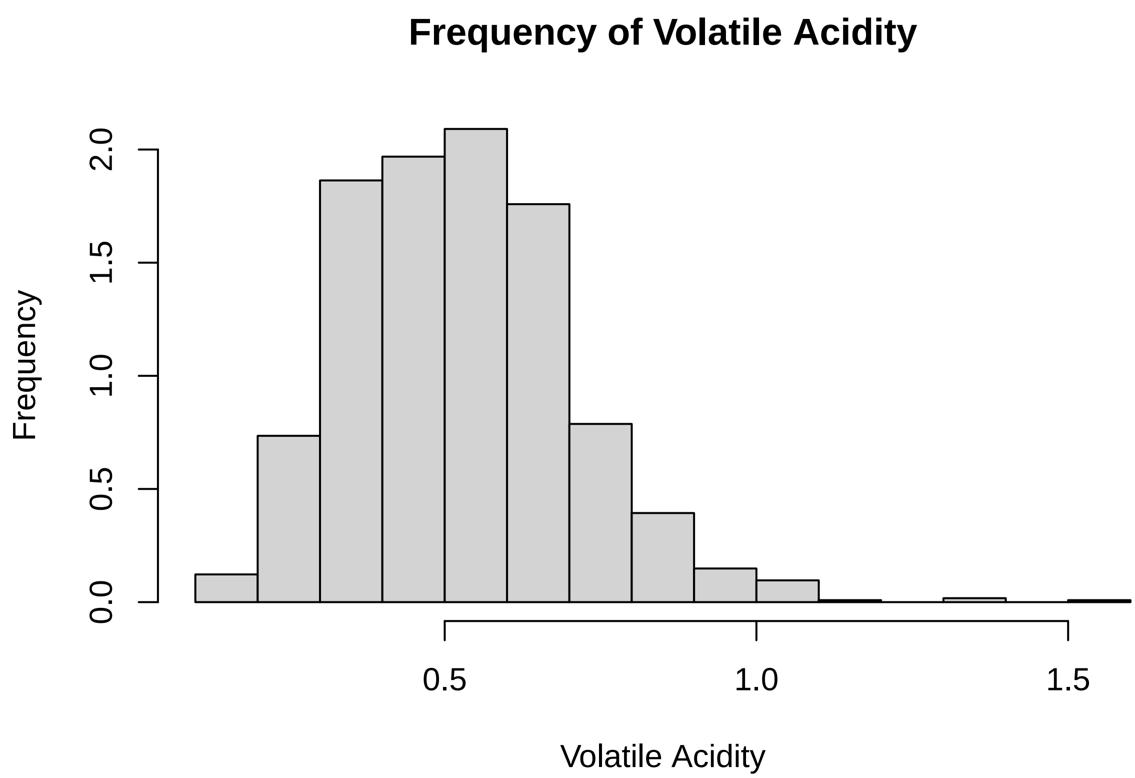
```
## Loading required package: MASS

## Loading required package: survival
```

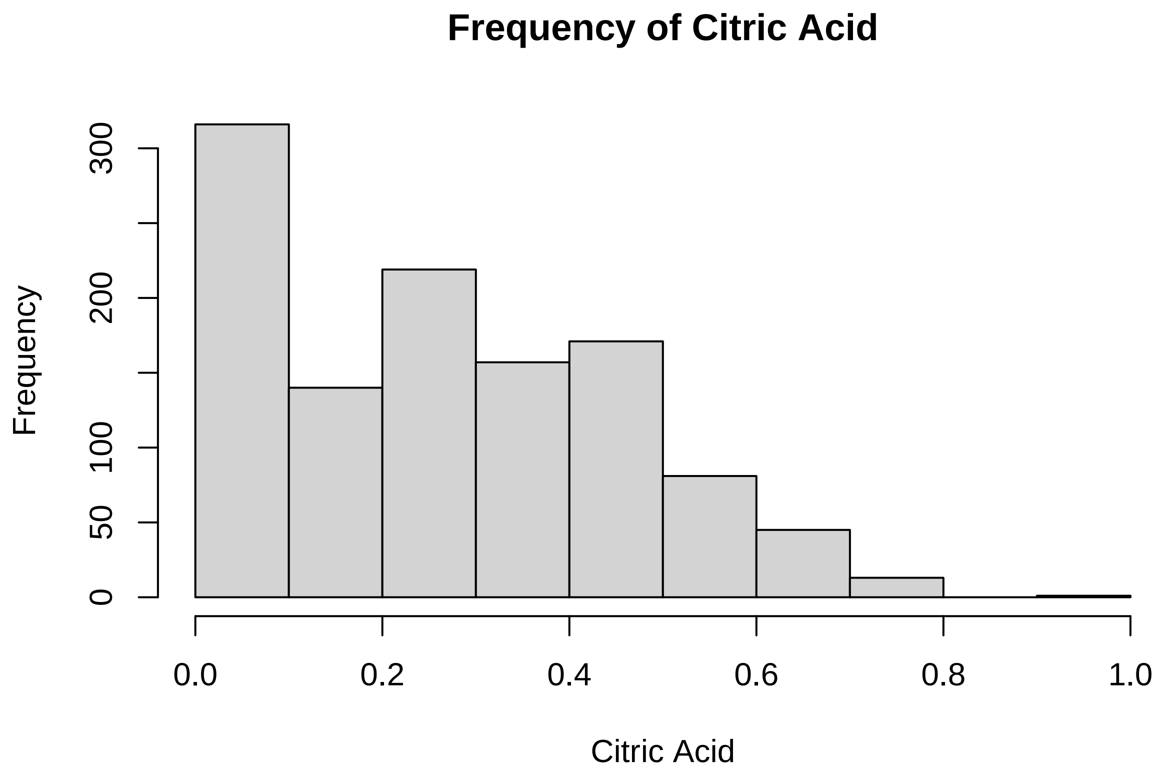
```
hist(Wine_Quality$fixed.acidity, prob = TRUE, xlab = "Fixed Acidity", ylab = "Frequency", main = "Frequ
```



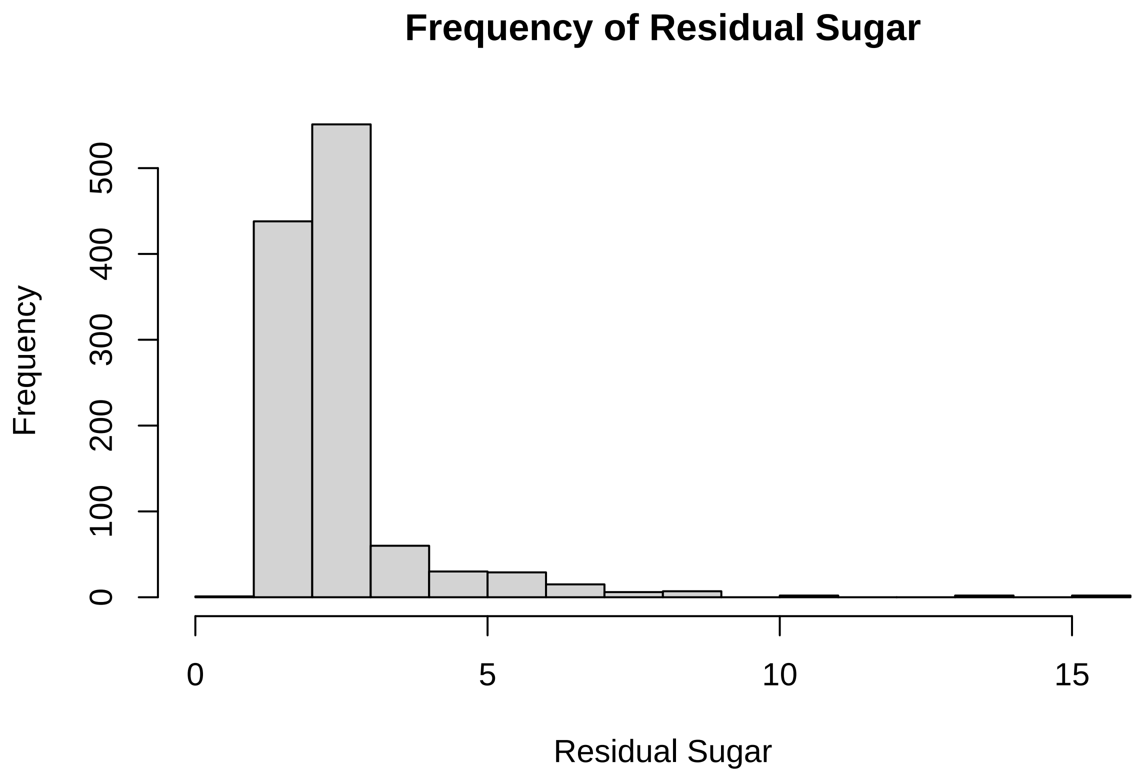

```
hist(Wine_Quality$volatile.acidity, prob = TRUE, xlab = "Volatile Acidity", ylab = "Frequency", main =
```



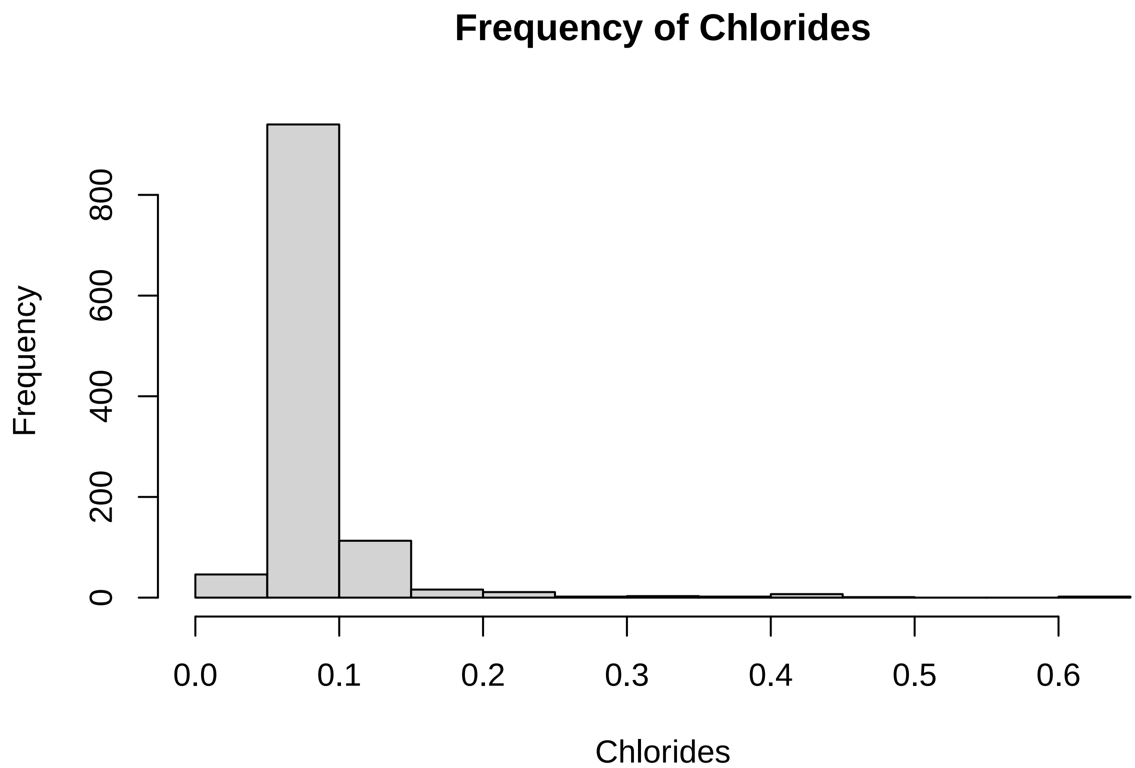

```
hist(Wine_Quality$citric.acid, xlab = "Citric Acid", ylab = "Frequency", main = "Frequency of Citric Aci
```



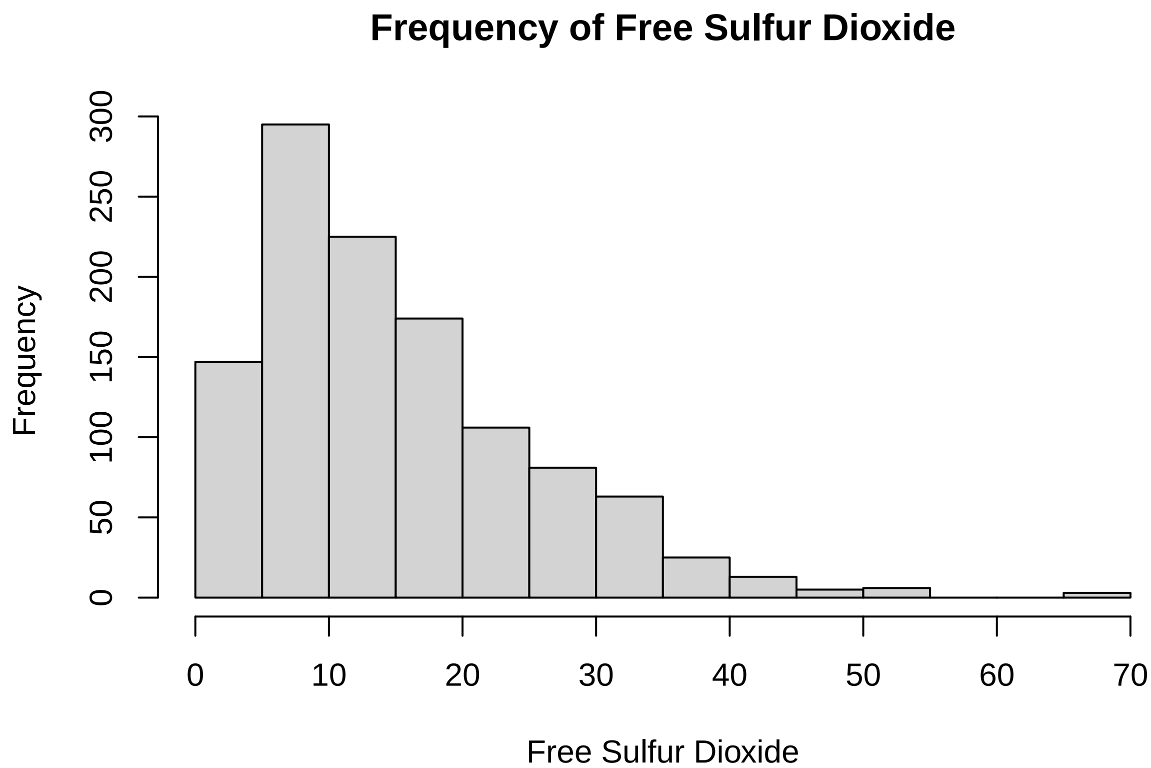

```
hist(Wine_Quality$residual.sugar, xlab = "Residual Sugar", ylab = "Frequency", main = "Frequency of Resi
```



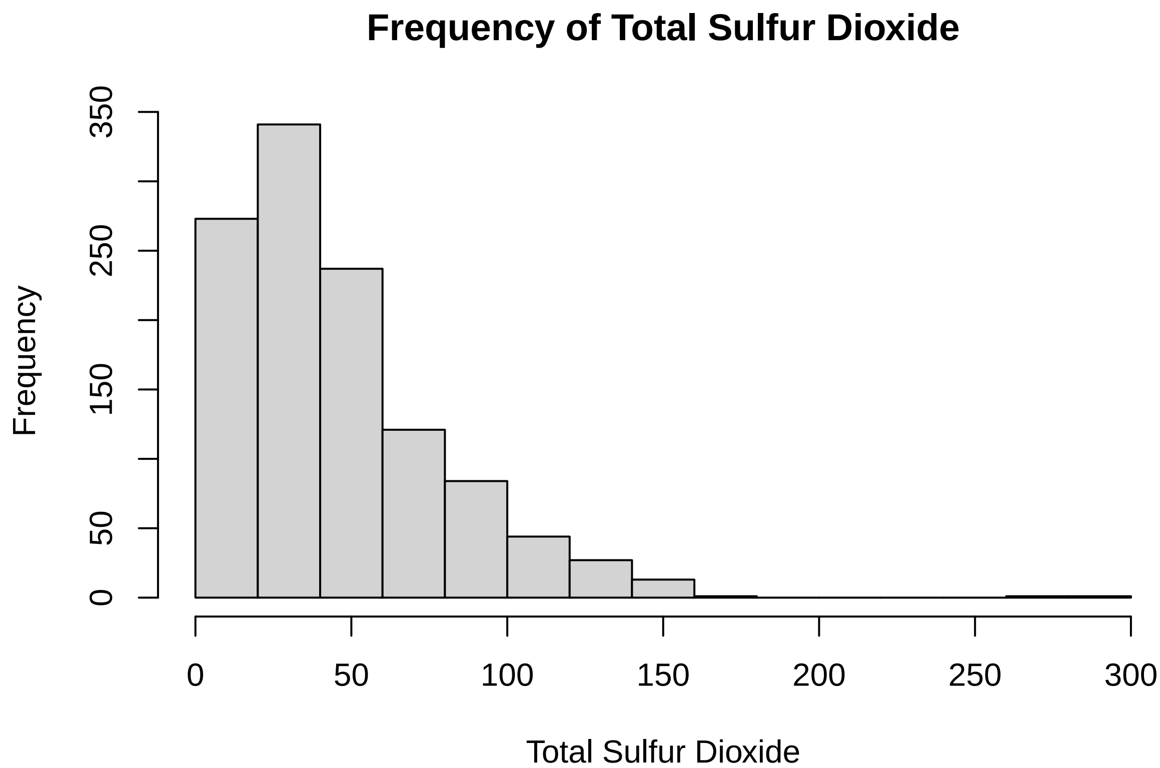

```
hist(Wine_Quality$chlorides, xlab = "Chlorides", ylab = "Frequency", main = "Frequency of Chlorides")
```



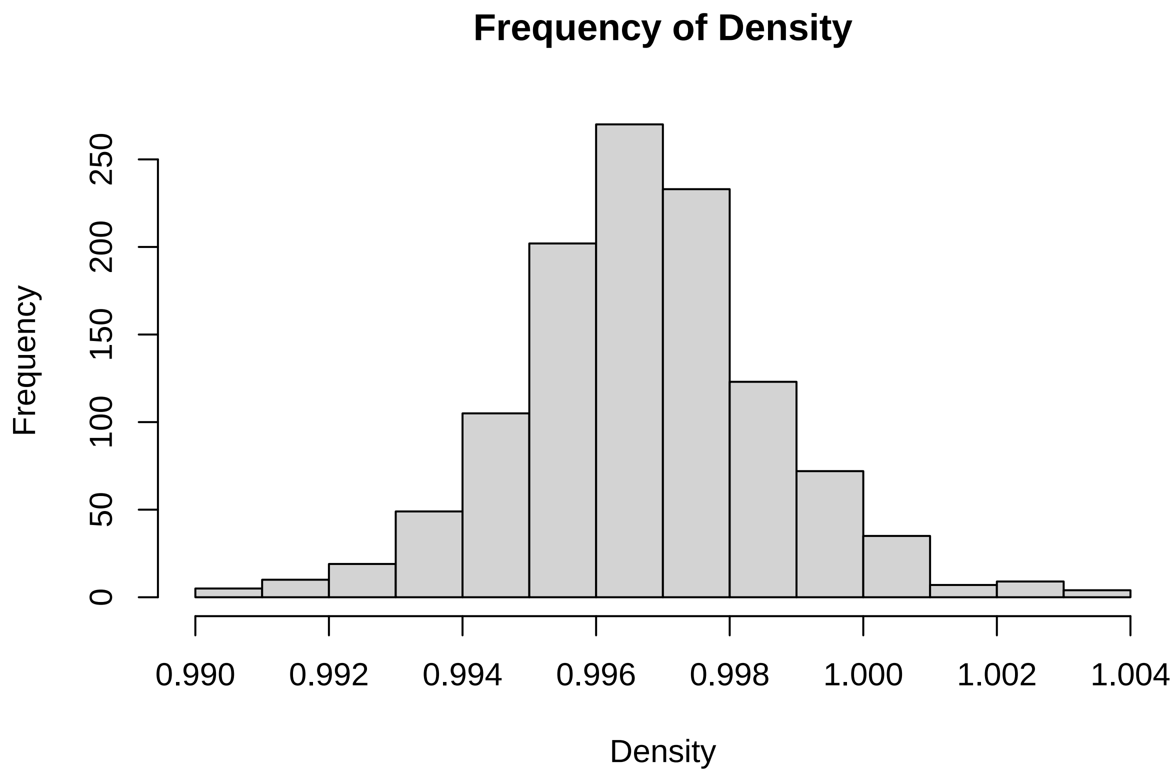

```
hist(Wine_Quality$free.sulfur.dioxide, xlab = "Free Sulfur Dioxide", ylab = "Frequency", main = "Frequen
```



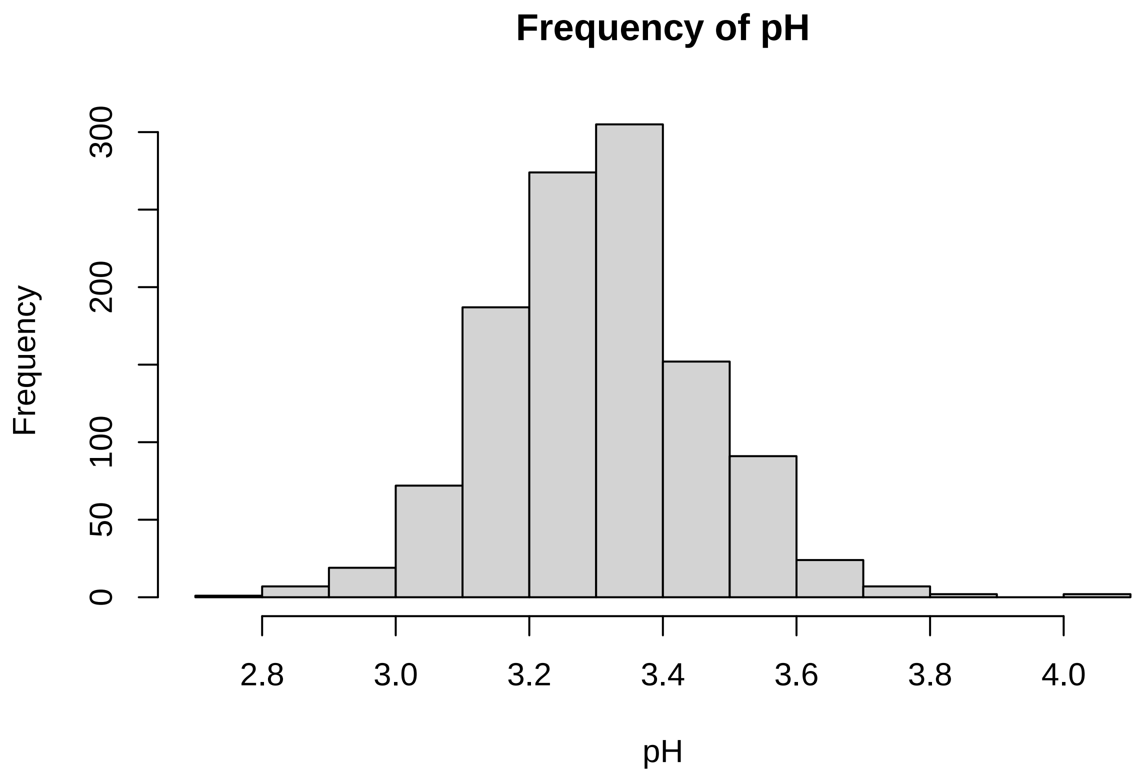

```
hist(Wine_Quality$total.sulfur.dioxide, xlab = "Total Sulfur Dioxide", ylab = "Frequency", main = "Frequ
```



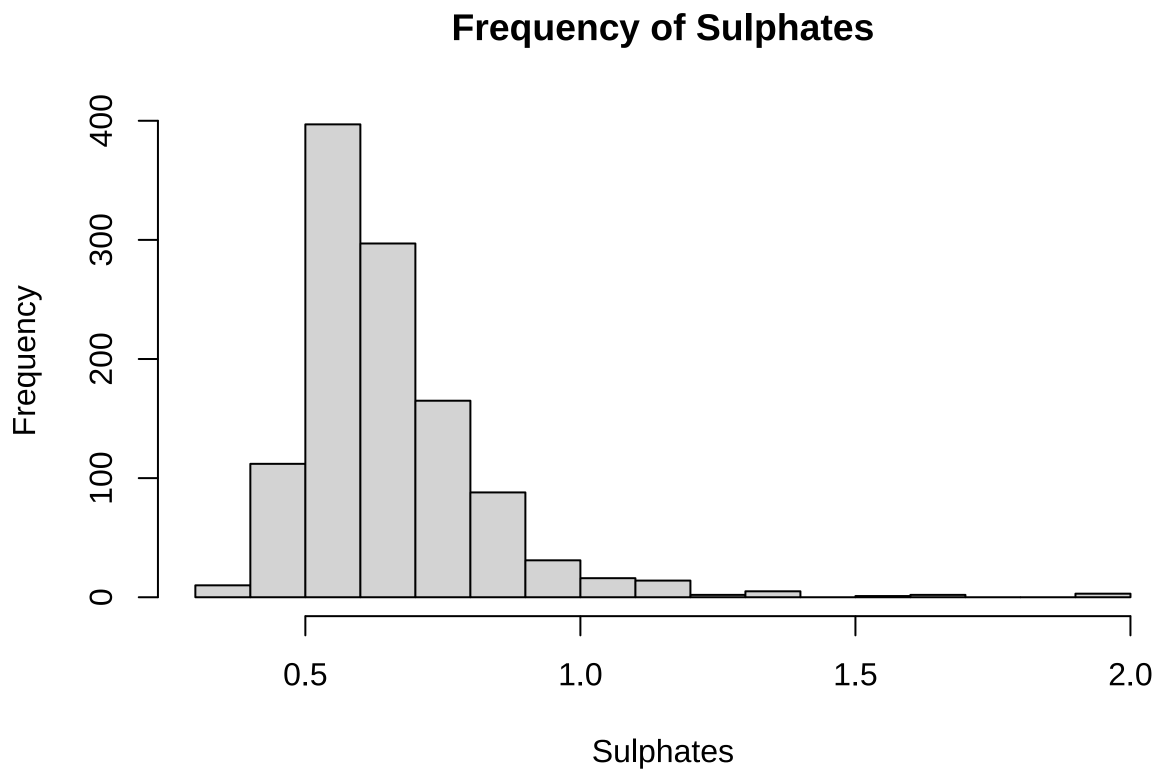

```
hist(Wine_Quality$density, xlab = "Density", ylab = "Frequency", main = "Frequency of Density")
```



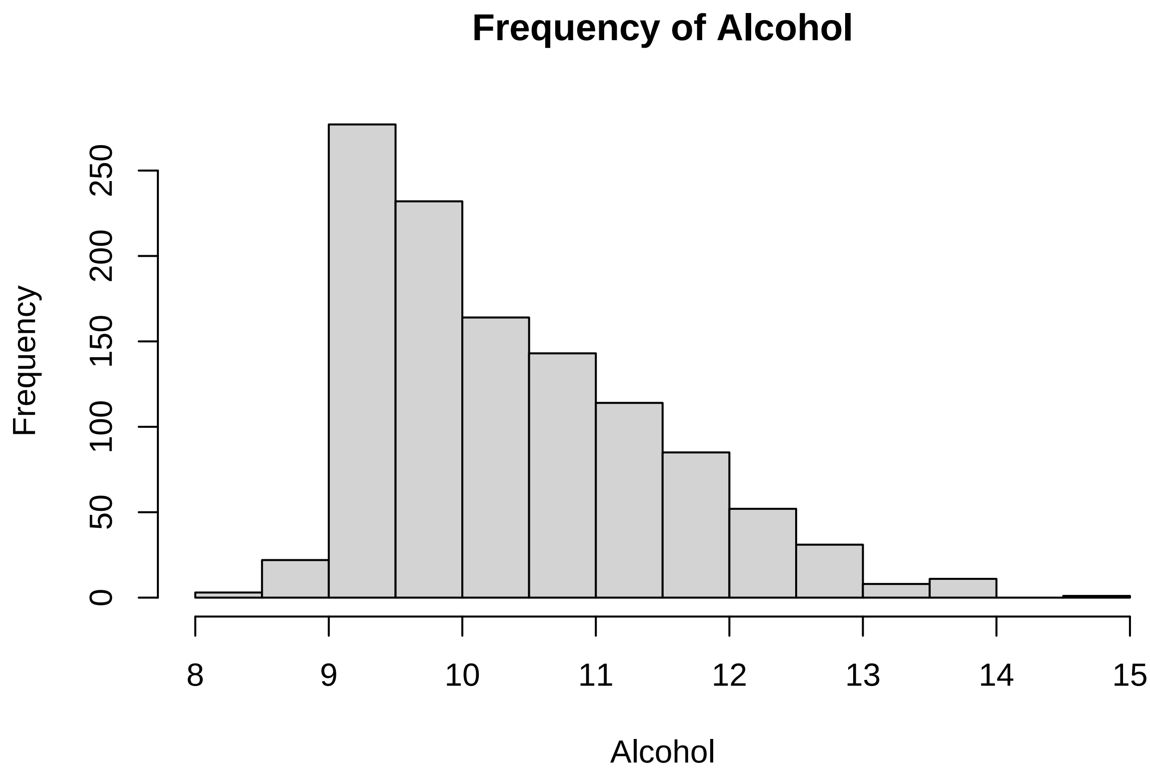

```
hist(Wine_Quality$pH, xlab = "pH", ylab = "Frequency", main = "Frequency of pH")
```



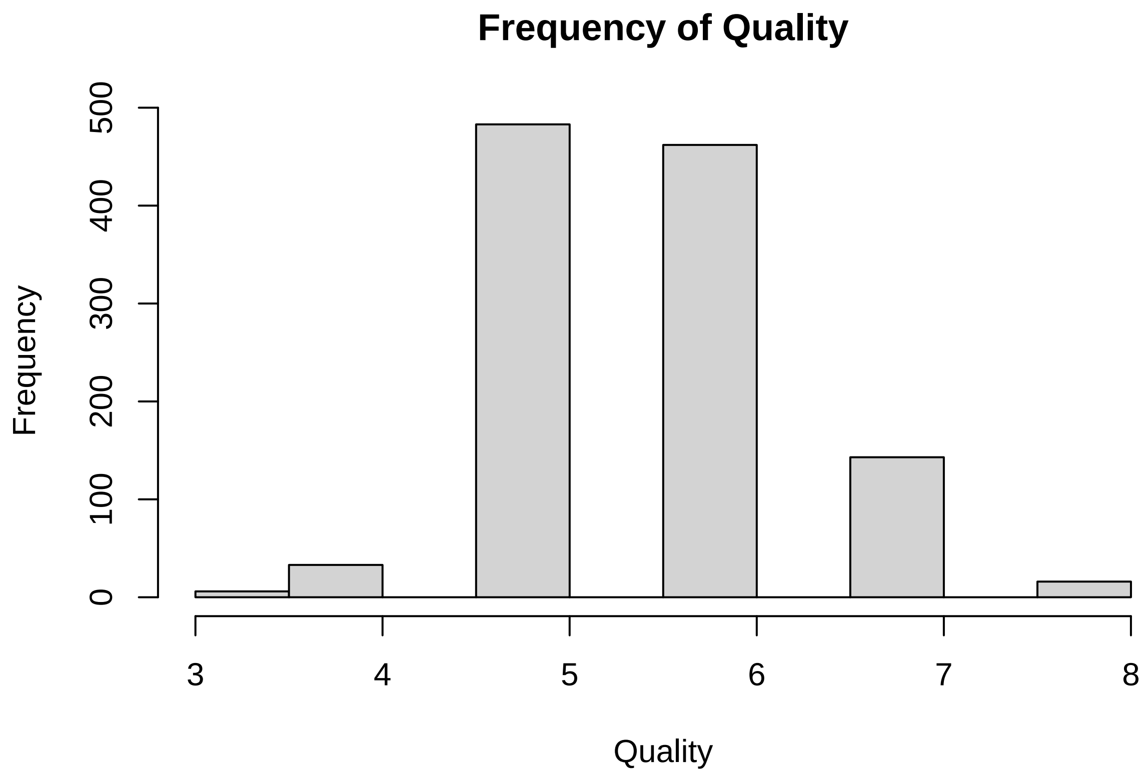

```
hist(Wine_Quality$sulphates, xlab = "Sulphates", ylab = "Frequency", main = "Frequency of Sulphates")
```




```
hist(Wine_Quality$alcohol, xlab = "Alcohol", ylab = "Frequency", main = "Frequency of Alcohol")
```




```
hist(Wine_Quality$quality, xlab = "Quality", ylab = "Frequency", main = "Frequency of Quality")
```



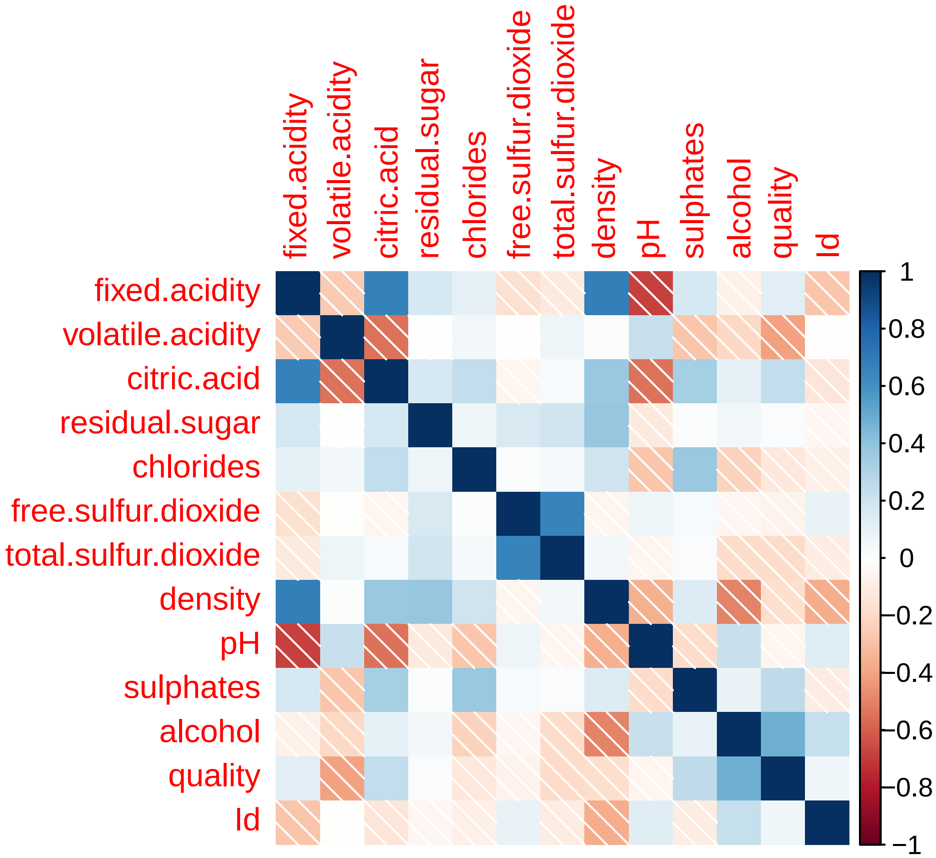
Correlation Plot

Based on the correlation plot above, there is a positive correlation between the quality of wine and alcohol, sulphates, citric acid, and fixed acidity. However, there is a negative correlation between wine quality and volatile acidity, chlorides, free sulfur dioxide, total sulfur dioxide, density, and pH.

```r
# Correlation Plot
# uses cor() to make correlation matrix
# uses corrplot() to make correlation plot
library(corrplot)
```

```
## corrplot 0.92 loaded
```

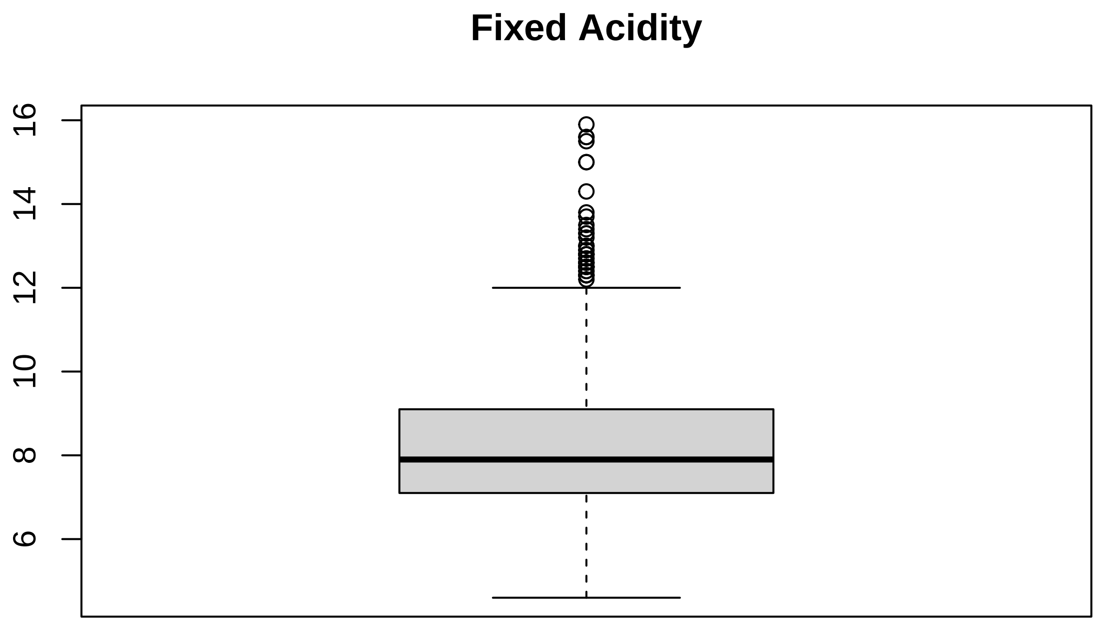
```r
M = cor(Wine_Quality)
corrplot(M, method = 'shade')
```



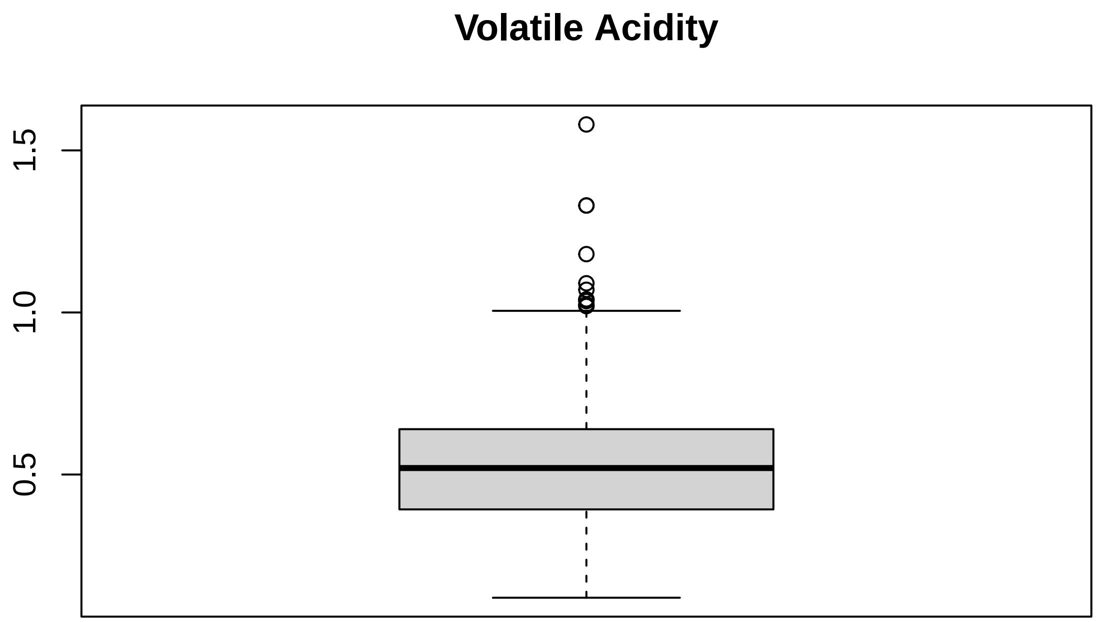
Box Plot

Similar to a histogram, a box plot provides insight into the distribution of a data set. The box is the interquartile range of data, showing the values between the first and third quartile. The length of the box shows how spread out or concentrated the data is. It is evident by the models above that the data is moderately concentrated with the predictor variable citric acid having the largest spread and chlorides having the smallest spread. The points outside the box are outliers. Besides the predictor variables citric acid and alcohol, the rest have many outliers which will be helpful in determining what variables are best are predicting or explaining the response variable, quality.

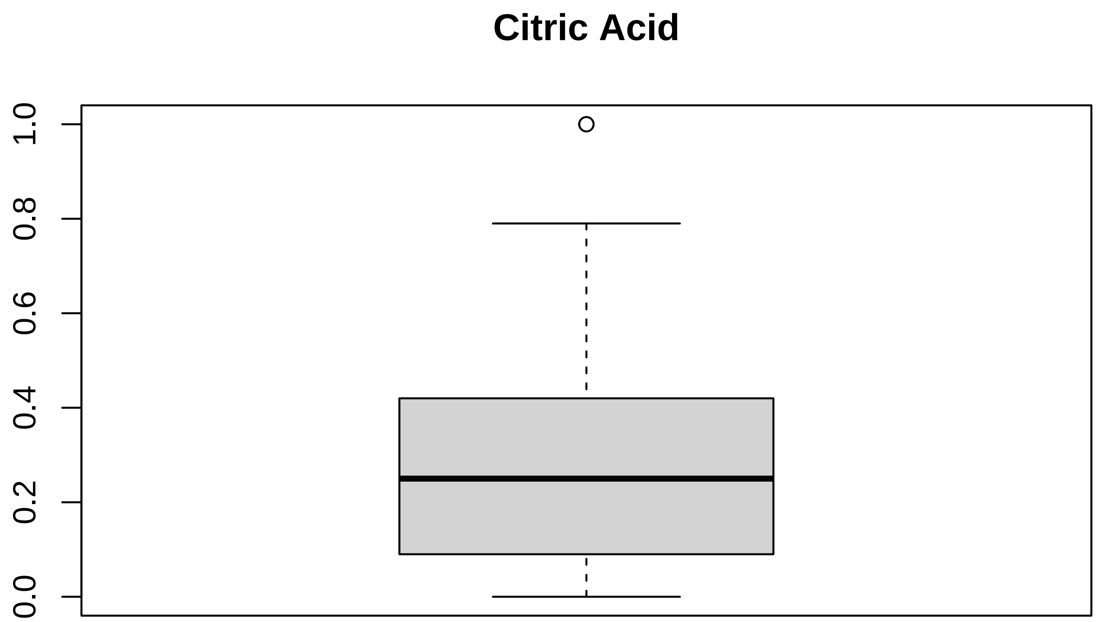
```
# Box Plot
# Uses boxplot() to create a boxplot
boxplot(Wine_Quality$fixed.acidity, main = "Fixed Acidity")
```



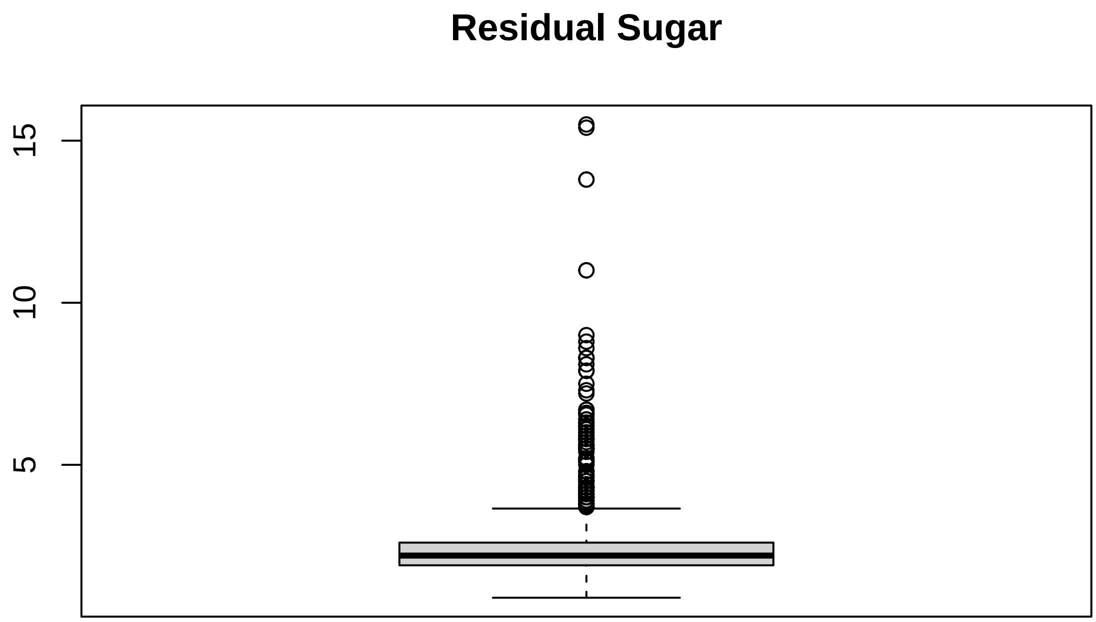

```
boxplot(Wine_Quality$volatile.acidity, main = "Volatile Acidity")
```



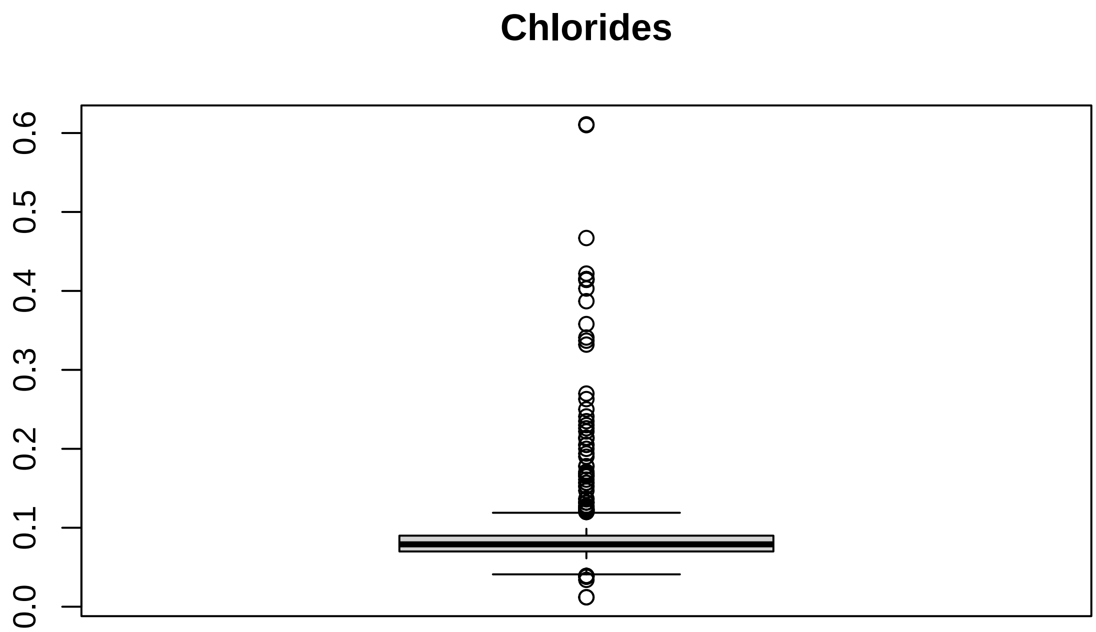

```
boxplot(Wine_Quality$citric.acid, main = "Citric Acid")
```



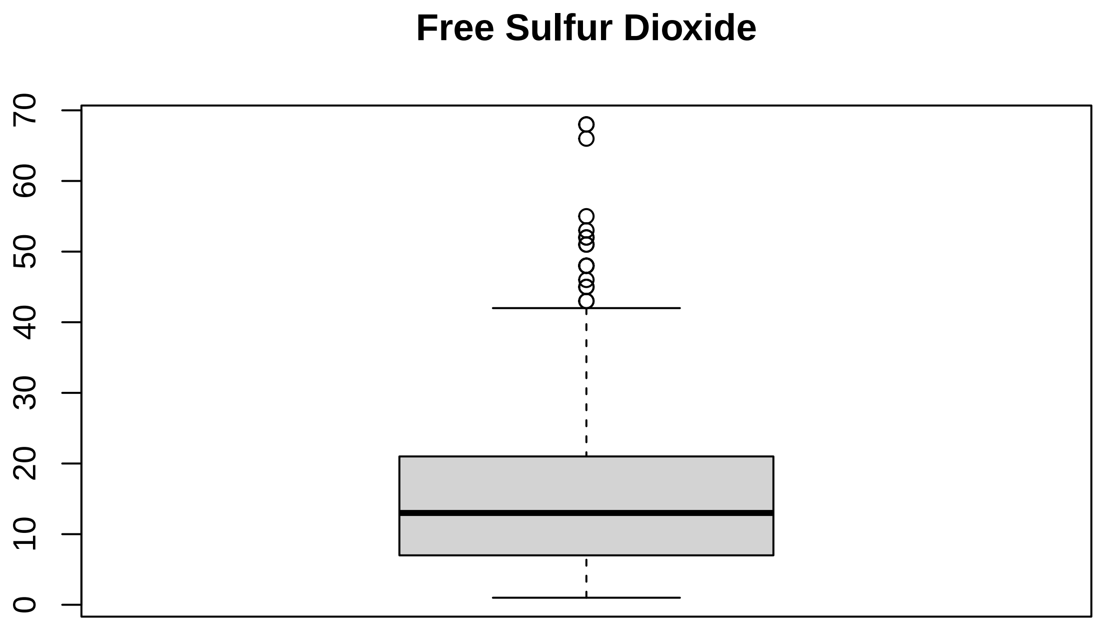

```
boxplot(Wine_Quality$residual.sugar, main = "Residual Sugar")
```



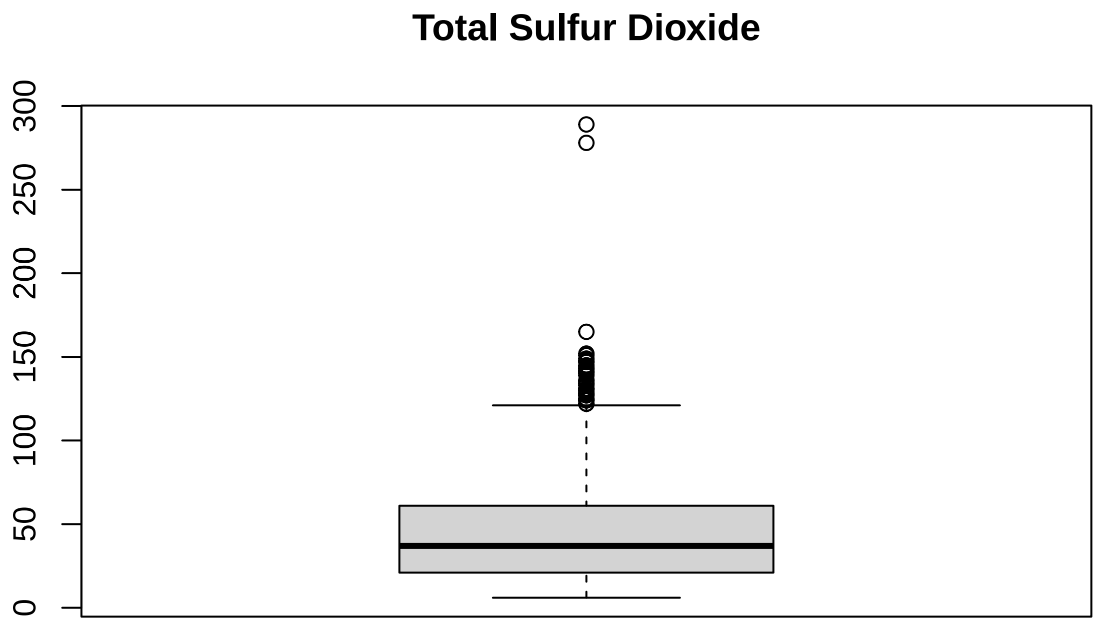

```
boxplot(Wine_Quality$chlorides, main = "Chlorides")
```



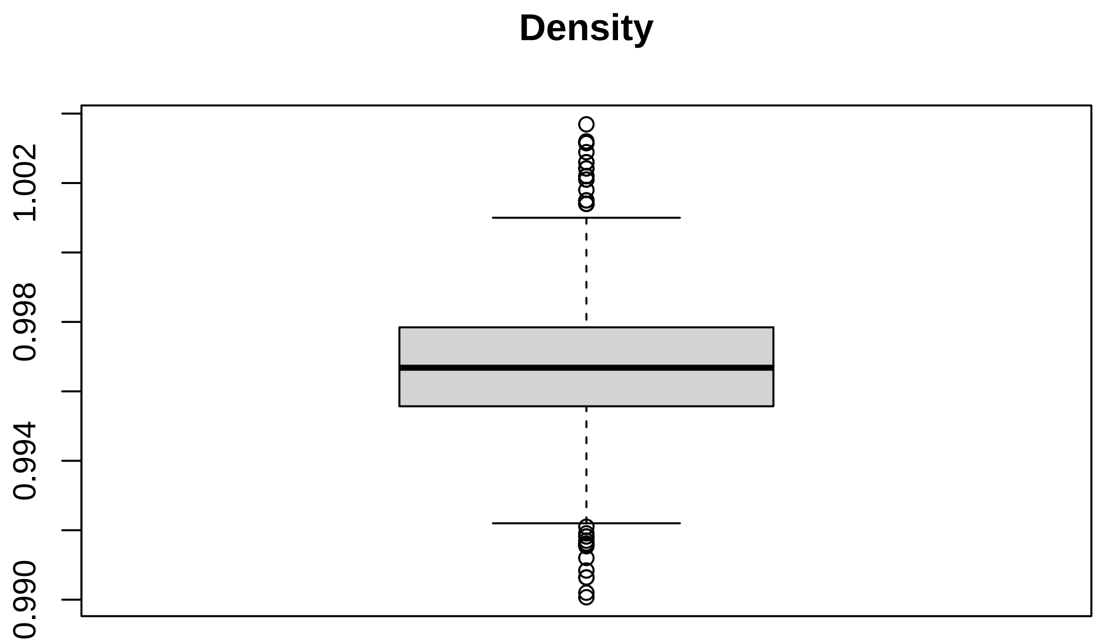

```r
boxplot(Wine_Quality$free.sulfur.dioxide, main = "Free Sulfur Dioxide")
```



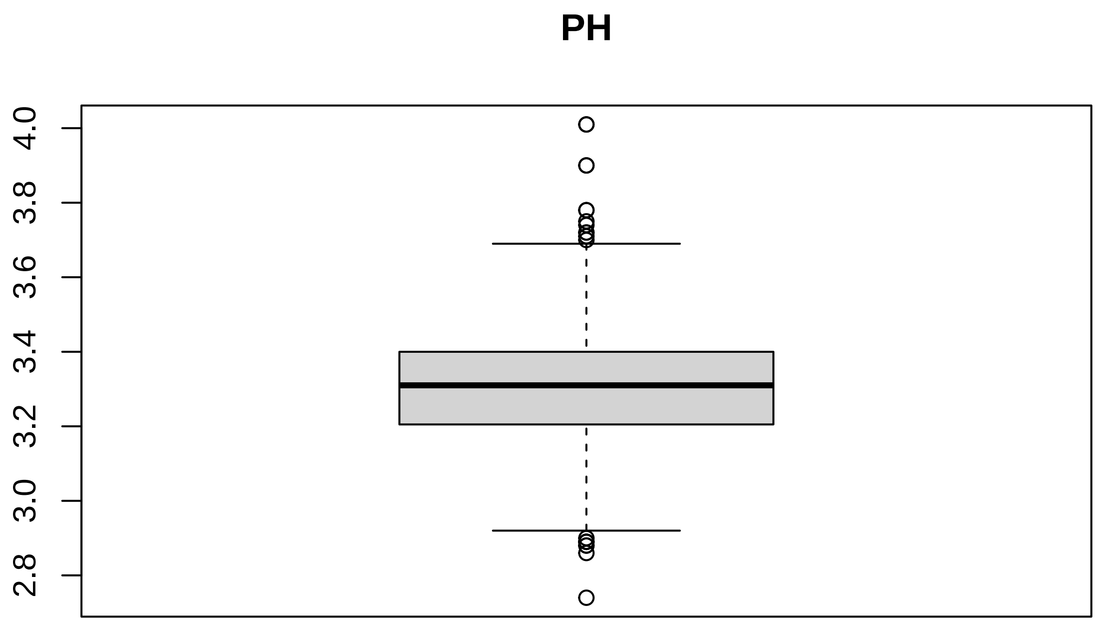

```
boxplot(Wine_Quality$total.sulfur.dioxide, main = "Total Sulfur Dioxide")
```



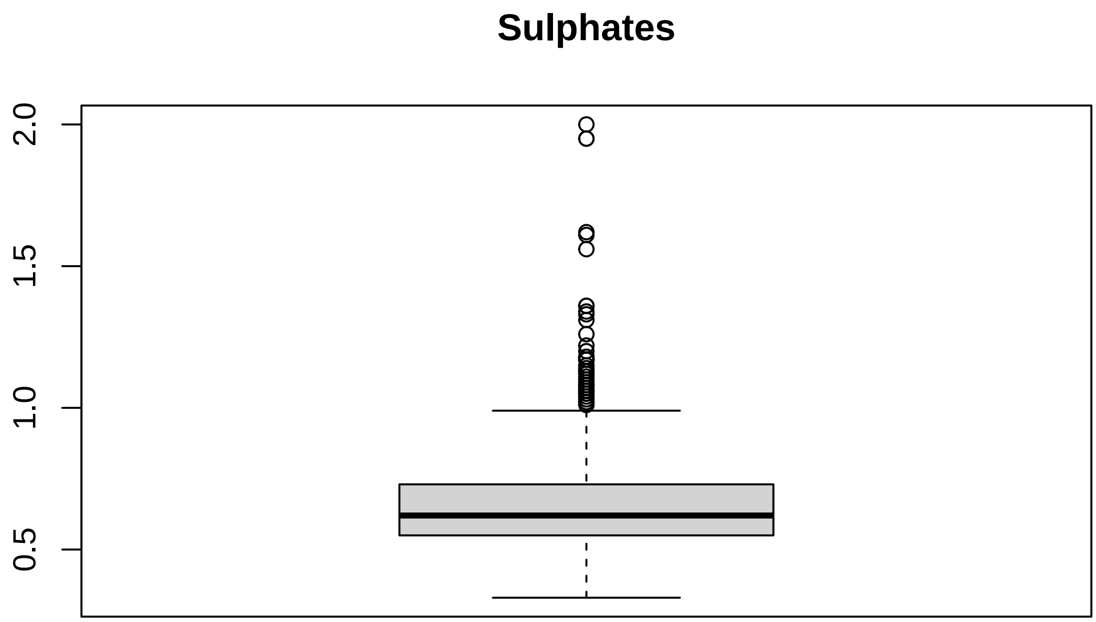

```
boxplot(Wine_Quality$density, main = "Density")
```



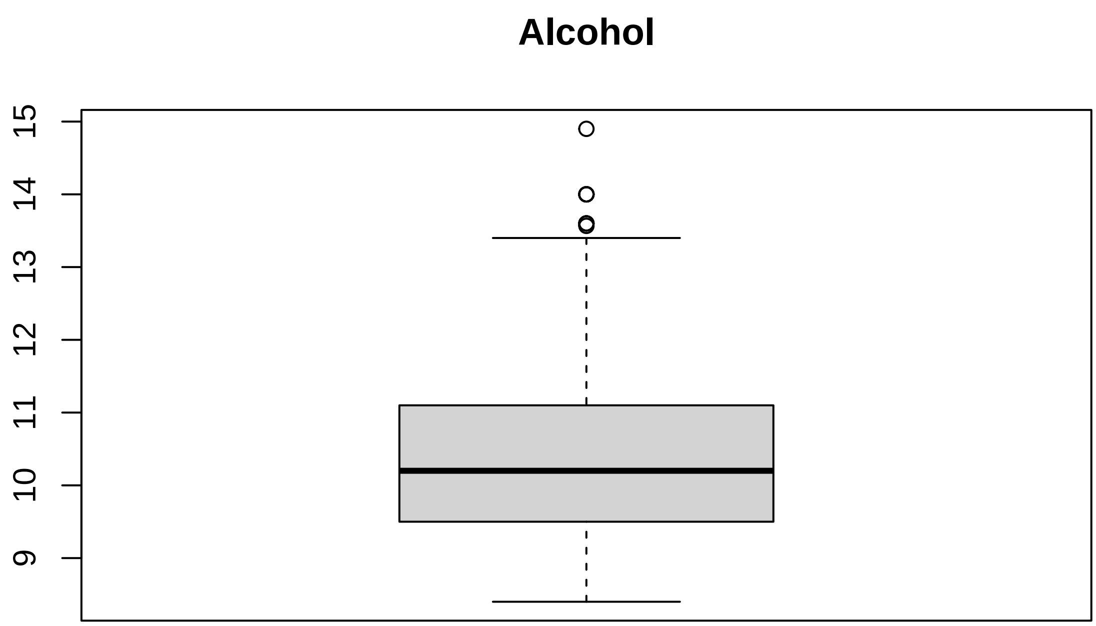

```
boxplot(Wine_Quality$pH, main = "PH")
```




```r
boxplot(Wine_Quality$sulphates, main = "Sulphates")
```



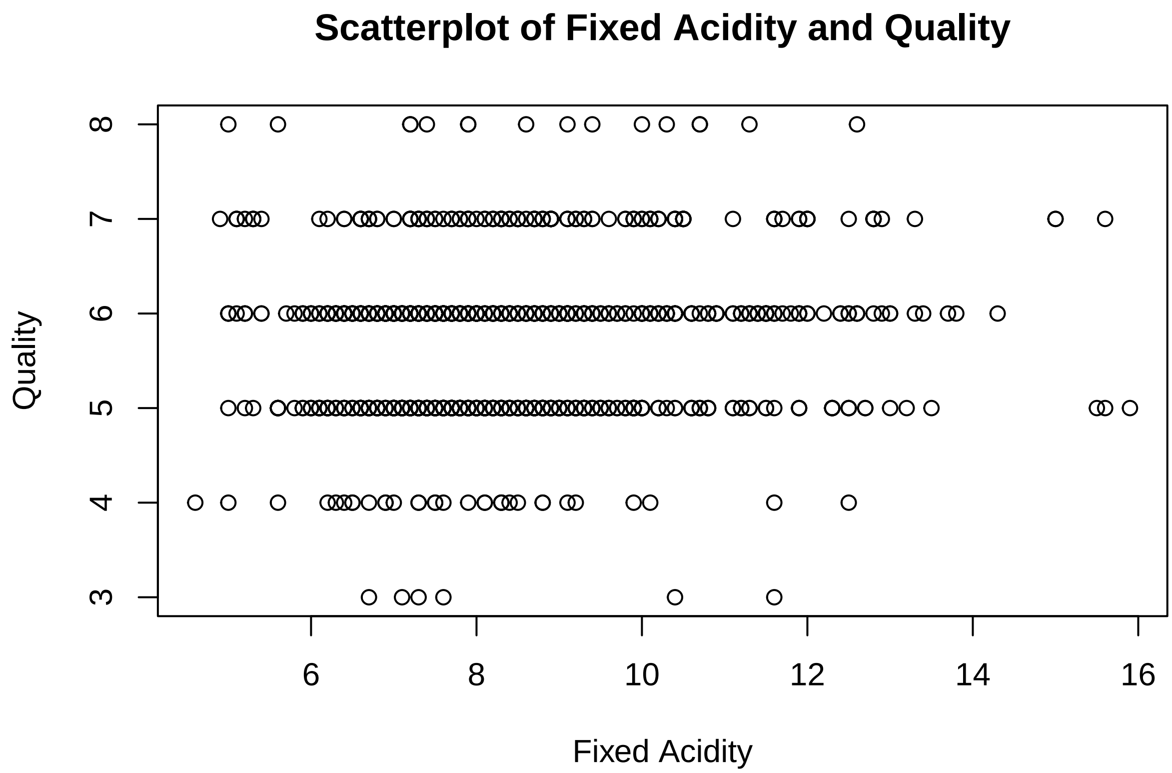

```
boxplot(Wine_Quality$alcohol, main = "Alcohol")
```



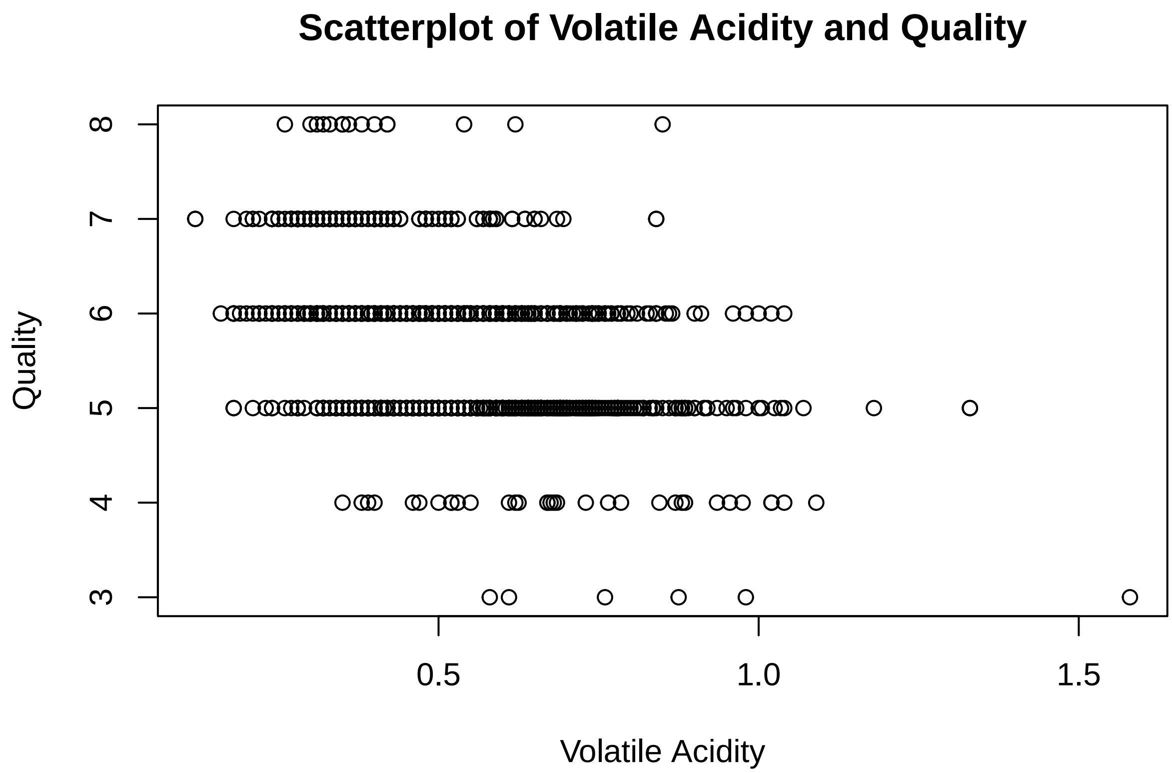
Scatter Plot

A scatterplot shows the relationship between two variables. In this data set, it's a visual of the relationship between the quality of wine and fixed acidity, pH, alcohol, etc… Variables with a strong relationship will have clusters of data points while variables with weak relationships will have data points that are spread out. From the scatter plots above, we can conclude that there is a strong relationship between wine quality and the other variables as of now.

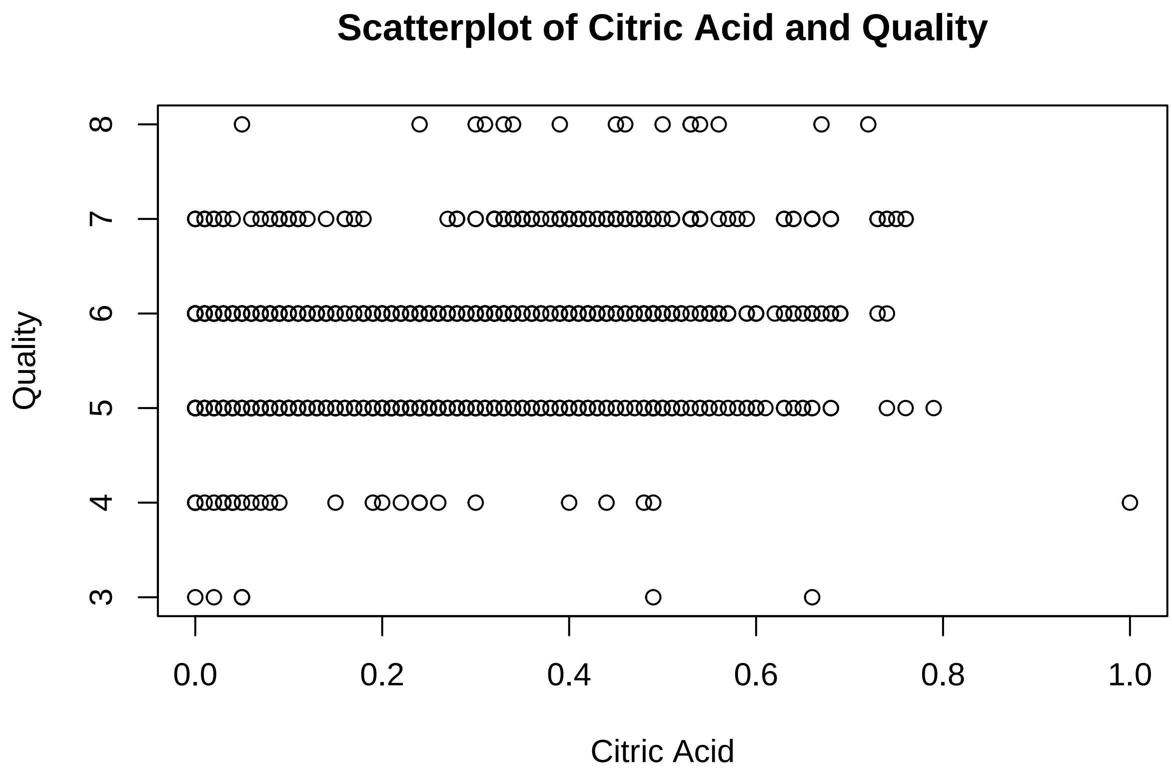
```
# Scatter Plot
# uses plot() to create the graph with type = "p" for scatter plot
plot(Wine_Quality$fixed.acidity, Wine_Quality$quality, xlab="Fixed Acidity", ylab="Quality", main = "Sca
```



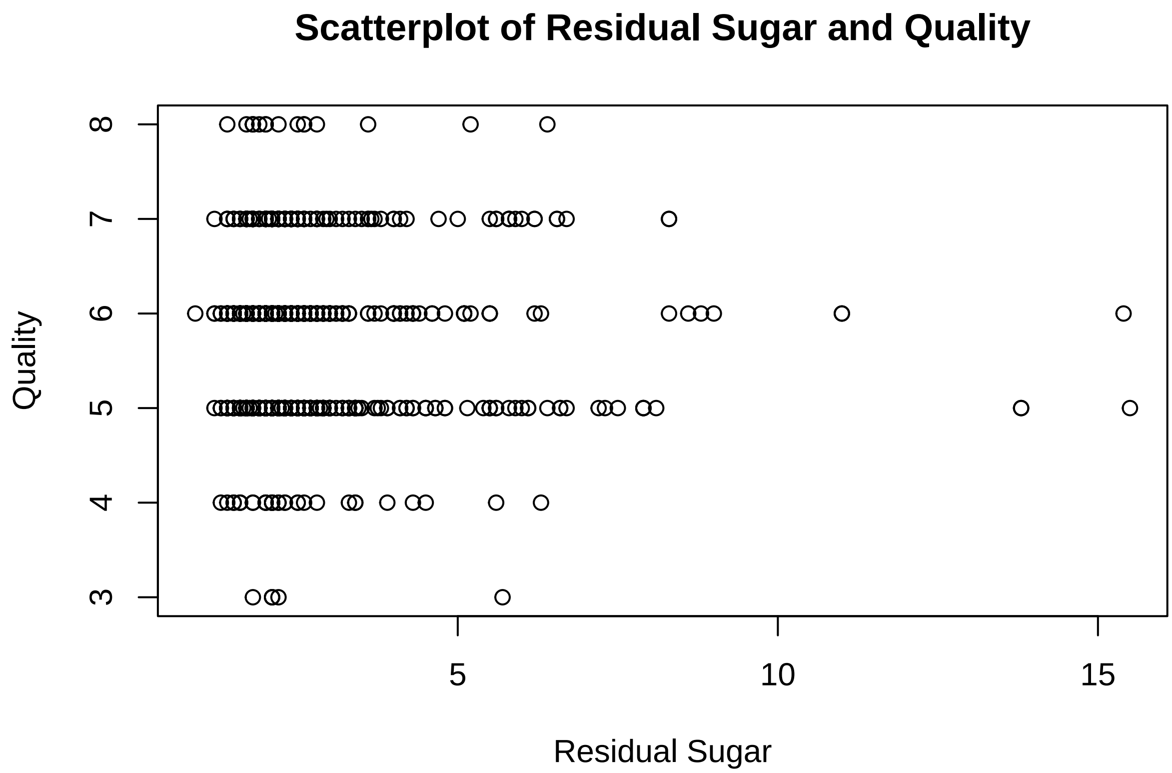

```
plot(Wine_Quality$volatile.acidity, Wine_Quality$quality, xlab="Volatile Acidity", ylab="Quality", main
```



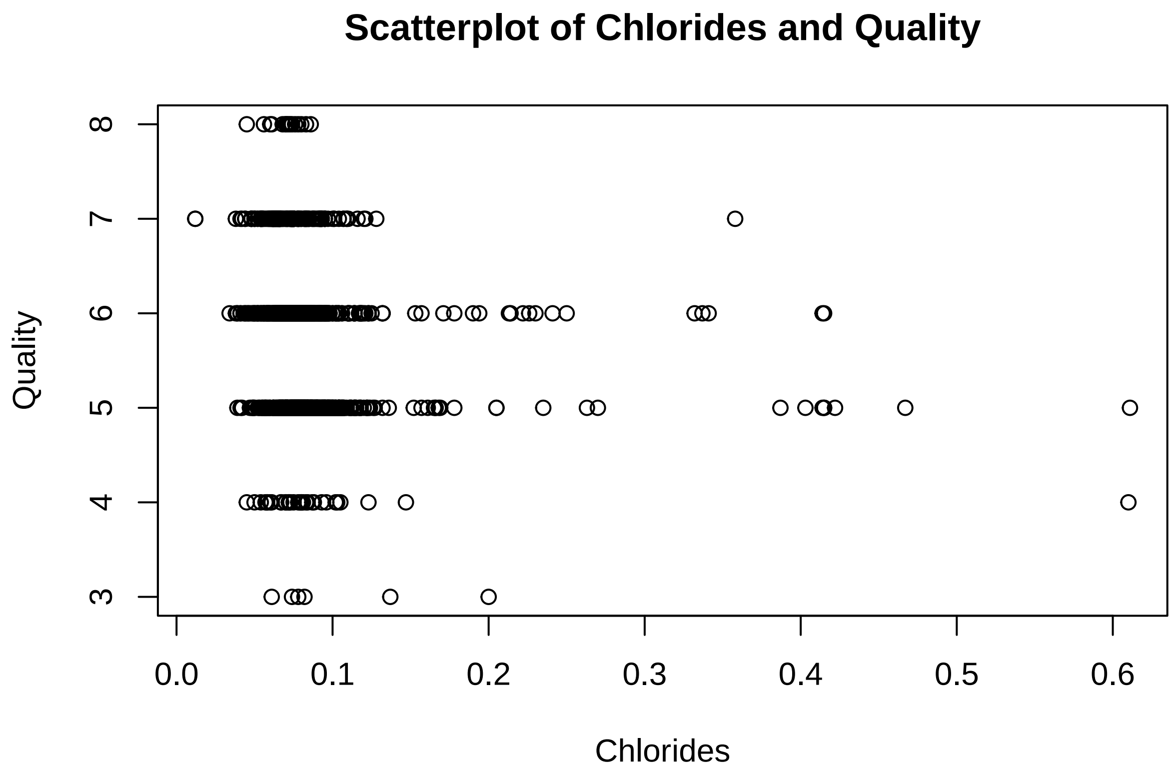

```
plot(Wine_Quality$citric.acid, Wine_Quality$quality, xlab="Citric Acid", ylab="Quality", main = "Scatte
```



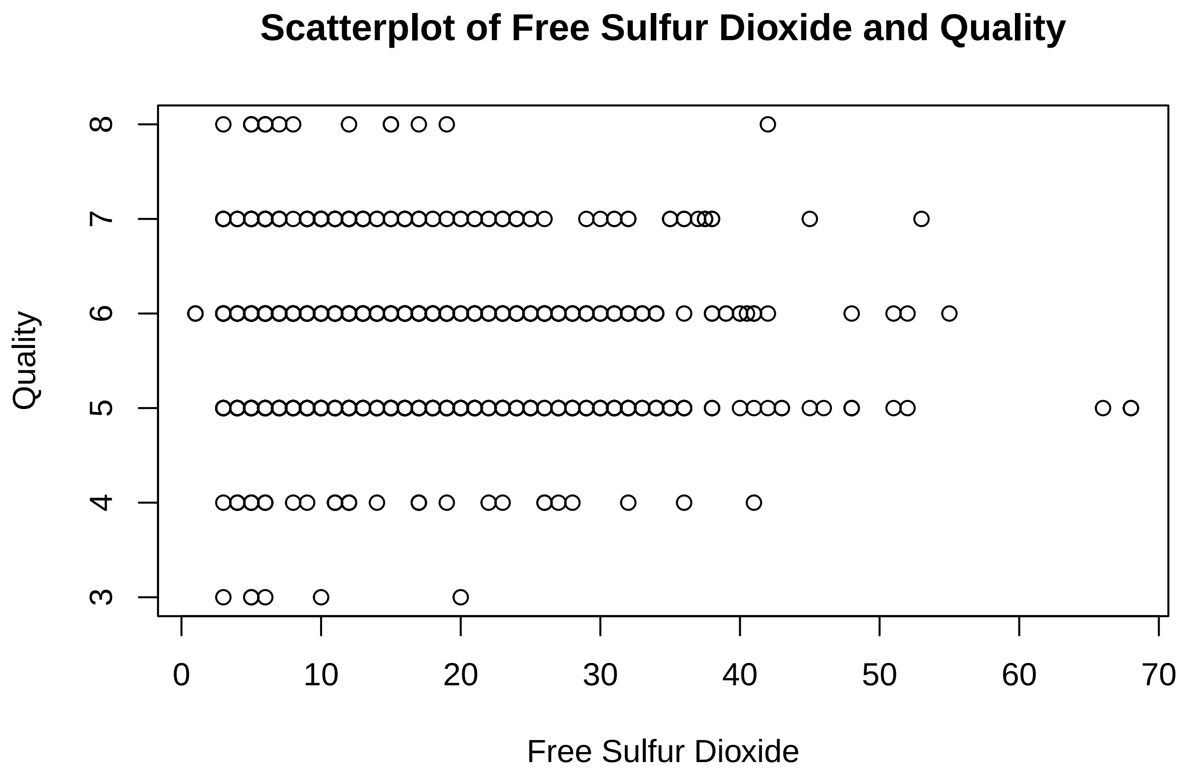

```
plot(Wine_Quality$residual.sugar, Wine_Quality$quality, xlab="Residual Sugar", ylab="Quality", main = "S
```



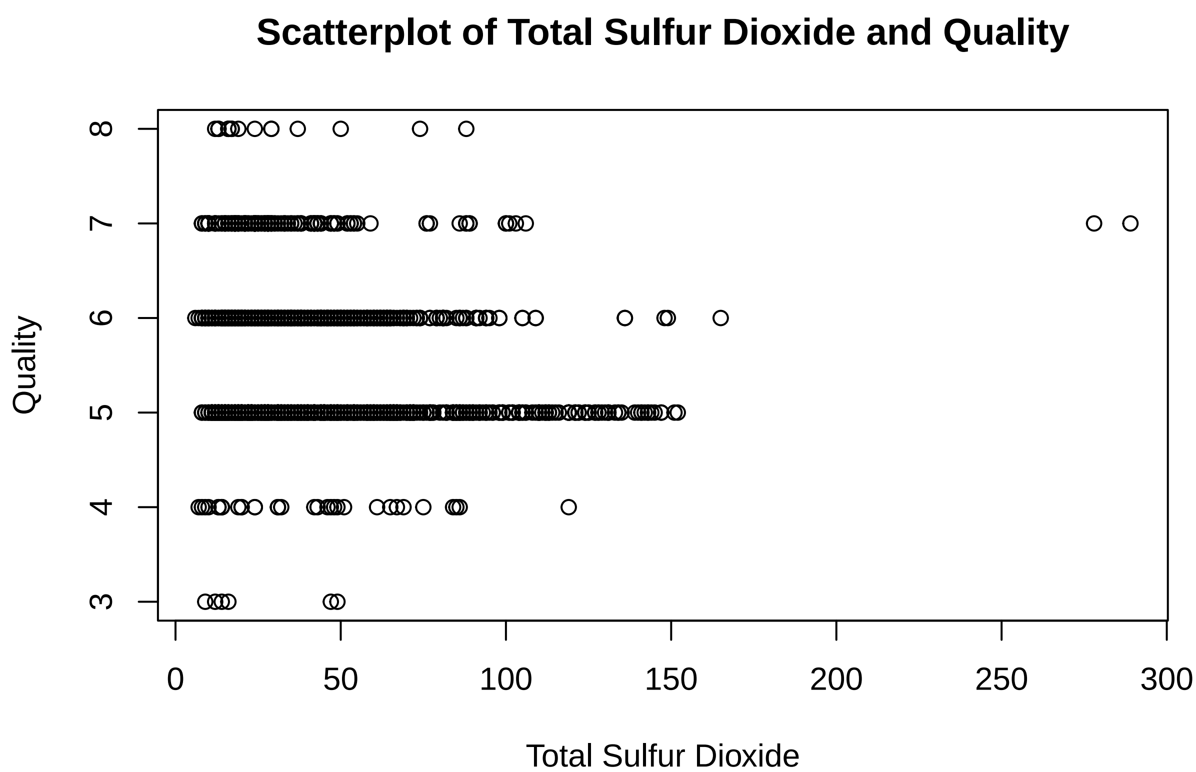

```
plot(Wine_Quality$chlorides, Wine_Quality$quality, xlab="Chlorides", ylab="Quality", main = "Scatterplo
```



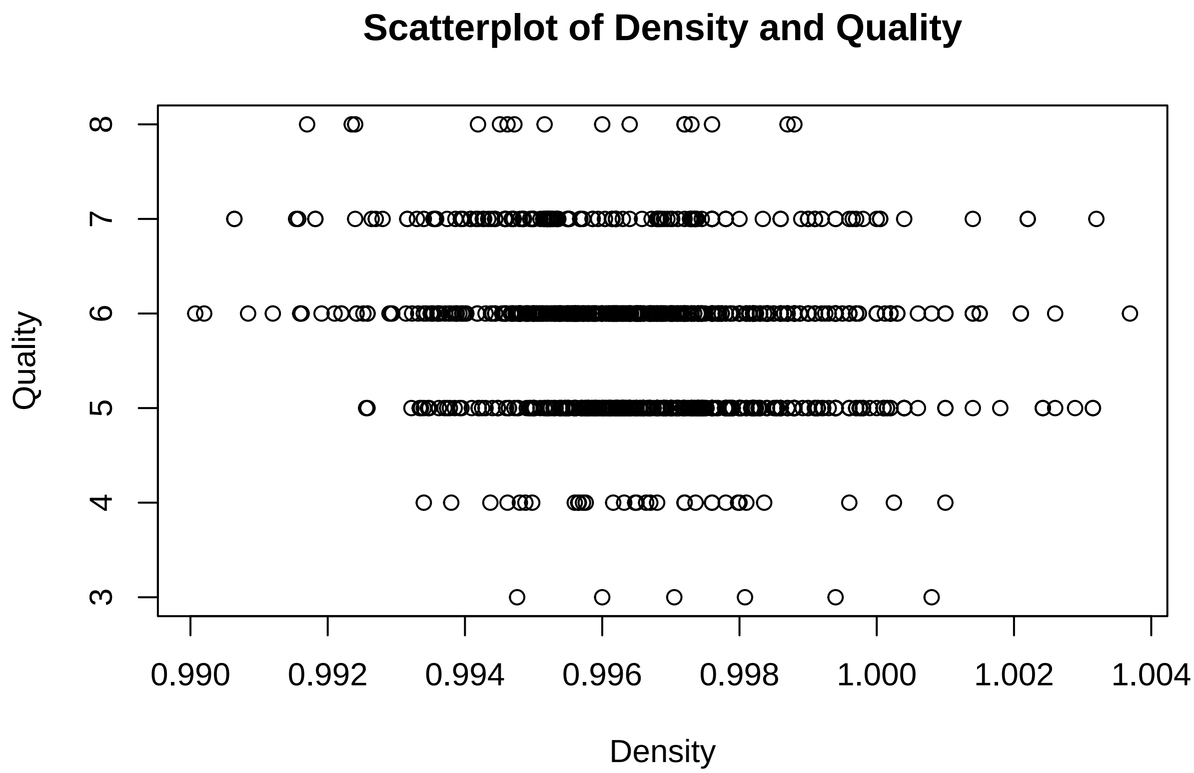

```
plot(Wine_Quality$free.sulfur.dioxide, Wine_Quality$quality, xlab="Free Sulfur Dioxide", ylab="Quality"
```



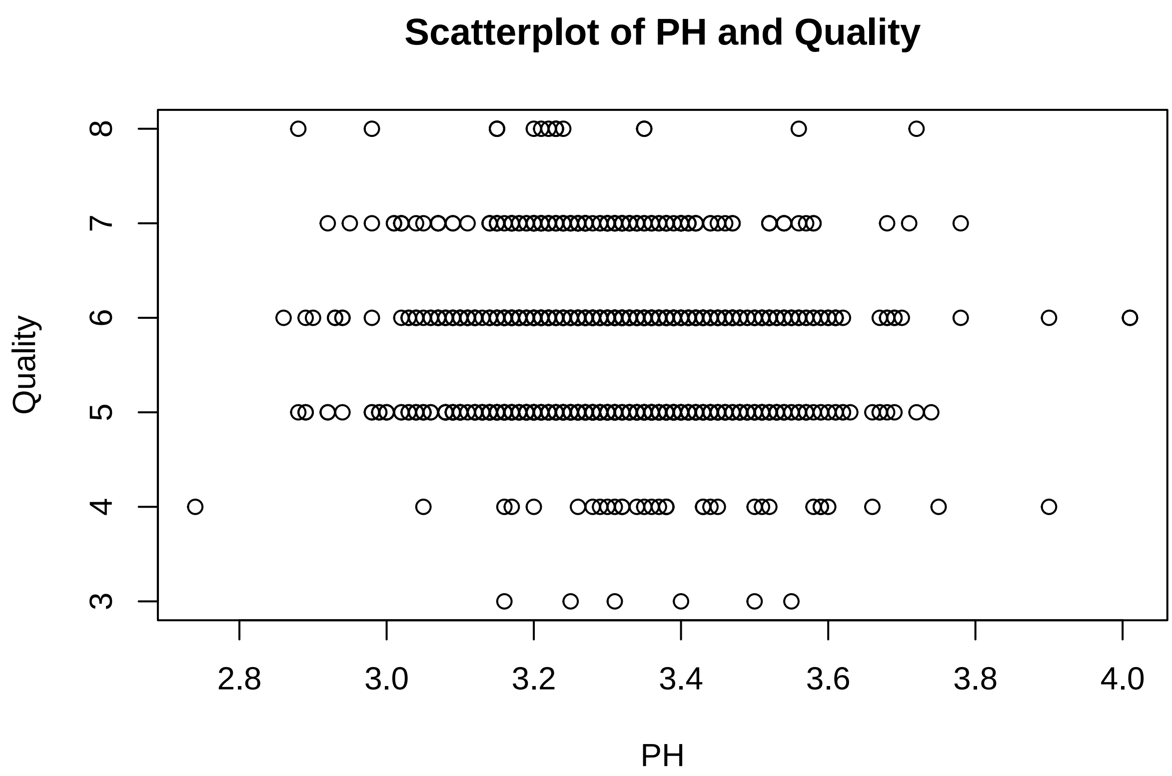

```
plot(Wine_Quality$total.sulfur.dioxide, Wine_Quality$quality, xlab="Total Sulfur Dioxide", ylab="Quality
```



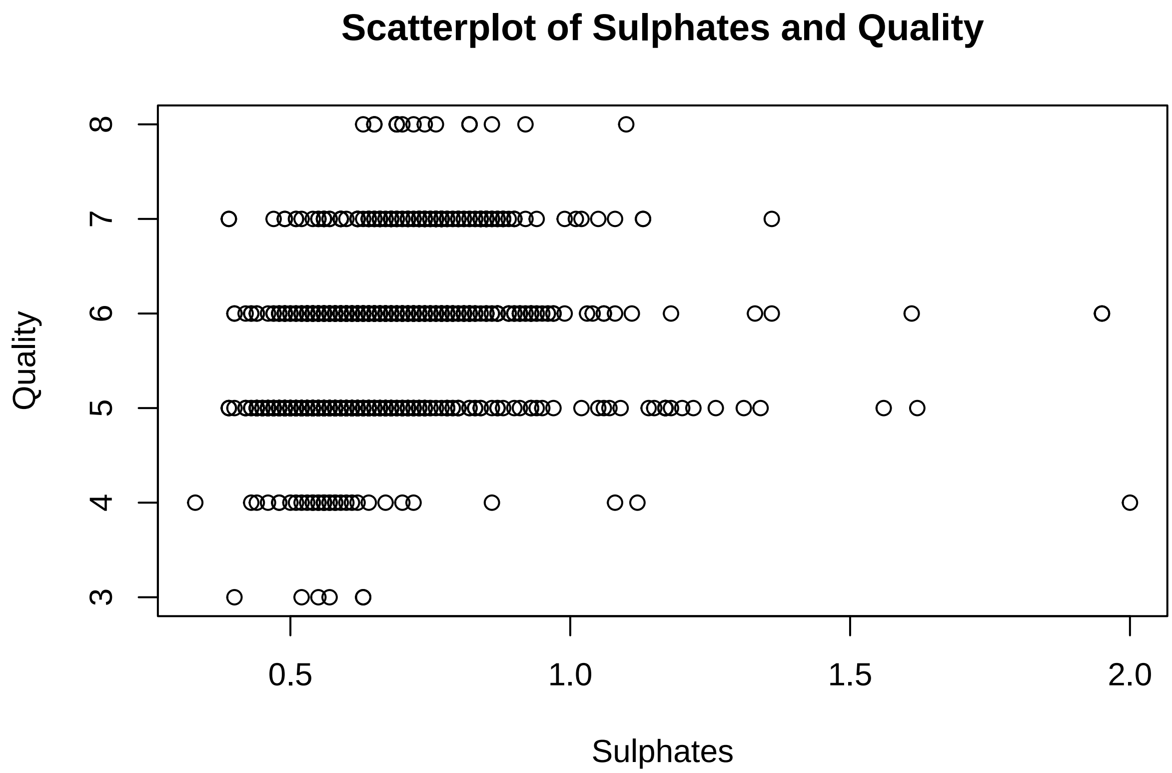

```
plot(Wine_Quality$density, Wine_Quality$quality, xlab="Density", ylab="Quality", main = "Scatterplot of
```



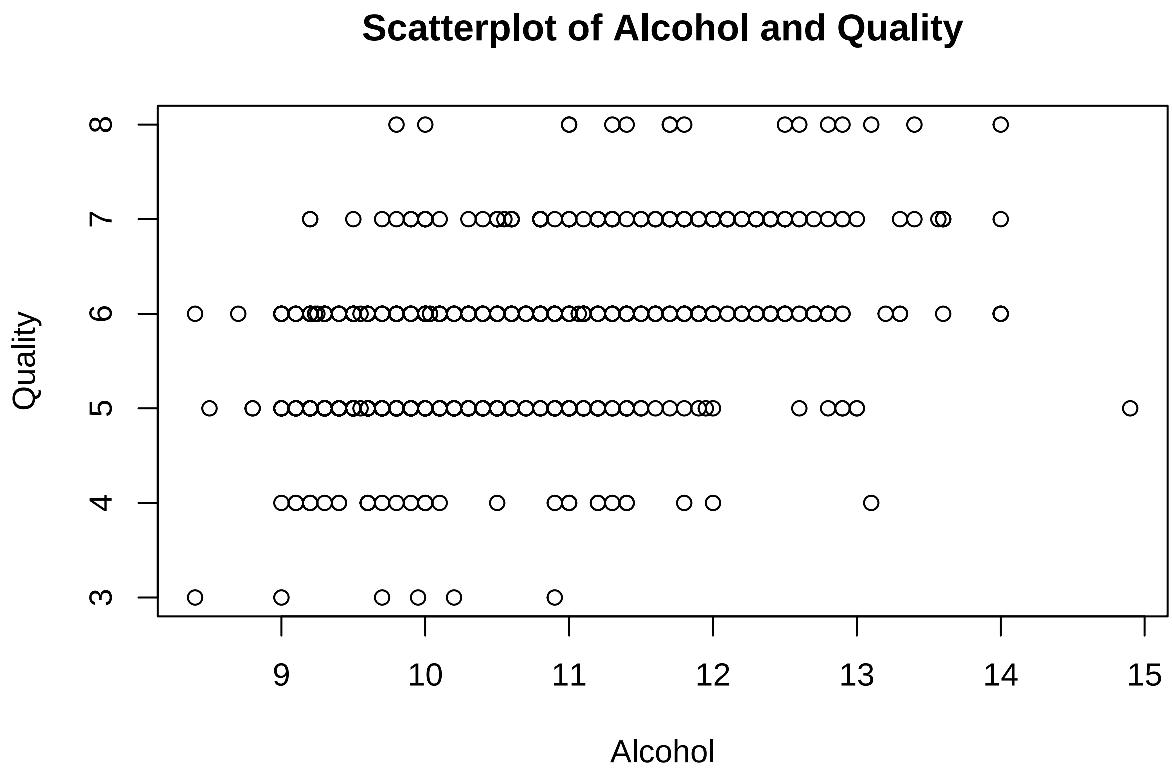

```
plot(Wine_Quality$pH, Wine_Quality$quality, xlab="PH", ylab="Quality", main = "Scatterplot of PH and Qua
```




```
plot(Wine_Quality$sulphates, Wine_Quality$quality, xlab="Sulphates", ylab="Quality", main = "Scatterplo
```




```
plot(Wine_Quality$alcohol, Wine_Quality$quality, xlab="Alcohol", ylab="Quality", main = "Scatterplot of
```



Statistical Summary

Summarizing the data set provides us information about the minimum, 1st quartile, mean, 3rd quartile, and maximum of each variable. Having the average value of each variable, the mean, helps us compare variables and identify outliers.

```
# Statistical Summary (Five-Number Summary)
# Uses summary() for statistical summary
summary(Wine_Quality)
```

```
##  fixed.acidity    volatile.acidity  citric.acid     residual.sugar
##  Min.   : 4.600   Min.   :0.1200   Min.   :0.0000   Min.   : 0.900
##  1st Qu.: 7.100   1st Qu.:0.3925   1st Qu.:0.0900   1st Qu.: 1.900
##  Median : 7.900   Median :0.5200   Median :0.2500   Median : 2.200
##  Mean   : 8.311   Mean   :0.5313   Mean   :0.2684   Mean   : 2.532
##  3rd Qu.: 9.100   3rd Qu.:0.6400   3rd Qu.:0.4200   3rd Qu.: 2.600
##  Max.   :15.900   Max.   :1.5800   Max.   :1.0000   Max.   :15.500
##    chlorides        free.sulfur.dioxide total.sulfur.dioxide    density
##  Min.   :0.01200   Min.   : 1.00       Min.   :  6.00       Min.   :0.9901
##  1st Qu.:0.07000   1st Qu.: 7.00       1st Qu.: 21.00       1st Qu.:0.9956
##  Median :0.07900   Median :13.00       Median : 37.00       Median :0.9967
##  Mean   :0.08693   Mean   :15.62       Mean   : 45.91       Mean   :0.9967
##  3rd Qu.:0.09000   3rd Qu.:21.00       3rd Qu.: 61.00       3rd Qu.:0.9978
##  Max.   :0.61100   Max.   :68.00       Max.   :289.00       Max.   :1.0037
##        pH          sulphates         alcohol          quality
```

```
##  Min.   :2.740   Min.   :0.3300   Min.   : 8.40   Min.   :3.000  
##  1st Qu.:3.205   1st Qu.:0.5500   1st Qu.: 9.50   1st Qu.:5.000  
##  Median :3.310   Median :0.6200   Median :10.20   Median :6.000  
##  Mean   :3.311   Mean   :0.6577   Mean   :10.44   Mean   :5.657  
##  3rd Qu.:3.400   3rd Qu.:0.7300   3rd Qu.:11.10   3rd Qu.:6.000  
##  Max.   :4.010   Max.   :2.0000   Max.   :14.90   Max.   :8.000  
##        Id      
##  Min.   :   0  
##  1st Qu.: 411  
##  Median : 794  
##  Mean   : 805  
##  3rd Qu.:1210  
##  Max.   :1597
```

Question 2

Multiple Linear Regression Model

```
# Estimating The Multiple Linear Regression Model
# uses lm() to create linear model
# uses summary() to display regression details
wineq <- lm(quality ~ fixed.acidity + volatile.acidity + citric.acid + residual.sugar + chlorides + free
summary(wineq)
```

```
## 
## Call:
## lm(formula = quality ~ fixed.acidity + volatile.acidity + citric.acid + 
##     residual.sugar + chlorides + free.sulfur.dioxide + total.sulfur.dioxide + 
##     density + pH + sulphates + alcohol, data = Wine_Quality)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -2.49977 -0.36903 -0.04658  0.43956  2.00117 
## 
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)    
## (Intercept)           2.155e+01  2.477e+01   0.870 0.384551    
## fixed.acidity         2.297e-02  3.025e-02   0.759 0.447770    
## volatile.acidity     -1.129e+00  1.407e-01  -8.023 2.56e-15 ***
## citric.acid          -1.319e-01  1.730e-01  -0.762 0.446105    
## residual.sugar        1.351e-02  1.846e-02   0.732 0.464278    
## chlorides            -1.708e+00  4.974e-01  -3.434 0.000616 ***
## free.sulfur.dioxide   2.369e-03  2.553e-03   0.928 0.353547    
## total.sulfur.dioxide -2.785e-03  8.386e-04  -3.321 0.000926 ***
## density              -1.745e+01  2.529e+01  -0.690 0.490284    
## pH                   -4.082e-01  2.229e-01  -1.832 0.067280 .  
## sulphates             8.752e-01  1.335e-01   6.555 8.44e-11 ***
## alcohol               2.801e-01  3.126e-02   8.963  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.6405 on 1131 degrees of freedom
```

```
## Multiple R-squared:  0.3742, Adjusted R-squared:  0.3682
## F-statistic: 61.49 on 11 and 1131 DF,  p-value: < 2.2e-16
```

Question 3

Outlier, High Leverage, and New Model

It is evident that there are outliers and high leverage observations by the values outputted by the boxplot.stats() and hats() codes. The values derived by performing these tests corroborate the outcomes of the histogram, boxplot, scatterplot, and summary functions. These values are important because they are capable of altering the model's best-fit line.

```
# Outlier
# uses boxplot.stats() with $out  to capture outliers
boxplot.stats(Wine_Quality$fixed.acidity)$out
```

```
##  [1] 12.8 12.8 15.0 15.0 12.5 13.3 13.4 12.5 13.8 13.5 12.6 12.5 12.8 12.8 13.7
## [16] 12.2 12.5 12.8 12.3 12.3 12.6 15.6 12.5 13.0 12.5 13.3 12.5 12.9 14.3 12.4
## [31] 15.5 15.6 13.0 12.7 13.0 12.7 12.3 12.3 12.4 13.2 15.9 12.9 12.6 12.6
```

```
boxplot.stats(Wine_Quality$volatile.acidity)$out
```

```
##  [1] 1.020 1.070 1.330 1.330 1.040 1.090 1.040 1.020 1.035 1.025 1.020 1.580
## [13] 1.180 1.040
```

```
boxplot.stats(Wine_Quality$citric.acid)$out
```

```
## [1] 1
```

```
boxplot.stats(Wine_Quality$residual.sugar)$out
```

```
##   [1]  5.50  5.90  4.65  4.65  5.50  5.50  5.50  7.30  7.20  5.60  4.00  4.00
##  [13]  4.00  4.00  6.40  5.60  5.60 11.00 11.00  4.50  4.80  5.80  5.80  6.20
##  [25]  4.20  7.90  7.90  4.50  6.70  6.60  3.70  5.20 15.50  8.30  6.55  6.55
##  [37]  6.10  4.30  5.80  5.15  6.30  4.20  4.60  4.20  4.30  4.30  7.90  5.10
##  [49]  5.60  8.60  7.50  6.00  3.90  4.20  4.00  6.60  6.00  3.80  9.00  8.80
##  [61]  5.00  3.80  4.10  5.90  4.10  6.20  4.00  3.90  4.00  8.10  6.40  8.30
##  [73]  8.30  4.70  5.50  5.50  4.30  5.50  3.70  6.20  5.60  4.60  5.80  4.10
##  [85]  4.30  4.80  6.30  4.50  4.50  4.30  3.80  5.40  6.10  5.10  5.10  3.90
##  [97] 15.40  4.80  5.20  3.75 13.80 13.80  5.70  4.30  4.10  4.10  4.40  3.70
## [109]  6.70  5.10
```

```
boxplot.stats(Wine_Quality$chlorides)$out
```

```
##  [1] 0.341 0.332 0.467 0.178 0.610 0.270 0.039 0.337 0.263 0.611 0.358 0.213
## [13] 0.214 0.121 0.128 0.120 0.122 0.122 0.121 0.127 0.152 0.125 0.122 0.200
## [25] 0.226 0.250 0.124 0.222 0.039 0.157 0.422 0.034 0.387 0.415 0.157 0.241
## [37] 0.190 0.132 0.126 0.038 0.165 0.147 0.012 0.012 0.194 0.132 0.161 0.120
## [49] 0.120 0.123 0.123 0.414 0.171 0.178 0.166 0.136 0.132 0.132 0.123 0.123
## [61] 0.403 0.137 0.414 0.166 0.168 0.415 0.153 0.415 0.123 0.214 0.169 0.205
## [73] 0.205 0.039 0.235 0.230 0.038
```

```r
boxplot.stats(Wine_Quality$free.sulfur.dioxide)$out
```

```
##  [1] 68 68 43 46 45 53 52 51 45 48 48 43 51 52 55 48 48 66
```

```r
boxplot.stats(Wine_Quality$total.sulfur.dioxide)$out
```

```
##  [1] 136 125 140 136 134 141 128 129 128 143 127 135 165 124 124 122 134 124 151
## [20] 142 149 147 145 148 152 122 125 127 139 143 144 130 278 289 141 133 147 131
## [39] 131 131
```

```r
boxplot.stats(Wine_Quality$density)$out
```

```
##  [1] 0.99160 0.99160 1.00140 1.00150 1.00150 1.00180 0.99120 1.00220 1.00220
## [10] 1.00140 1.00140 1.00320 1.00260 1.00140 1.00315 1.00315 1.00210 1.00210
## [19] 0.99170 1.00260 0.99210 0.99154 0.99064 0.99064 1.00289 0.99162 0.99007
## [28] 0.99020 0.99157 0.99084 0.99191 1.00369 1.00242 0.99182 1.00242 0.99182
```

```r
boxplot.stats(Wine_Quality$pH)$out
```

```
##  [1] 3.90 3.75 2.74 2.88 2.86 3.74 3.72 2.89 2.89 3.90 3.71 2.89 3.78 3.70 3.78
## [16] 4.01 2.90 4.01 2.88 3.72
```

```r
boxplot.stats(Wine_Quality$sulphates)$out
```

```
##  [1] 1.56 1.08 1.20 1.12 1.95 1.22 1.95 1.31 2.00 1.08 1.02 1.61 1.09 1.26 1.08
## [16] 1.36 1.13 1.04 1.11 1.13 1.07 1.06 1.06 1.05 1.02 1.14 1.36 1.05 1.17 1.62
## [31] 1.06 1.18 1.34 1.15 1.17 1.17 1.33 1.18 1.17 1.03 1.17 1.10 1.01
```

```r
boxplot.stats(Wine_Quality$alcohol)$out
```

```
##  [1] 14.00000 14.00000 14.00000 14.00000 14.90000 14.00000 13.60000 13.60000
##  [9] 13.60000 14.00000 13.56667 13.60000
```

```r
# High Leverage
# creates a dataframe named hats
# uses hatvalues to be able to see high leverage
hats <- as.data.frame(hatvalues(wineq))
hats
```

```
##      hatvalues(wineq)
## 1         0.005079901
## 2         0.008111271
## 3         0.004023940
## 4         0.007623793
## 5         0.005079901
## 6         0.005195357
## 7         0.004376642
## 8         0.006281831
```

```
## 9           0.004199146
## 10          0.004587509
## 11          0.007865355
## 12          0.039475498
## 13          0.009102351
## 14          0.031016001
## 15          0.007029088
## 16          0.006842187
## 17          0.007758928
## 18          0.004464620
## 19          0.006623544
## 20          0.004968706
## 21          0.003894204
## 22          0.004424527
## 23          0.004086631
## 24          0.010734583
## 25          0.012192743
## 26          0.011429170
## 27          0.005009245
## 28          0.004019111
## 29          0.013214273
## 30          0.003247586
## 31          0.033958794
## 32          0.018021960
## 33          0.023179000
## 34          0.017739270
## 35          0.006858603
## 36          0.005920293
## 37          0.006436876
## 38          0.007741004
## 39          0.006880550
## 40          0.011403010
## 41          0.002198632
## 42          0.006725434
## 43          0.007631385
## 44          0.012291719
## 45          0.002939521
## 46          0.008984319
## 47          0.008984319
## 48          0.005352136
## 49          0.009143397
## 50          0.005917860
## 51          0.003420589
## 52          0.014484615
## 53          0.005104983
## 54          0.005348206
## 55          0.018567518
## 56          0.007867479
## 57          0.008761552
## 58          0.010479424
## 59          0.002094741
## 60          0.081831903
## 61          0.003620478
## 62          0.017934258
```

```
## 63         0.005626477
## 64         0.011034703
## 65         0.081831903
## 66         0.003620478
## 67         0.026058577
## 68         0.004965231
## 69         0.006924950
## 70         0.003190962
## 71         0.002627188
## 72         0.003190962
## 73         0.004031028
## 74         0.005401992
## 75         0.004031028
## 76         0.062431247
## 77         0.009445839
## 78         0.006613036
## 79         0.013188200
## 80         0.005808918
## 81         0.006613036
## 82         0.005807121
## 83         0.002636436
## 84         0.017216599
## 85         0.010090365
## 86         0.006487944
## 87         0.009381146
## 88         0.025899899
## 89         0.025814195
## 90         0.008154838
## 91         0.017073414
## 92         0.016196634
## 93         0.011667638
## 94         0.006802741
## 95         0.007359562
## 96         0.006802741
## 97         0.019198679
## 98         0.007868277
## 99         0.019198679
## 100        0.016013518
## 101        0.008720091
## 102        0.002624015
## 103        0.008817519
## 104        0.133177620
## 105        0.009118347
## 106        0.009118347
## 107        0.012727589
## 108        0.012756503
## 109        0.012727589
## 110        0.004855619
## 111        0.009480488
## 112        0.009656767
## 113        0.018813497
## 114        0.003852312
## 115        0.018054168
## 116        0.017540038
```

```
## 117         0.010572574
## 118         0.006444366
## 119         0.006811906
## 120         0.003491123
## 121         0.008754605
## 122         0.004811525
## 123         0.006578860
## 124         0.005091483
## 125         0.005738819
## 126         0.009100582
## 127         0.005299619
## 128         0.003903896
## 129         0.003903896
## 130         0.019719613
## 131         0.003667121
## 132         0.006932116
## 133         0.007178859
## 134         0.008580822
## 135         0.008959370
## 136         0.016851147
## 137         0.008874103
## 138         0.008743930
## 139         0.003114941
## 140         0.003114941
## 141         0.003397219
## 142         0.010101226
## 143         0.014083877
## 144         0.005132741
## 145         0.006620674
## 146         0.010819629
## 147         0.010819629
## 148         0.005353649
## 149         0.012565433
## 150         0.003784830
## 151         0.004905335
## 152         0.005009256
## 153         0.015115312
## 154         0.006786415
## 155         0.006552290
## 156         0.004272601
## 157         0.005587752
## 158         0.005362801
## 159         0.006347747
## 160         0.003965990
## 161         0.003260961
## 162         0.047373878
## 163         0.013709180
## 164         0.008068169
## 165         0.004527924
## 166         0.012228389
## 167         0.004337255
## 168         0.020350076
## 169         0.006367275
## 170         0.012445977
```

```
## 171         0.038927348
## 172         0.038927348
## 173         0.013354946
## 174         0.002880585
## 175         0.013354946
## 176         0.005447317
## 177         0.004486328
## 178         0.005743384
## 179         0.013168467
## 180         0.009846312
## 181         0.005307292
## 182         0.005441965
## 183         0.118854325
## 184         0.006671231
## 185         0.004892508
## 186         0.009623948
## 187         0.005224927
## 188         0.004748625
## 189         0.010272145
## 190         0.013920363
## 191         0.015867654
## 192         0.016450816
## 193         0.015948092
## 194         0.005741738
## 195         0.005741738
## 196         0.016450816
## 197         0.015948092
## 198         0.014611609
## 199         0.008730115
## 200         0.013429971
## 201         0.039029734
## 202         0.008730115
## 203         0.012499295
## 204         0.012499295
## 205         0.008724965
## 206         0.005379783
## 207         0.006233517
## 208         0.012040431
## 209         0.006233517
## 210         0.006000866
## 211         0.005537018
## 212         0.010874625
## 213         0.008722790
## 214         0.009601031
## 215         0.010585386
## 216         0.013886143
## 217         0.003703577
## 218         0.010377901
## 219         0.007113945
## 220         0.014361988
## 221         0.005800270
## 222         0.013157968
## 223         0.012623822
## 224         0.008455737
```

```
## 225         0.006137896
## 226         0.009964636
## 227         0.004332168
## 228         0.007314865
## 229         0.007856334
## 230         0.006368845
## 231         0.045802556
## 232         0.045802556
## 233         0.008834126
## 234         0.006942294
## 235         0.014390546
## 236         0.009720319
## 237         0.007599918
## 238         0.012673264
## 239         0.006153202
## 240         0.005348831
## 241         0.026551003
## 242         0.009479648
## 243         0.009479648
## 244         0.009438034
## 245         0.015746980
## 246         0.005781693
## 247         0.009408379
## 248         0.006095757
## 249         0.028558323
## 250         0.026248601
## 251         0.011641691
## 252         0.010141486
## 253         0.006726683
## 254         0.006654301
## 255         0.008701215
## 256         0.005024658
## 257         0.004753529
## 258         0.009562148
## 259         0.010455555
## 260         0.013921791
## 261         0.013921791
## 262         0.011758921
## 263         0.011547039
## 264         0.013992321
## 265         0.010405945
## 266         0.012926691
## 267         0.011547039
## 268         0.023843261
## 269         0.006974247
## 270         0.004847490
## 271         0.005092769
## 272         0.020007777
## 273         0.010750765
## 274         0.006550467
## 275         0.023650110
## 276         0.039783183
## 277         0.006765622
## 278         0.007956123
```

```
## 279        0.039783183
## 280        0.007861485
## 281        0.005216551
## 282        0.005167776
## 283        0.003596048
## 284        0.006890454
## 285        0.021423679
## 286        0.005482052
## 287        0.004727162
## 288        0.013054666
## 289        0.007306184
## 290        0.023402575
## 291        0.007871348
## 292        0.010072335
## 293        0.007953246
## 294        0.011574141
## 295        0.012706800
## 296        0.010430451
## 297        0.006622259
## 298        0.012706800
## 299        0.003944442
## 300        0.008553293
## 301        0.003698456
## 302        0.017075068
## 303        0.006622259
## 304        0.005273926
## 305        0.010108172
## 306        0.007040437
## 307        0.010108172
## 308        0.011261060
## 309        0.009360822
## 310        0.011490732
## 311        0.011537910
## 312        0.025358646
## 313        0.010417620
## 314        0.010046370
## 315        0.004379162
## 316        0.003219880
## 317        0.008076105
## 318        0.008076105
## 319        0.004270310
## 320        0.011048877
## 321        0.007651280
## 322        0.019567842
## 323        0.005388295
## 324        0.004917417
## 325        0.013032216
## 326        0.005976044
## 327        0.004078504
## 328        0.007996837
## 329        0.008754485
## 330        0.012433077
## 331        0.009526742
## 332        0.016791331
```

```
## 333          0.008973606
## 334          0.004982638
## 335          0.009640338
## 336          0.007973323
## 337          0.004951452
## 338          0.009118691
## 339          0.006120230
## 340          0.106565971
## 341          0.007306222
## 342          0.009940647
## 343          0.006553434
## 344          0.006943406
## 345          0.003293674
## 346          0.008400204
## 347          0.010253720
## 348          0.020589839
## 349          0.024727263
## 350          0.004409090
## 351          0.004947392
## 352          0.013087617
## 353          0.004409090
## 354          0.020589839
## 355          0.004947392
## 356          0.020551793
## 357          0.020551793
## 358          0.009875256
## 359          0.009846301
## 360          0.008868427
## 361          0.009615620
## 362          0.012181753
## 363          0.008574148
## 364          0.004609210
## 365          0.009893982
## 366          0.009893982
## 367          0.025941965
## 368          0.013859403
## 369          0.011810765
## 370          0.010962999
## 371          0.012043933
## 372          0.007779451
## 373          0.010138358
## 374          0.005522090
## 375          0.013897454
## 376          0.007779723
## 377          0.007779723
## 378          0.014623105
## 379          0.008180976
## 380          0.013897454
## 381          0.005522090
## 382          0.007608308
## 383          0.019157753
## 384          0.012549933
## 385          0.002669846
## 386          0.012289740
```

```
## 387          0.005982028
## 388          0.012811628
## 389          0.018746333
## 390          0.011146179
## 391          0.005947660
## 392          0.005432180
## 393          0.009596262
## 394          0.005086123
## 395          0.003389547
## 396          0.004417586
## 397          0.030825023
## 398          0.025866299
## 399          0.019690144
## 400          0.026132260
## 401          0.029248156
## 402          0.010039624
## 403          0.012731858
## 404          0.007694591
## 405          0.029248156
## 406          0.010039624
## 407          0.013477930
## 408          0.017124899
## 409          0.006930447
## 410          0.009566124
## 411          0.004912524
## 412          0.006744248
## 413          0.008704061
## 414          0.008420771
## 415          0.006231876
## 416          0.010115137
## 417          0.010115137
## 418          0.009559295
## 419          0.006806976
## 420          0.015828703
## 421          0.005841226
## 422          0.025544368
## 423          0.005841226
## 424          0.009191441
## 425          0.018638090
## 426          0.008576398
## 427          0.003954989
## 428          0.006229862
## 429          0.006010567
## 430          0.005674210
## 431          0.019888309
## 432          0.013569976
## 433          0.011202074
## 434          0.010302514
## 435          0.003932326
## 436          0.013615173
## 437          0.008909637
## 438          0.007270941
## 439          0.008210024
## 440          0.007443318
```

```
## 441          0.004748573
## 442          0.015410233
## 443          0.005628613
## 444          0.005628613
## 445          0.002867840
## 446          0.004271219
## 447          0.002867840
## 448          0.007069472
## 449          0.004140436
## 450          0.023923626
## 451          0.012181585
## 452          0.012794476
## 453          0.016363669
## 454          0.015786810
## 455          0.022727770
## 456          0.007195552
## 457          0.007195552
## 458          0.007195552
## 459          0.006907687
## 460          0.001791692
## 461          0.037970561
## 462          0.013166343
## 463          0.064431145
## 464          0.017399250
## 465          0.013168482
## 466          0.012250051
## 467          0.013166343
## 468          0.012702911
## 469          0.008022115
## 470          0.008022115
## 471          0.006411230
## 472          0.004665163
## 473          0.007553608
## 474          0.014709885
## 475          0.008890054
## 476          0.008890054
## 477          0.006281198
## 478          0.006281198
## 479          0.005332526
## 480          0.004426889
## 481          0.013874626
## 482          0.011200291
## 483          0.004629409
## 484          0.006649564
## 485          0.006866976
## 486          0.021461370
## 487          0.006866976
## 488          0.006273689
## 489          0.010854356
## 490          0.014800555
## 491          0.047290280
## 492          0.009511356
## 493          0.009359931
## 494          0.021617918
```

```
## 495        0.004578662
## 496        0.006762835
## 497        0.005876937
## 498        0.018529768
## 499        0.004578662
## 500        0.004729266
## 501        0.007112874
## 502        0.005939253
## 503        0.015524387
## 504        0.004577662
## 505        0.002813320
## 506        0.004385304
## 507        0.018084401
## 508        0.011204375
## 509        0.005966568
## 510        0.006388212
## 511        0.007729637
## 512        0.005196377
## 513        0.007729637
## 514        0.009955767
## 515        0.062110237
## 516        0.003325647
## 517        0.014533859
## 518        0.006651515
## 519        0.006651515
## 520        0.012517657
## 521        0.046096151
## 522        0.014948332
## 523        0.004961432
## 524        0.008680905
## 525        0.005776592
## 526        0.008964746
## 527        0.012297773
## 528        0.006784455
## 529        0.008975675
## 530        0.011444870
## 531        0.010515204
## 532        0.005503159
## 533        0.007021380
## 534        0.005101395
## 535        0.005503159
## 536        0.003212240
## 537        0.003628557
## 538        0.003212240
## 539        0.048494966
## 540        0.011129079
## 541        0.010087377
## 542        0.005922074
## 543        0.005922074
## 544        0.008646773
## 545        0.009862676
## 546        0.005025752
## 547        0.005335595
## 548        0.004264042
```

```
## 549        0.004668932
## 550        0.005471347
## 551        0.008754771
## 552        0.004580484
## 553        0.008754771
## 554        0.015048777
## 555        0.015798171
## 556        0.009343257
## 557        0.010665412
## 558        0.016569613
## 559        0.003670652
## 560        0.008044206
## 561        0.012141367
## 562        0.010582744
## 563        0.007808802
## 564        0.005815284
## 565        0.006908585
## 566        0.006908585
## 567        0.011906876
## 568        0.005876660
## 569        0.010342706
## 570        0.006894703
## 571        0.004542554
## 572        0.008739116
## 573        0.024894062
## 574        0.004867659
## 575        0.008150074
## 576        0.005269406
## 577        0.005914954
## 578        0.018010025
## 579        0.004221806
## 580        0.003050112
## 581        0.007545450
## 582        0.008451368
## 583        0.007954023
## 584        0.005222558
## 585        0.008375444
## 586        0.005558538
## 587        0.009957454
## 588        0.007680146
## 589        0.003830349
## 590        0.016560130
## 591        0.005996954
## 592        0.009899322
## 593        0.005945645
## 594        0.004389160
## 595        0.004096525
## 596        0.007534837
## 597        0.027532005
## 598        0.023159059
## 599        0.023159059
## 600        0.005537560
## 601        0.004678989
## 602        0.009460657
```

```
## 603         0.010083623
## 604         0.007014766
## 605         0.004399480
## 606         0.007014766
## 607         0.005678085
## 608         0.005678085
## 609         0.015898135
## 610         0.015898135
## 611         0.015898135
## 612         0.004278369
## 613         0.010689713
## 614         0.007361671
## 615         0.007292338
## 616         0.006532395
## 617         0.006710544
## 618         0.009943311
## 619         0.006108955
## 620         0.004470924
## 621         0.011588365
## 622         0.008686844
## 623         0.013956713
## 624         0.005593725
## 625         0.004229023
## 626         0.005172338
## 627         0.006156458
## 628         0.018335219
## 629         0.007604259
## 630         0.006330111
## 631         0.007604259
## 632         0.007962758
## 633         0.004663281
## 634         0.005123541
## 635         0.004663281
## 636         0.005123541
## 637         0.005592100
## 638         0.005072712
## 639         0.005072712
## 640         0.011886791
## 641         0.005127564
## 642         0.006409780
## 643         0.005798826
## 644         0.013979715
## 645         0.029481967
## 646         0.005616815
## 647         0.004899745
## 648         0.015101840
## 649         0.027337598
## 650         0.004858245
## 651         0.014737887
## 652         0.003340743
## 653         0.014737887
## 654         0.005127724
## 655         0.022672262
## 656         0.025034732
```

```
## 657          0.004624759
## 658          0.005127724
## 659          0.006174217
## 660          0.004267040
## 661          0.006067928
## 662          0.009680245
## 663          0.006573470
## 664          0.006853510
## 665          0.014892666
## 666          0.002548746
## 667          0.006516621
## 668          0.009393250
## 669          0.007482932
## 670          0.008340855
## 671          0.008340855
## 672          0.007482932
## 673          0.004246215
## 674          0.009124761
## 675          0.006547528
## 676          0.005937867
## 677          0.005151231
## 678          0.006096341
## 679          0.004860436
## 680          0.005465453
## 681          0.006096341
## 682          0.008863624
## 683          0.008471542
## 684          0.008266779
## 685          0.010812274
## 686          0.008678351
## 687          0.008965891
## 688          0.007292361
## 689          0.002747866
## 690          0.007653995
## 691          0.023051842
## 692          0.002747866
## 693          0.004804278
## 694          0.010378502
## 695          0.009155375
## 696          0.006392998
## 697          0.009155375
## 698          0.008920039
## 699          0.008920039
## 700          0.002764909
## 701          0.019219638
## 702          0.011802889
## 703          0.004135039
## 704          0.005030545
## 705          0.009109378
## 706          0.005546230
## 707          0.005321142
## 708          0.005124043
## 709          0.008621481
## 710          0.005039610
```

```
## 711          0.010650066
## 712          0.004685276
## 713          0.006446435
## 714          0.005116012
## 715          0.037193878
## 716          0.003705626
## 717          0.012398413
## 718          0.012398413
## 719          0.004613892
## 720          0.008728782
## 721          0.004418403
## 722          0.006891000
## 723          0.007360142
## 724          0.007540516
## 725          0.004418403
## 726          0.007561172
## 727          0.003902842
## 728          0.004367703
## 729          0.010138885
## 730          0.013850561
## 731          0.003970697
## 732          0.003302993
## 733          0.008382766
## 734          0.011754239
## 735          0.010450933
## 736          0.004613816
## 737          0.003276449
## 738          0.002617161
## 739          0.047571012
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## 741          0.006233330
## 742          0.008664132
## 743          0.008826964
## 744          0.008977899
## 745          0.009641035
## 746          0.008826964
## 747          0.008784370
## 748          0.006763227
## 749          0.004798463
## 750          0.008005156
## 751          0.007573492
## 752          0.002551232
## 753          0.006226865
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## 755          0.006037430
## 756          0.007769967
## 757          0.021496085
## 758          0.004195011
## 759          0.006913759
## 760          0.019281406
## 761          0.089029480
## 762          0.095333756
## 763          0.006697048
## 764          0.007236448
```

```
## 765        0.017810722
## 766        0.007846673
## 767        0.006942330
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## 769        0.009289638
## 770        0.018742820
## 771        0.004364177
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## 776        0.005412347
## 777        0.010235618
## 778        0.003822216
## 779        0.003822216
## 780        0.005802851
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## 782        0.004998834
## 783        0.005931687
## 784        0.009606222
## 785        0.006697282
## 786        0.014417906
## 787        0.007216853
## 788        0.027627730
## 789        0.004558150
## 790        0.004558150
## 791        0.004558150
## 792        0.012757563
## 793        0.009191437
## 794        0.012024577
## 795        0.010704968
## 796        0.007835479
## 797        0.007625080
## 798        0.005199395
## 799        0.010553850
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## 801        0.010452715
## 802        0.004665440
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## 804        0.004674711
## 805        0.011672394
## 806        0.004438053
## 807        0.006514514
## 808        0.003629586
## 809        0.006709971
## 810        0.012330594
## 811        0.007194722
## 812        0.012898719
## 813        0.009701620
## 814        0.013093002
## 815        0.003990406
## 816        0.006197804
## 817        0.007483683
## 818        0.004173496
```

```
## 819          0.002737305
## 820          0.004173496
## 821          0.014153490
## 822          0.016977571
## 823          0.006346128
## 824          0.005572442
## 825          0.007641965
## 826          0.011957933
## 827          0.003436975
## 828          0.003748221
## 829          0.005517165
## 830          0.006234082
## 831          0.006234082
## 832          0.017923817
## 833          0.014336139
## 834          0.018309636
## 835          0.003222426
## 836          0.008037051
## 837          0.004086144
## 838          0.013385955
## 839          0.013385955
## 840          0.011413265
## 841          0.009772217
## 842          0.012428795
## 843          0.008200358
## 844          0.004964455
## 845          0.005907860
## 846          0.007866858
## 847          0.009381935
## 848          0.004291227
## 849          0.007866858
## 850          0.005777311
## 851          0.003882772
## 852          0.013347722
## 853          0.007340511
## 854          0.007340511
## 855          0.017916361
## 856          0.007340511
## 857          0.010277925
## 858          0.003905776
## 859          0.003905776
## 860          0.006995281
## 861          0.005288668
## 862          0.008493557
## 863          0.008638927
## 864          0.008638927
## 865          0.008233760
## 866          0.010019829
## 867          0.017050344
## 868          0.002621911
## 869          0.020759346
## 870          0.003511985
## 871          0.007124679
## 872          0.014832368
```

```
## 873         0.003511985
## 874         0.012187564
## 875         0.005997235
## 876         0.012187564
## 877         0.004994217
## 878         0.007185641
## 879         0.015181362
## 880         0.006830291
## 881         0.002563129
## 882         0.008144575
## 883         0.004273200
## 884         0.003339793
## 885         0.005519616
## 886         0.009700822
## 887         0.003826460
## 888         0.020086082
## 889         0.006830141
## 890         0.005634777
## 891         0.046122909
## 892         0.013010326
## 893         0.006951249
## 894         0.006126386
## 895         0.002497264
## 896         0.002497264
## 897         0.007575138
## 898         0.006021742
## 899         0.018905110
## 900         0.005276989
## 901         0.006283172
## 902         0.015716002
## 903         0.005107362
## 904         0.004848527
## 905         0.015368149
## 906         0.005228561
## 907         0.004848527
## 908         0.007584605
## 909         0.003365236
## 910         0.003365236
## 911         0.005236537
## 912         0.001877579
## 913         0.002849964
## 914         0.007120236
## 915         0.026538314
## 916         0.026538314
## 917         0.005422678
## 918         0.004734643
## 919         0.007231775
## 920         0.021971806
## 921         0.005169192
## 922         0.007628939
## 923         0.047768638
## 924         0.011494110
## 925         0.009422035
## 926         0.004957766
```

```
## 927          0.009577336
## 928          0.003879554
## 929          0.005450121
## 930          0.005267120
## 931          0.008242147
## 932          0.021044316
## 933          0.004681003
## 934          0.033641770
## 935          0.054385943
## 936          0.029995529
## 937          0.007875178
## 938          0.003027281
## 939          0.003027281
## 940          0.003027281
## 941          0.003027281
## 942          0.004345945
## 943          0.004385165
## 944          0.004385165
## 945          0.012743047
## 946          0.011224403
## 947          0.004479022
## 948          0.009402272
## 949          0.008177478
## 950          0.006777982
## 951          0.006777982
## 952          0.006777982
## 953          0.004575394
## 954          0.004575394
## 955          0.004575394
## 956          0.004575394
## 957          0.006582772
## 958          0.002768126
## 959          0.004827404
## 960          0.005468801
## 961          0.008940215
## 962          0.005699530
## 963          0.009227998
## 964          0.009022129
## 965          0.007243487
## 966          0.020860507
## 967          0.009449523
## 968          0.004216894
## 969          0.009449523
## 970          0.006914380
## 971          0.004572422
## 972          0.025291476
## 973          0.006120700
## 974          0.050782777
## 975          0.016836973
## 976          0.050782777
## 977          0.012388005
## 978          0.004291488
## 979          0.003876094
## 980          0.003876094
```

```
## 981        0.005562992
## 982        0.005562992
## 983        0.006838611
## 984        0.011306016
## 985        0.003407028
## 986        0.014139366
## 987        0.008846999
## 988        0.002977304
## 989        0.008650269
## 990        0.002457176
## 991        0.004277336
## 992        0.004265547
## 993        0.006642425
## 994        0.004227717
## 995        0.010434008
## 996        0.008498004
## 997        0.016563469
## 998        0.007719376
## 999        0.013094985
## 1000       0.014300420
## 1001       0.014296550
## 1002       0.014300420
## 1003       0.011699119
## 1004       0.010026880
## 1005       0.013094985
## 1006       0.013380879
## 1007       0.008564196
## 1008       0.009311933
## 1009       0.008564196
## 1010       0.006010877
## 1011       0.009466940
## 1012       0.009754534
## 1013       0.009466940
## 1014       0.004333159
## 1015       0.012976820
## 1016       0.006887978
## 1017       0.004163175
## 1018       0.007270451
## 1019       0.012229035
## 1020       0.002137596
## 1021       0.004602682
## 1022       0.005219989
## 1023       0.100657718
## 1024       0.011401571
## 1025       0.017162004
## 1026       0.005148163
## 1027       0.007450408
## 1028       0.005529905
## 1029       0.011039110
## 1030       0.005545873
## 1031       0.006414147
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## 1033       0.007273566
## 1034       0.010897408
```

```
## 1035       0.007933250
## 1036       0.008118296
## 1037       0.006720748
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## 1041       0.006796964
## 1042       0.003637300
## 1043       0.007267395
## 1044       0.010331102
## 1045       0.013004286
## 1046       0.010331102
## 1047       0.009725129
## 1048       0.016651301
## 1049       0.009556038
## 1050       0.010965160
## 1051       0.004121891
## 1052       0.080368069
## 1053       0.017867082
## 1054       0.080368069
## 1055       0.018006516
## 1056       0.013056016
## 1057       0.010954447
## 1058       0.008572701
## 1059       0.010954447
## 1060       0.006161560
## 1061       0.003469087
## 1062       0.004306460
## 1063       0.005670430
## 1064       0.009149252
## 1065       0.010275511
## 1066       0.006373657
## 1067       0.013400156
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## 1069       0.006314284
## 1070       0.004778359
## 1071       0.004778359
## 1072       0.011772503
## 1073       0.008776780
## 1074       0.011901712
## 1075       0.006302878
## 1076       0.005612852
## 1077       0.011901712
## 1078       0.023428407
## 1079       0.007392169
## 1080       0.008050690
## 1081       0.004495425
## 1082       0.011886690
## 1083       0.011819245
## 1084       0.007532963
## 1085       0.006979356
## 1086       0.003694112
## 1087       0.010434640
## 1088       0.004252869
```

```
## 1089       0.004228476
## 1090       0.004648992
## 1091       0.005828117
## 1092       0.012049071
## 1093       0.004107560
## 1094       0.005915030
## 1095       0.007033434
## 1096       0.007600662
## 1097       0.003315827
## 1098       0.002520473
## 1099       0.006012693
## 1100       0.007989529
## 1101       0.006163778
## 1102       0.009398812
## 1103       0.003385163
## 1104       0.005424882
## 1105       0.005053118
## 1106       0.005072068
## 1107       0.007527427
## 1108       0.004830059
## 1109       0.003093297
## 1110       0.007083670
## 1111       0.006426557
## 1112       0.042366306
## 1113       0.009048258
## 1114       0.009048258
## 1115       0.009048258
## 1116       0.004001436
## 1117       0.004001436
## 1118       0.006964141
## 1119       0.015697848
## 1120       0.004001436
## 1121       0.005027675
## 1122       0.003300393
## 1123       0.018848437
## 1124       0.004737355
## 1125       0.008976918
## 1126       0.007382903
## 1127       0.007546168
## 1128       0.009940292
## 1129       0.009669513
## 1130       0.004433224
## 1131       0.004351128
## 1132       0.006943704
## 1133       0.007538937
## 1134       0.011571489
## 1135       0.008082932
## 1136       0.004367800
## 1137       0.008429891
## 1138       0.008319370
## 1139       0.007128400
## 1140       0.008351917
## 1141       0.006225574
## 1142       0.010764761
```

```
## 1143      0.008912159
```

```r
hats[order(hats['hatvalues(wineq)']), ]
```

```
## Warning in xtfrm.data.frame(x): cannot xtfrm data frames
```

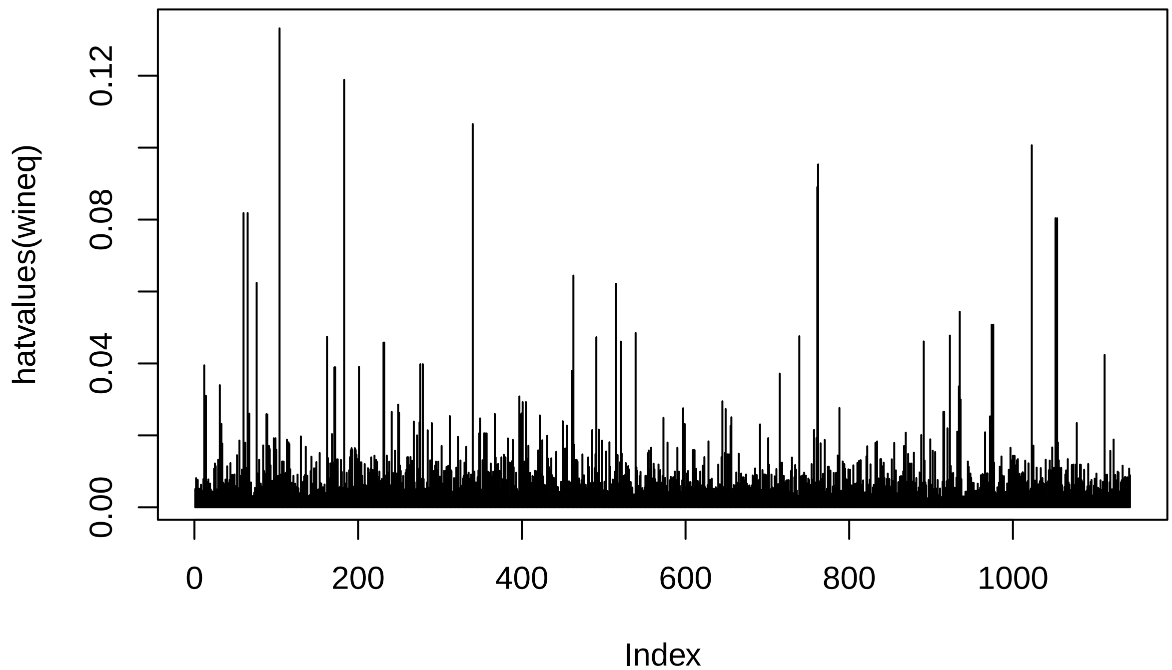
```
##    [1] 0.001791692 0.001877579 0.002094741 0.002137596 0.002198632 0.002457176
##    [7] 0.002497264 0.002497264 0.002520473 0.002548746 0.002551232 0.002563129
##   [13] 0.002617161 0.002621911 0.002624015 0.002627188 0.002636436 0.002669846
##   [19] 0.002737305 0.002747866 0.002747866 0.002764909 0.002768126 0.002813320
##   [25] 0.002849964 0.002867840 0.002867840 0.002880585 0.002939521 0.002977304
##   [31] 0.003027281 0.003027281 0.003027281 0.003027281 0.003050112 0.003093297
##   [37] 0.003114941 0.003114941 0.003190962 0.003190962 0.003212240 0.003212240
##   [43] 0.003219880 0.003222426 0.003247586 0.003260961 0.003276449 0.003293674
##   [49] 0.003300393 0.003302993 0.003315827 0.003325647 0.003339793 0.003340743
##   [55] 0.003365236 0.003365236 0.003385163 0.003389547 0.003397219 0.003407028
##   [61] 0.003420589 0.003436975 0.003469087 0.003475888 0.003491123 0.003511985
##   [67] 0.003511985 0.003596048 0.003620478 0.003620478 0.003628557 0.003629586
##   [73] 0.003637300 0.003667121 0.003670652 0.003694112 0.003698456 0.003703577
##   [79] 0.003705626 0.003748221 0.003784830 0.003822216 0.003822216 0.003826460
##   [85] 0.003830349 0.003852312 0.003876094 0.003876094 0.003879554 0.003882772
##   [91] 0.003894204 0.003902842 0.003903896 0.003903896 0.003905776 0.003905776
##   [97] 0.003932326 0.003944442 0.003954989 0.003965990 0.003970697 0.003990406
##  [103] 0.004001436 0.004001436 0.004001436 0.004019111 0.004023940 0.004031028
##  [109] 0.004031028 0.004078504 0.004086144 0.004086631 0.004096525 0.004107560
##  [115] 0.004121891 0.004135039 0.004140436 0.004163175 0.004173496 0.004173496
##  [121] 0.004195011 0.004199146 0.004216894 0.004221806 0.004227717 0.004228476
##  [127] 0.004229023 0.004246215 0.004252869 0.004264042 0.004265547 0.004267040
##  [133] 0.004270310 0.004271219 0.004272601 0.004273200 0.004277336 0.004278369
##  [139] 0.004291227 0.004291488 0.004306460 0.004332168 0.004333159 0.004337255
##  [145] 0.004345945 0.004351128 0.004364177 0.004367703 0.004367800 0.004376642
##  [151] 0.004379162 0.004385165 0.004385165 0.004385304 0.004389160 0.004399480
##  [157] 0.004409090 0.004409090 0.004417586 0.004418403 0.004418403 0.004424527
##  [163] 0.004426889 0.004433224 0.004438053 0.004464620 0.004470924 0.004479022
##  [169] 0.004486328 0.004495425 0.004527924 0.004542554 0.004558150 0.004558150
##  [175] 0.004558150 0.004572422 0.004575394 0.004575394 0.004575394 0.004575394
##  [181] 0.004577662 0.004578662 0.004578662 0.004580484 0.004587509 0.004602682
##  [187] 0.004609210 0.004613816 0.004613892 0.004624759 0.004629409 0.004648992
##  [193] 0.004663281 0.004663281 0.004665163 0.004665440 0.004668932 0.004674711
##  [199] 0.004678989 0.004681003 0.004685276 0.004727162 0.004729266 0.004734643
##  [205] 0.004737355 0.004748573 0.004748625 0.004753529 0.004778359 0.004778359
##  [211] 0.004798463 0.004804278 0.004811525 0.004827404 0.004830059 0.004847490
##  [217] 0.004848527 0.004848527 0.004855619 0.004858245 0.004860436 0.004867659
##  [223] 0.004892508 0.004899745 0.004905335 0.004912524 0.004917417 0.004947392
##  [229] 0.004947392 0.004951452 0.004957766 0.004961432 0.004964455 0.004965231
##  [235] 0.004968706 0.004982638 0.004994217 0.004998834 0.005009245 0.005009256
##  [241] 0.005024658 0.005025752 0.005027675 0.005030545 0.005039610 0.005053118
##  [247] 0.005072068 0.005072712 0.005072712 0.005079901 0.005079901 0.005086123
##  [253] 0.005091483 0.005092769 0.005101395 0.005104983 0.005107362 0.005111624
##  [259] 0.005116012 0.005123541 0.005123541 0.005124043 0.005127564 0.005127724
##  [265] 0.005127724 0.005132741 0.005148163 0.005151231 0.005167776 0.005169192
##  [271] 0.005172338 0.005188055 0.005195357 0.005196377 0.005199395 0.005216551
##  [277] 0.005219989 0.005222558 0.005224927 0.005228561 0.005236537 0.005267120
```

```
## [283] 0.005269406 0.005273926 0.005276989 0.005288668 0.005299619 0.005307292
## [289] 0.005321142 0.005332526 0.005335595 0.005348206 0.005348831 0.005352136
## [295] 0.005353649 0.005362801 0.005379783 0.005388295 0.005401992 0.005412347
## [301] 0.005422678 0.005424882 0.005432180 0.005441965 0.005447317 0.005450121
## [307] 0.005465453 0.005468801 0.005471347 0.005482052 0.005503159 0.005503159
## [313] 0.005517165 0.005519616 0.005522090 0.005522090 0.005529905 0.005537018
## [319] 0.005537560 0.005545873 0.005546230 0.005558538 0.005562992 0.005562992
## [325] 0.005572442 0.005587752 0.005592100 0.005593725 0.005612852 0.005616815
## [331] 0.005626477 0.005628613 0.005628613 0.005634777 0.005670430 0.005674210
## [337] 0.005678085 0.005678085 0.005699530 0.005738819 0.005741738 0.005741738
## [343] 0.005743384 0.005776592 0.005777311 0.005781693 0.005798826 0.005800270
## [349] 0.005802851 0.005807121 0.005808918 0.005815284 0.005828117 0.005841226
## [355] 0.005841226 0.005876660 0.005876937 0.005907860 0.005914954 0.005915030
## [361] 0.005917860 0.005920293 0.005922074 0.005922074 0.005931687 0.005937867
## [367] 0.005939253 0.005945645 0.005947660 0.005966568 0.005976044 0.005982028
## [373] 0.005996954 0.005997235 0.006000866 0.006010567 0.006010877 0.006012693
## [379] 0.006021742 0.006037430 0.006067928 0.006095757 0.006096341 0.006096341
## [385] 0.006103196 0.006108955 0.006112586 0.006120230 0.006120700 0.006126386
## [391] 0.006137896 0.006153202 0.006156458 0.006161560 0.006163778 0.006174217
## [397] 0.006197804 0.006225574 0.006226865 0.006229862 0.006231876 0.006233330
## [403] 0.006233517 0.006233517 0.006234082 0.006234082 0.006273689 0.006281198
## [409] 0.006281198 0.006281831 0.006283172 0.006302878 0.006314284 0.006330111
## [415] 0.006346128 0.006347747 0.006367275 0.006368845 0.006373657 0.006388212
## [421] 0.006392998 0.006409780 0.006411230 0.006414147 0.006426557 0.006436876
## [427] 0.006444366 0.006446435 0.006487944 0.006489302 0.006514514 0.006516621
## [433] 0.006532395 0.006547528 0.006550467 0.006552290 0.006553434 0.006573470
## [439] 0.006578860 0.006582772 0.006613036 0.006613036 0.006620674 0.006622259
## [445] 0.006622259 0.006623544 0.006642425 0.006649564 0.006651515 0.006651515
## [451] 0.006654301 0.006671231 0.006697048 0.006697282 0.006709971 0.006710544
## [457] 0.006720748 0.006725434 0.006726683 0.006744248 0.006762835 0.006763227
## [463] 0.006765622 0.006777982 0.006777982 0.006777982 0.006784455 0.006786415
## [469] 0.006796964 0.006802741 0.006802741 0.006806976 0.006811906 0.006830141
## [475] 0.006830291 0.006838611 0.006842187 0.006853510 0.006858603 0.006866976
## [481] 0.006866976 0.006880550 0.006887978 0.006890454 0.006891000 0.006894703
## [487] 0.006907687 0.006908585 0.006908585 0.006913126 0.006913759 0.006914380
## [493] 0.006924950 0.006930447 0.006932116 0.006942294 0.006942330 0.006943406
## [499] 0.006943704 0.006951249 0.006964141 0.006974247 0.006979356 0.006995281
## [505] 0.007014766 0.007014766 0.007021380 0.007029088 0.007033434 0.007040437
## [511] 0.007069472 0.007083670 0.007112874 0.007113945 0.007120236 0.007124679
## [517] 0.007128400 0.007178859 0.007185641 0.007194722 0.007195552 0.007195552
## [523] 0.007195552 0.007216853 0.007231775 0.007236448 0.007243487 0.007267395
## [529] 0.007270451 0.007270941 0.007273566 0.007292338 0.007292361 0.007306184
## [535] 0.007306222 0.007314865 0.007340511 0.007340511 0.007340511 0.007359562
## [541] 0.007360142 0.007361671 0.007382903 0.007392169 0.007443318 0.007450408
## [547] 0.007482932 0.007482932 0.007483683 0.007527427 0.007532963 0.007534837
## [553] 0.007538937 0.007540516 0.007545450 0.007546168 0.007553608 0.007561172
## [559] 0.007573492 0.007575138 0.007584605 0.007599918 0.007600662 0.007604259
## [565] 0.007604259 0.007608308 0.007623793 0.007625080 0.007628939 0.007631385
## [571] 0.007641965 0.007651280 0.007653995 0.007680146 0.007694591 0.007719376
## [577] 0.007729637 0.007729637 0.007741004 0.007758928 0.007769967 0.007779451
## [583] 0.007779723 0.007779723 0.007808802 0.007835479 0.007846673 0.007856334
## [589] 0.007861485 0.007865355 0.007866858 0.007866858 0.007867479 0.007868277
## [595] 0.007871348 0.007875178 0.007933250 0.007953246 0.007954023 0.007956123
## [601] 0.007962758 0.007973323 0.007989529 0.007996837 0.008005156 0.008022115
```

```
## [607] 0.008022115 0.008037051 0.008044206 0.008050690 0.008068169 0.008076105
## [613] 0.008076105 0.008082932 0.008111271 0.008118296 0.008144575 0.008150074
## [619] 0.008154838 0.008177478 0.008180976 0.008200358 0.008210024 0.008233760
## [625] 0.008242147 0.008266779 0.008319370 0.008340855 0.008340855 0.008351917
## [631] 0.008375444 0.008382766 0.008400204 0.008420771 0.008429891 0.008451368
## [637] 0.008455737 0.008471542 0.008493557 0.008498004 0.008553293 0.008564196
## [643] 0.008564196 0.008572701 0.008574148 0.008576398 0.008580822 0.008599023
## [649] 0.008621481 0.008638927 0.008638927 0.008646773 0.008650269 0.008664132
## [655] 0.008678351 0.008680905 0.008686844 0.008701215 0.008704061 0.008720091
## [661] 0.008722790 0.008724965 0.008728782 0.008730115 0.008730115 0.008739116
## [667] 0.008743930 0.008754485 0.008754605 0.008754771 0.008754771 0.008761552
## [673] 0.008776780 0.008784370 0.008817519 0.008826964 0.008826964 0.008834126
## [679] 0.008846999 0.008863624 0.008868427 0.008874103 0.008890054 0.008890054
## [685] 0.008909637 0.008912159 0.008920039 0.008920039 0.008940215 0.008959370
## [691] 0.008964746 0.008965891 0.008973606 0.008975675 0.008976918 0.008977899
## [697] 0.008984319 0.008984319 0.009022129 0.009048258 0.009048258 0.009048258
## [703] 0.009100582 0.009102351 0.009109378 0.009118347 0.009118347 0.009118691
## [709] 0.009124761 0.009143397 0.009149252 0.009155375 0.009155375 0.009191437
## [715] 0.009191441 0.009227998 0.009289638 0.009289638 0.009311933 0.009343257
## [721] 0.009359931 0.009360822 0.009381146 0.009381935 0.009393250 0.009398812
## [727] 0.009402272 0.009408379 0.009422035 0.009438034 0.009445839 0.009449523
## [733] 0.009449523 0.009460657 0.009466940 0.009466940 0.009479648 0.009479648
## [739] 0.009480488 0.009511356 0.009526742 0.009555612 0.009556038 0.009559295
## [745] 0.009562148 0.009566124 0.009577336 0.009596262 0.009601031 0.009606222
## [751] 0.009615620 0.009623948 0.009640338 0.009641035 0.009656767 0.009669513
## [757] 0.009680245 0.009700822 0.009701620 0.009720319 0.009725129 0.009754534
## [763] 0.009772217 0.009846301 0.009846312 0.009862676 0.009875256 0.009893982
## [769] 0.009893982 0.009899322 0.009940292 0.009940647 0.009943311 0.009955767
## [775] 0.009957454 0.009964636 0.010019829 0.010026880 0.010039624 0.010039624
## [781] 0.010046370 0.010072335 0.010083623 0.010087377 0.010090365 0.010101226
## [787] 0.010108172 0.010108172 0.010115137 0.010115137 0.010138358 0.010138885
## [793] 0.010141486 0.010235618 0.010253720 0.010272145 0.010275511 0.010277925
## [799] 0.010285306 0.010302514 0.010331102 0.010331102 0.010342706 0.010377901
## [805] 0.010378502 0.010405945 0.010417620 0.010430451 0.010434008 0.010434640
## [811] 0.010450933 0.010452715 0.010455555 0.010479424 0.010515204 0.010553850
## [817] 0.010572574 0.010582744 0.010585386 0.010650066 0.010665412 0.010689713
## [823] 0.010704968 0.010734583 0.010750765 0.010764761 0.010812274 0.010819629
## [829] 0.010819629 0.010854356 0.010874625 0.010897408 0.010954447 0.010954447
## [835] 0.010962999 0.010965160 0.011034703 0.011039110 0.011048877 0.011129079
## [841] 0.011146179 0.011200291 0.011202074 0.011204375 0.011224403 0.011261060
## [847] 0.011283793 0.011283793 0.011306016 0.011401571 0.011403010 0.011413265
## [853] 0.011429170 0.011444870 0.011490732 0.011494110 0.011537910 0.011547039
## [859] 0.011547039 0.011571489 0.011574141 0.011588365 0.011641691 0.011667638
## [865] 0.011672394 0.011699119 0.011754239 0.011758921 0.011772503 0.011802889
## [871] 0.011810765 0.011819245 0.011886690 0.011886791 0.011901712 0.011901712
## [877] 0.011906876 0.011957933 0.012010198 0.012024577 0.012040431 0.012043933
## [883] 0.012049071 0.012141367 0.012181585 0.012181753 0.012187564 0.012187564
## [889] 0.012192743 0.012228389 0.012229035 0.012250051 0.012289740 0.012291719
## [895] 0.012297773 0.012330594 0.012388005 0.012398413 0.012398413 0.012428795
## [901] 0.012433077 0.012445977 0.012499295 0.012499295 0.012517657 0.012549933
## [907] 0.012565433 0.012623822 0.012673264 0.012702911 0.012706800 0.012706800
## [913] 0.012727589 0.012727589 0.012731858 0.012743047 0.012756503 0.012757563
## [919] 0.012794476 0.012811628 0.012898719 0.012926691 0.012976820 0.013004286
## [925] 0.013010326 0.013032216 0.013054666 0.013056016 0.013087617 0.013093002
```

```
##  [931] 0.013094985 0.013094985 0.013157968 0.013166343 0.013166343 0.013168467
##  [937] 0.013168482 0.013188200 0.013214273 0.013228564 0.013347722 0.013354946
##  [943] 0.013354946 0.013380879 0.013385955 0.013385955 0.013400156 0.013429971
##  [949] 0.013477930 0.013569976 0.013615173 0.013709180 0.013850561 0.013859403
##  [955] 0.013874626 0.013886143 0.013897454 0.013897454 0.013920363 0.013921791
##  [961] 0.013921791 0.013956713 0.013979715 0.013992321 0.014083877 0.014139366
##  [967] 0.014153490 0.014296550 0.014300420 0.014300420 0.014336139 0.014361988
##  [973] 0.014390546 0.014417906 0.014484615 0.014533859 0.014611609 0.014623105
##  [979] 0.014709885 0.014737887 0.014737887 0.014800555 0.014832368 0.014892666
##  [985] 0.014948332 0.015048777 0.015101840 0.015115312 0.015181362 0.015368149
##  [991] 0.015410233 0.015524387 0.015697848 0.015716002 0.015746980 0.015786810
##  [997] 0.015798171 0.015828703 0.015867654 0.015898135 0.015898135 0.015898135
## [1003] 0.015948092 0.015948092 0.016013518 0.016196634 0.016363669 0.016450816
## [1009] 0.016450816 0.016560130 0.016563469 0.016569613 0.016651301 0.016791331
## [1015] 0.016836973 0.016851147 0.016977571 0.017050344 0.017073414 0.017075068
## [1021] 0.017124899 0.017162004 0.017216599 0.017399250 0.017540038 0.017739270
## [1027] 0.017810722 0.017867082 0.017916361 0.017923817 0.017934258 0.018006516
## [1033] 0.018010025 0.018021960 0.018054168 0.018084401 0.018309636 0.018335219
## [1039] 0.018529768 0.018567518 0.018638090 0.018742820 0.018746333 0.018813497
## [1045] 0.018848437 0.018905110 0.019157753 0.019198679 0.019198679 0.019219638
## [1051] 0.019281406 0.019567842 0.019690144 0.019719613 0.019888309 0.020007777
## [1057] 0.020086082 0.020350076 0.020551793 0.020551793 0.020589839 0.020589839
## [1063] 0.020759346 0.020860507 0.021044316 0.021423679 0.021461370 0.021496085
## [1069] 0.021617918 0.021971806 0.022672262 0.022727770 0.023051842 0.023159059
## [1075] 0.023159059 0.023179000 0.023402575 0.023428407 0.023650110 0.023843261
## [1081] 0.023923626 0.024727263 0.024894062 0.025034732 0.025291476 0.025358646
## [1087] 0.025544368 0.025814195 0.025866299 0.025899899 0.025941965 0.026058577
## [1093] 0.026132260 0.026248601 0.026538314 0.026538314 0.026551003 0.027337598
## [1099] 0.027532005 0.027627730 0.028558323 0.029248156 0.029248156 0.029481967
## [1105] 0.029995529 0.030825023 0.031016001 0.033641770 0.033958794 0.037193878
## [1111] 0.037970561 0.038927348 0.038927348 0.039029734 0.039475498 0.039783183
## [1117] 0.039783183 0.042366306 0.045802556 0.045802556 0.046096151 0.046122909
## [1123] 0.047290280 0.047373878 0.047571012 0.047768638 0.048494966 0.050782777
## [1129] 0.050782777 0.054385943 0.062110237 0.062431247 0.064431145 0.080368069
## [1135] 0.080368069 0.081831903 0.081831903 0.089029480 0.095333756 0.100657718
## [1141] 0.106565971 0.118854325 0.133177620
```

```r
plot(hatvalues(wineq), type = 'h')
```




```r
# New Regression Model
wineq2 <- lm(quality ~ volatile.acidity + chlorides + free.sulfur.dioxide + total.sulfur.dioxide + pH +
summary(wineq2)
```

```
##
## Call:
## lm(formula = quality ~ volatile.acidity + chlorides + free.sulfur.dioxide +
##     total.sulfur.dioxide + pH + sulphates + alcohol, data = Wine_Quality)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.39463 -0.36932 -0.04649  0.44290  2.00640
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           4.466534   0.461804   9.672  < 2e-16 ***
## volatile.acidity     -1.082354   0.117570  -9.206  < 2e-16 ***
## chlorides            -1.837430   0.465909  -3.944 8.51e-05 ***
## free.sulfur.dioxide   0.002845   0.002510   1.134 0.257199
## total.sulfur.dioxide -0.002937   0.000796  -3.689 0.000236 ***
## pH                   -0.485507   0.135380  -3.586 0.000350 ***
## sulphates             0.845081   0.128332   6.585 6.93e-11 ***
## alcohol               0.293758   0.019382  15.156  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.6398 on 1135 degrees of freedom
## Multiple R-squared:  0.3736, Adjusted R-squared:  0.3697
## F-statistic: 96.69 on 7 and 1135 DF,  p-value: < 2.2e-16
```

Question 4

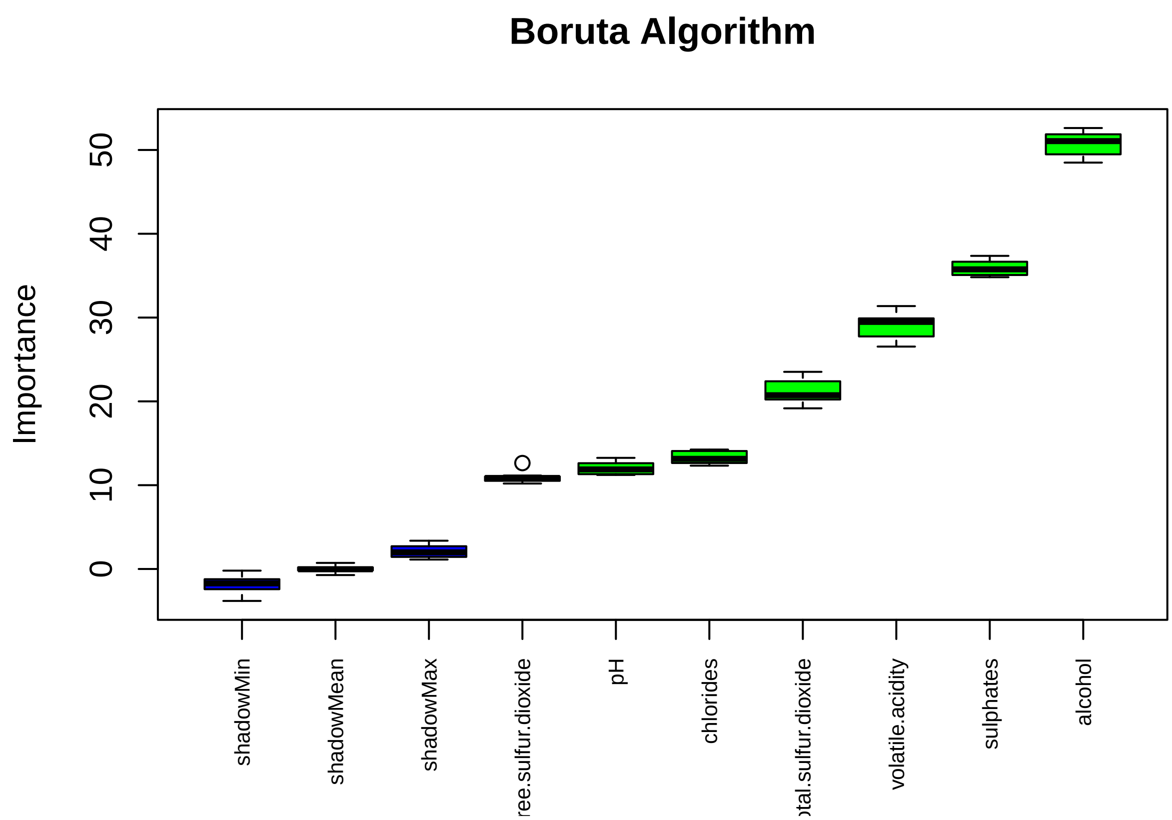
Boruta Algorithm

We first used the Boruta Algorithm to filter out which response variables were insignificant before also conducting the Mallows CP test. With the Boruta Algorithm we were able to see that alcohol was the best variable, as it had the highest importance. Also, we could eliminate variables that reported "rejected" from this test which meant they had no significance. Next, we took only the results of the significant variables from the Boruta algorithm and ran the Mallows CP test. Here, the smaller the value, the better the model. Evidently, the best model had the predictors volatile acidity, chlorides, total sulfur dioxide, sulphates, and alcohol. The models to the right of the model are higher and therefore, insignificant. These two tests helped us create a better model by determining and eliminating what variables were insignificant.

```r
library(Boruta)
# Uses Boruta() for Boruta algorithm process
Bor.res <- Boruta(quality ~ volatile.acidity + chlorides + free.sulfur.dioxide + total.sulfur.dioxide +
```

```
##  1. run of importance source...

##  2. run of importance source...

##  3. run of importance source...

##  4. run of importance source...

##  5. run of importance source...

##  6. run of importance source...

##  7. run of importance source...

##  8. run of importance source...

##  9. run of importance source...

##  10. run of importance source...

## After 10 iterations, +4.9 secs:

##  confirmed 7 attributes: alcohol, chlorides, free.sulfur.dioxide, pH, sulphates and 2 more;

##  no more attributes left.
```

```
plot(Bor.res, xlab = "", xaxt = "n", main="Boruta Algorithm")
Lz <- lapply(1:ncol(Bor.res$ImpHistory),function(i) Bor.res$ImpHistory[is.finite(Bor.res$ImpHistory[,i]
names (Lz) <- colnames(Bor.res$ImpHistory)
Labels <- sort(sapply(Lz,median))
axis(side = 1,las=2,labels = names(Labels),
at = 1:ncol(Bor.res$ImpHistory), cex.axis = 0.7)
```




```
boruta_signif <- names(Bor.res$finalDecision[Bor.res$finalDecision %in% c("Confirmed", "Tentative")])
boruta_signif_Conf <- names(Bor.res$finalDecision[Bor.res$finalDecision %in% c("Confirmed")])
boruta_signif_Tent <- names(Bor.res$finalDecision[Bor.res$finalDecision %in% c("Tentative")])
boruta_signif_Reject <- names(Bor.res$finalDecision[Bor.res$finalDecision %in% c("Rejected")])
print(boruta_signif_Conf)
```

```
## [1] "alcohol"              "sulphates"            "pH"
## [4] "total.sulfur.dioxide" "free.sulfur.dioxide"  "chlorides"
## [7] "volatile.acidity"
```

```
attStats(Bor.res)
```

```
##                       meanImp medianImp   minImp   maxImp normHits  decision
## alcohol              50.75096  51.05660 48.49180 52.61674        1 Confirmed
## sulphates            35.89367  35.75507 34.81560 37.36271        1 Confirmed
## pH                   12.03961  11.88174 11.21923 13.25784        1 Confirmed
## total.sulfur.dioxide 21.19278  20.73419 19.17124 23.52463        1 Confirmed
```

```
## free.sulfur.dioxide  10.92329  10.86146 10.19757 12.63947        1 Confirmed
## chlorides            13.25104  13.14339 12.33071 14.25105        1 Confirmed
## volatile.acidity     29.18301  29.48177 26.53715 31.36999        1 Confirmed
```

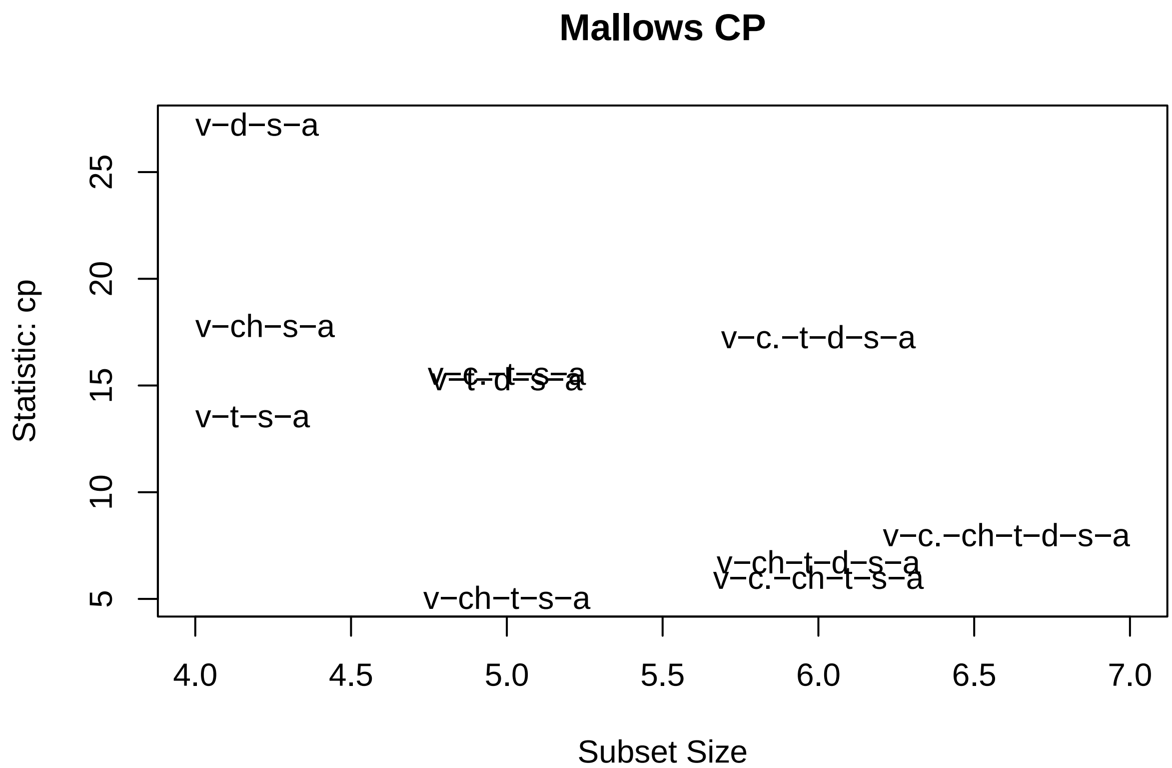
```
sorted_vars = attStats(Bor.res)[order(-attStats(Bor.res)$meanImp),]
print(sorted_vars)
```

```
##                       meanImp medianImp   minImp   maxImp normHits  decision
## alcohol              50.75096  51.05660 48.49180 52.61674        1 Confirmed
## sulphates            35.89367  35.75507 34.81560 37.36271        1 Confirmed
## volatile.acidity     29.18301  29.48177 26.53715 31.36999        1 Confirmed
## total.sulfur.dioxide 21.19278  20.73419 19.17124 23.52463        1 Confirmed
## chlorides            13.25104  13.14339 12.33071 14.25105        1 Confirmed
## pH                   12.03961  11.88174 11.21923 13.25784        1 Confirmed
## free.sulfur.dioxide  10.92329  10.86146 10.19757 12.63947        1 Confirmed
```

```
# Mallows CP
library(AER)
```

```
## Loading required package: car

## Loading required package: carData

## Loading required package: lmtest

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: sandwich
```

```
library(leaps)
# Codes use subsets and regsubset to perform the Mallows CP
mcp <- lm(quality ~ volatile.acidity + citric.acid + chlorides + total.sulfur.dioxide + density + sulpha
ss =regsubsets(quality ~ volatile.acidity + citric.acid  + chlorides  + total.sulfur.dioxide + density
subsets(ss, statistic = "cp", legend =F , main = "Mallows CP", col = "green", min.size = 4)
```




```
##                        Abbreviation
## volatile.acidity                  v
## citric.acid                      c.
## chlorides                        ch
## total.sulfur.dioxide              t
## density                           d
## sulphates                         s
## alcohol                           a
```

Question 5

Multi Collinearity with VIF

With the output for the vif(), all of the predictor variables are in the range of [1,2]. This means there are no variables that are highly correlated with another. Therefore the model with the removed variables won't have significant issues with multicollinearity.

```
# Multi Collinearity
vifFunction<- lm(quality ~ volatile.acidity  + chlorides +  total.sulfur.dioxide + sulphates + alcohol,
vif(vifFunction)
```

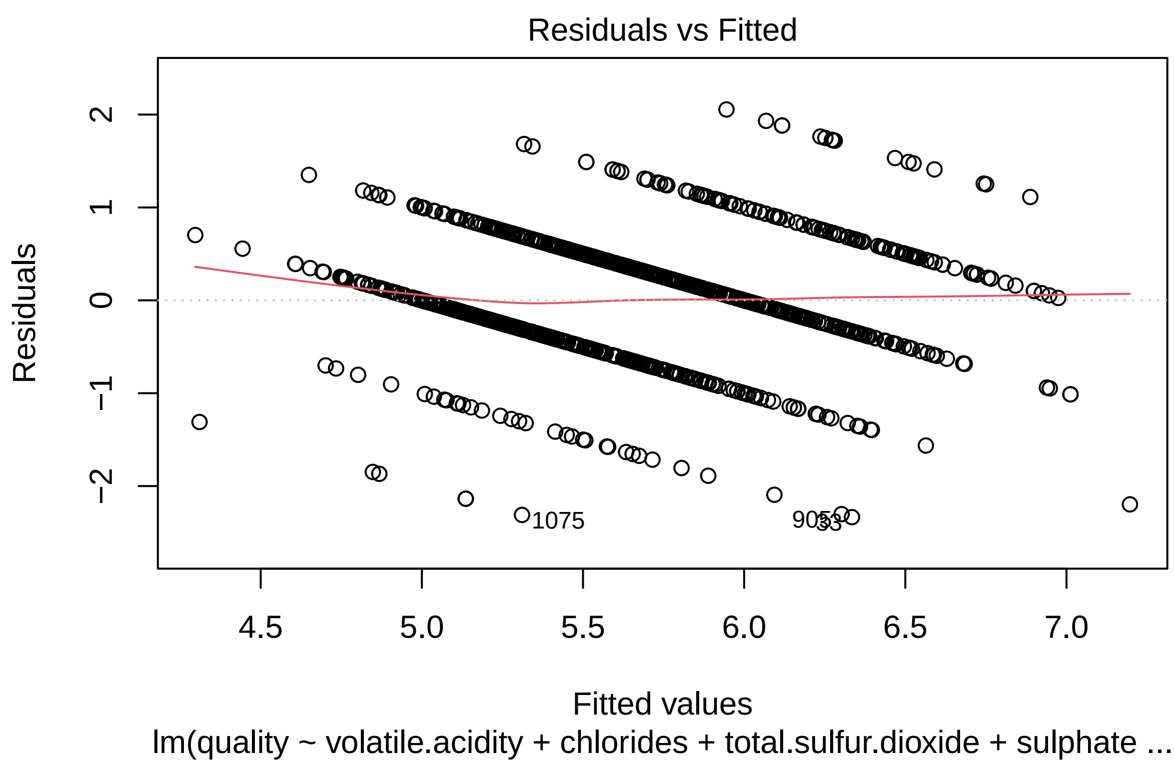
```
##     volatile.acidity            chlorides total.sulfur.dioxide
##             1.145466             1.291971             1.042551
##            sulphates              alcohol
##             1.331226             1.159650
```

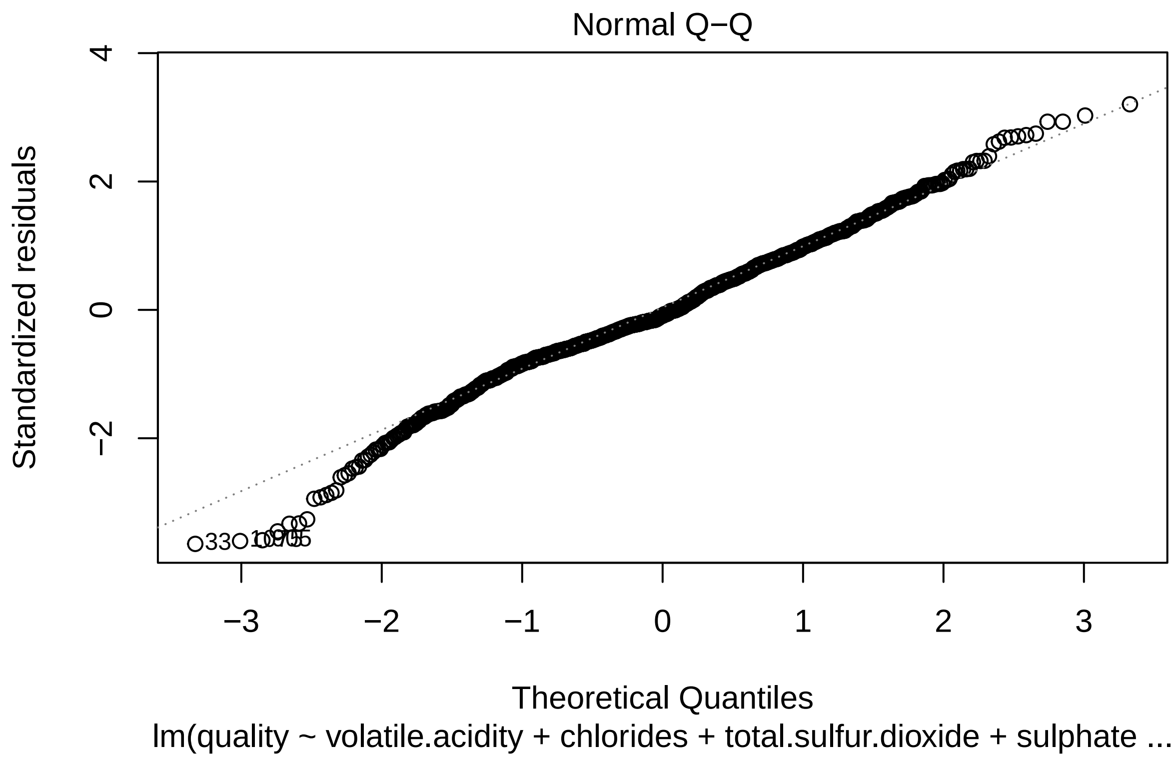
Question 6

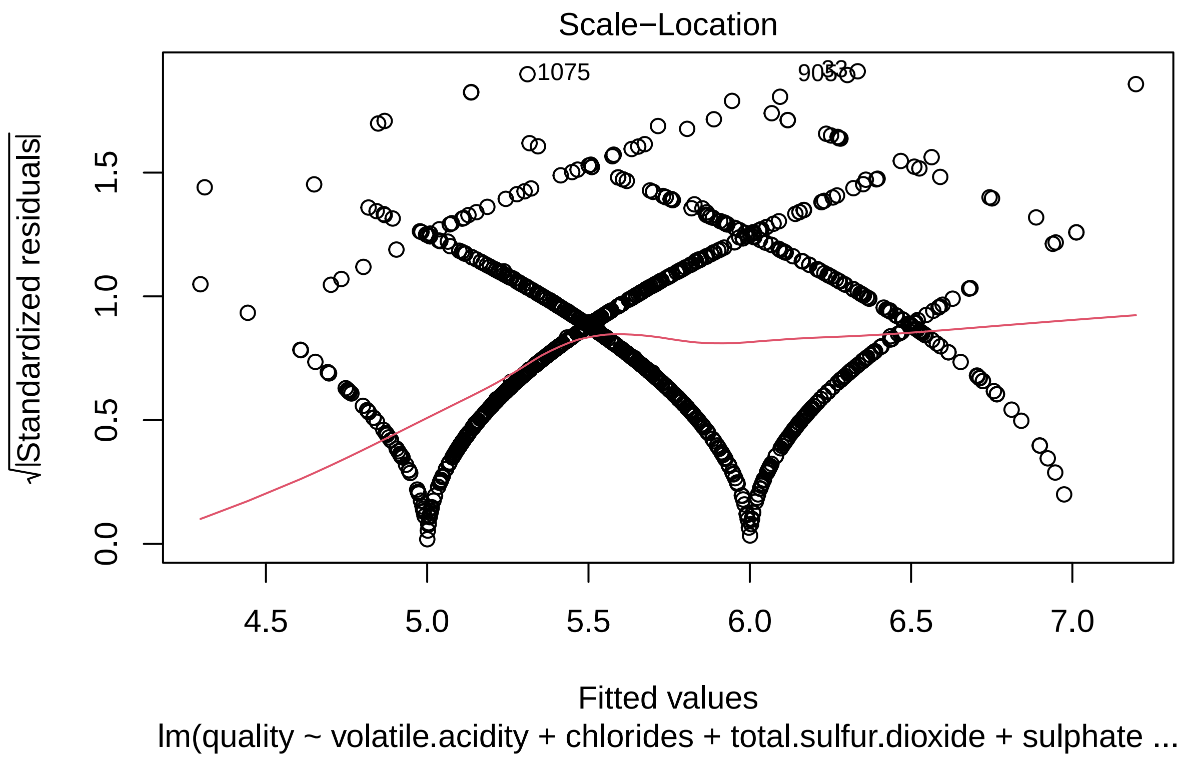
Plotting The Respective Residuals

The graph with residuals versus fitted shows a red line that tries to capture all of the residuals. There are more values that are above the red line than below the red line. The residuals vs y-hat plot lets us visualize whether heteroskedasticity is present. Our results show that there is a spread in the variance, which allows us to conclude that heteroskedasticity is present in our model.

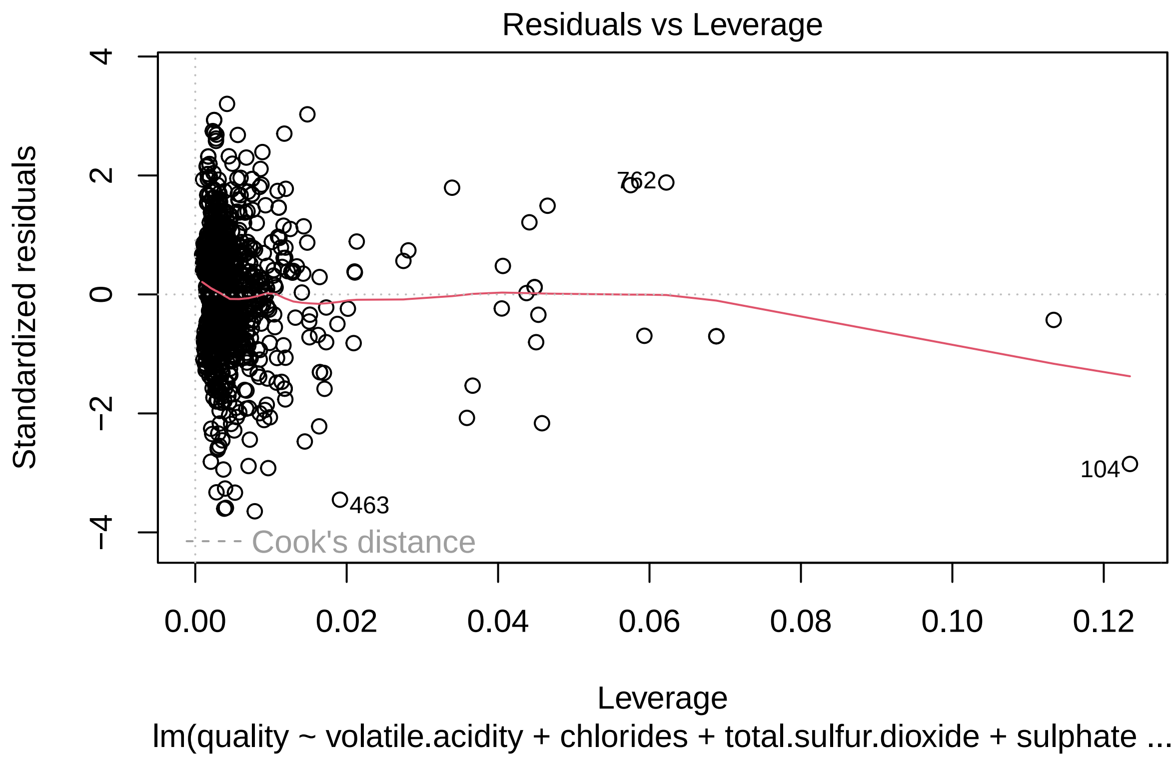
```
plot(vifFunction)
```



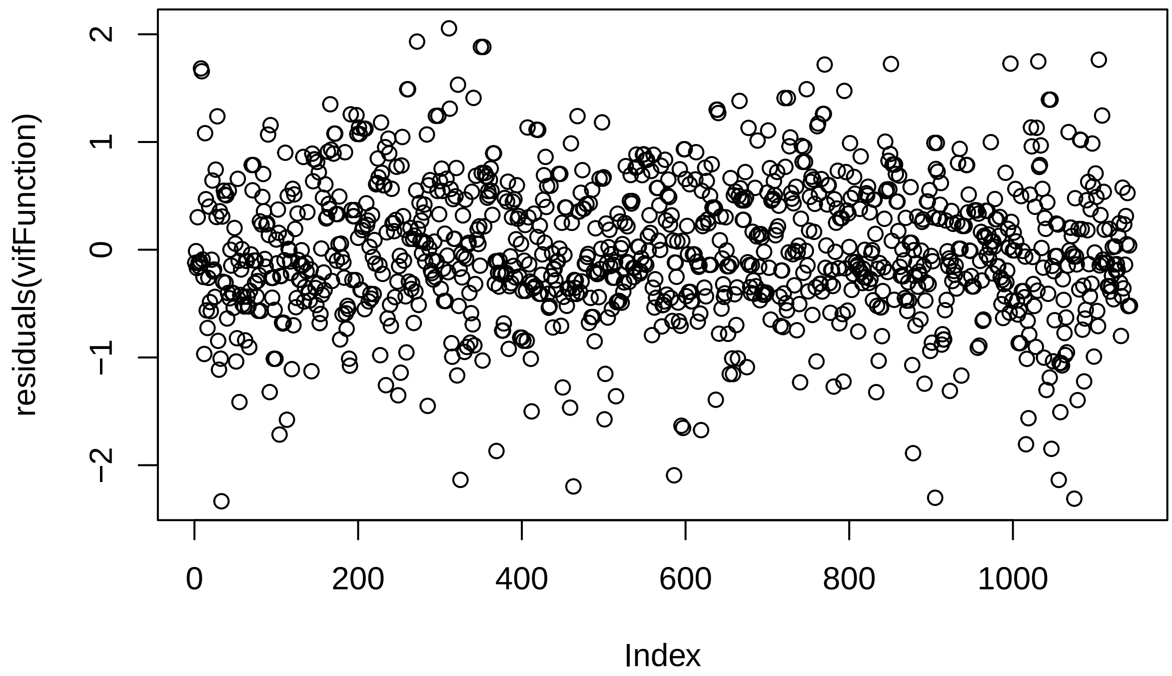

Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(quality ~ volatile.acidity + chlorides + total.sulfur.dioxide + sulphate ...



Scale−Location
1075
905
33
√|Standardized residuals|
0.0
0.5
1.0
1.5
4.5
5.0
5.5
6.0
6.5
7.0
Fitted values
lm(quality ~ volatile.acidity + chlorides + total.sulfur.dioxide + sulphate ...




```
plot(residuals(vifFunction))
```



Question 7

Heteroskedasticity

Using the Breusch Pagan test, we reject the null and conclude there is heteroskedasticity present. To fix this, weapply robust standard errors. With this, all of our variables are significant and the standard errors decreased resultingi n a better model.

```
reg.mod = lm(quality ~ volatile.acidity  + chlorides +  total.sulfur.dioxide + sulphates + alcohol, data
# BP Test Short Way
bptest(reg.mod)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  reg.mod
## BP = 35.162, df = 5, p-value = 1.397e-06
```

```
# BP Test Long Way
alpha <- 0.05
ressq <- resid(reg.mod)^2
modres <- lm(ressq~volatile.acidity  + chlorides +  total.sulfur.dioxide + sulphates + alcohol, data = W
summary(modres)
```

```
##
## Call:
## lm(formula = ressq ~ volatile.acidity + chlorides + total.sulfur.dioxide +
##     sulphates + alcohol, data = Wine_Quality)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.7400 -0.3492 -0.2046  0.0790  4.8907
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)          -0.6402296  0.2461118  -2.601  0.00941 **
## volatile.acidity      0.1707468  0.1183714   1.442  0.14945
## chlorides            -0.2438618  0.4777576  -0.510  0.60985
## total.sulfur.dioxide -0.0014706  0.0006188  -2.377  0.01764 *
## sulphates             0.3664716  0.1345247   2.724  0.00654 **
## alcohol               0.0774116  0.0197697   3.916 9.55e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6714 on 1137 degrees of freedom
## Multiple R-squared:  0.03076,    Adjusted R-squared:  0.0265
## F-statistic: 7.217 on 5 and 1137 DF,  p-value: 1.156e-06
```

```r
#Robust Standard Errors
cov1<-hccm(reg.mod, type="hc1")
coeftest(reg.mod, vcov.=cov1)
```

```
##
## t test of coefficients:
##
##                        Estimate  Std. Error t value  Pr(>|t|)
## (Intercept)           3.03976345  0.27457600 11.0708 < 2.2e-16 ***
## volatile.acidity     -1.20284003  0.12700674 -9.4707 < 2.2e-16 ***
## chlorides            -1.48112808  0.51586067 -2.8712 0.0041653 **
## total.sulfur.dioxide -0.00227580  0.00062456 -3.6439 0.0002807 ***
## sulphates             0.86674198  0.14302592  6.0600 1.848e-09 ***
## alcohol               0.27959699  0.02268052 12.3276 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Question 8

AIC/BIC Model

Visualizing the plot of the respective residuals and y-hats in question 6, it seemed that the best fit could be a log linear form. With AIC/BIC we are able to see if our hypothesis is correct. We included different models with varying predictors, the best model we have found previously, and in the best model in terms of log. From the results, the best model is still the best model but better as a log linear with the same predictors obtained from previous tests.

```r
#AIC Model
# For the new model, it is in log linear form.
bestMOD<- lm(log(quality) ~ chlorides + total.sulfur.dioxide + sulphates + alcohol + volatile.acidity, d
bestMOD2<- lm(quality ~ chlorides + total.sulfur.dioxide + sulphates + alcohol + volatile.acidity, data=
MOD_1 <- lm(log(quality) ~ sulphates + alcohol,data = Wine_Quality)
MOD_2 <- lm(log(quality) ~ pH + sulphates + alcohol,data = Wine_Quality)
MOD_3 <-  lm(log(quality) ~ density + pH + sulphates + alcohol,data = Wine_Quality)
MOD_4 <- lm(log(quality) ~  total.sulfur.dioxide + density + pH + sulphates + alcohol,data = Wine_Quali
MOD_5 <- lm(log(quality) ~ free.sulfur.dioxide + total.sulfur.dioxide + density + pH + sulphates + alco
MOD_6 <- lm(log(quality) ~ chlorides + free.sulfur.dioxide + total.sulfur.dioxide + density + pH + sulp
MOD_7 <-lm(log(quality) ~ residual.sugar + chlorides + free.sulfur.dioxide + total.sulfur.dioxide + dens
MOD_8 <-lm(log(quality) ~ citric.acid + residual.sugar + chlorides + free.sulfur.dioxide + total.sulfur
MOD_9 <- lm(log(quality) ~ volatile.acidity + citric.acid + residual.sugar + chlorides + free.sulfur.di
MOD_10<- lm(log(quality) ~ fixed.acidity + volatile.acidity + citric.acid + residual.sugar + chlorides +
AIC(MOD_1, MOD_2, MOD_3, MOD_4, MOD_5, MOD_6, MOD_7, MOD_8, MOD_9, MOD_10, bestMOD, bestMOD2)
```

```
##          df       AIC
## MOD_1     4 -1526.656
## MOD_2     5 -1547.205
## MOD_3     6 -1545.221
## MOD_4     7 -1558.177
## MOD_5     8 -1559.779
## MOD_6     9 -1586.914
## MOD_7    10 -1584.923
## MOD_8    11 -1599.442
## MOD_9    12 -1670.418
## MOD_10   13 -1668.687
## bestMOD   7 -1667.423
## bestMOD2  7  2241.781
```

```r
BIC(MOD_1, MOD_2, MOD_3, MOD_4, MOD_5, MOD_6, MOD_7, MOD_8, MOD_9, MOD_10, bestMOD, bestMOD2)
```

```
##          df       BIC
## MOD_1     4 -1506.491
## MOD_2     5 -1521.998
## MOD_3     6 -1514.973
## MOD_4     7 -1522.887
## MOD_5     8 -1519.448
## MOD_6     9 -1541.541
## MOD_7    10 -1534.509
## MOD_8    11 -1543.987
## MOD_9    12 -1609.921
## MOD_10   13 -1603.149
## bestMOD   7 -1632.133
## bestMOD2  7  2277.070
```

Question 9

Cross-Validation

We performed a 5-fold cross validation and obstained an RMSE of .11608. This means that on average the predicted value is off by .11608. We also split the data into testing/training and calculated the RMSE for both and got small numbers. Overall, we can conclude our model is a good fit for our data and the results are accurate.

```
# train for training sample
# test for testing sample
set.seed(1)
row.number <- sample(1:nrow(Wine_Quality), 0.66*nrow(Wine_Quality))
train = Wine_Quality[row.number,]
test = Wine_Quality[-row.number,]
reg.mod=lm(log(quality) ~ volatile.acidity  + chlorides +  total.sulfur.dioxide + sulphates + alcohol, d
#RMSE
sqrt(mean(log(test$quality)-predict(reg.mod,test))^2)
```

```
## [1] 0.009056762
```

```
sqrt(mean(log(train$quality)-predict(reg.mod,train))^2)
```

```
## [1] 1.582641e-15
```

```
# Cross Validation
library(lmvar)
fit= lm(log(quality) ~ volatile.acidity  + chlorides +  total.sulfur.dioxide + sulphates + alcohol,x = T
cv.lm(fit, k = 5)
```

```
## Mean absolute error        :  0.08922306
## Sample standard deviation  :  0.007942391
##
## Mean squared error         :  0.01370225
## Sample standard deviation  :  0.003211446
##
## Root mean squared error    :  0.1164704
## Sample standard deviation  :  0.01308115
```