Quality of Wine

Introduction

How much of volatile acidity, citric acid, residual sugar, chlorides, free sulfur dioxide, total sulfur dioxide, density, pH, sulfphates, and alcohol help explain the quality of wine? A linear regression will estimate the quality of wine based on these 11 predictors. The following series of tests will filter out the best model to get the most accurate results for this question.

Downloading Wine Quality Data Set

```
Wine_Quality<- read.csv("~/Downloads/WineQT.csv")
View(Wine_Quality)
```

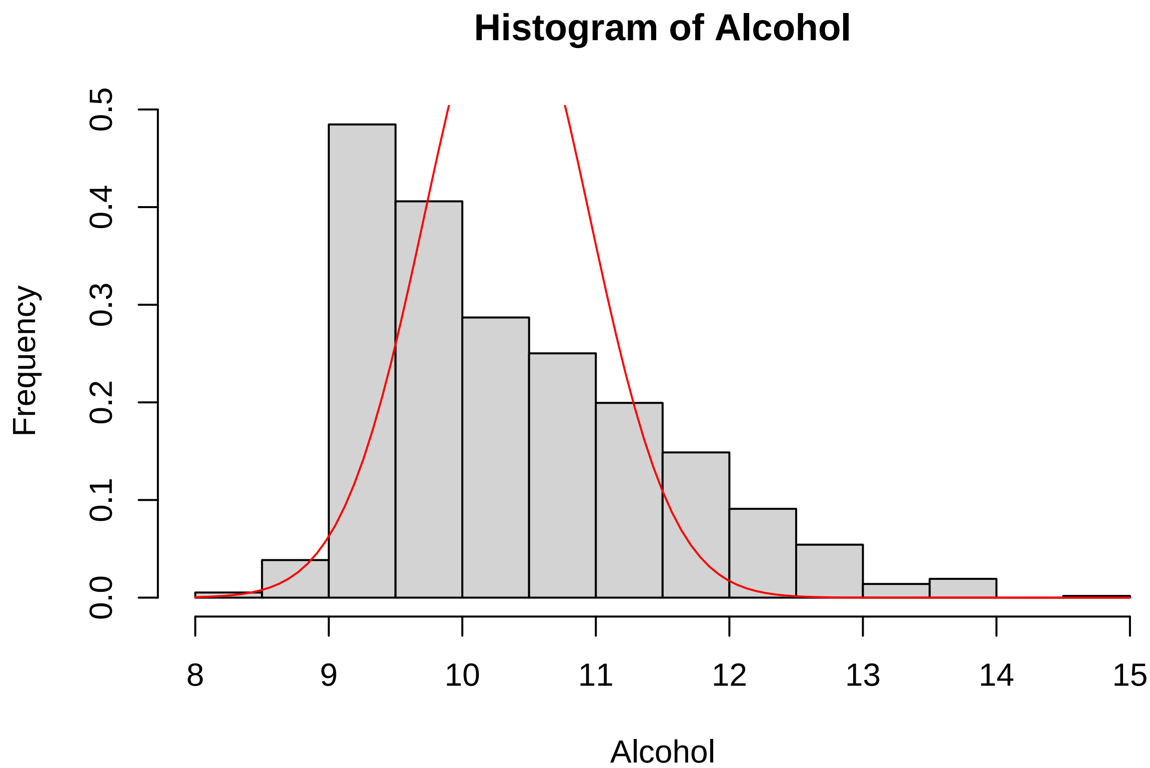
Histograms

In histograms 1 (fixed acidity), 2 (volatile acidity), 4 (residual sugar), 5 (chlorides), 6 (free sulfur dioxide), 7 (total sulfur dioxide), 10 (sulphates), and 11 (alcohol) the models are right-skewed, meaning there is a positive distribution. Histograms 8 (pH) and 9 (density) have a normal distribution.

```
library(MASS)
hist(Wine_Quality$alcohol, prob = TRUE, xlab = "Alcohol", ylab = "Frequency", main = "Histogram of Alcol
fit1<-fitdistr(Wine_Quality$alcohol, densfun="logistic")
```

```
## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced
```

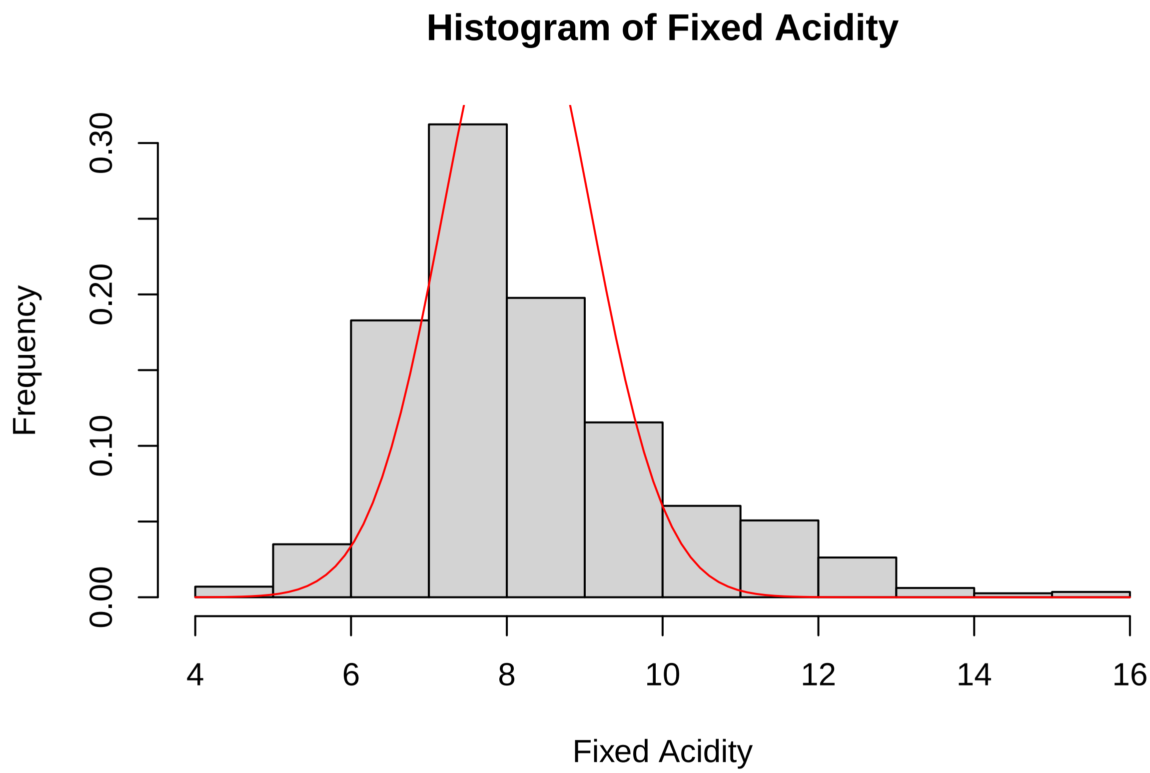
```
curve(dnorm(x,fit1$estimate[1], fit1$estimate[2]), col="red", add=T)
```




```
hist(Wine_Quality$fixed.acidity, prob = TRUE, xlab = "Fixed Acidity", ylab = "Frequency", main = "Histog
fit1<-fitdistr(Wine_Quality$fixed.acidity, densfun="logistic")
```

```
## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced
```

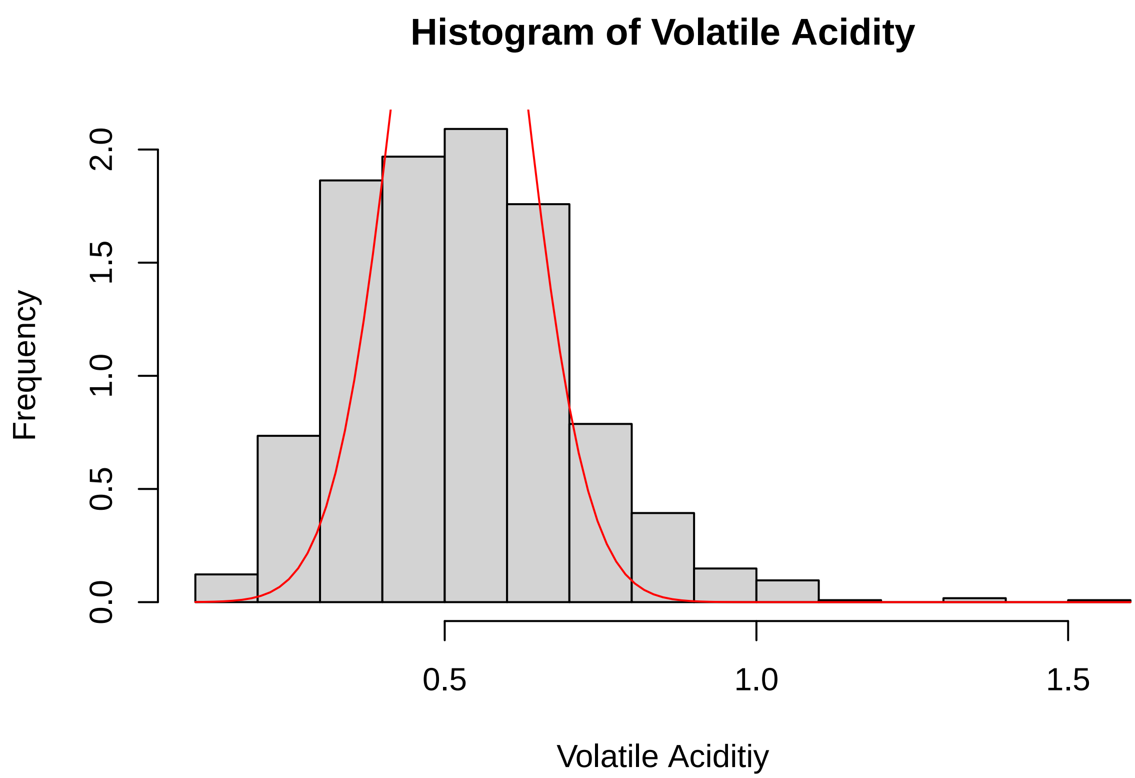
```
curve(dnorm(x,fit1$estimate[1], fit1$estimate[2]), col="red", add=T)
```




```
hist(Wine_Quality$volatile.acidity, prob = TRUE, xlab = "Volatile Aciditiy", ylab = "Frequency", main =
fit1<-fitdistr(Wine_Quality$volatile.acidity, densfun="logistic")
```

```
## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced
```

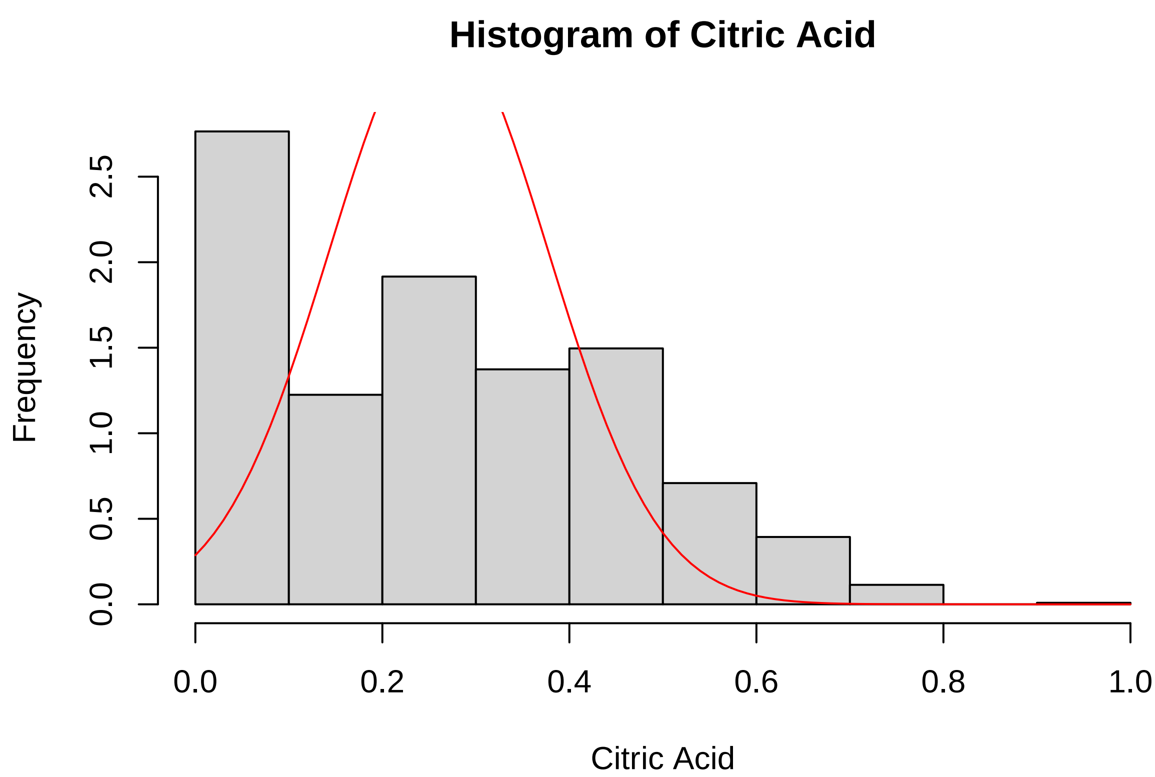
```
curve(dnorm(x,fit1$estimate[1], fit1$estimate[2]), col="red", add=T)
```




```
hist(Wine_Quality$citric.acid, prob = TRUE, xlab = "Citric Acid", ylab = "Frequency", main = "Histogram
fit1<-fitdistr(Wine_Quality$citric.acid, densfun="logistic")
```

```
## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced
```

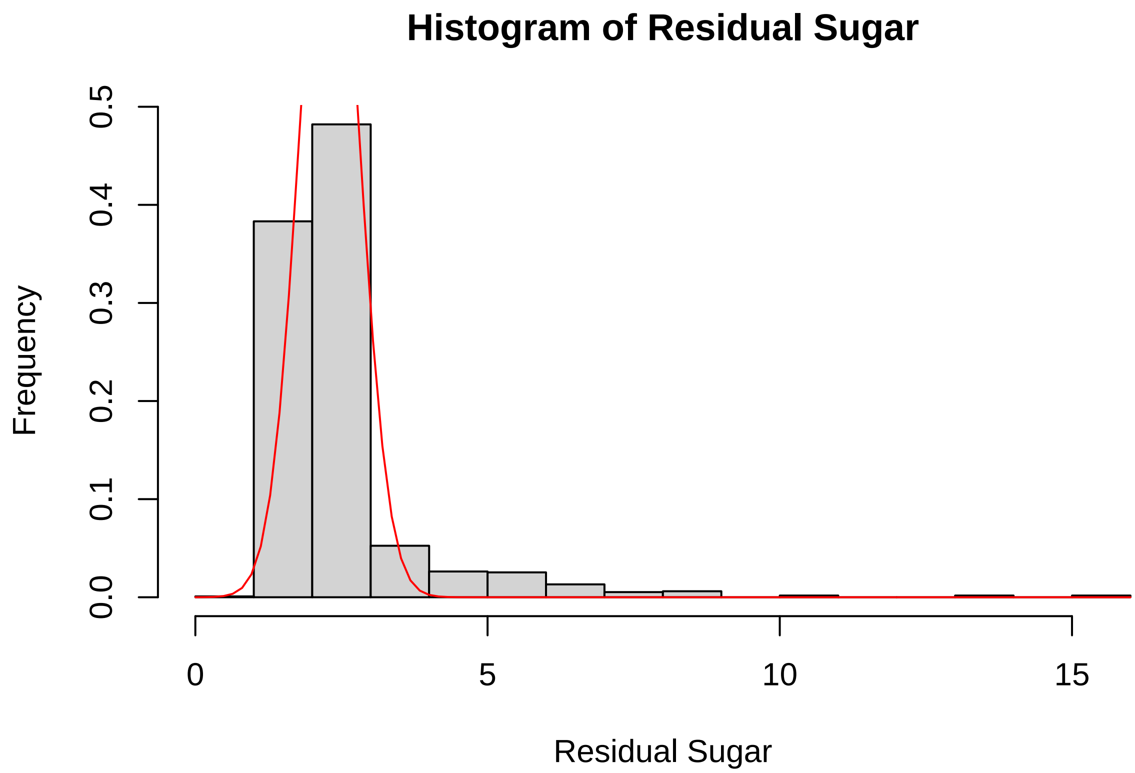
```
curve(dnorm(x,fit1$estimate[1], fit1$estimate[2]), col="red", add=T)
```




```
hist(Wine_Quality$residual.sugar, prob = TRUE, xlab = "Residual Sugar", ylab = "Frequency", main = "His
fit1<-fitdistr(Wine_Quality$residual.sugar, densfun="logistic")
```

```
## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced
```

```
curve(dnorm(x,fit1$estimate[1], fit1$estimate[2]), col="red", add=T)
```

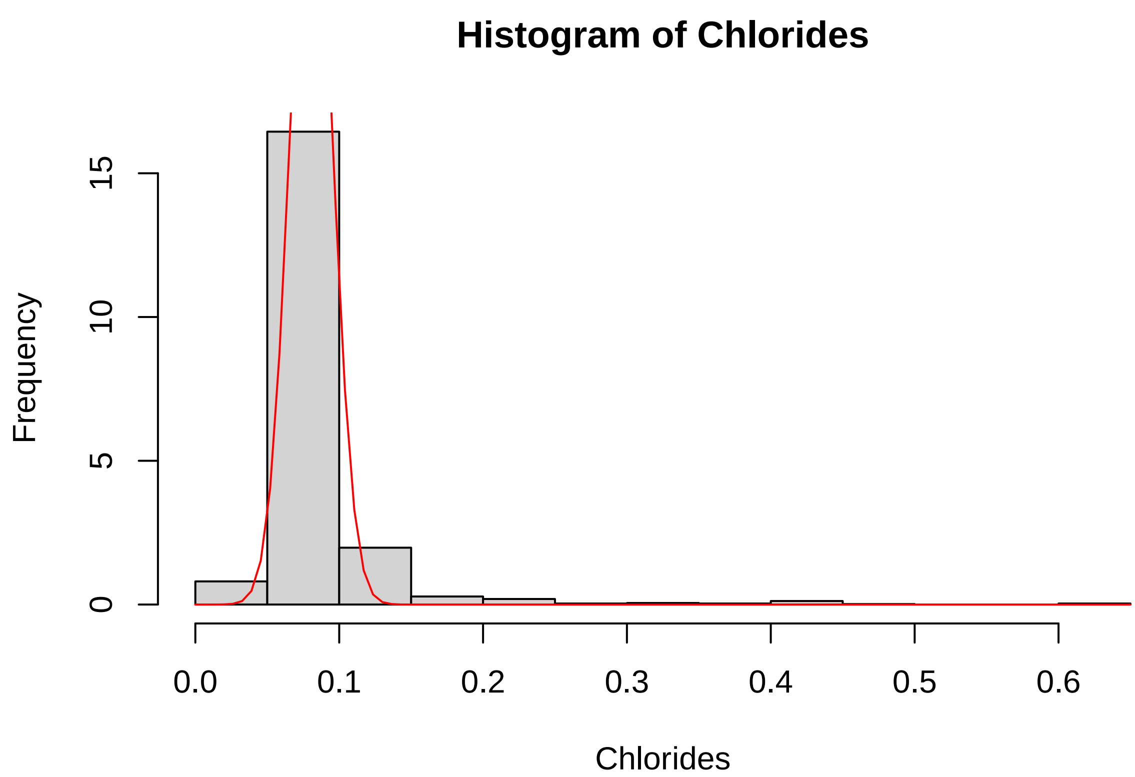



```r
hist(Wine_Quality$chlorides, prob = TRUE, xlab = "Chlorides", ylab = "Frequency", main = "Histogram of C
fit1<-fitdistr(Wine_Quality$chlorides, densfun="logistic")
```

```
## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced
```

```
## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced
```

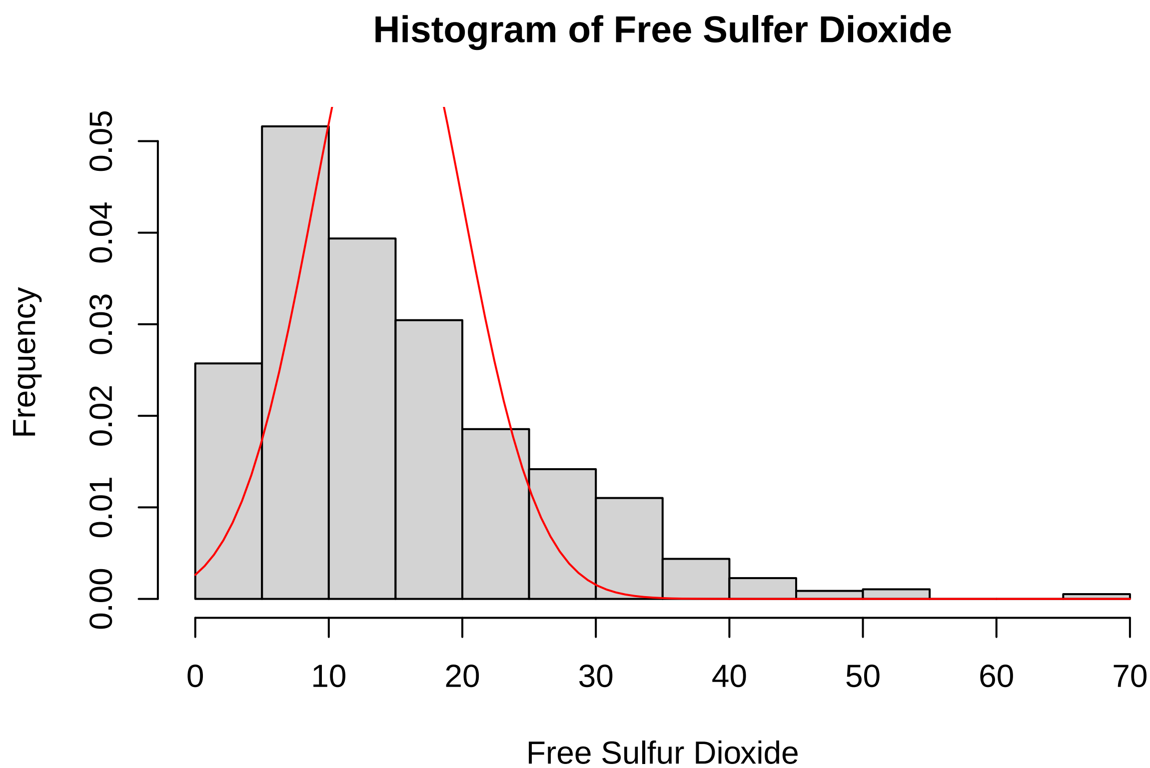
```
curve(dnorm(x,fit1$estimate[1], fit1$estimate[2]), col="red", add=T)
```



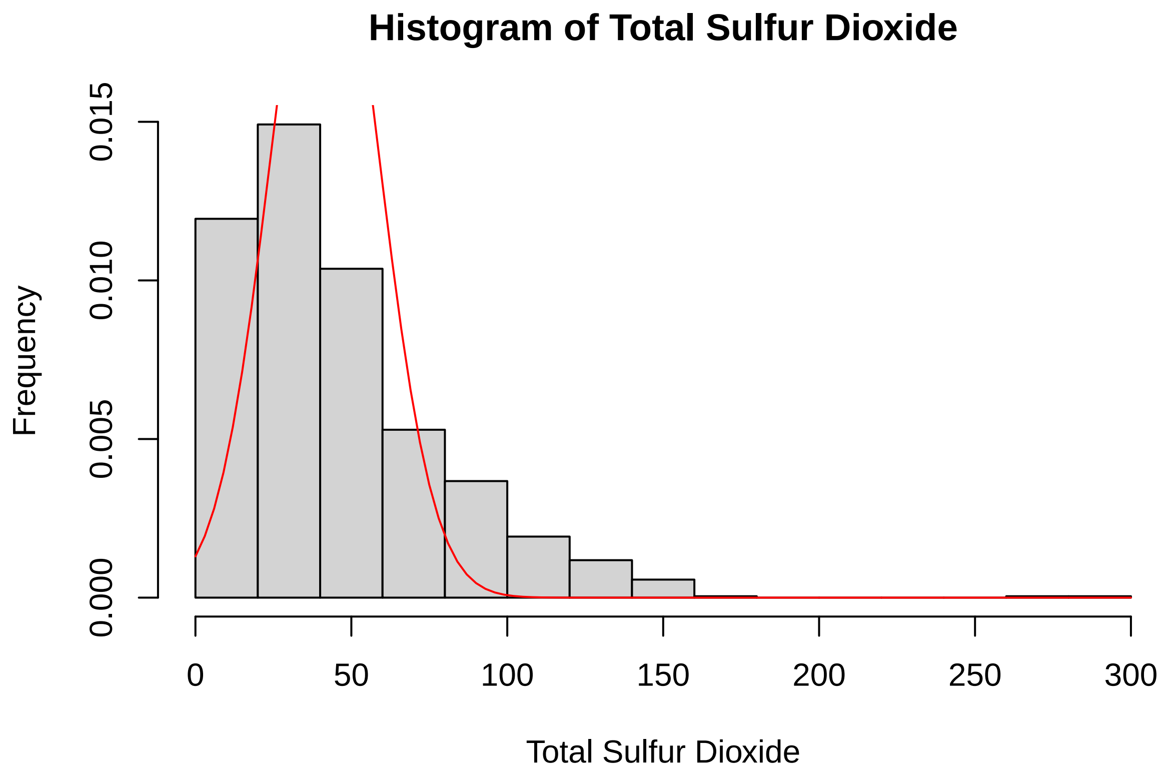

```
hist(Wine_Quality$free.sulfur.dioxide, prob = TRUE, xlab = "Free Sulfur Dioxide", ylab = "Frequency", ma
fit1<-fitdistr(Wine_Quality$free.sulfur.dioxide, densfun="logistic")
```

```
## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced
```

```r
curve(dnorm(x,fit1$estimate[1], fit1$estimate[2]), col="red", add=T)
```



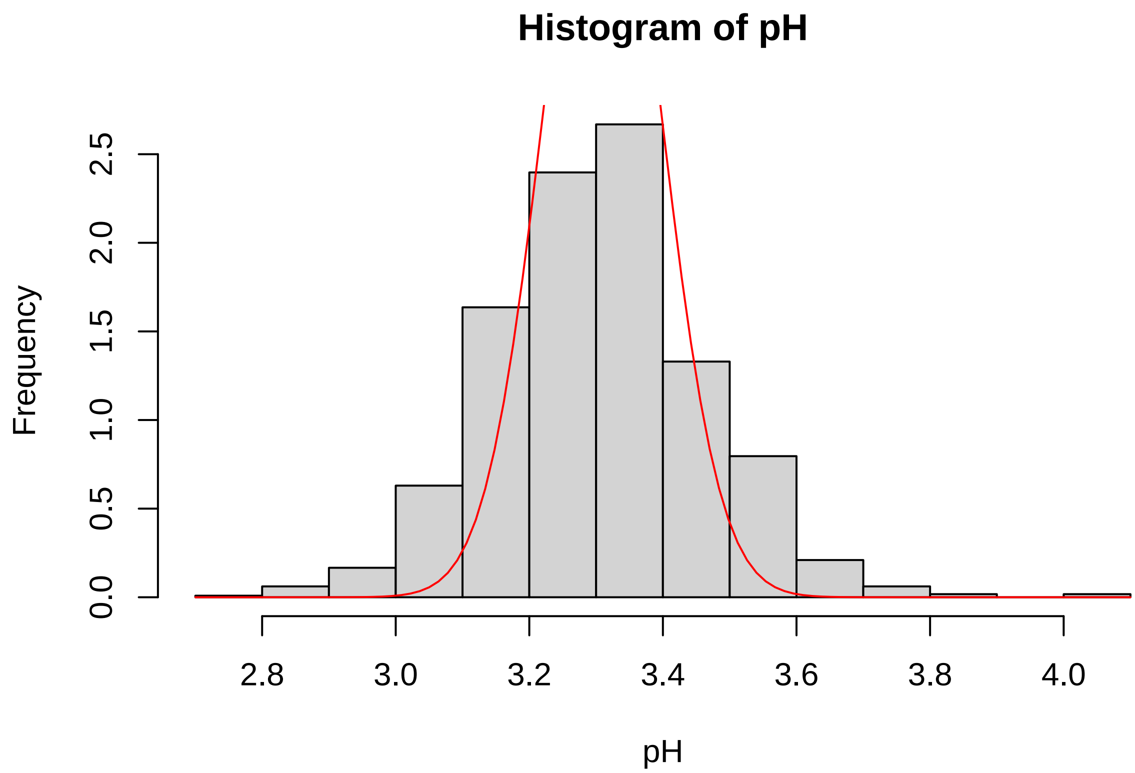

```r
hist(Wine_Quality$total.sulfur.dioxide, prob = TRUE, xlab = "Total Sulfur Dioxide", ylab = "Frequency",
fit1<-fitdistr(Wine_Quality$total.sulfur.dioxide, densfun="logistic")
curve(dnorm(x,fit1$estimate[1], fit1$estimate[2]), col="red", add=T)
```




```
hist(Wine_Quality$pH, prob = TRUE, xlab = "pH", ylab = "Frequency", main = "Histogram of pH")
fit1<-fitdistr(Wine_Quality$pH, densfun="logistic")
```

```
## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced
```

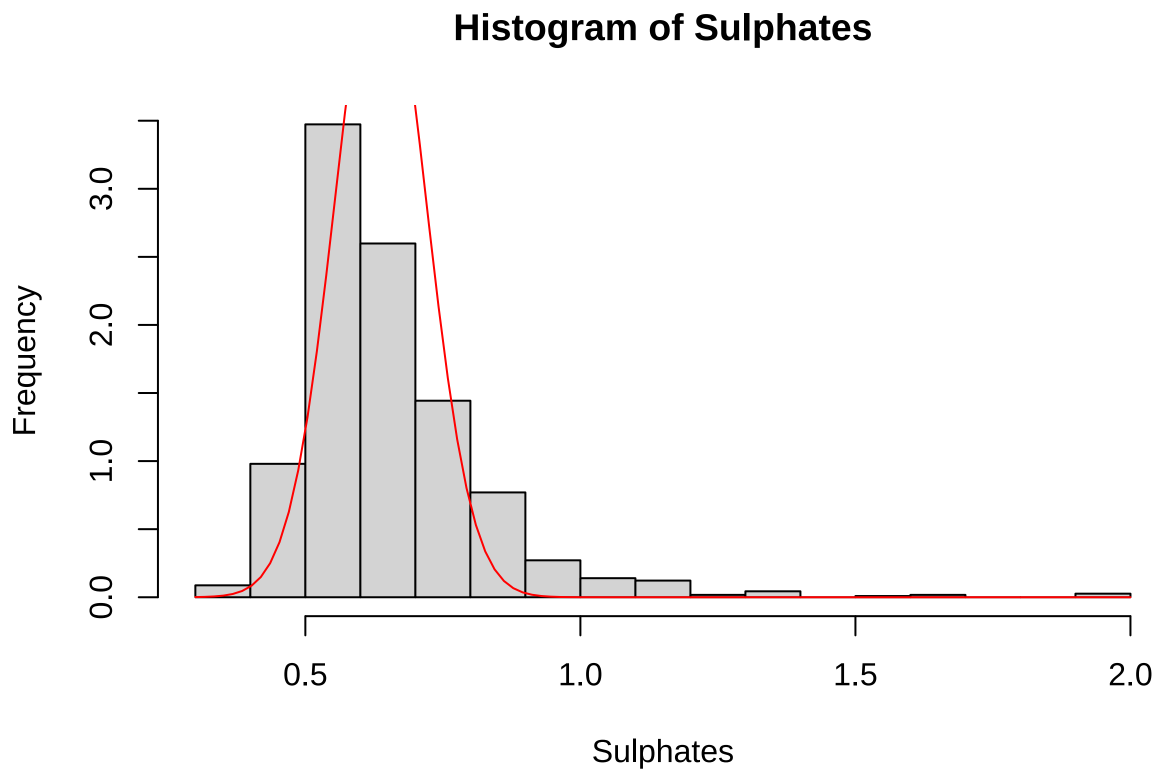
```
curve(dnorm(x,fit1$estimate[1], fit1$estimate[2]), col="red", add=T)
```




```
hist(Wine_Quality$sulphates, prob = TRUE, xlab = "Sulphates", ylab = "Frequency", main = "Histogram of S
fit1<-fitdistr(Wine_Quality$sulphates, densfun="logistic")
```

```
## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced

## Warning in densfun(x, parm[1], parm[2], ...): NaNs produced
```

```
curve(dnorm(x,fit1$estimate[1], fit1$estimate[2]), col="red", add=T)
```



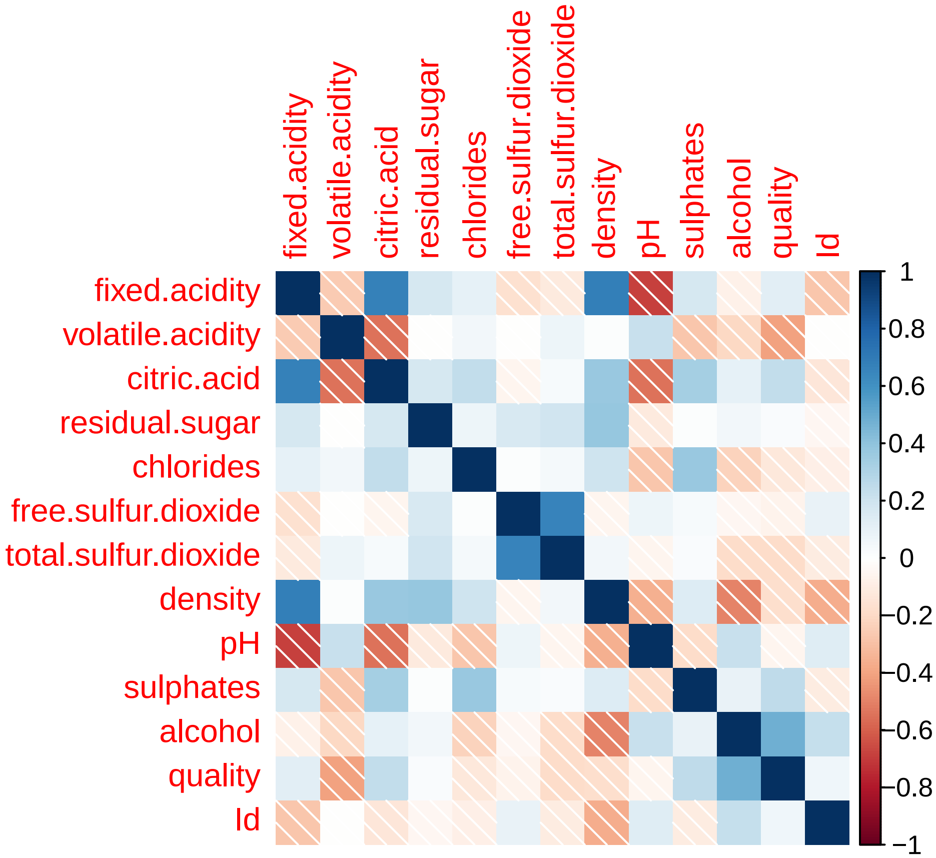
Correlation Plot

Based on the correlation plot above, there is a positive correlation between the quality of wine and alcohol, sulphates, citric acid, and fixed acidity. However, there is a negative correlation between wine quality and volatile acidity, chlorides, free sulfur dioxide, total sulfur dioxide, density, and pH.

```
library(corrplot)
```

```
## corrplot 0.92 loaded
```

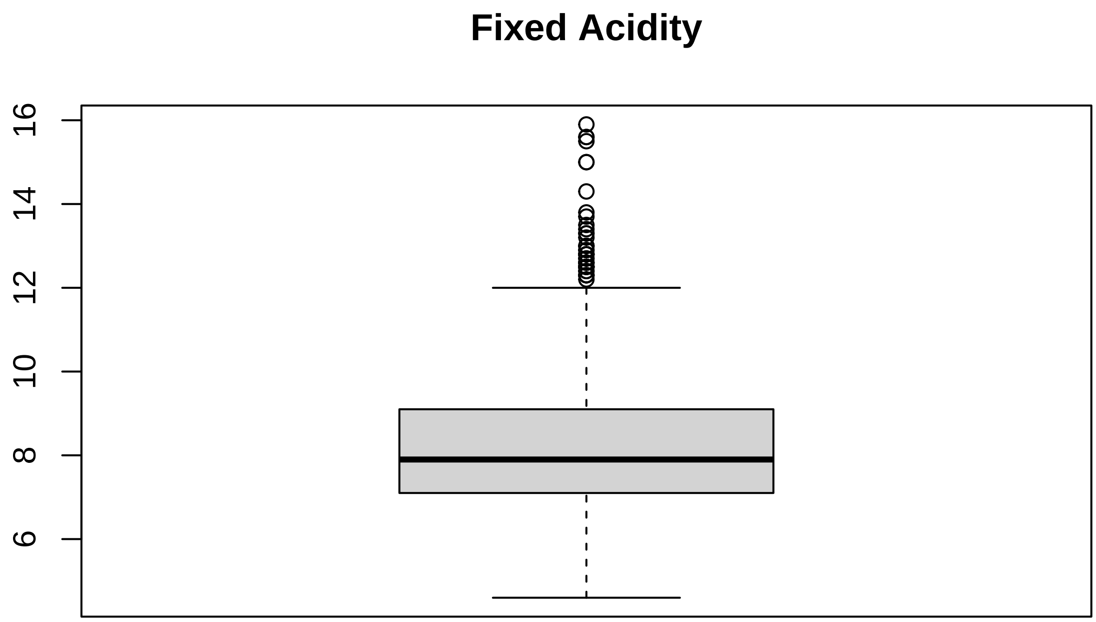
```
M = cor(Wine_Quality)
corrplot(M, method = 'shade')
```



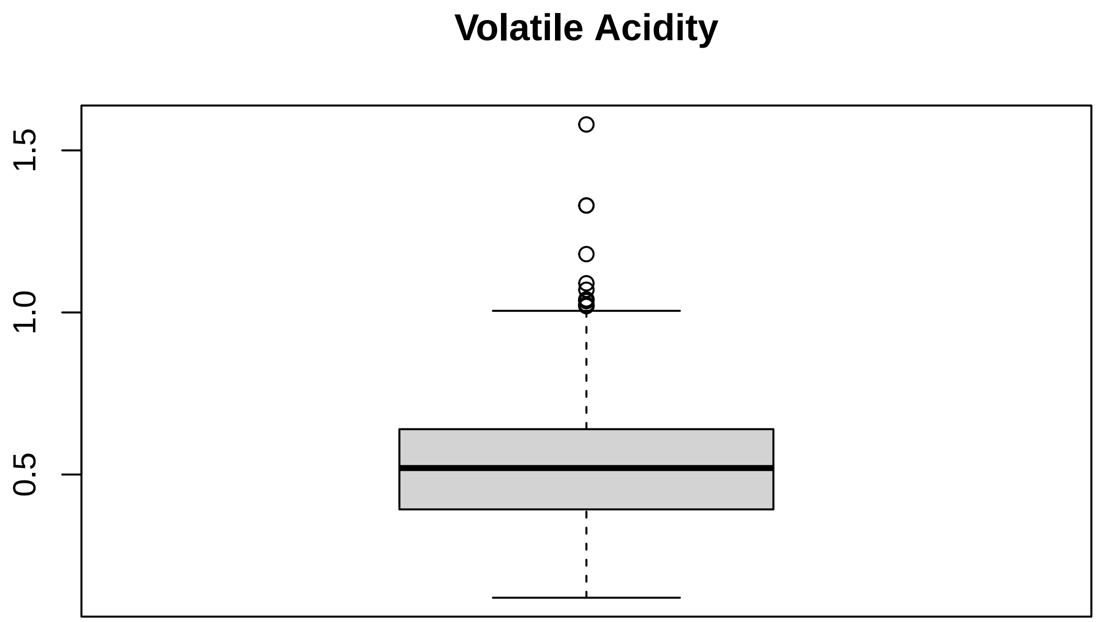
Box Plot

It is evident by the models above that the data is moderately concentrated with the predictor variable citric acid having the largest spread and chlorides having the smallest spread. It is important to note when there are many outliers, as it may skews results.

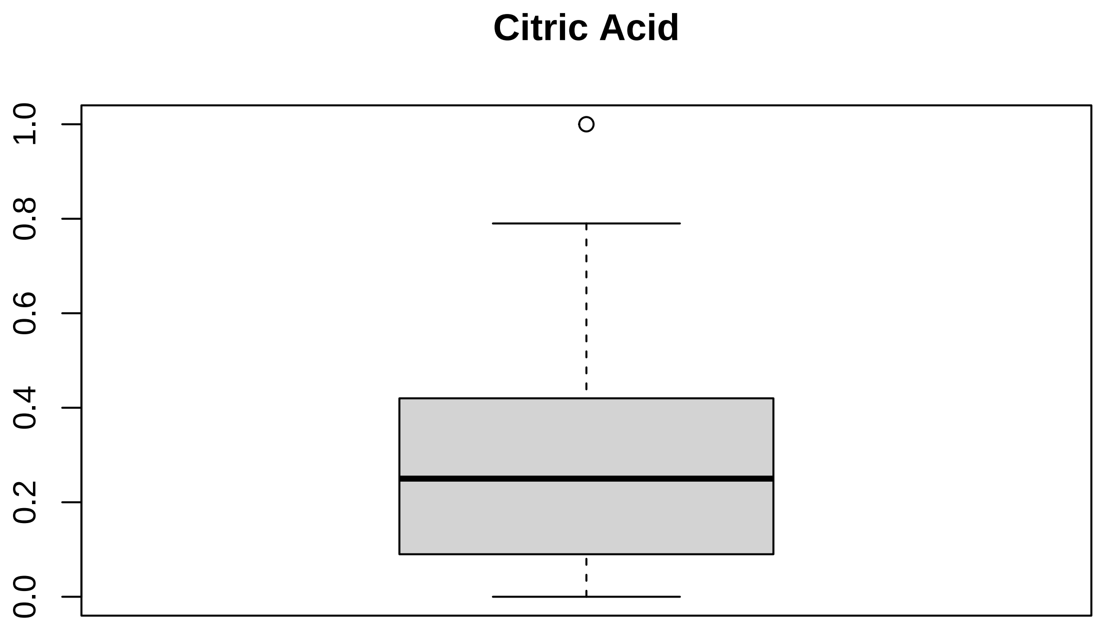
```
boxplot(Wine_Quality$fixed.acidity, main = "Fixed Acidity")
```



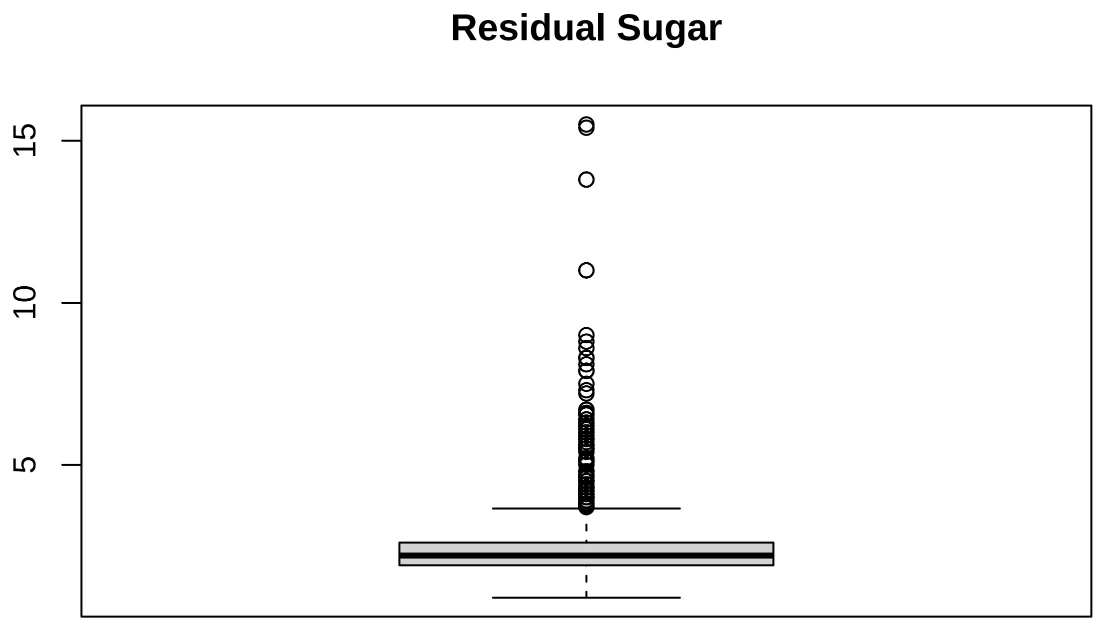

```r
boxplot(Wine_Quality$volatile.acidity, main = "Volatile Acidity")
```




```
boxplot(Wine_Quality$citric.acid, main = "Citric Acid")
```

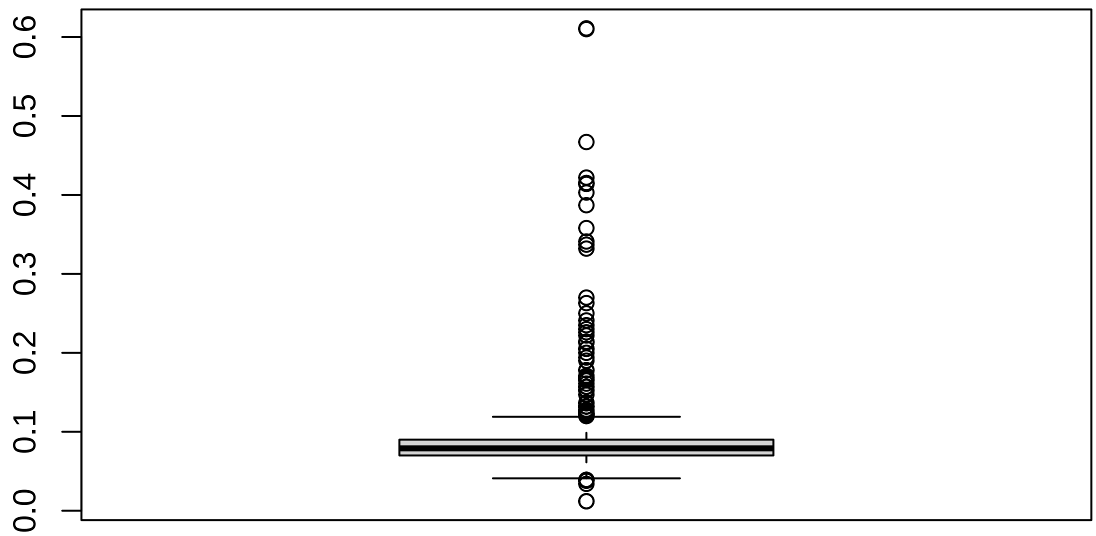



```
boxplot(Wine_Quality$residual.sugar, main = "Residual Sugar")
```

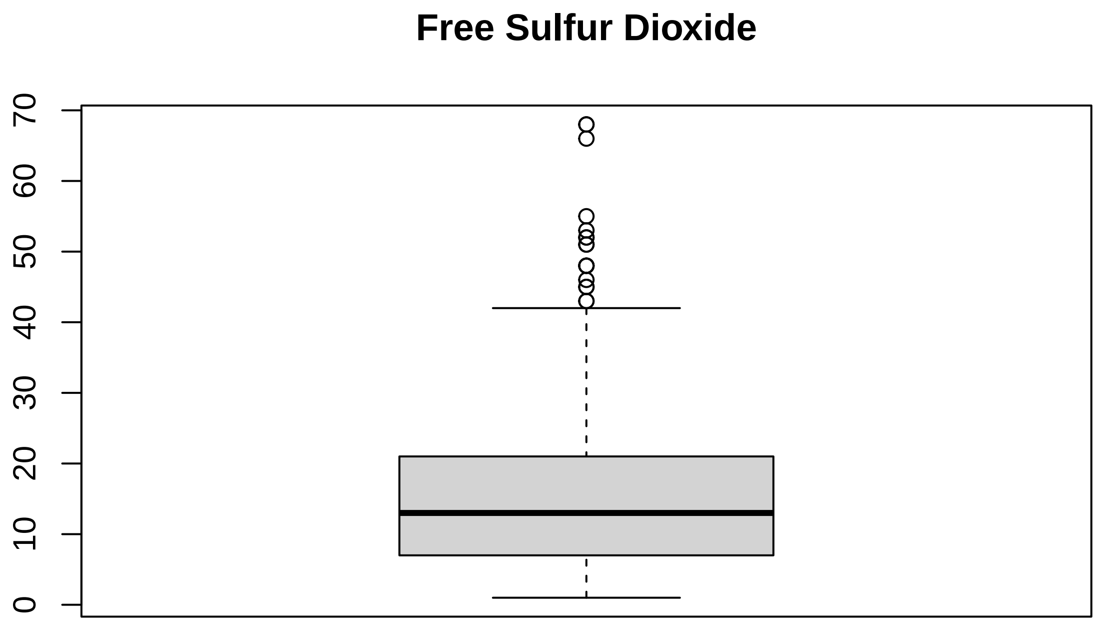



```
boxplot(Wine_Quality$chlorides, main = "Chlorides")
```

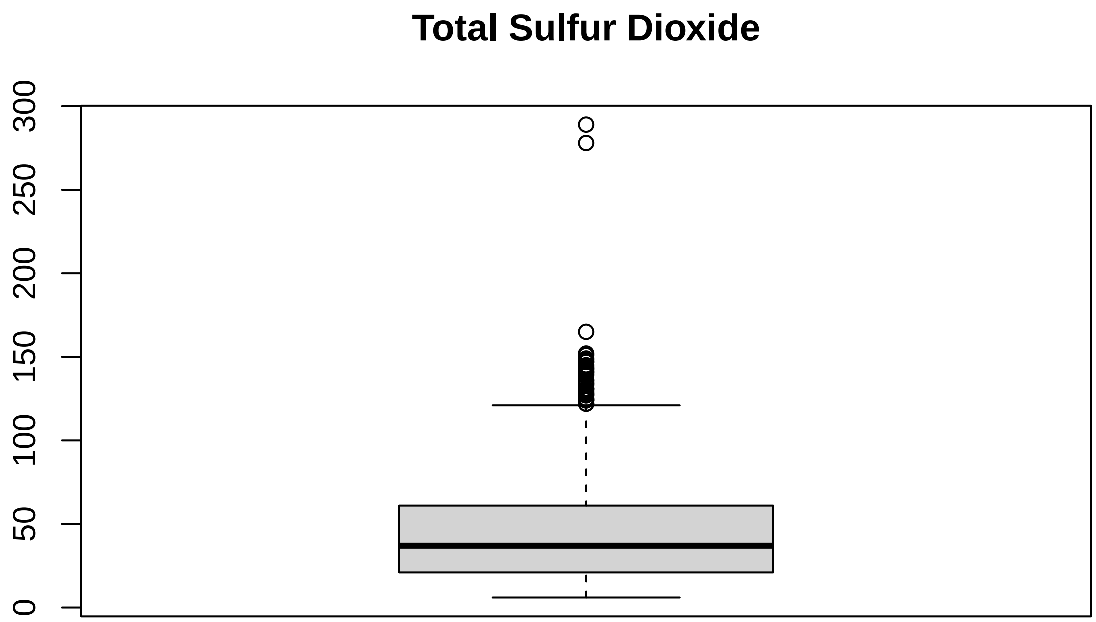
Chlorides



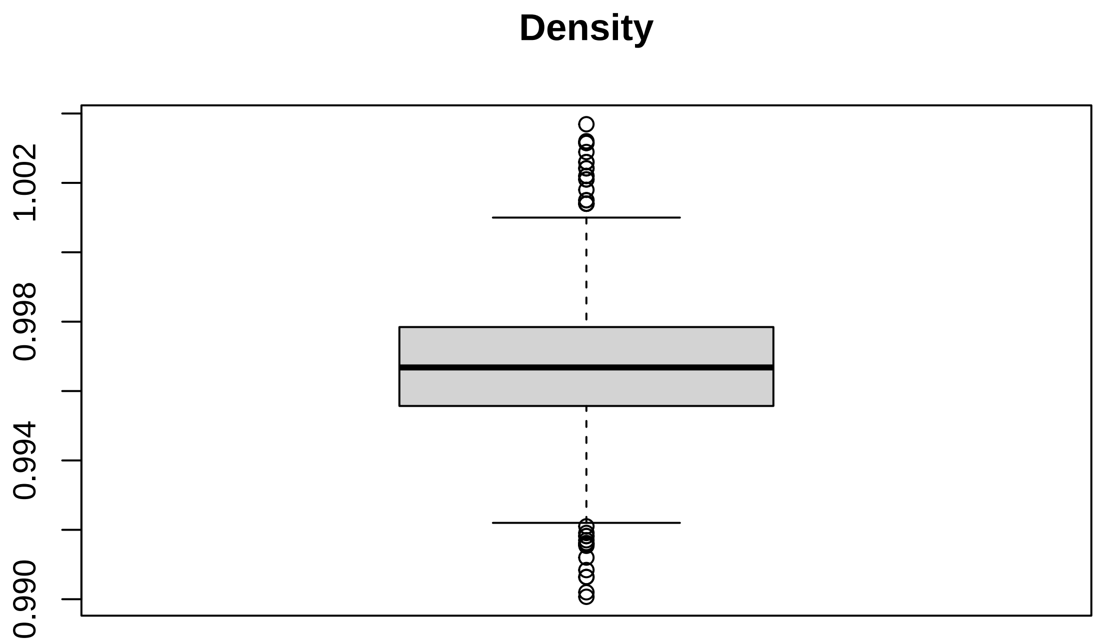

```r
boxplot(Wine_Quality$free.sulfur.dioxide, main = "Free Sulfur Dioxide")
```



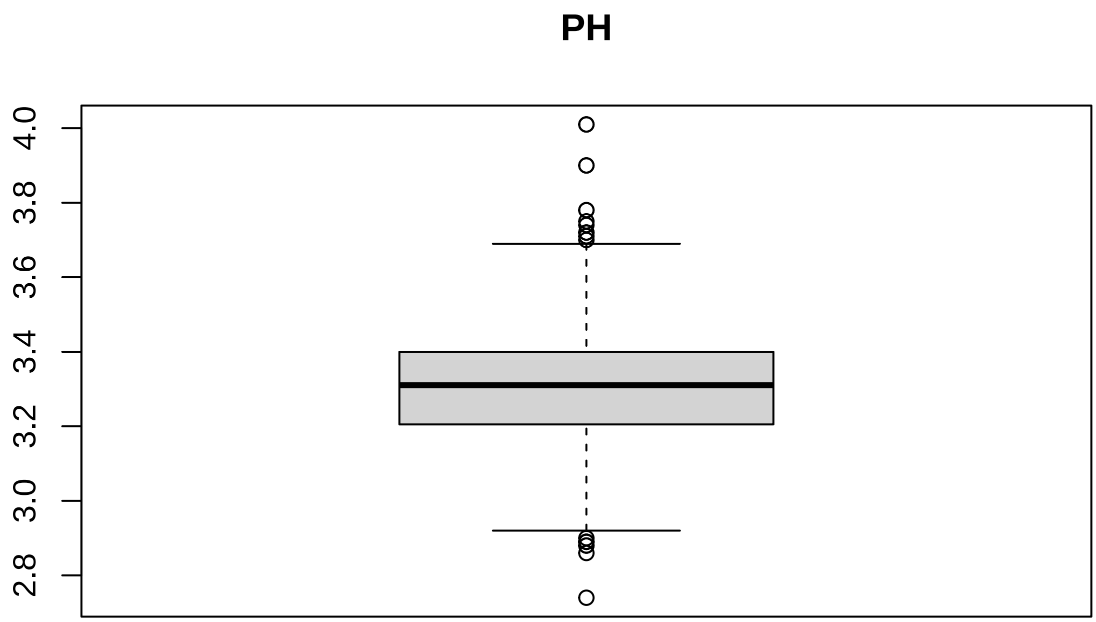

```
boxplot(Wine_Quality$total.sulfur.dioxide, main = "Total Sulfur Dioxide")
```



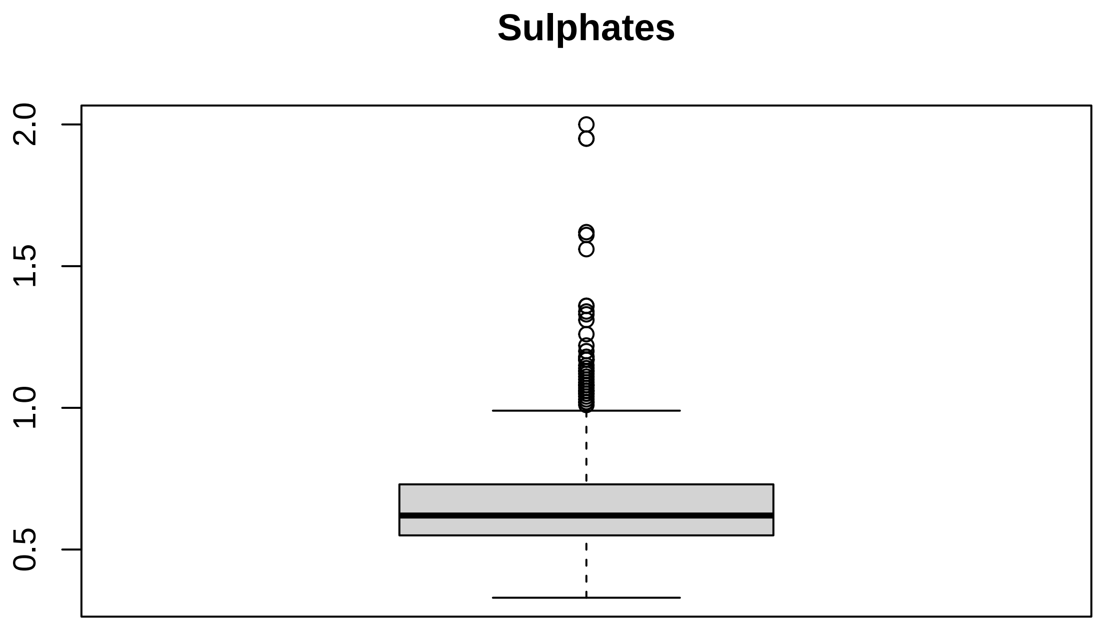

```r
boxplot(Wine_Quality$density, main = "Density")
```




```r
boxplot(Wine_Quality$pH, main = "PH")
```




```r
boxplot(Wine_Quality$sulphates, main = "Sulphates")
```

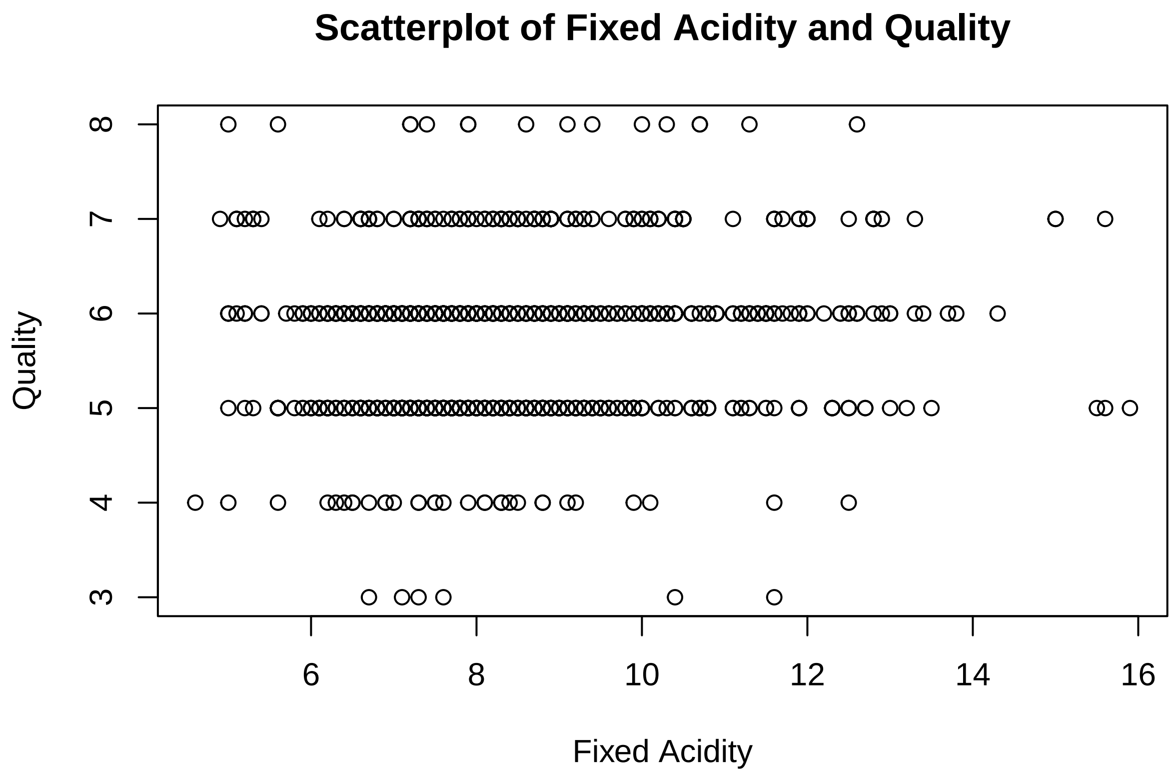



```
boxplot(Wine_Quality$alcohol, main = "Alcohol")
```

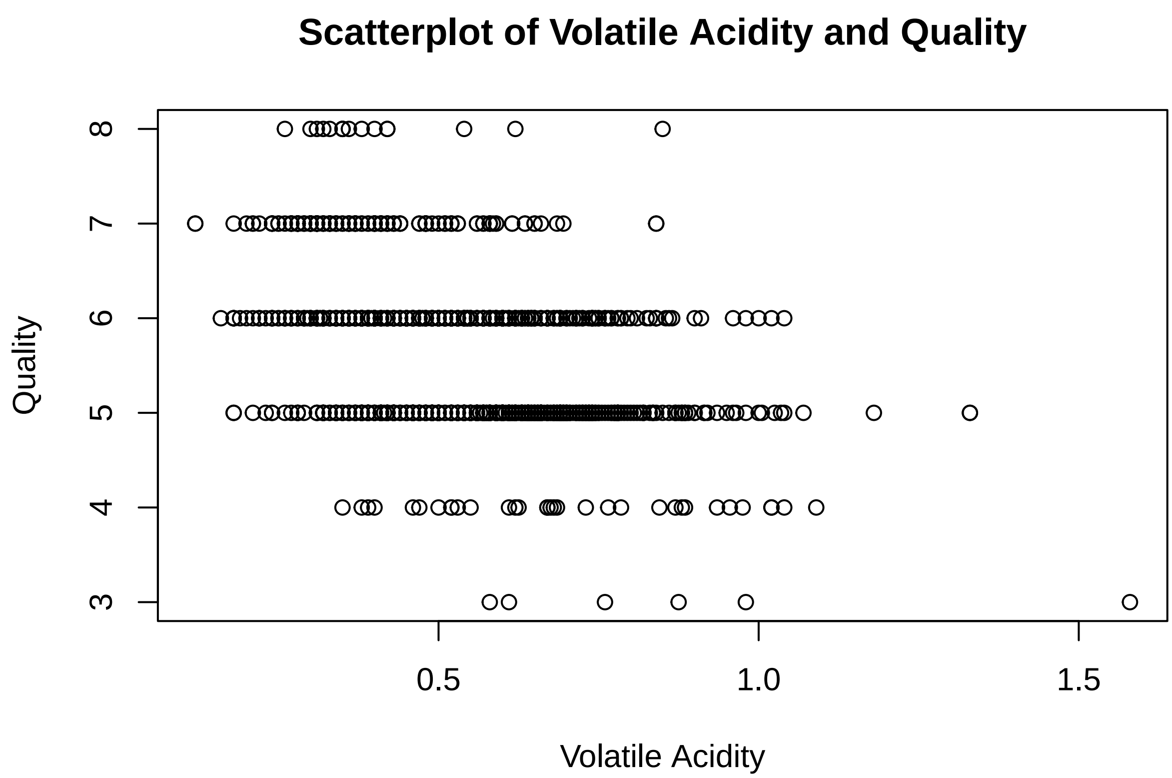
Scatter Plot

A scatterplot shows the relationship between two variables. In this data set, it's a visual of the relationship between the quality of wine and fixed acidity, pH, alcohol, etc… Variables with a strong relationship will have clusters of data points while variables with weak relationships will have data points that are spread out. From the scatter plots, it can be said there is a strong relationship between wine quality and the other variables as of now.

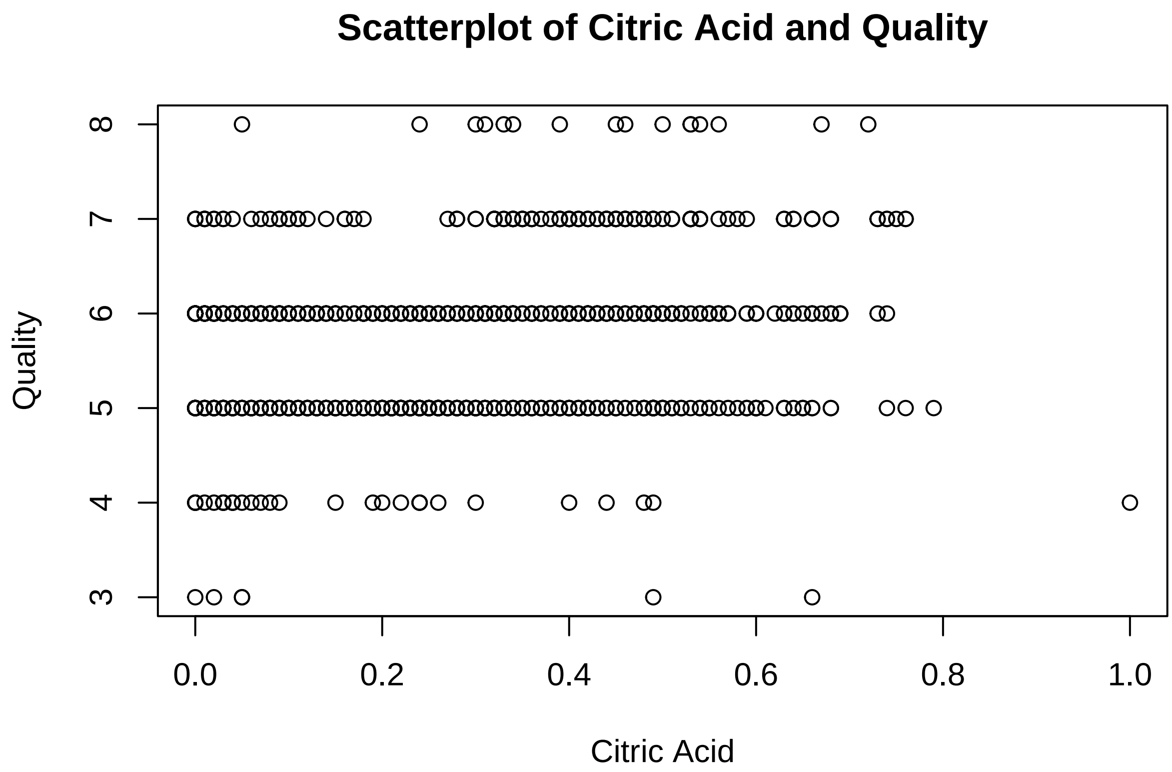
```
plot(Wine_Quality$fixed.acidity, Wine_Quality$quality, xlab="Fixed Acidity", ylab="Quality", main = "Sca
```



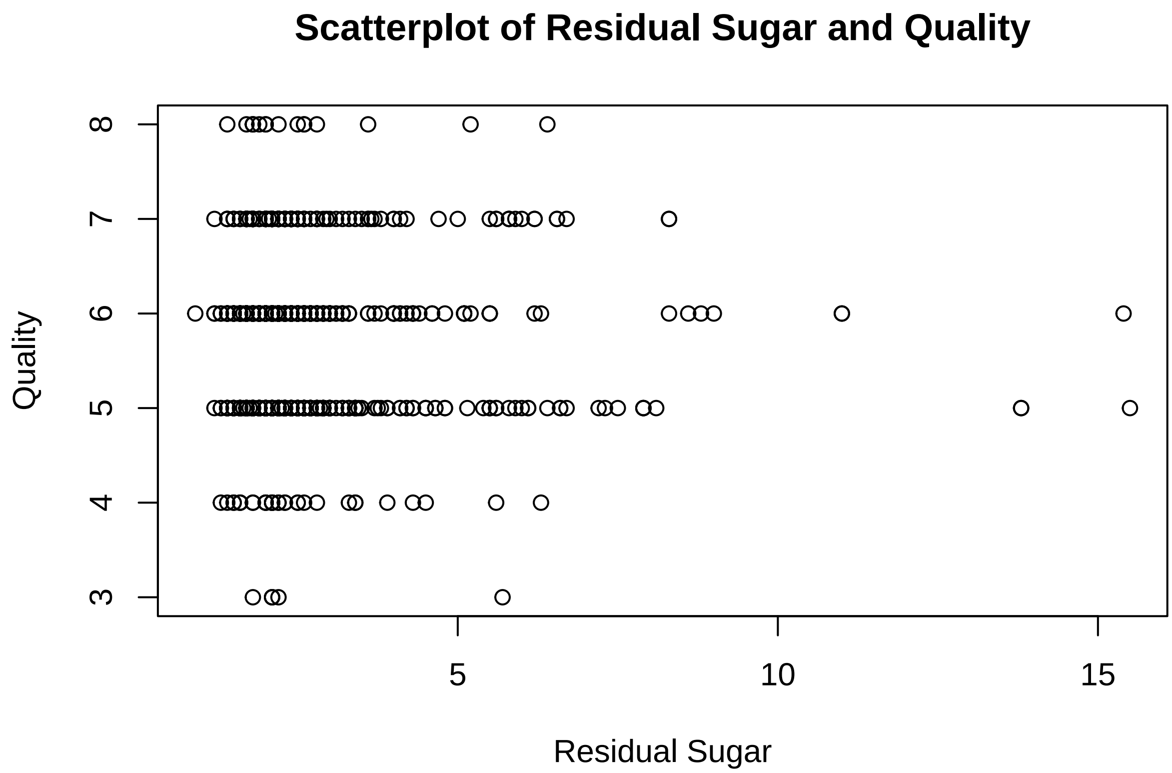

```
plot(Wine_Quality$volatile.acidity, Wine_Quality$quality, xlab="Volatile Acidity", ylab="Quality", main
```



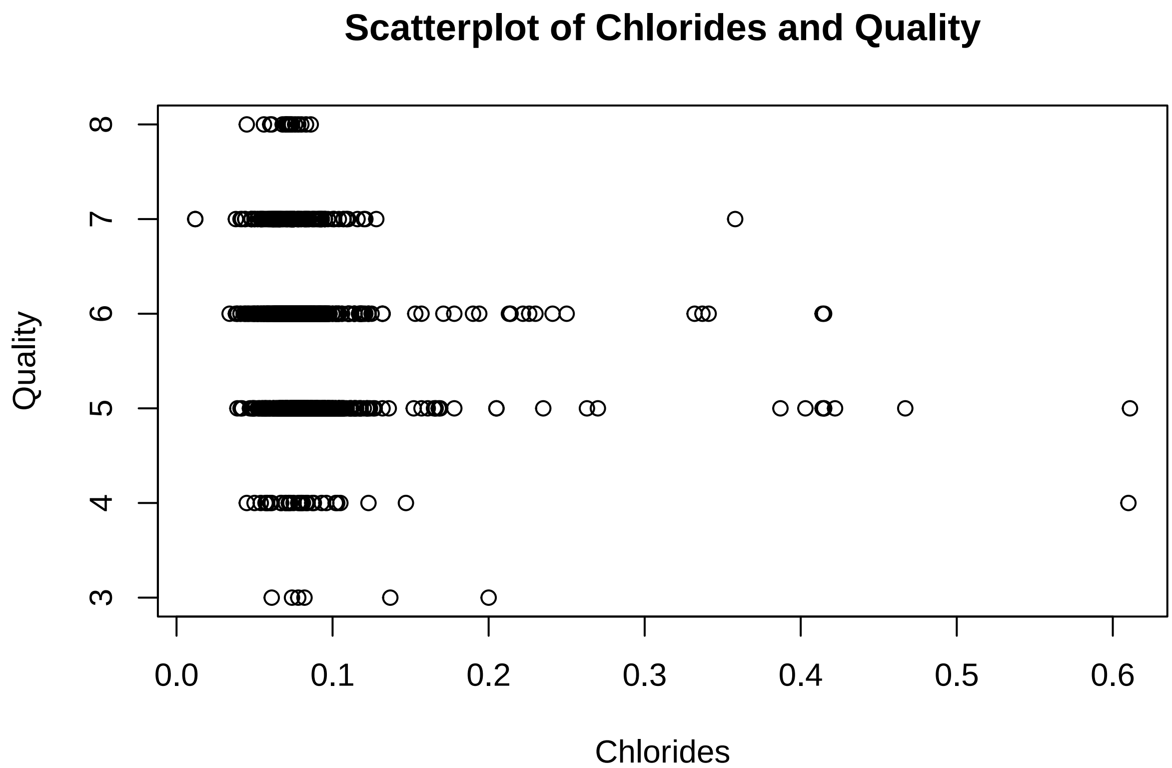

```
plot(Wine_Quality$citric.acid, Wine_Quality$quality, xlab="Citric Acid", ylab="Quality", main = "Scatte
```



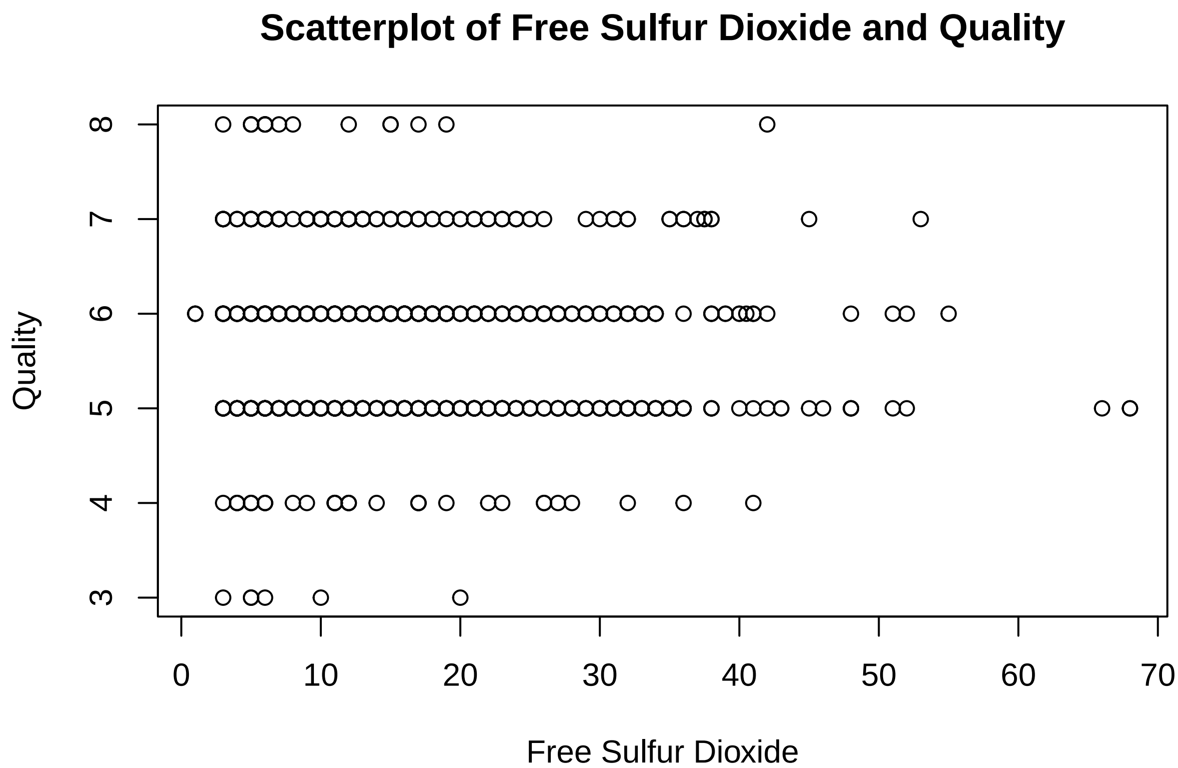

```
plot(Wine_Quality$residual.sugar, Wine_Quality$quality, xlab="Residual Sugar", ylab="Quality", main = "S
```



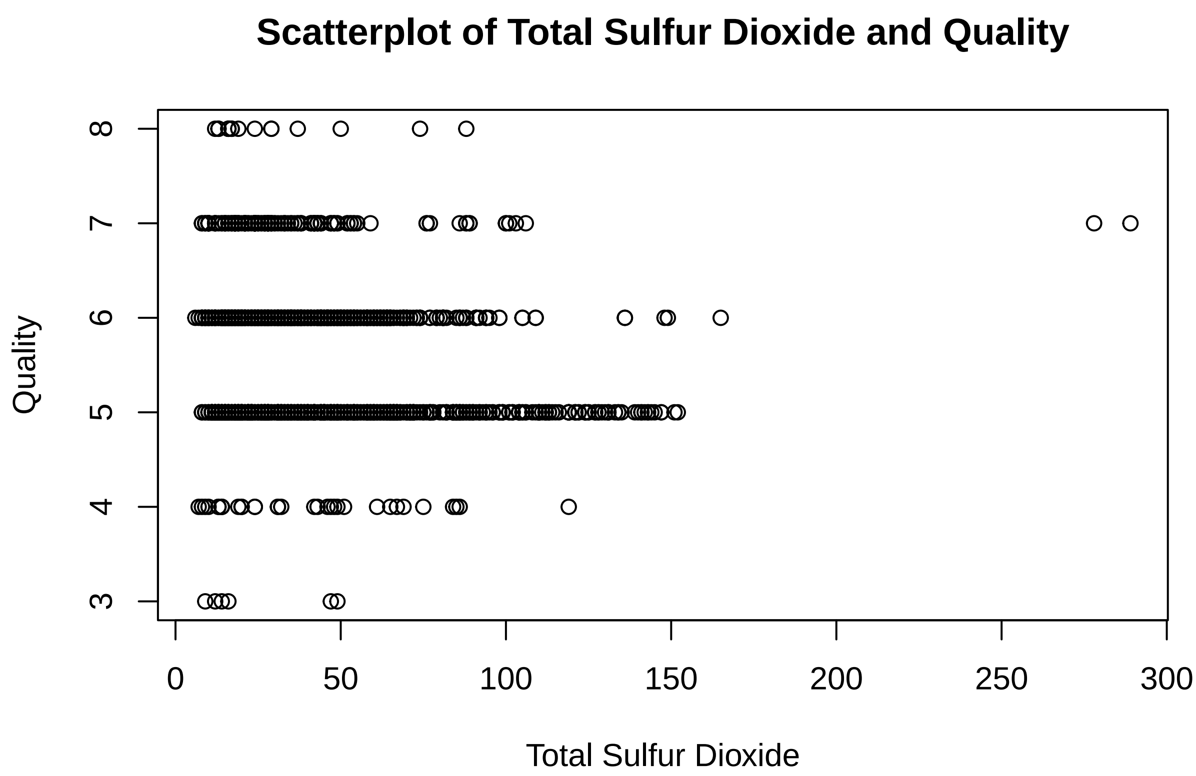

```
plot(Wine_Quality$chlorides, Wine_Quality$quality, xlab="Chlorides", ylab="Quality", main = "Scatterplo
```



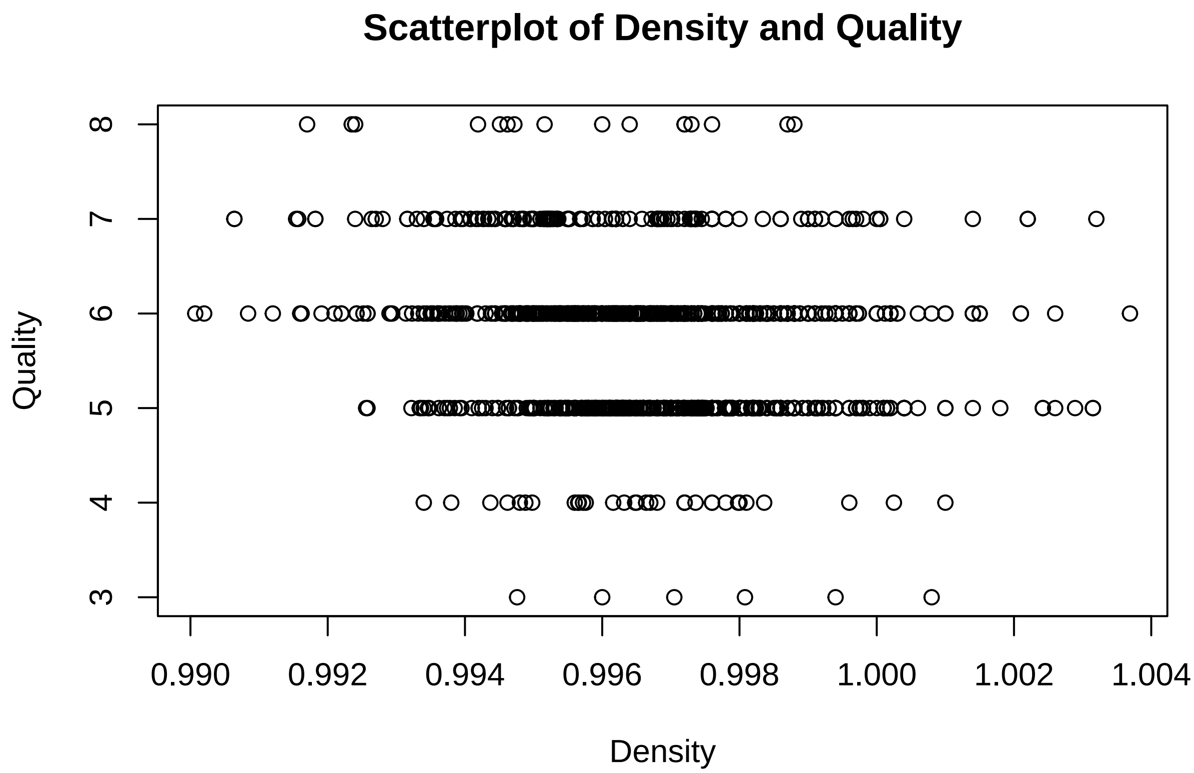

```
plot(Wine_Quality$free.sulfur.dioxide, Wine_Quality$quality, xlab="Free Sulfur Dioxide", ylab="Quality"
```



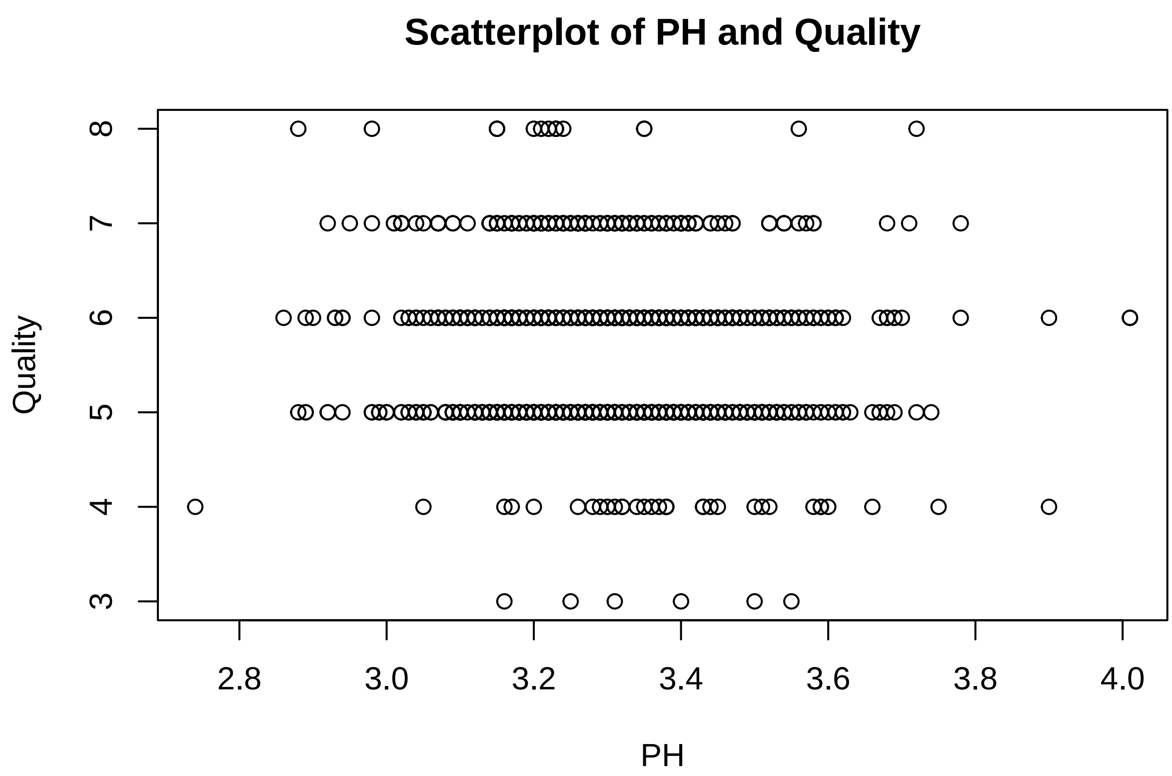

```
plot(Wine_Quality$total.sulfur.dioxide, Wine_Quality$quality, xlab="Total Sulfur Dioxide", ylab="Qualit
```



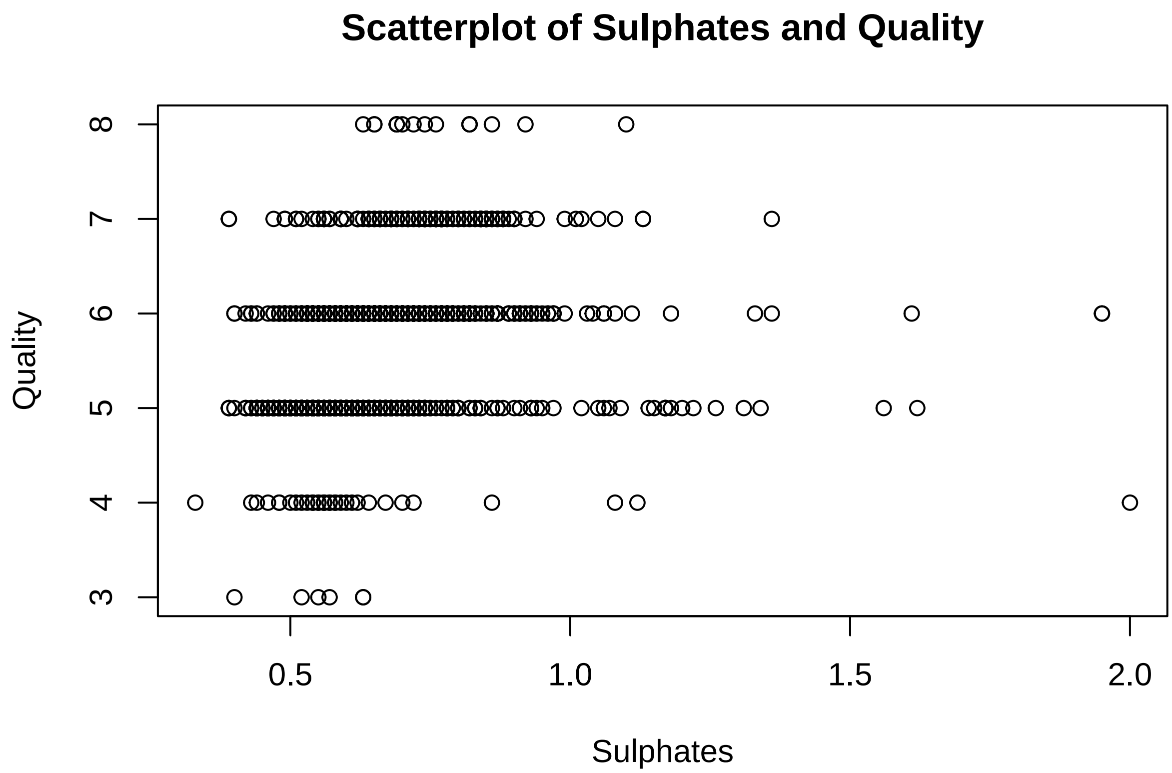

```
plot(Wine_Quality$density, Wine_Quality$quality, xlab="Density", ylab="Quality", main = "Scatterplot of
```



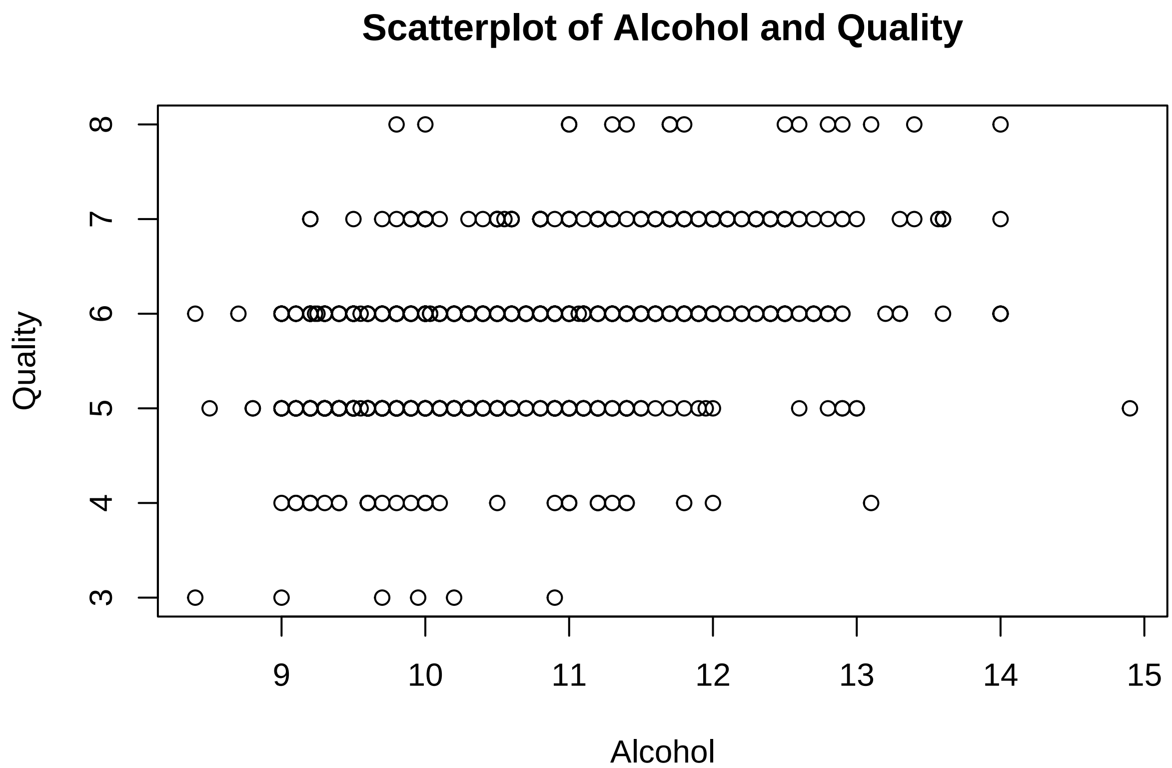

```
plot(Wine_Quality$pH, Wine_Quality$quality, xlab="PH", ylab="Quality", main = "Scatterplot of PH and Qua
```




```
plot(Wine_Quality$sulphates, Wine_Quality$quality, xlab="Sulphates", ylab="Quality", main = "Scatterplo
```




```
plot(Wine_Quality$alcohol, Wine_Quality$quality, xlab="Alcohol", ylab="Quality", main = "Scatterplot of
```



Statistical Summary

```
summary(Wine_Quality)
```

```
##  fixed.acidity    volatile.acidity  citric.acid     residual.sugar
##  Min.   : 4.600   Min.   :0.1200   Min.   :0.0000   Min.   : 0.900
##  1st Qu.: 7.100   1st Qu.:0.3925   1st Qu.:0.0900   1st Qu.: 1.900
##  Median : 7.900   Median :0.5200   Median :0.2500   Median : 2.200
##  Mean   : 8.311   Mean   :0.5313   Mean   :0.2684   Mean   : 2.532
##  3rd Qu.: 9.100   3rd Qu.:0.6400   3rd Qu.:0.4200   3rd Qu.: 2.600
##  Max.   :15.900   Max.   :1.5800   Max.   :1.0000   Max.   :15.500
##    chlorides       free.sulfur.dioxide total.sulfur.dioxide    density
##  Min.   :0.01200   Min.   : 1.00       Min.   :  6.00       Min.   :0.9901
##  1st Qu.:0.07000   1st Qu.: 7.00       1st Qu.: 21.00       1st Qu.:0.9956
##  Median :0.07900   Median :13.00       Median : 37.00       Median :0.9967
##  Mean   :0.08693   Mean   :15.62       Mean   : 45.91       Mean   :0.9967
##  3rd Qu.:0.09000   3rd Qu.:21.00       3rd Qu.: 61.00       3rd Qu.:0.9978
##  Max.   :0.61100   Max.   :68.00       Max.   :289.00       Max.   :1.0037
##        pH          sulphates         alcohol         quality
##  Min.   :2.740   Min.   :0.3300   Min.   : 8.40   Min.   :3.000
##  1st Qu.:3.205   1st Qu.:0.5500   1st Qu.: 9.50   1st Qu.:5.000
##  Median :3.310   Median :0.6200   Median :10.20   Median :6.000
##  Mean   :3.311   Mean   :0.6577   Mean   :10.44   Mean   :5.657
##  3rd Qu.:3.400   3rd Qu.:0.7300   3rd Qu.:11.10   3rd Qu.:6.000
```

```
## Max.   :4.010   Max.   :2.0000   Max.   :14.90   Max.   :8.000
##        Id
## Min.   :   0
## 1st Qu.: 411
## Median : 794
## Mean   : 805
## 3rd Qu.:1210
## Max.   :1597
```

Question 2

Multiple Linear Regression Model

```
wineq <- lm(quality ~ fixed.acidity + volatile.acidity + citric.acid + residual.sugar + chlorides + free
summary(wineq)
```

```
##
## Call:
## lm(formula = quality ~ fixed.acidity + volatile.acidity + citric.acid +
##     residual.sugar + chlorides + free.sulfur.dioxide + total.sulfur.dioxide +
##     density + pH + sulphates + alcohol, data = Wine_Quality)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.49977 -0.36903 -0.04658  0.43956  2.00117
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           2.155e+01  2.477e+01   0.870 0.384551
## fixed.acidity         2.297e-02  3.025e-02   0.759 0.447770
## volatile.acidity     -1.129e+00  1.407e-01  -8.023 2.56e-15 ***
## citric.acid          -1.319e-01  1.730e-01  -0.762 0.446105
## residual.sugar        1.351e-02  1.846e-02   0.732 0.464278
## chlorides            -1.708e+00  4.974e-01  -3.434 0.000616 ***
## free.sulfur.dioxide   2.369e-03  2.553e-03   0.928 0.353547
## total.sulfur.dioxide -2.785e-03  8.386e-04  -3.321 0.000926 ***
## density              -1.745e+01  2.529e+01  -0.690 0.490284
## pH                   -4.082e-01  2.229e-01  -1.832 0.067280 .
## sulphates             8.752e-01  1.335e-01   6.555 8.44e-11 ***
## alcohol               2.801e-01  3.126e-02   8.963  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6405 on 1131 degrees of freedom
## Multiple R-squared:  0.3742, Adjusted R-squared:  0.3682
## F-statistic: 61.49 on 11 and 1131 DF,  p-value: < 2.2e-16
```

Outlier, High Leverage, and New Model

It is evident that there are outliers and high leverage observations by the values outputted by the boxplot.stats() and hats() codes. The values derived by performing these tests confirm the outcomes of the histogram, boxplot, scatterplot, and summary functions. These values are important because they are capable of altering the model's best-fit line.

```
# Outlier
# uses boxplot.stats() with $out  to capture outliers
boxplot.stats(Wine_Quality$fixed.acidity)$out
```

```
##  [1] 12.8 12.8 15.0 15.0 12.5 13.3 13.4 12.5 13.8 13.5 12.6 12.5 12.8 12.8 13.7
## [16] 12.2 12.5 12.8 12.3 12.3 12.6 15.6 12.5 13.0 12.5 13.3 12.5 12.9 14.3 12.4
## [31] 15.5 15.6 13.0 12.7 13.0 12.7 12.3 12.3 12.4 13.2 15.9 12.9 12.6 12.6
```

```
boxplot.stats(Wine_Quality$volatile.acidity)$out
```

```
##  [1] 1.020 1.070 1.330 1.330 1.040 1.090 1.040 1.020 1.035 1.025 1.020 1.580
## [13] 1.180 1.040
```

```
boxplot.stats(Wine_Quality$citric.acid)$out
```

```
## [1] 1
```

```
boxplot.stats(Wine_Quality$residual.sugar)$out
```

```
##   [1]  5.50  5.90  4.65  4.65  5.50  5.50  5.50  7.30  7.20  5.60  4.00  4.00
##  [13]  4.00  4.00  6.40  5.60  5.60 11.00 11.00  4.50  4.80  5.80  5.80  6.20
##  [25]  4.20  7.90  7.90  4.50  6.70  6.60  3.70  5.20 15.50  8.30  6.55  6.55
##  [37]  6.10  4.30  5.80  5.15  6.30  4.20  4.60  4.20  4.30  4.30  7.90  5.10
##  [49]  5.60  8.60  7.50  6.00  3.90  4.20  4.00  6.60  6.00  3.80  9.00  8.80
##  [61]  5.00  3.80  4.10  5.90  4.10  6.20  4.00  3.90  4.00  8.10  6.40  8.30
##  [73]  8.30  4.70  5.50  5.50  4.30  5.50  3.70  6.20  5.60  4.60  5.80  4.10
##  [85]  4.30  4.80  6.30  4.50  4.50  4.30  3.80  5.40  6.10  5.10  5.10  3.90
##  [97] 15.40  4.80  5.20  3.75 13.80 13.80  5.70  4.30  4.10  4.10  4.40  3.70
## [109]  6.70  5.10
```

```
boxplot.stats(Wine_Quality$chlorides)$out
```

```
##  [1] 0.341 0.332 0.467 0.178 0.610 0.270 0.039 0.337 0.263 0.611 0.358 0.213
## [13] 0.214 0.121 0.128 0.120 0.122 0.122 0.121 0.127 0.152 0.125 0.122 0.200
## [25] 0.226 0.250 0.124 0.222 0.039 0.157 0.422 0.034 0.387 0.415 0.157 0.241
## [37] 0.190 0.132 0.126 0.038 0.165 0.147 0.012 0.012 0.194 0.132 0.161 0.120
## [49] 0.120 0.123 0.123 0.414 0.171 0.178 0.166 0.136 0.132 0.132 0.123 0.123
## [61] 0.403 0.137 0.414 0.166 0.168 0.415 0.153 0.415 0.123 0.214 0.169 0.205
## [73] 0.205 0.039 0.235 0.230 0.038
```

```
boxplot.stats(Wine_Quality$free.sulfur.dioxide)$out
```

```
##  [1] 68 68 43 46 45 53 52 51 45 48 48 43 51 52 55 48 48 66
```

```r
boxplot.stats(Wine_Quality$total.sulfur.dioxide)$out
```

```
##  [1] 136 125 140 136 134 141 128 129 128 143 127 135 165 124 124 122 134 124 151
## [20] 142 149 147 145 148 152 122 125 127 139 143 144 130 278 289 141 133 147 131
## [39] 131 131
```

```r
boxplot.stats(Wine_Quality$density)$out
```

```
##  [1] 0.99160 0.99160 1.00140 1.00150 1.00150 1.00180 0.99120 1.00220 1.00220
## [10] 1.00140 1.00140 1.00320 1.00260 1.00140 1.00315 1.00315 1.00210 1.00210
## [19] 0.99170 1.00260 0.99210 0.99154 0.99064 0.99064 1.00289 0.99162 0.99007
## [28] 0.99020 0.99157 0.99084 0.99191 1.00369 1.00242 0.99182 1.00242 0.99182
```

```r
boxplot.stats(Wine_Quality$pH)$out
```

```
##  [1] 3.90 3.75 2.74 2.88 2.86 3.74 3.72 2.89 2.89 3.90 3.71 2.89 3.78 3.70 3.78
## [16] 4.01 2.90 4.01 2.88 3.72
```

```r
boxplot.stats(Wine_Quality$sulphates)$out
```

```
##  [1] 1.56 1.08 1.20 1.12 1.95 1.22 1.95 1.31 2.00 1.08 1.02 1.61 1.09 1.26 1.08
## [16] 1.36 1.13 1.04 1.11 1.13 1.07 1.06 1.06 1.05 1.02 1.14 1.36 1.05 1.17 1.62
## [31] 1.06 1.18 1.34 1.15 1.17 1.17 1.33 1.18 1.17 1.03 1.17 1.10 1.01
```

```r
boxplot.stats(Wine_Quality$alcohol)$out
```

```
##  [1] 14.00000 14.00000 14.00000 14.00000 14.90000 14.00000 13.60000 13.60000
##  [9] 13.60000 14.00000 13.56667 13.60000
```

```r
# High Leverage
# creates a dataframe named hats
# uses hatvalues to be able to see high leverage
hats <- as.data.frame(hatvalues(wineq))
hats
```

```
##      hatvalues(wineq)
## 1         0.005079901
## 2         0.008111271
## 3         0.004023940
## 4         0.007623793
## 5         0.005079901
## 6         0.005195357
## 7         0.004376642
## 8         0.006281831
## 9         0.004199146
## 10        0.004587509
## 11        0.007865355
## 12        0.039475498
## 13        0.009102351
```

```
## 14          0.031016001
## 15          0.007029088
## 16          0.006842187
## 17          0.007758928
## 18          0.004464620
## 19          0.006623544
## 20          0.004968706
## 21          0.003894204
## 22          0.004424527
## 23          0.004086631
## 24          0.010734583
## 25          0.012192743
## 26          0.011429170
## 27          0.005009245
## 28          0.004019111
## 29          0.013214273
## 30          0.003247586
## 31          0.033958794
## 32          0.018021960
## 33          0.023179000
## 34          0.017739270
## 35          0.006858603
## 36          0.005920293
## 37          0.006436876
## 38          0.007741004
## 39          0.006880550
## 40          0.011403010
## 41          0.002198632
## 42          0.006725434
## 43          0.007631385
## 44          0.012291719
## 45          0.002939521
## 46          0.008984319
## 47          0.008984319
## 48          0.005352136
## 49          0.009143397
## 50          0.005917860
## 51          0.003420589
## 52          0.014484615
## 53          0.005104983
## 54          0.005348206
## 55          0.018567518
## 56          0.007867479
## 57          0.008761552
## 58          0.010479424
## 59          0.002094741
## 60          0.081831903
## 61          0.003620478
## 62          0.017934258
## 63          0.005626477
## 64          0.011034703
## 65          0.081831903
## 66          0.003620478
## 67          0.026058577
```

```
## 68          0.004965231
## 69          0.006924950
## 70          0.003190962
## 71          0.002627188
## 72          0.003190962
## 73          0.004031028
## 74          0.005401992
## 75          0.004031028
## 76          0.062431247
## 77          0.009445839
## 78          0.006613036
## 79          0.013188200
## 80          0.005808918
## 81          0.006613036
## 82          0.005807121
## 83          0.002636436
## 84          0.017216599
## 85          0.010090365
## 86          0.006487944
## 87          0.009381146
## 88          0.025899899
## 89          0.025814195
## 90          0.008154838
## 91          0.017073414
## 92          0.016196634
## 93          0.011667638
## 94          0.006802741
## 95          0.007359562
## 96          0.006802741
## 97          0.019198679
## 98          0.007868277
## 99          0.019198679
## 100         0.016013518
## 101         0.008720091
## 102         0.002624015
## 103         0.008817519
## 104         0.133177620
## 105         0.009118347
## 106         0.009118347
## 107         0.012727589
## 108         0.012756503
## 109         0.012727589
## 110         0.004855619
## 111         0.009480488
## 112         0.009656767
## 113         0.018813497
## 114         0.003852312
## 115         0.018054168
## 116         0.017540038
## 117         0.010572574
## 118         0.006444366
## 119         0.006811906
## 120         0.003491123
## 121         0.008754605
```

```
## 122        0.004811525
## 123        0.006578860
## 124        0.005091483
## 125        0.005738819
## 126        0.009100582
## 127        0.005299619
## 128        0.003903896
## 129        0.003903896
## 130        0.019719613
## 131        0.003667121
## 132        0.006932116
## 133        0.007178859
## 134        0.008580822
## 135        0.008959370
## 136        0.016851147
## 137        0.008874103
## 138        0.008743930
## 139        0.003114941
## 140        0.003114941
## 141        0.003397219
## 142        0.010101226
## 143        0.014083877
## 144        0.005132741
## 145        0.006620674
## 146        0.010819629
## 147        0.010819629
## 148        0.005353649
## 149        0.012565433
## 150        0.003784830
## 151        0.004905335
## 152        0.005009256
## 153        0.015115312
## 154        0.006786415
## 155        0.006552290
## 156        0.004272601
## 157        0.005587752
## 158        0.005362801
## 159        0.006347747
## 160        0.003965990
## 161        0.003260961
## 162        0.047373878
## 163        0.013709180
## 164        0.008068169
## 165        0.004527924
## 166        0.012228389
## 167        0.004337255
## 168        0.020350076
## 169        0.006367275
## 170        0.012445977
## 171        0.038927348
## 172        0.038927348
## 173        0.013354946
## 174        0.002880585
## 175        0.013354946
```

```
## 176         0.005447317
## 177         0.004486328
## 178         0.005743384
## 179         0.013168467
## 180         0.009846312
## 181         0.005307292
## 182         0.005441965
## 183         0.118854325
## 184         0.006671231
## 185         0.004892508
## 186         0.009623948
## 187         0.005224927
## 188         0.004748625
## 189         0.010272145
## 190         0.013920363
## 191         0.015867654
## 192         0.016450816
## 193         0.015948092
## 194         0.005741738
## 195         0.005741738
## 196         0.016450816
## 197         0.015948092
## 198         0.014611609
## 199         0.008730115
## 200         0.013429971
## 201         0.039029734
## 202         0.008730115
## 203         0.012499295
## 204         0.012499295
## 205         0.008724965
## 206         0.005379783
## 207         0.006233517
## 208         0.012040431
## 209         0.006233517
## 210         0.006000866
## 211         0.005537018
## 212         0.010874625
## 213         0.008722790
## 214         0.009601031
## 215         0.010585386
## 216         0.013886143
## 217         0.003703577
## 218         0.010377901
## 219         0.007113945
## 220         0.014361988
## 221         0.005800270
## 222         0.013157968
## 223         0.012623822
## 224         0.008455737
## 225         0.006137896
## 226         0.009964636
## 227         0.004332168
## 228         0.007314865
## 229         0.007856334
```

```
## 230         0.006368845
## 231         0.045802556
## 232         0.045802556
## 233         0.008834126
## 234         0.006942294
## 235         0.014390546
## 236         0.009720319
## 237         0.007599918
## 238         0.012673264
## 239         0.006153202
## 240         0.005348831
## 241         0.026551003
## 242         0.009479648
## 243         0.009479648
## 244         0.009438034
## 245         0.015746980
## 246         0.005781693
## 247         0.009408379
## 248         0.006095757
## 249         0.028558323
## 250         0.026248601
## 251         0.011641691
## 252         0.010141486
## 253         0.006726683
## 254         0.006654301
## 255         0.008701215
## 256         0.005024658
## 257         0.004753529
## 258         0.009562148
## 259         0.010455555
## 260         0.013921791
## 261         0.013921791
## 262         0.011758921
## 263         0.011547039
## 264         0.013992321
## 265         0.010405945
## 266         0.012926691
## 267         0.011547039
## 268         0.023843261
## 269         0.006974247
## 270         0.004847490
## 271         0.005092769
## 272         0.020007777
## 273         0.010750765
## 274         0.006550467
## 275         0.023650110
## 276         0.039783183
## 277         0.006765622
## 278         0.007956123
## 279         0.039783183
## 280         0.007861485
## 281         0.005216551
## 282         0.005167776
## 283         0.003596048
```

```
## 284        0.006890454
## 285        0.021423679
## 286        0.005482052
## 287        0.004727162
## 288        0.013054666
## 289        0.007306184
## 290        0.023402575
## 291        0.007871348
## 292        0.010072335
## 293        0.007953246
## 294        0.011574141
## 295        0.012706800
## 296        0.010430451
## 297        0.006622259
## 298        0.012706800
## 299        0.003944442
## 300        0.008553293
## 301        0.003698456
## 302        0.017075068
## 303        0.006622259
## 304        0.005273926
## 305        0.010108172
## 306        0.007040437
## 307        0.010108172
## 308        0.011261060
## 309        0.009360822
## 310        0.011490732
## 311        0.011537910
## 312        0.025358646
## 313        0.010417620
## 314        0.010046370
## 315        0.004379162
## 316        0.003219880
## 317        0.008076105
## 318        0.008076105
## 319        0.004270310
## 320        0.011048877
## 321        0.007651280
## 322        0.019567842
## 323        0.005388295
## 324        0.004917417
## 325        0.013032216
## 326        0.005976044
## 327        0.004078504
## 328        0.007996837
## 329        0.008754485
## 330        0.012433077
## 331        0.009526742
## 332        0.016791331
## 333        0.008973606
## 334        0.004982638
## 335        0.009640338
## 336        0.007973323
## 337        0.004951452
```

```
## 338         0.009118691
## 339         0.006120230
## 340         0.106565971
## 341         0.007306222
## 342         0.009940647
## 343         0.006553434
## 344         0.006943406
## 345         0.003293674
## 346         0.008400204
## 347         0.010253720
## 348         0.020589839
## 349         0.024727263
## 350         0.004409090
## 351         0.004947392
## 352         0.013087617
## 353         0.004409090
## 354         0.020589839
## 355         0.004947392
## 356         0.020551793
## 357         0.020551793
## 358         0.009875256
## 359         0.009846301
## 360         0.008868427
## 361         0.009615620
## 362         0.012181753
## 363         0.008574148
## 364         0.004609210
## 365         0.009893982
## 366         0.009893982
## 367         0.025941965
## 368         0.013859403
## 369         0.011810765
## 370         0.010962999
## 371         0.012043933
## 372         0.007779451
## 373         0.010138358
## 374         0.005522090
## 375         0.013897454
## 376         0.007779723
## 377         0.007779723
## 378         0.014623105
## 379         0.008180976
## 380         0.013897454
## 381         0.005522090
## 382         0.007608308
## 383         0.019157753
## 384         0.012549933
## 385         0.002669846
## 386         0.012289740
## 387         0.005982028
## 388         0.012811628
## 389         0.018746333
## 390         0.011146179
## 391         0.005947660
```

```
## 392         0.005432180
## 393         0.009596262
## 394         0.005086123
## 395         0.003389547
## 396         0.004417586
## 397         0.030825023
## 398         0.025866299
## 399         0.019690144
## 400         0.026132260
## 401         0.029248156
## 402         0.010039624
## 403         0.012731858
## 404         0.007694591
## 405         0.029248156
## 406         0.010039624
## 407         0.013477930
## 408         0.017124899
## 409         0.006930447
## 410         0.009566124
## 411         0.004912524
## 412         0.006744248
## 413         0.008704061
## 414         0.008420771
## 415         0.006231876
## 416         0.010115137
## 417         0.010115137
## 418         0.009559295
## 419         0.006806976
## 420         0.015828703
## 421         0.005841226
## 422         0.025544368
## 423         0.005841226
## 424         0.009191441
## 425         0.018638090
## 426         0.008576398
## 427         0.003954989
## 428         0.006229862
## 429         0.006010567
## 430         0.005674210
## 431         0.019888309
## 432         0.013569976
## 433         0.011202074
## 434         0.010302514
## 435         0.003932326
## 436         0.013615173
## 437         0.008909637
## 438         0.007270941
## 439         0.008210024
## 440         0.007443318
## 441         0.004748573
## 442         0.015410233
## 443         0.005628613
## 444         0.005628613
## 445         0.002867840
```

```
## 446        0.004271219
## 447        0.002867840
## 448        0.007069472
## 449        0.004140436
## 450        0.023923626
## 451        0.012181585
## 452        0.012794476
## 453        0.016363669
## 454        0.015786810
## 455        0.022727770
## 456        0.007195552
## 457        0.007195552
## 458        0.007195552
## 459        0.006907687
## 460        0.001791692
## 461        0.037970561
## 462        0.013166343
## 463        0.064431145
## 464        0.017399250
## 465        0.013168482
## 466        0.012250051
## 467        0.013166343
## 468        0.012702911
## 469        0.008022115
## 470        0.008022115
## 471        0.006411230
## 472        0.004665163
## 473        0.007553608
## 474        0.014709885
## 475        0.008890054
## 476        0.008890054
## 477        0.006281198
## 478        0.006281198
## 479        0.005332526
## 480        0.004426889
## 481        0.013874626
## 482        0.011200291
## 483        0.004629409
## 484        0.006649564
## 485        0.006866976
## 486        0.021461370
## 487        0.006866976
## 488        0.006273689
## 489        0.010854356
## 490        0.014800555
## 491        0.047290280
## 492        0.009511356
## 493        0.009359931
## 494        0.021617918
## 495        0.004578662
## 496        0.006762835
## 497        0.005876937
## 498        0.018529768
## 499        0.004578662
```

```
## 500         0.004729266
## 501         0.007112874
## 502         0.005939253
## 503         0.015524387
## 504         0.004577662
## 505         0.002813320
## 506         0.004385304
## 507         0.018084401
## 508         0.011204375
## 509         0.005966568
## 510         0.006388212
## 511         0.007729637
## 512         0.005196377
## 513         0.007729637
## 514         0.009955767
## 515         0.062110237
## 516         0.003325647
## 517         0.014533859
## 518         0.006651515
## 519         0.006651515
## 520         0.012517657
## 521         0.046096151
## 522         0.014948332
## 523         0.004961432
## 524         0.008680905
## 525         0.005776592
## 526         0.008964746
## 527         0.012297773
## 528         0.006784455
## 529         0.008975675
## 530         0.011444870
## 531         0.010515204
## 532         0.005503159
## 533         0.007021380
## 534         0.005101395
## 535         0.005503159
## 536         0.003212240
## 537         0.003628557
## 538         0.003212240
## 539         0.048494966
## 540         0.011129079
## 541         0.010087377
## 542         0.005922074
## 543         0.005922074
## 544         0.008646773
## 545         0.009862676
## 546         0.005025752
## 547         0.005335595
## 548         0.004264042
## 549         0.004668932
## 550         0.005471347
## 551         0.008754771
## 552         0.004580484
## 553         0.008754771
```

```
## 554         0.015048777
## 555         0.015798171
## 556         0.009343257
## 557         0.010665412
## 558         0.016569613
## 559         0.003670652
## 560         0.008044206
## 561         0.012141367
## 562         0.010582744
## 563         0.007808802
## 564         0.005815284
## 565         0.006908585
## 566         0.006908585
## 567         0.011906876
## 568         0.005876660
## 569         0.010342706
## 570         0.006894703
## 571         0.004542554
## 572         0.008739116
## 573         0.024894062
## 574         0.004867659
## 575         0.008150074
## 576         0.005269406
## 577         0.005914954
## 578         0.018010025
## 579         0.004221806
## 580         0.003050112
## 581         0.007545450
## 582         0.008451368
## 583         0.007954023
## 584         0.005222558
## 585         0.008375444
## 586         0.005558538
## 587         0.009957454
## 588         0.007680146
## 589         0.003830349
## 590         0.016560130
## 591         0.005996954
## 592         0.009899322
## 593         0.005945645
## 594         0.004389160
## 595         0.004096525
## 596         0.007534837
## 597         0.027532005
## 598         0.023159059
## 599         0.023159059
## 600         0.005537560
## 601         0.004678989
## 602         0.009460657
## 603         0.010083623
## 604         0.007014766
## 605         0.004399480
## 606         0.007014766
## 607         0.005678085
```

```
## 608         0.005678085
## 609         0.015898135
## 610         0.015898135
## 611         0.015898135
## 612         0.004278369
## 613         0.010689713
## 614         0.007361671
## 615         0.007292338
## 616         0.006532395
## 617         0.006710544
## 618         0.009943311
## 619         0.006108955
## 620         0.004470924
## 621         0.011588365
## 622         0.008686844
## 623         0.013956713
## 624         0.005593725
## 625         0.004229023
## 626         0.005172338
## 627         0.006156458
## 628         0.018335219
## 629         0.007604259
## 630         0.006330111
## 631         0.007604259
## 632         0.007962758
## 633         0.004663281
## 634         0.005123541
## 635         0.004663281
## 636         0.005123541
## 637         0.005592100
## 638         0.005072712
## 639         0.005072712
## 640         0.011886791
## 641         0.005127564
## 642         0.006409780
## 643         0.005798826
## 644         0.013979715
## 645         0.029481967
## 646         0.005616815
## 647         0.004899745
## 648         0.015101840
## 649         0.027337598
## 650         0.004858245
## 651         0.014737887
## 652         0.003340743
## 653         0.014737887
## 654         0.005127724
## 655         0.022672262
## 656         0.025034732
## 657         0.004624759
## 658         0.005127724
## 659         0.006174217
## 660         0.004267040
## 661         0.006067928
```

```
## 662         0.009680245
## 663         0.006573470
## 664         0.006853510
## 665         0.014892666
## 666         0.002548746
## 667         0.006516621
## 668         0.009393250
## 669         0.007482932
## 670         0.008340855
## 671         0.008340855
## 672         0.007482932
## 673         0.004246215
## 674         0.009124761
## 675         0.006547528
## 676         0.005937867
## 677         0.005151231
## 678         0.006096341
## 679         0.004860436
## 680         0.005465453
## 681         0.006096341
## 682         0.008863624
## 683         0.008471542
## 684         0.008266779
## 685         0.010812274
## 686         0.008678351
## 687         0.008965891
## 688         0.007292361
## 689         0.002747866
## 690         0.007653995
## 691         0.023051842
## 692         0.002747866
## 693         0.004804278
## 694         0.010378502
## 695         0.009155375
## 696         0.006392998
## 697         0.009155375
## 698         0.008920039
## 699         0.008920039
## 700         0.002764909
## 701         0.019219638
## 702         0.011802889
## 703         0.004135039
## 704         0.005030545
## 705         0.009109378
## 706         0.005546230
## 707         0.005321142
## 708         0.005124043
## 709         0.008621481
## 710         0.005039610
## 711         0.010650066
## 712         0.004685276
## 713         0.006446435
## 714         0.005116012
## 715         0.037193878
```

```
## 716         0.003705626
## 717         0.012398413
## 718         0.012398413
## 719         0.004613892
## 720         0.008728782
## 721         0.004418403
## 722         0.006891000
## 723         0.007360142
## 724         0.007540516
## 725         0.004418403
## 726         0.007561172
## 727         0.003902842
## 728         0.004367703
## 729         0.010138885
## 730         0.013850561
## 731         0.003970697
## 732         0.003302993
## 733         0.008382766
## 734         0.011754239
## 735         0.010450933
## 736         0.004613816
## 737         0.003276449
## 738         0.002617161
## 739         0.047571012
## 740         0.008599023
## 741         0.006233330
## 742         0.008664132
## 743         0.008826964
## 744         0.008977899
## 745         0.009641035
## 746         0.008826964
## 747         0.008784370
## 748         0.006763227
## 749         0.004798463
## 750         0.008005156
## 751         0.007573492
## 752         0.002551232
## 753         0.006226865
## 754         0.012010198
## 755         0.006037430
## 756         0.007769967
## 757         0.021496085
## 758         0.004195011
## 759         0.006913759
## 760         0.019281406
## 761         0.089029480
## 762         0.095333756
## 763         0.006697048
## 764         0.007236448
## 765         0.017810722
## 766         0.007846673
## 767         0.006942330
## 768         0.009289638
## 769         0.009289638
```

```
## 770          0.018742820
## 771          0.004364177
## 772          0.003475888
## 773          0.005188055
## 774          0.011283793
## 775          0.011283793
## 776          0.005412347
## 777          0.010235618
## 778          0.003822216
## 779          0.003822216
## 780          0.005802851
## 781          0.009555612
## 782          0.004998834
## 783          0.005931687
## 784          0.009606222
## 785          0.006697282
## 786          0.014417906
## 787          0.007216853
## 788          0.027627730
## 789          0.004558150
## 790          0.004558150
## 791          0.004558150
## 792          0.012757563
## 793          0.009191437
## 794          0.012024577
## 795          0.010704968
## 796          0.007835479
## 797          0.007625080
## 798          0.005199395
## 799          0.010553850
## 800          0.010285306
## 801          0.010452715
## 802          0.004665440
## 803          0.006103196
## 804          0.004674711
## 805          0.011672394
## 806          0.004438053
## 807          0.006514514
## 808          0.003629586
## 809          0.006709971
## 810          0.012330594
## 811          0.007194722
## 812          0.012898719
## 813          0.009701620
## 814          0.013093002
## 815          0.003990406
## 816          0.006197804
## 817          0.007483683
## 818          0.004173496
## 819          0.002737305
## 820          0.004173496
## 821          0.014153490
## 822          0.016977571
## 823          0.006346128
```

```
## 824         0.005572442
## 825         0.007641965
## 826         0.011957933
## 827         0.003436975
## 828         0.003748221
## 829         0.005517165
## 830         0.006234082
## 831         0.006234082
## 832         0.017923817
## 833         0.014336139
## 834         0.018309636
## 835         0.003222426
## 836         0.008037051
## 837         0.004086144
## 838         0.013385955
## 839         0.013385955
## 840         0.011413265
## 841         0.009772217
## 842         0.012428795
## 843         0.008200358
## 844         0.004964455
## 845         0.005907860
## 846         0.007866858
## 847         0.009381935
## 848         0.004291227
## 849         0.007866858
## 850         0.005777311
## 851         0.003882772
## 852         0.013347722
## 853         0.007340511
## 854         0.007340511
## 855         0.017916361
## 856         0.007340511
## 857         0.010277925
## 858         0.003905776
## 859         0.003905776
## 860         0.006995281
## 861         0.005288668
## 862         0.008493557
## 863         0.008638927
## 864         0.008638927
## 865         0.008233760
## 866         0.010019829
## 867         0.017050344
## 868         0.002621911
## 869         0.020759346
## 870         0.003511985
## 871         0.007124679
## 872         0.014832368
## 873         0.003511985
## 874         0.012187564
## 875         0.005997235
## 876         0.012187564
## 877         0.004994217
```

```
## 878        0.007185641
## 879        0.015181362
## 880        0.006830291
## 881        0.002563129
## 882        0.008144575
## 883        0.004273200
## 884        0.003339793
## 885        0.005519616
## 886        0.009700822
## 887        0.003826460
## 888        0.020086082
## 889        0.006830141
## 890        0.005634777
## 891        0.046122909
## 892        0.013010326
## 893        0.006951249
## 894        0.006126386
## 895        0.002497264
## 896        0.002497264
## 897        0.007575138
## 898        0.006021742
## 899        0.018905110
## 900        0.005276989
## 901        0.006283172
## 902        0.015716002
## 903        0.005107362
## 904        0.004848527
## 905        0.015368149
## 906        0.005228561
## 907        0.004848527
## 908        0.007584605
## 909        0.003365236
## 910        0.003365236
## 911        0.005236537
## 912        0.001877579
## 913        0.002849964
## 914        0.007120236
## 915        0.026538314
## 916        0.026538314
## 917        0.005422678
## 918        0.004734643
## 919        0.007231775
## 920        0.021971806
## 921        0.005169192
## 922        0.007628939
## 923        0.047768638
## 924        0.011494110
## 925        0.009422035
## 926        0.004957766
## 927        0.009577336
## 928        0.003879554
## 929        0.005450121
## 930        0.005267120
## 931        0.008242147
```

```
## 932         0.021044316
## 933         0.004681003
## 934         0.033641770
## 935         0.054385943
## 936         0.029995529
## 937         0.007875178
## 938         0.003027281
## 939         0.003027281
## 940         0.003027281
## 941         0.003027281
## 942         0.004345945
## 943         0.004385165
## 944         0.004385165
## 945         0.012743047
## 946         0.011224403
## 947         0.004479022
## 948         0.009402272
## 949         0.008177478
## 950         0.006777982
## 951         0.006777982
## 952         0.006777982
## 953         0.004575394
## 954         0.004575394
## 955         0.004575394
## 956         0.004575394
## 957         0.006582772
## 958         0.002768126
## 959         0.004827404
## 960         0.005468801
## 961         0.008940215
## 962         0.005699530
## 963         0.009227998
## 964         0.009022129
## 965         0.007243487
## 966         0.020860507
## 967         0.009449523
## 968         0.004216894
## 969         0.009449523
## 970         0.006914380
## 971         0.004572422
## 972         0.025291476
## 973         0.006120700
## 974         0.050782777
## 975         0.016836973
## 976         0.050782777
## 977         0.012388005
## 978         0.004291488
## 979         0.003876094
## 980         0.003876094
## 981         0.005562992
## 982         0.005562992
## 983         0.006838611
## 984         0.011306016
## 985         0.003407028
```

```
## 986        0.014139366
## 987        0.008846999
## 988        0.002977304
## 989        0.008650269
## 990        0.002457176
## 991        0.004277336
## 992        0.004265547
## 993        0.006642425
## 994        0.004227717
## 995        0.010434008
## 996        0.008498004
## 997        0.016563469
## 998        0.007719376
## 999        0.013094985
## 1000       0.014300420
## 1001       0.014296550
## 1002       0.014300420
## 1003       0.011699119
## 1004       0.010026880
## 1005       0.013094985
## 1006       0.013380879
## 1007       0.008564196
## 1008       0.009311933
## 1009       0.008564196
## 1010       0.006010877
## 1011       0.009466940
## 1012       0.009754534
## 1013       0.009466940
## 1014       0.004333159
## 1015       0.012976820
## 1016       0.006887978
## 1017       0.004163175
## 1018       0.007270451
## 1019       0.012229035
## 1020       0.002137596
## 1021       0.004602682
## 1022       0.005219989
## 1023       0.100657718
## 1024       0.011401571
## 1025       0.017162004
## 1026       0.005148163
## 1027       0.007450408
## 1028       0.005529905
## 1029       0.011039110
## 1030       0.005545873
## 1031       0.006414147
## 1032       0.006112586
## 1033       0.007273566
## 1034       0.010897408
## 1035       0.007933250
## 1036       0.008118296
## 1037       0.006720748
## 1038       0.005111624
## 1039       0.006489302
```

```
## 1040       0.013228564
## 1041       0.006796964
## 1042       0.003637300
## 1043       0.007267395
## 1044       0.010331102
## 1045       0.013004286
## 1046       0.010331102
## 1047       0.009725129
## 1048       0.016651301
## 1049       0.009556038
## 1050       0.010965160
## 1051       0.004121891
## 1052       0.080368069
## 1053       0.017867082
## 1054       0.080368069
## 1055       0.018006516
## 1056       0.013056016
## 1057       0.010954447
## 1058       0.008572701
## 1059       0.010954447
## 1060       0.006161560
## 1061       0.003469087
## 1062       0.004306460
## 1063       0.005670430
## 1064       0.009149252
## 1065       0.010275511
## 1066       0.006373657
## 1067       0.013400156
## 1068       0.006913126
## 1069       0.006314284
## 1070       0.004778359
## 1071       0.004778359
## 1072       0.011772503
## 1073       0.008776780
## 1074       0.011901712
## 1075       0.006302878
## 1076       0.005612852
## 1077       0.011901712
## 1078       0.023428407
## 1079       0.007392169
## 1080       0.008050690
## 1081       0.004495425
## 1082       0.011886690
## 1083       0.011819245
## 1084       0.007532963
## 1085       0.006979356
## 1086       0.003694112
## 1087       0.010434640
## 1088       0.004252869
## 1089       0.004228476
## 1090       0.004648992
## 1091       0.005828117
## 1092       0.012049071
## 1093       0.004107560
```

```
## 1094      0.005915030
## 1095      0.007033434
## 1096      0.007600662
## 1097      0.003315827
## 1098      0.002520473
## 1099      0.006012693
## 1100      0.007989529
## 1101      0.006163778
## 1102      0.009398812
## 1103      0.003385163
## 1104      0.005424882
## 1105      0.005053118
## 1106      0.005072068
## 1107      0.007527427
## 1108      0.004830059
## 1109      0.003093297
## 1110      0.007083670
## 1111      0.006426557
## 1112      0.042366306
## 1113      0.009048258
## 1114      0.009048258
## 1115      0.009048258
## 1116      0.004001436
## 1117      0.004001436
## 1118      0.006964141
## 1119      0.015697848
## 1120      0.004001436
## 1121      0.005027675
## 1122      0.003300393
## 1123      0.018848437
## 1124      0.004737355
## 1125      0.008976918
## 1126      0.007382903
## 1127      0.007546168
## 1128      0.009940292
## 1129      0.009669513
## 1130      0.004433224
## 1131      0.004351128
## 1132      0.006943704
## 1133      0.007538937
## 1134      0.011571489
## 1135      0.008082932
## 1136      0.004367800
## 1137      0.008429891
## 1138      0.008319370
## 1139      0.007128400
## 1140      0.008351917
## 1141      0.006225574
## 1142      0.010764761
## 1143      0.008912159
```

```r
hats[order(hats['hatvalues(wineq)']), ]
```

```
## Warning in xtfrm.data.frame(x): cannot xtfrm data frames
```

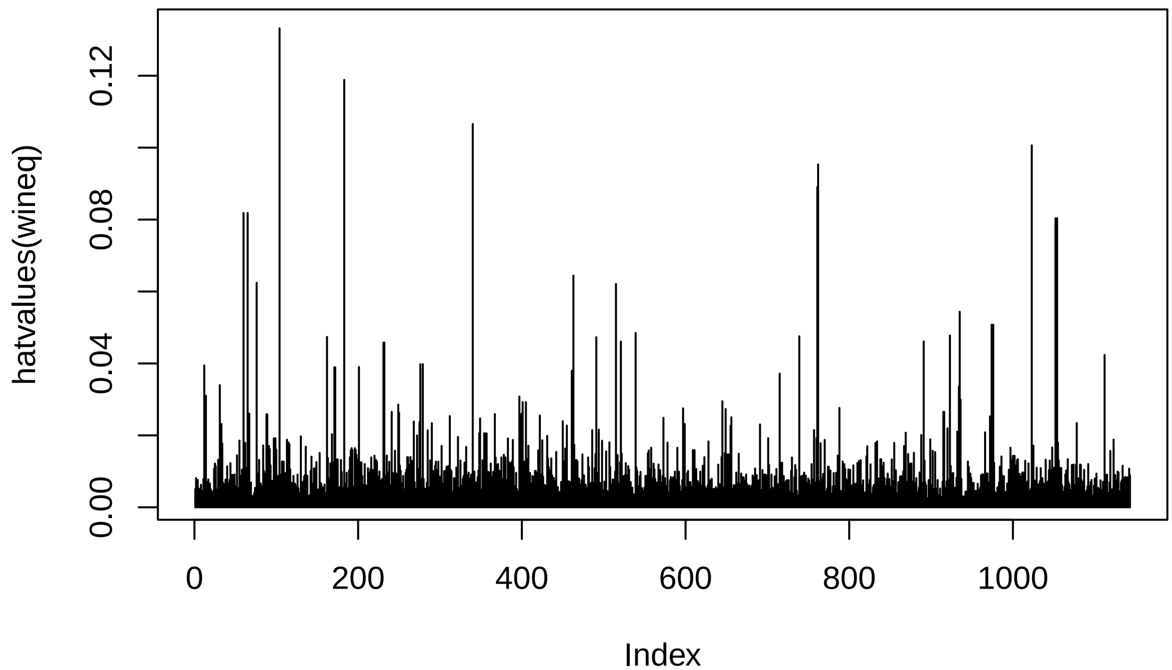
```
##   [1] 0.001791692 0.001877579 0.002094741 0.002137596 0.002198632 0.002457176
##   [7] 0.002497264 0.002497264 0.002520473 0.002548746 0.002551232 0.002563129
##  [13] 0.002617161 0.002621911 0.002624015 0.002627188 0.002636436 0.002669846
##  [19] 0.002737305 0.002747866 0.002747866 0.002764909 0.002768126 0.002813320
##  [25] 0.002849964 0.002867840 0.002867840 0.002880585 0.002939521 0.002977304
##  [31] 0.003027281 0.003027281 0.003027281 0.003027281 0.003050112 0.003093297
##  [37] 0.003114941 0.003114941 0.003190962 0.003190962 0.003212240 0.003212240
##  [43] 0.003219880 0.003222426 0.003247586 0.003260961 0.003276449 0.003293674
##  [49] 0.003300393 0.003302993 0.003315827 0.003325647 0.003339793 0.003340743
##  [55] 0.003365236 0.003365236 0.003385163 0.003389547 0.003397219 0.003407028
##  [61] 0.003420589 0.003436975 0.003469087 0.003475888 0.003491123 0.003511985
##  [67] 0.003511985 0.003596048 0.003620478 0.003620478 0.003628557 0.003629586
##  [73] 0.003637300 0.003667121 0.003670652 0.003694112 0.003698456 0.003703577
##  [79] 0.003705626 0.003748221 0.003784830 0.003822216 0.003822216 0.003826460
##  [85] 0.003830349 0.003852312 0.003876094 0.003876094 0.003879554 0.003882772
##  [91] 0.003894204 0.003902842 0.003903896 0.003903896 0.003905776 0.003905776
##  [97] 0.003932326 0.003944442 0.003954989 0.003965990 0.003970697 0.003990406
## [103] 0.004001436 0.004001436 0.004001436 0.004019111 0.004023940 0.004031028
## [109] 0.004031028 0.004078504 0.004086144 0.004086631 0.004096525 0.004107560
## [115] 0.004121891 0.004135039 0.004140436 0.004163175 0.004173496 0.004173496
## [121] 0.004195011 0.004199146 0.004216894 0.004221806 0.004227717 0.004228476
## [127] 0.004229023 0.004246215 0.004252869 0.004264042 0.004265547 0.004267040
## [133] 0.004270310 0.004271219 0.004272601 0.004273200 0.004277336 0.004278369
## [139] 0.004291227 0.004291488 0.004306460 0.004332168 0.004333159 0.004337255
## [145] 0.004345945 0.004351128 0.004364177 0.004367703 0.004367800 0.004376642
## [151] 0.004379162 0.004385165 0.004385165 0.004385304 0.004389160 0.004399480
## [157] 0.004409090 0.004409090 0.004417586 0.004418403 0.004418403 0.004424527
## [163] 0.004426889 0.004433224 0.004438053 0.004464620 0.004470924 0.004479022
## [169] 0.004486328 0.004495425 0.004527924 0.004542554 0.004558150 0.004558150
## [175] 0.004558150 0.004572422 0.004575394 0.004575394 0.004575394 0.004575394
## [181] 0.004577662 0.004578662 0.004578662 0.004580484 0.004587509 0.004602682
## [187] 0.004609210 0.004613816 0.004613892 0.004624759 0.004629409 0.004648992
## [193] 0.004663281 0.004663281 0.004665163 0.004665440 0.004668932 0.004674711
## [199] 0.004678989 0.004681003 0.004685276 0.004727162 0.004729266 0.004734643
## [205] 0.004737355 0.004748573 0.004748625 0.004753529 0.004778359 0.004778359
## [211] 0.004798463 0.004804278 0.004811525 0.004827404 0.004830059 0.004847490
## [217] 0.004848527 0.004848527 0.004855619 0.004858245 0.004860436 0.004867659
## [223] 0.004892508 0.004899745 0.004905335 0.004912524 0.004917417 0.004947392
## [229] 0.004947392 0.004951452 0.004957766 0.004961432 0.004964455 0.004965231
## [235] 0.004968706 0.004982638 0.004994217 0.004998834 0.005009245 0.005009256
## [241] 0.005024658 0.005025752 0.005027675 0.005030545 0.005039610 0.005053118
## [247] 0.005072068 0.005072712 0.005072712 0.005079901 0.005079901 0.005086123
## [253] 0.005091483 0.005092769 0.005101395 0.005104983 0.005107362 0.005111624
## [259] 0.005116012 0.005123541 0.005123541 0.005124043 0.005127564 0.005127724
## [265] 0.005127724 0.005132741 0.005148163 0.005151231 0.005167776 0.005169192
## [271] 0.005172338 0.005188055 0.005195357 0.005196377 0.005199395 0.005216551
## [277] 0.005219989 0.005222558 0.005224927 0.005228561 0.005236537 0.005267120
## [283] 0.005269406 0.005273926 0.005276989 0.005288668 0.005299619 0.005307292
## [289] 0.005321142 0.005332526 0.005335595 0.005348206 0.005348831 0.005352136
## [295] 0.005353649 0.005362801 0.005379783 0.005388295 0.005401992 0.005412347
## [301] 0.005422678 0.005424882 0.005432180 0.005441965 0.005447317 0.005450121
## [307] 0.005465453 0.005468801 0.005471347 0.005482052 0.005503159 0.005503159
## [313] 0.005517165 0.005519616 0.005522090 0.005522090 0.005529905 0.005537018
## [319] 0.005537560 0.005545873 0.005546230 0.005558538 0.005562992 0.005562992
```

```
## [325] 0.005572442 0.005587752 0.005592100 0.005593725 0.005612852 0.005616815
## [331] 0.005626477 0.005628613 0.005628613 0.005634777 0.005670430 0.005674210
## [337] 0.005678085 0.005678085 0.005699530 0.005738819 0.005741738 0.005741738
## [343] 0.005743384 0.005776592 0.005777311 0.005781693 0.005798826 0.005800270
## [349] 0.005802851 0.005807121 0.005808918 0.005815284 0.005828117 0.005841226
## [355] 0.005841226 0.005876660 0.005876937 0.005907860 0.005914954 0.005915030
## [361] 0.005917860 0.005920293 0.005922074 0.005922074 0.005931687 0.005937867
## [367] 0.005939253 0.005945645 0.005947660 0.005966568 0.005976044 0.005982028
## [373] 0.005996954 0.005997235 0.006000866 0.006010567 0.006010877 0.006012693
## [379] 0.006021742 0.006037430 0.006067928 0.006095757 0.006096341 0.006096341
## [385] 0.006103196 0.006108955 0.006112586 0.006120230 0.006120700 0.006126386
## [391] 0.006137896 0.006153202 0.006156458 0.006161560 0.006163778 0.006174217
## [397] 0.006197804 0.006225574 0.006226865 0.006229862 0.006231876 0.006233330
## [403] 0.006233517 0.006233517 0.006234082 0.006234082 0.006273689 0.006281198
## [409] 0.006281198 0.006281831 0.006283172 0.006302878 0.006314284 0.006330111
## [415] 0.006346128 0.006347747 0.006367275 0.006368845 0.006373657 0.006388212
## [421] 0.006392998 0.006409780 0.006411230 0.006414147 0.006426557 0.006436876
## [427] 0.006444366 0.006446435 0.006487944 0.006489302 0.006514514 0.006516621
## [433] 0.006532395 0.006547528 0.006550467 0.006552290 0.006553434 0.006573470
## [439] 0.006578860 0.006582772 0.006613036 0.006613036 0.006620674 0.006622259
## [445] 0.006622259 0.006623544 0.006642425 0.006649564 0.006651515 0.006651515
## [451] 0.006654301 0.006671231 0.006697048 0.006697282 0.006709971 0.006710544
## [457] 0.006720748 0.006725434 0.006726683 0.006744248 0.006762835 0.006763227
## [463] 0.006765622 0.006777982 0.006777982 0.006777982 0.006784455 0.006786415
## [469] 0.006796964 0.006802741 0.006802741 0.006806976 0.006811906 0.006830141
## [475] 0.006830291 0.006838611 0.006842187 0.006853510 0.006858603 0.006866976
## [481] 0.006866976 0.006880550 0.006887978 0.006890454 0.006891000 0.006894703
## [487] 0.006907687 0.006908585 0.006908585 0.006913126 0.006913759 0.006914380
## [493] 0.006924950 0.006930447 0.006932116 0.006942294 0.006942330 0.006943406
## [499] 0.006943704 0.006951249 0.006964141 0.006974247 0.006979356 0.006995281
## [505] 0.007014766 0.007014766 0.007021380 0.007029088 0.007033434 0.007040437
## [511] 0.007069472 0.007083670 0.007112874 0.007113945 0.007120236 0.007124679
## [517] 0.007128400 0.007178859 0.007185641 0.007194722 0.007195552 0.007195552
## [523] 0.007195552 0.007216853 0.007231775 0.007236448 0.007243487 0.007267395
## [529] 0.007270451 0.007270941 0.007273566 0.007292338 0.007292361 0.007306184
## [535] 0.007306222 0.007314865 0.007340511 0.007340511 0.007340511 0.007359562
## [541] 0.007360142 0.007361671 0.007382903 0.007392169 0.007443318 0.007450408
## [547] 0.007482932 0.007482932 0.007483683 0.007527427 0.007532963 0.007534837
## [553] 0.007538937 0.007540516 0.007545450 0.007546168 0.007553608 0.007561172
## [559] 0.007573492 0.007575138 0.007584605 0.007599918 0.007600662 0.007604259
## [565] 0.007604259 0.007608308 0.007623793 0.007625080 0.007628939 0.007631385
## [571] 0.007641965 0.007651280 0.007653995 0.007680146 0.007694591 0.007719376
## [577] 0.007729637 0.007729637 0.007741004 0.007758928 0.007769967 0.007779451
## [583] 0.007779723 0.007779723 0.007808802 0.007835479 0.007846673 0.007856334
## [589] 0.007861485 0.007865355 0.007866858 0.007866858 0.007867479 0.007868277
## [595] 0.007871348 0.007875178 0.007933250 0.007953246 0.007954023 0.007956123
## [601] 0.007962758 0.007973323 0.007989529 0.007996837 0.008005156 0.008022115
## [607] 0.008022115 0.008037051 0.008044206 0.008050690 0.008068169 0.008076105
## [613] 0.008076105 0.008082932 0.008111271 0.008118296 0.008144575 0.008150074
## [619] 0.008154838 0.008177478 0.008180976 0.008200358 0.008210024 0.008233760
## [625] 0.008242147 0.008266779 0.008319370 0.008340855 0.008340855 0.008351917
## [631] 0.008375444 0.008382766 0.008400204 0.008420771 0.008429891 0.008451368
## [637] 0.008455737 0.008471542 0.008493557 0.008498004 0.008553293 0.008564196
## [643] 0.008564196 0.008572701 0.008574148 0.008576398 0.008580822 0.008599023
```

```
## [649] 0.008621481 0.008638927 0.008638927 0.008646773 0.008650269 0.008664132
## [655] 0.008678351 0.008680905 0.008686844 0.008701215 0.008704061 0.008720091
## [661] 0.008722790 0.008724965 0.008728782 0.008730115 0.008730115 0.008739116
## [667] 0.008743930 0.008754485 0.008754605 0.008754771 0.008754771 0.008761552
## [673] 0.008776780 0.008784370 0.008817519 0.008826964 0.008826964 0.008834126
## [679] 0.008846999 0.008863624 0.008868427 0.008874103 0.008890054 0.008890054
## [685] 0.008909637 0.008912159 0.008920039 0.008920039 0.008940215 0.008959370
## [691] 0.008964746 0.008965891 0.008973606 0.008975675 0.008976918 0.008977899
## [697] 0.008984319 0.008984319 0.009022129 0.009048258 0.009048258 0.009048258
## [703] 0.009100582 0.009102351 0.009109378 0.009118347 0.009118347 0.009118691
## [709] 0.009124761 0.009143397 0.009149252 0.009155375 0.009155375 0.009191437
## [715] 0.009191441 0.009227998 0.009289638 0.009289638 0.009311933 0.009343257
## [721] 0.009359931 0.009360822 0.009381146 0.009381935 0.009393250 0.009398812
## [727] 0.009402272 0.009408379 0.009422035 0.009438034 0.009445839 0.009449523
## [733] 0.009449523 0.009460657 0.009466940 0.009466940 0.009479648 0.009479648
## [739] 0.009480488 0.009511356 0.009526742 0.009555612 0.009556038 0.009559295
## [745] 0.009562148 0.009566124 0.009577336 0.009596262 0.009601031 0.009606222
## [751] 0.009615620 0.009623948 0.009640338 0.009641035 0.009656767 0.009669513
## [757] 0.009680245 0.009700822 0.009701620 0.009720319 0.009725129 0.009754534
## [763] 0.009772217 0.009846301 0.009846312 0.009862676 0.009875256 0.009893982
## [769] 0.009893982 0.009899322 0.009940292 0.009940647 0.009943311 0.009955767
## [775] 0.009957454 0.009964636 0.010019829 0.010026880 0.010039624 0.010039624
## [781] 0.010046370 0.010072335 0.010083623 0.010087377 0.010090365 0.010101226
## [787] 0.010108172 0.010108172 0.010115137 0.010115137 0.010138358 0.010138885
## [793] 0.010141486 0.010235618 0.010253720 0.010272145 0.010275511 0.010277925
## [799] 0.010285306 0.010302514 0.010331102 0.010331102 0.010342706 0.010377901
## [805] 0.010378502 0.010405945 0.010417620 0.010430451 0.010434008 0.010434640
## [811] 0.010450933 0.010452715 0.010455555 0.010479424 0.010515204 0.010553850
## [817] 0.010572574 0.010582744 0.010585386 0.010650066 0.010665412 0.010689713
## [823] 0.010704968 0.010734583 0.010750765 0.010764761 0.010812274 0.010819629
## [829] 0.010819629 0.010854356 0.010874625 0.010897408 0.010954447 0.010954447
## [835] 0.010962999 0.010965160 0.011034703 0.011039110 0.011048877 0.011129079
## [841] 0.011146179 0.011200291 0.011202074 0.011204375 0.011224403 0.011261060
## [847] 0.011283793 0.011283793 0.011306016 0.011401571 0.011403010 0.011413265
## [853] 0.011429170 0.011444870 0.011490732 0.011494110 0.011537910 0.011547039
## [859] 0.011547039 0.011571489 0.011574141 0.011588365 0.011641691 0.011667638
## [865] 0.011672394 0.011699119 0.011754239 0.011758921 0.011772503 0.011802889
## [871] 0.011810765 0.011819245 0.011886690 0.011886791 0.011901712 0.011901712
## [877] 0.011906876 0.011957933 0.012010198 0.012024577 0.012040431 0.012043933
## [883] 0.012049071 0.012141367 0.012181585 0.012181753 0.012187564 0.012187564
## [889] 0.012192743 0.012228389 0.012229035 0.012250051 0.012289740 0.012291719
## [895] 0.012297773 0.012330594 0.012388005 0.012398413 0.012398413 0.012428795
## [901] 0.012433077 0.012445977 0.012499295 0.012499295 0.012517657 0.012549933
## [907] 0.012565433 0.012623822 0.012673264 0.012702911 0.012706800 0.012706800
## [913] 0.012727589 0.012727589 0.012731858 0.012743047 0.012756503 0.012757563
## [919] 0.012794476 0.012811628 0.012898719 0.012926691 0.012976820 0.013004286
## [925] 0.013010326 0.013032216 0.013054666 0.013056016 0.013087617 0.013093002
## [931] 0.013094985 0.013094985 0.013157968 0.013166343 0.013166343 0.013168467
## [937] 0.013168482 0.013188200 0.013214273 0.013228564 0.013347722 0.013354946
## [943] 0.013354946 0.013380879 0.013385955 0.013385955 0.013400156 0.013429971
## [949] 0.013477930 0.013569976 0.013615173 0.013709180 0.013850561 0.013859403
## [955] 0.013874626 0.013886143 0.013897454 0.013897454 0.013920363 0.013921791
## [961] 0.013921791 0.013956713 0.013979715 0.013992321 0.014083877 0.014139366
## [967] 0.014153490 0.014296550 0.014300420 0.014300420 0.014336139 0.014361988
```

```
##  [973] 0.014390546 0.014417906 0.014484615 0.014533859 0.014611609 0.014623105
##  [979] 0.014709885 0.014737887 0.014737887 0.014800555 0.014832368 0.014892666
##  [985] 0.014948332 0.015048777 0.015101840 0.015115312 0.015181362 0.015368149
##  [991] 0.015410233 0.015524387 0.015697848 0.015716002 0.015746980 0.015786810
##  [997] 0.015798171 0.015828703 0.015867654 0.015898135 0.015898135 0.015898135
## [1003] 0.015948092 0.015948092 0.016013518 0.016196634 0.016363669 0.016450816
## [1009] 0.016450816 0.016560130 0.016563469 0.016569613 0.016651301 0.016791331
## [1015] 0.016836973 0.016851147 0.016977571 0.017050344 0.017073414 0.017075068
## [1021] 0.017124899 0.017162004 0.017216599 0.017399250 0.017540038 0.017739270
## [1027] 0.017810722 0.017867082 0.017916361 0.017923817 0.017934258 0.018006516
## [1033] 0.018010025 0.018021960 0.018054168 0.018084401 0.018309636 0.018335219
## [1039] 0.018529768 0.018567518 0.018638090 0.018742820 0.018746333 0.018813497
## [1045] 0.018848437 0.018905110 0.019157753 0.019198679 0.019198679 0.019219638
## [1051] 0.019281406 0.019567842 0.019690144 0.019719613 0.019888309 0.020007777
## [1057] 0.020086082 0.020350076 0.020551793 0.020551793 0.020589839 0.020589839
## [1063] 0.020759346 0.020860507 0.021044316 0.021423679 0.021461370 0.021496085
## [1069] 0.021617918 0.021971806 0.022672262 0.022727770 0.023051842 0.023159059
## [1075] 0.023159059 0.023179000 0.023402575 0.023428407 0.023650110 0.023843261
## [1081] 0.023923626 0.024727263 0.024894062 0.025034732 0.025291476 0.025358646
## [1087] 0.025544368 0.025814195 0.025866299 0.025899899 0.025941965 0.026058577
## [1093] 0.026132260 0.026248601 0.026538314 0.026538314 0.026551003 0.027337598
## [1099] 0.027532005 0.027627730 0.028558323 0.029248156 0.029248156 0.029481967
## [1105] 0.029995529 0.030825023 0.031016001 0.033641770 0.033958794 0.037193878
## [1111] 0.037970561 0.038927348 0.038927348 0.039029734 0.039475498 0.039783183
## [1117] 0.039783183 0.042366306 0.045802556 0.045802556 0.046096151 0.046122909
## [1123] 0.047290280 0.047373878 0.047571012 0.047768638 0.048494966 0.050782777
## [1129] 0.050782777 0.054385943 0.062110237 0.062431247 0.064431145 0.080368069
## [1135] 0.080368069 0.081831903 0.081831903 0.089029480 0.095333756 0.100657718
## [1141] 0.106565971 0.118854325 0.133177620
```

```r
plot(hatvalues(wineq), type = 'h')
```




```
# New Regression Model
wineq2 <- lm(quality ~ volatile.acidity + chlorides + free.sulfur.dioxide + total.sulfur.dioxide + pH +
summary(wineq2)
```

```
##
## Call:
## lm(formula = quality ~ volatile.acidity + chlorides + free.sulfur.dioxide +
##     total.sulfur.dioxide + pH + sulphates + alcohol, data = Wine_Quality)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.39463 -0.36932 -0.04649  0.44290  2.00640
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           4.466534   0.461804   9.672  < 2e-16 ***
## volatile.acidity     -1.082354   0.117570  -9.206  < 2e-16 ***
## chlorides            -1.837430   0.465909  -3.944 8.51e-05 ***
## free.sulfur.dioxide   0.002845   0.002510   1.134 0.257199
## total.sulfur.dioxide -0.002937   0.000796  -3.689 0.000236 ***
## pH                   -0.485507   0.135380  -3.586 0.000350 ***
## sulphates             0.845081   0.128332   6.585 6.93e-11 ***
## alcohol               0.293758   0.019382  15.156  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.6398 on 1135 degrees of freedom
## Multiple R-squared:  0.3736, Adjusted R-squared:  0.3697
## F-statistic: 96.69 on 7 and 1135 DF,  p-value: < 2.2e-16
```

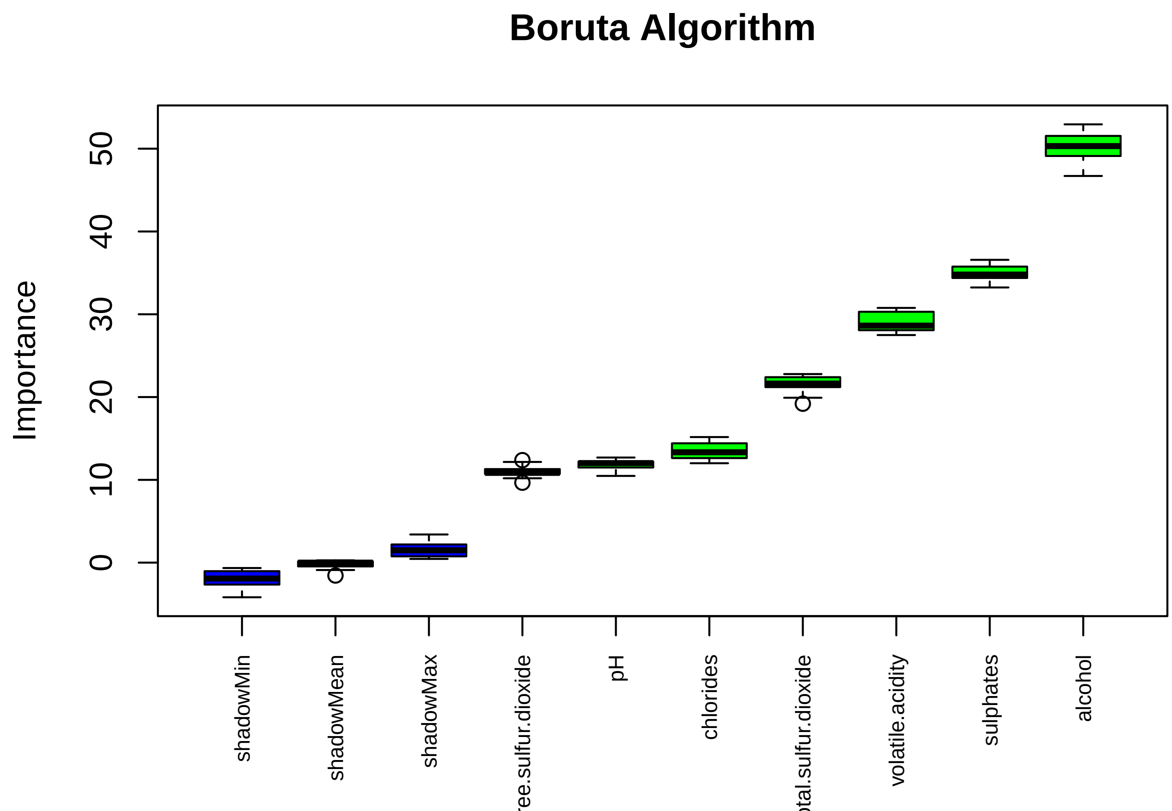
Boruta Algorithm, Mallows CP

First, the Boruta Algorithm is applied to filter out which response variables were insignificant before also conducting the Mallows CP test. The Boruta Algorithm shows alcohol is the best variable, as it had the highest importance. Also, this test allows the report "rejected" on some variables which means they are insiginifcant and they can be eliminated. Next, with the results of the significant variables from the Boruta algorithm, the Mallows CP test is run. Here, the smaller the value, the better the model. Evidently, the best model had the predictors volatile acidity, chlorides, total sulfur dioxide, sulphates, and alcohol. The models to the right of the model are higher and therefore, insignificant.

```r
library(Boruta)
Bor.res <- Boruta(quality ~ volatile.acidity + chlorides + free.sulfur.dioxide + total.sulfur.dioxide +
```

```
##  1. run of importance source...

##  2. run of importance source...

##  3. run of importance source...

##  4. run of importance source...

##  5. run of importance source...

##  6. run of importance source...

##  7. run of importance source...

##  8. run of importance source...

##  9. run of importance source...

##  10. run of importance source...

## After 10 iterations, +9.3 secs:

##  confirmed 7 attributes: alcohol, chlorides, free.sulfur.dioxide, pH, sulphates and 2 more;

##  no more attributes left.
```

```r
plot(Bor.res, xlab = "", xaxt = "n", main="Boruta Algorithm")
Lz <- lapply(1:ncol(Bor.res$ImpHistory),function(i) Bor.res$ImpHistory[is.finite(Bor.res$ImpHistory[,i]
names (Lz) <- colnames(Bor.res$ImpHistory)
Labels <- sort(sapply(Lz,median))
axis(side = 1,las=2,labels = names(Labels),
at = 1:ncol(Bor.res$ImpHistory), cex.axis = 0.7)
```




```
boruta_signif <- names(Bor.res$finalDecision[Bor.res$finalDecision %in% c("Confirmed", "Tentative")])
boruta_signif_Conf <- names(Bor.res$finalDecision[Bor.res$finalDecision %in% c("Confirmed")])
boruta_signif_Tent <- names(Bor.res$finalDecision[Bor.res$finalDecision %in% c("Tentative")])
boruta_signif_Reject <- names(Bor.res$finalDecision[Bor.res$finalDecision %in% c("Rejected")])
print(boruta_signif_Conf)
```

```
## [1] "alcohol"              "sulphates"            "pH"
## [4] "total.sulfur.dioxide" "free.sulfur.dioxide"  "chlorides"
## [7] "volatile.acidity"
```

```
attStats(Bor.res)
```

```
##                       meanImp medianImp    minImp   maxImp normHits  decision
## alcohol              50.21424  50.31270 46.704855 52.94055        1 Confirmed
## sulphates            34.90733  34.79388 33.239216 36.57690        1 Confirmed
## pH                   11.81983  12.02327 10.476230 12.69449        1 Confirmed
## total.sulfur.dioxide 21.49372  21.62699 19.198927 22.77689        1 Confirmed
## free.sulfur.dioxide  11.00015  10.84284  9.656997 12.37755        1 Confirmed
## chlorides            13.48343  13.33111 12.009345 15.16746        1 Confirmed
## volatile.acidity     29.03553  28.62633 27.489466 30.76679        1 Confirmed
```

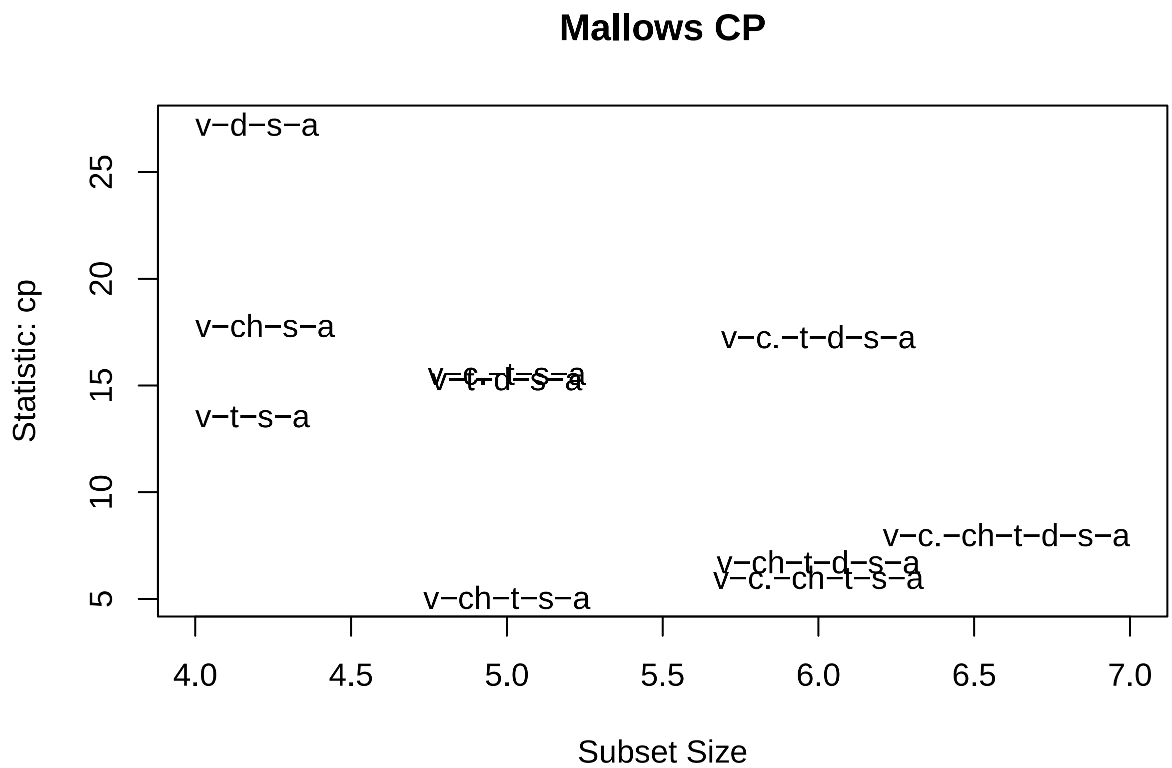
```
sorted_vars = attStats(Bor.res)[order(-attStats(Bor.res)$meanImp),]
print(sorted_vars)
```

```
##                       meanImp medianImp    minImp   maxImp normHits  decision
## alcohol              50.21424  50.31270 46.704855 52.94055        1 Confirmed
## sulphates            34.90733  34.79388 33.239216 36.57690        1 Confirmed
## volatile.acidity     29.03553  28.62633 27.489466 30.76679        1 Confirmed
## total.sulfur.dioxide 21.49372  21.62699 19.198927 22.77689        1 Confirmed
## chlorides            13.48343  13.33111 12.009345 15.16746        1 Confirmed
## pH                   11.81983  12.02327 10.476230 12.69449        1 Confirmed
## free.sulfur.dioxide  11.00015  10.84284  9.656997 12.37755        1 Confirmed
```

```r
#mallowscp
library(AER)
```

```
## Loading required package: car

## Loading required package: carData

## Loading required package: lmtest

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: sandwich

## Loading required package: survival
```

```r
library(leaps)
mcp <- lm(quality ~ volatile.acidity + citric.acid + chlorides + total.sulfur.dioxide + density + sulpha
ss =regsubsets(quality ~ volatile.acidity + citric.acid  + chlorides  + total.sulfur.dioxide + density
subsets(ss, statistic = "cp", legend =F , main = "Mallows CP", col = "green", min.size = 4)
```




```
##                      Abbreviation
## volatile.acidity                v
## citric.acid                    c.
## chlorides                      ch
## total.sulfur.dioxide            t
## density                         d
## sulphates                       s
## alcohol                         a
```

Question 5

Multi Collinearity with VIF

With the output for the vif(), all of the predictor variables are in the range of [1,2]. This means there are no variables that are highly correlated with another. Therefore the model with the removed variables won't have significant issues with multicollinearity.

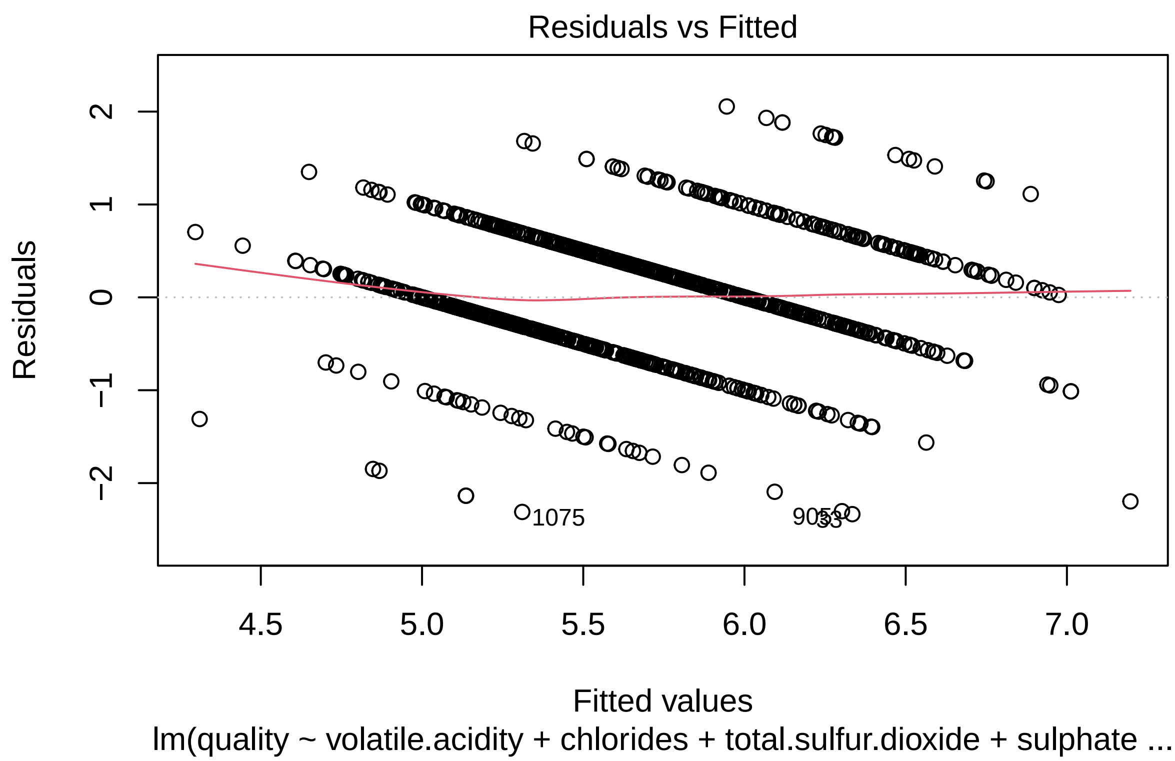
```
# Multi Collinearity
vifFunction<- lm(quality ~ volatile.acidity  + chlorides +  total.sulfur.dioxide + sulphates + alcohol,
vif(vifFunction)
```

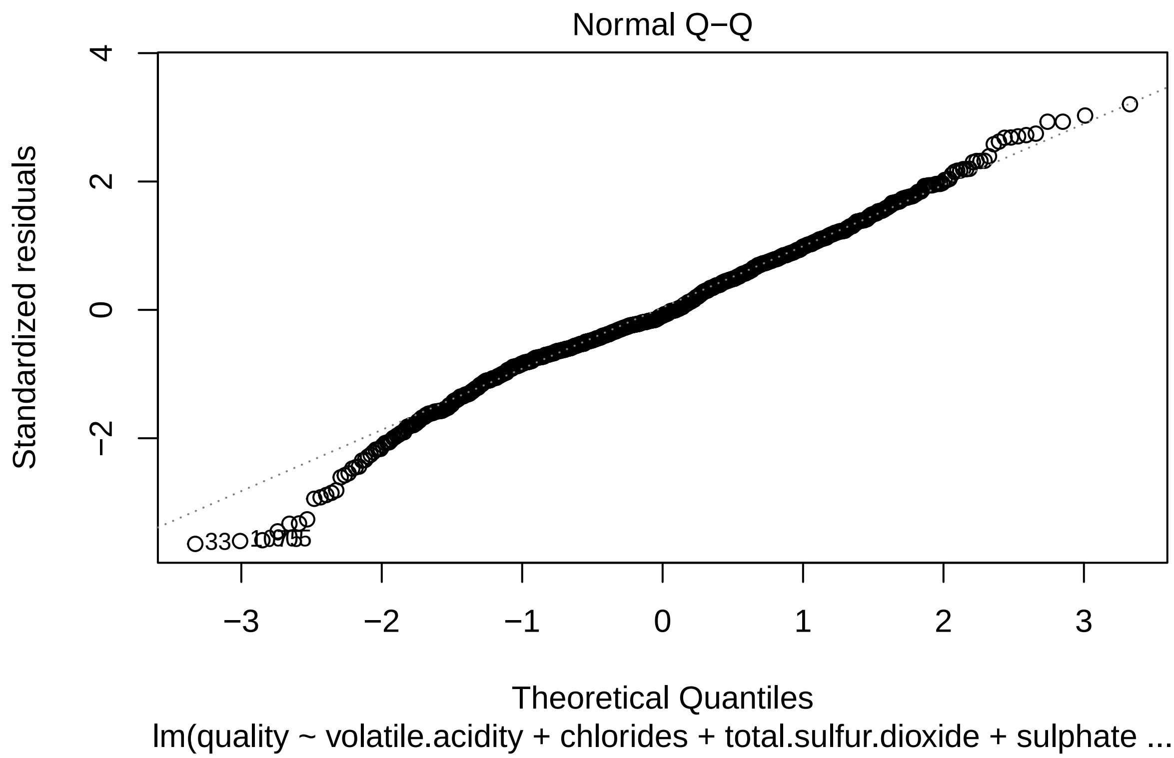
```
##     volatile.acidity            chlorides total.sulfur.dioxide
##             1.145466             1.291971             1.042551
##            sulphates              alcohol
##             1.331226             1.159650
```

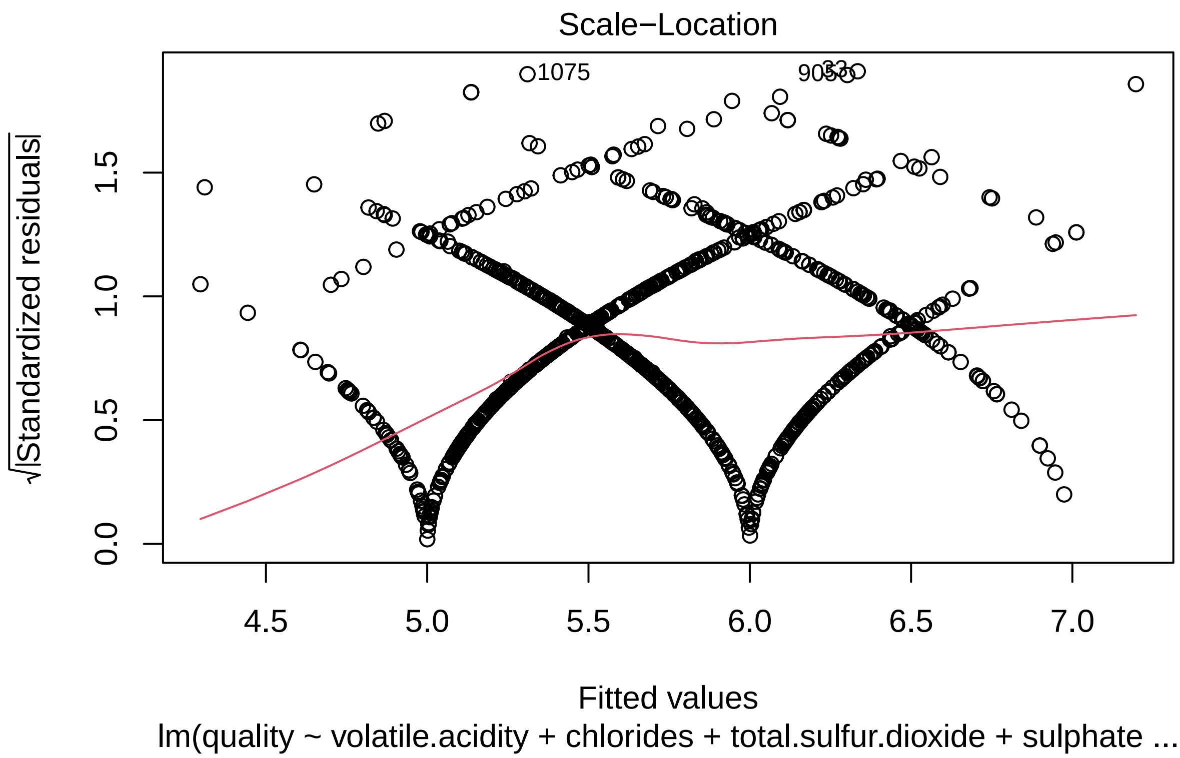
Plotting The Respective Residuals

The graph with residuals versus fitted shows a red line that tries to capture all of the residuals. There are more values that are above the red line than below the red line. The residuals vs y-hat plot visualize whether heteroskedasticity is present. The results show that there is a spread in the variance, so it is present.

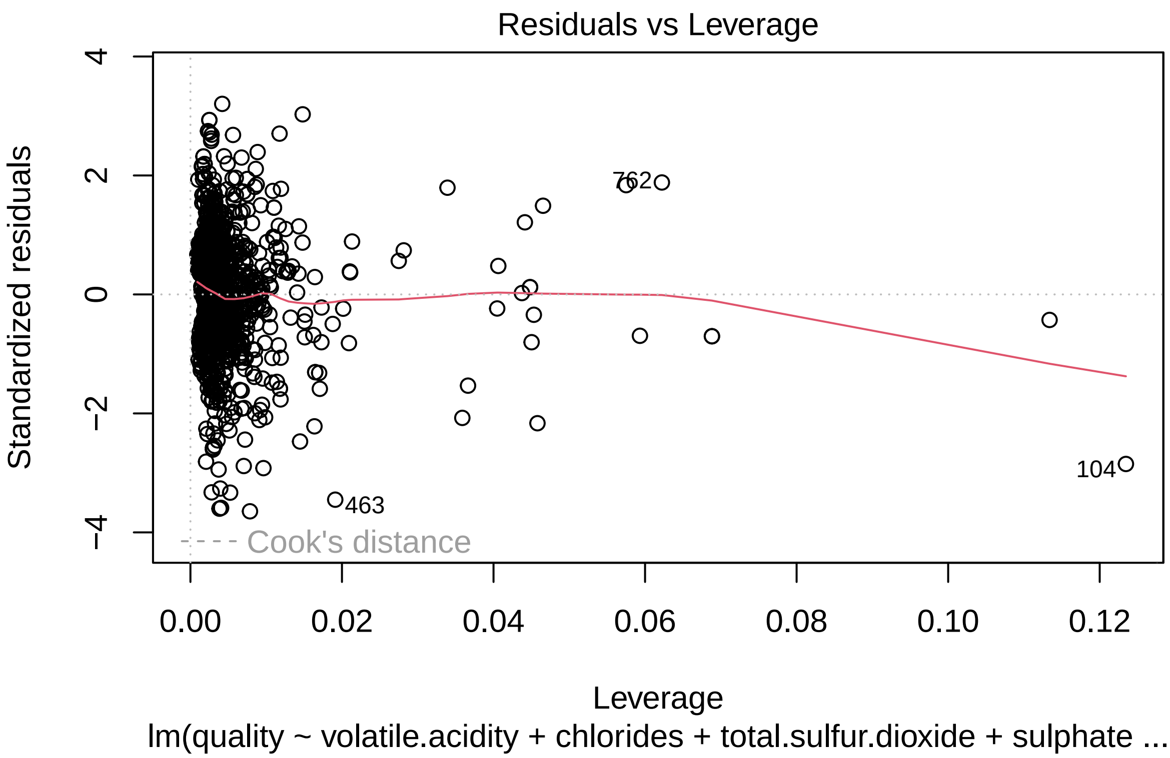
```
# Plotting Respective Residuals/VIF Function
plot(vifFunction)
```



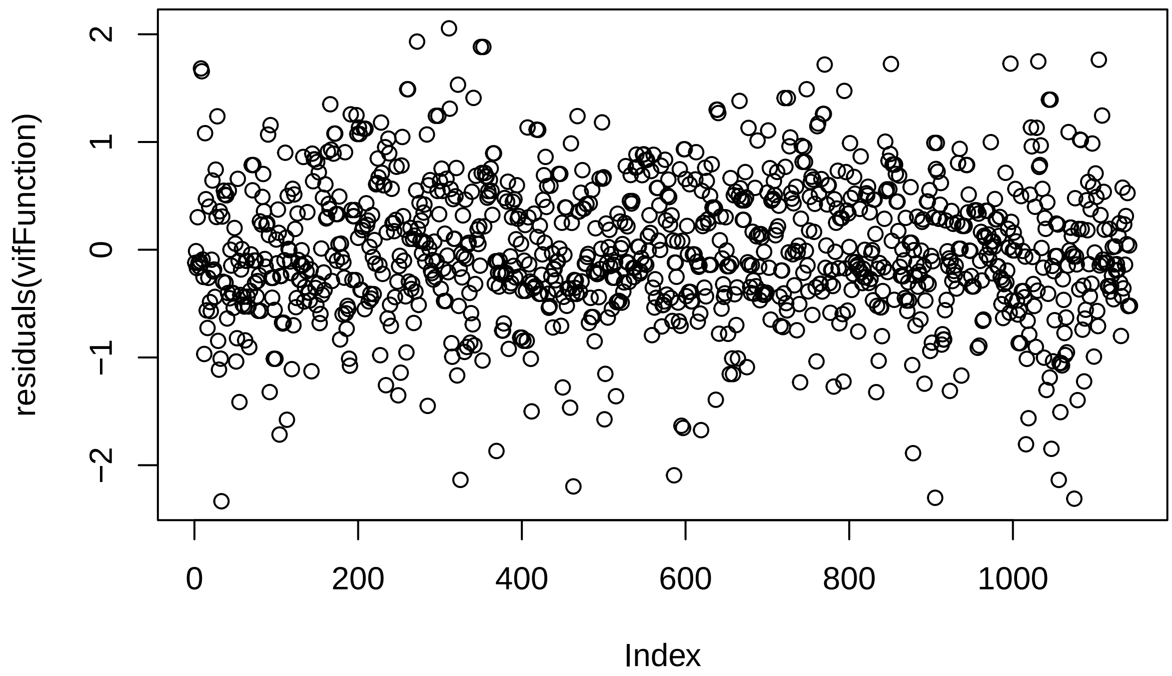

Normal Q−Q
Standardized residuals
4
2
0
−2
−3
−2
−1
0
1
2
3
Theoretical Quantiles
lm(quality ~ volatile.acidity + chlorides + total.sulfur.dioxide + sulphate ...



Scale−Location
1075
903
33
√|Standardized residuals|
0.0
0.5
1.0
1.5
4.5
5.0
5.5
6.0
6.5
7.0
Fitted values
lm(quality ~ volatile.acidity + chlorides + total.sulfur.dioxide + sulphate ...




```
# Just to see more of the actual residuals
plot(residuals(vifFunction))
```



Heteroskedasticity

Using the Breusch Pagan test, it is confirmed heteroskedasticity is present. The robust standard errors fix this. With this, all of the variables are significant and the standard errors decreased resulting in a better model.

```
reg.mod = lm(quality ~ volatile.acidity  + chlorides +  total.sulfur.dioxide + sulphates + alcohol, data
# BP Test Short Way
bptest(reg.mod)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  reg.mod
## BP = 35.162, df = 5, p-value = 1.397e-06
```

```
# BP Test Long Way
alpha <- 0.05
ressq <- resid(reg.mod)^2
modres <- lm(ressq~volatile.acidity  + chlorides +  total.sulfur.dioxide + sulphates + alcohol, data = W
summary(modres)
```

```
##
## Call:
```

```
## lm(formula = ressq ~ volatile.acidity + chlorides + total.sulfur.dioxide +
##     sulphates + alcohol, data = Wine_Quality)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.7400 -0.3492 -0.2046  0.0790  4.8907
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)          -0.6402296  0.2461118  -2.601  0.00941 **
## volatile.acidity      0.1707468  0.1183714   1.442  0.14945
## chlorides            -0.2438618  0.4777576  -0.510  0.60985
## total.sulfur.dioxide -0.0014706  0.0006188  -2.377  0.01764 *
## sulphates             0.3664716  0.1345247   2.724  0.00654 **
## alcohol               0.0774116  0.0197697   3.916 9.55e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6714 on 1137 degrees of freedom
## Multiple R-squared:  0.03076,    Adjusted R-squared:  0.0265
## F-statistic: 7.217 on 5 and 1137 DF,  p-value: 1.156e-06
```

```
#Robust Standard Errors
cov1<-hccm(reg.mod, type="hc1")
coeftest(reg.mod, vcov.=cov1)
```

```
##
## t test of coefficients:
##
##                        Estimate  Std. Error t value  Pr(>|t|)
## (Intercept)           3.03976345  0.27457600 11.0708 < 2.2e-16 ***
## volatile.acidity     -1.20284003  0.12700674 -9.4707 < 2.2e-16 ***
## chlorides            -1.48112808  0.51586067 -2.8712 0.0041653 **
## total.sulfur.dioxide -0.00227580  0.00062456 -3.6439 0.0002807 ***
## sulphates             0.86674198  0.14302592  6.0600 1.848e-09 ***
## alcohol               0.27959699  0.02268052 12.3276 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

AIC/BIC Model

Visualizing the plot of the respective residuals and y-hats in question 6, it seemed that the best fit could be a log linear form. AIC/BIC helps test if this hypothesis is correct. The AIC/BIC test included different models with varying predictors, the best model found previously, and the best model in terms of log. From the results, the best model is still the best model but better as a log linear with the same predictors obtained from previous tests.

```
#AIC Model
# For the new model, it is in log linear form.
bestMOD<- lm(log(quality) ~ chlorides + total.sulfur.dioxide + sulphates + alcohol + volatile.acidity, 
bestMOD2<- lm(quality ~ chlorides + total.sulfur.dioxide + sulphates + alcohol + volatile.acidity, data=
MOD_1 <- lm(log(quality) ~ sulphates + alcohol,data = Wine_Quality)
MOD_2 <- lm(log(quality) ~ pH + sulphates + alcohol,data = Wine_Quality)
```

```
MOD_3 <-  lm(log(quality) ~ density + pH + sulphates + alcohol,data = Wine_Quality)
MOD_4 <- lm(log(quality) ~  total.sulfur.dioxide + density + pH + sulphates + alcohol,data = Wine_Quali
MOD_5 <- lm(log(quality) ~ free.sulfur.dioxide + total.sulfur.dioxide + density + pH + sulphates + alco
MOD_6 <- lm(log(quality) ~ chlorides + free.sulfur.dioxide + total.sulfur.dioxide + density + pH + sulp
MOD_7 <-lm(log(quality) ~ residual.sugar + chlorides + free.sulfur.dioxide + total.sulfur.dioxide + dens
MOD_8 <-lm(log(quality) ~ citric.acid + residual.sugar + chlorides + free.sulfur.dioxide + total.sulfur
MOD_9 <- lm(log(quality) ~ volatile.acidity + citric.acid + residual.sugar + chlorides + free.sulfur.di
MOD_10<- lm(log(quality) ~ fixed.acidity + volatile.acidity + citric.acid + residual.sugar + chlorides +
AIC(MOD_1, MOD_2, MOD_3, MOD_4, MOD_5, MOD_6, MOD_7, MOD_8, MOD_9, MOD_10, bestMOD, bestMOD2)
```

```
##          df       AIC
## MOD_1     4 -1526.656
## MOD_2     5 -1547.205
## MOD_3     6 -1545.221
## MOD_4     7 -1558.177
## MOD_5     8 -1559.779
## MOD_6     9 -1586.914
## MOD_7    10 -1584.923
## MOD_8    11 -1599.442
## MOD_9    12 -1670.418
## MOD_10   13 -1668.687
## bestMOD   7 -1667.423
## bestMOD2  7  2241.781
```

```
BIC(MOD_1, MOD_2, MOD_3, MOD_4, MOD_5, MOD_6, MOD_7, MOD_8, MOD_9, MOD_10, bestMOD, bestMOD2)
```

```
##          df       BIC
## MOD_1     4 -1506.491
## MOD_2     5 -1521.998
## MOD_3     6 -1514.973
## MOD_4     7 -1522.887
## MOD_5     8 -1519.448
## MOD_6     9 -1541.541
## MOD_7    10 -1534.509
## MOD_8    11 -1543.987
## MOD_9    12 -1609.921
## MOD_10   13 -1603.149
## bestMOD   7 -1632.133
## bestMOD2  7  2277.070
```

Cross-Validation

A 5-fold cross validation resulted in an RMSE of .11608. This means that on average the predicted value is off by .11608. In addition, splitting the data into testing/training to calculate the RMSE for both subsets and also got small numbers. Thus, the model is a good fit for the data and the results are accurate.

```
# train for training sample
# test for testing sample
set.seed(1)
row.number <- sample(1:nrow(Wine_Quality), 0.66*nrow(Wine_Quality))
train = Wine_Quality[row.number,]
test = Wine_Quality[-row.number,]
```

```
reg.mod=lm(log(quality) ~ volatile.acidity  + chlorides +  total.sulfur.dioxide + sulphates + alcohol, d
#RMSE
sqrt(mean(log(test$quality)-predict(reg.mod,test))^2)
```

```
## [1] 0.009056762
```

```
sqrt(mean(log(train$quality)-predict(reg.mod,train))^2)
```

```
## [1] 1.582641e-15
```

```
# Cross Validation
library(lmvar)
fit= lm(log(quality) ~ volatile.acidity  + chlorides +  total.sulfur.dioxide + sulphates + alcohol,x = T
cv.lm(fit, k = 5)
```

```
## Mean absolute error        :  0.08922306
## Sample standard deviation  :  0.007942391
##
## Mean squared error         :  0.01370225
## Sample standard deviation  :  0.003211446
##
## Root mean squared error    :  0.1164704
## Sample standard deviation  :  0.01308115
```