MATHEMATICS

1. if the binary operation $*$ on the set Z of integers is defined by $a * b = a + b - 5$,then writen the identity element for the operation $*$ in Z.

2. write the value of $\cot(\tan^{-1} a + \cos^{-1} a)$

3. if A is a square matrix such that $A^2 = A$, then writen the value of $(I + A)^2 - 3A$

4. if
$$x \begin{pmatrix} 2 \\ 3 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$
writen the value of x.

5. write the following determinant :
$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

6. if $\int \left(\frac{x-1}{x^2}\right) e^x dx = f(x)e^x + c$ then write the value of $f(x)$.

7. if $\int_a^0 3x^2 dx = 8$ then write the value of $'a'$.

8. write the value of
$$(\hat{i} * \hat{j}) \cdot \hat{k} + (\hat{j} * \hat{k}) \cdot \hat{i}$$

9. write the value of the area of the parallelogram determined by the vectors $2\hat{i}$ and $3\hat{j}$

10. write the direction cosines of a line parallel to z-axis

11. if
$$f(x) = \frac{4x + 3}{6x - 4}$$

12. x $\neq \frac{2}{3}$, show that $fof(x) = x$ all $x \neq \frac{2}{3}$.what is the inverse of f?

13. prove that and solve tha :

$$\sin^{-1}(\frac{63}{65}) = \sin^{-1}(\frac{5}{13}) + \cos^{-1}(\frac{3}{5})2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x), x \neq \frac{\pi}{2}$$

14. using properties of determinants, prove that

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3C \end{vmatrix} = a^3$$

15. if $x^m\ y^n = (x+y)^{m+n}$, prove that

$$\frac{dy}{dx} = \frac{y}{x}$$

16. if

$$y = e^{a\cos^{-1}x}, -1 \leq x < 1 showthat :$$

$$(1-x^2)\frac{d^2(y)}{d^2(x)} - x\frac{dy}{dx} - a^2y = 0$$

17. if $x\sqrt{1+y} + y\sqrt{1+x} = 0 - 1 < 1, x \neq y$ prove that $\frac{dy}{dx} = -\frac{1}{(1+x^2)}$

18. show that $y = \log(1+x) - \frac{2x}{2+x}\ \ x > -1$ is an increasing function of x throughout its domain.

19. find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$

20. Evaluate:

$$\int x^2 \tan^{-1} x dx$$

$$\int \frac{3x-1}{(x+2)^2} dx$$

21. solve the following differential equation : $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dy}{dx} = 1, x \neq 0$

22. solve the following differential equation :

$$3e^x \tan(y)dx + (2-e^x)\sec^2(y)dy = 0$$

given that when $x = 0$, $y = \frac{\pi}{4}$

23. if $\overrightarrow{\alpha} = 3\hat{j} + 4\hat{j} + 5\hat{k}$ and $\overrightarrow{\beta} = 2\hat{i} + \hat{j} - 4\hat{k}$, then express $\overrightarrow{\beta}$ in the from $\overrightarrow{\beta} = \overrightarrow{\beta_1} + \overrightarrow{\beta_2}$,where $\overrightarrow{\beta_1}$ is parallel of $\overrightarrow{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\overrightarrow{\alpha}$.

24. Find the vector and cartesian equations of the line passing through the point $P(1,2,3)$and parallel to the planes $\mathbf{r}(\hat{i}-\hat{j}+2\hat{k}) = 5$ and $\mathbf{r}\cdot(3\hat{i}+\hat{j}+\hat{k}) = 6$

25. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes and hence find its mean.

26. Using matrices, solve the system equation

$$\begin{aligned} x - y + z &= 4 \\ 2x + y - 3z &= 0 \\ x + y + z &= 2 \end{aligned}$$

27. if

$$\mathbf{A}^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & -6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

find $(\mathbf{AB})^{-1}$.

28. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius is $\frac{4R}{3}$.

29. Find the area of the region in the first quadrant enclosed by x-axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

30. evaluat tha : $\int_1^3 (x^2 + x)dx$

31. evaluate the : $\int_0^{\frac{\pi}{4}} \frac{\cos^2(x)}{\cos^2(x)+4\sin^2(x)} dx$

32. find the vector equation of the plane passing through the points $(2,1,-1)$ and $(-1,3,4)$ and perpendicular to the plane $x - 2y + 4z = 10$. Also show that the plane thus obtain contains the line $\overrightarrow{r} = \hat{i}+3\hat{j}+4\hat{k}+\lambda(3\hat{i}-2\hat{j}-5\hat{k})$.

33. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs — 10. The supplier T has a

packet of mix of 1 unit of A and 1 unit of B that costs — 4. How many packets of mixes from S and T should the company purchase to honour the contract requirement and yet minimize cost ? Make a LPP and solve graphically.

34. In a certain college, 4% of boys and 1%of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl