London Moon

Assignment 6

1)

$u_1 = v_1$

$u_2 = v_2 - \text{proj}_{\text{span}\, u_1} v_2$

$\text{proj}_{\text{span}\, u_1} v_2 = \frac{v_2 \cdot v_1}{v_1 \cdot v_1} v_1$

$= \frac{27}{30} v_1 = \frac{9}{10} v_1$

$z = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$ $v_1 = \begin{bmatrix} 3 \\ -1 \\ 2 \\ -4 \end{bmatrix}$ $v_2 = \begin{bmatrix} 2 \\ -5 \\ -2 \\ -5 \end{bmatrix}$

$u_2 = v_2 - \frac{9}{10} v_1 = \begin{bmatrix} -\frac{7}{10} \\ -\frac{41}{10} \\ -\frac{19}{5} \\ -\frac{7}{5} \end{bmatrix}$ $v_2 = u_2 + \frac{9}{10} v_1$

$\hat{z} = \text{proj}_{\text{span}\{v\}} z$

$= \frac{z \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{z \cdot (v_2 - \frac{9}{10} v_1)}{(v_2 - \frac{9}{10} v_1) \cdot (v_2 - \frac{9}{10} v_1)} u_2$

$= \frac{-8}{30} v_1 + \frac{-79/5}{\frac{337}{10}} u_2$

$= \frac{-8}{30} v_1 + \frac{158}{337} u_2$

$\frac{-8}{30} v_1 \quad \frac{158}{337}(v_2 - \frac{9}{10} v_1)$

$= \frac{-8}{30} v_1 + \frac{158}{337} v_2 - \frac{9}{10} \frac{158}{337} v_1$

$\frac{-3481}{5055} v_1 + \frac{158}{337} v_2$

Gram-Schmidt

Projection

Conversion

2) $\hat{x} = \text{proj}_{\text{span}\mathcal{V}} x = \frac{x \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{x \cdot u_2}{u_2 u_2} u_2$

$u_2 = v_2 - \frac{v_2 \cdot v_1}{v_1 \cdot v_1} v_1 = \frac{-9}{38} v_1 + \frac{56561}{722} u_2$

$= \frac{15}{38} v_1$

$u_2 = \begin{bmatrix} 61/38 \\ -235/38 \\ -2 \\ -110/19 \end{bmatrix}$ gram-schmidt ← projection

↓ convert back to v_2 if you want, i'm not tho (not required)

3) $A = \begin{bmatrix} 1 & 0 & 4 \\ -2 & 3 & -2 \\ -2 & 0 & 6 \end{bmatrix}$ $R = \begin{bmatrix} -3 & 2 & 1.\overline{33} \\ 0 & -2.23 & 3.87 \\ 0 & 0 & 6.26 \end{bmatrix}$ Given

$Q = \begin{bmatrix} -.\overline{3} & -.29 & .89 \\ .\overline{6} & -.74 & 0 \\ .\overline{6} & .59 & .45 \end{bmatrix}$

Matches ✓

$R = Q^T A = \begin{bmatrix} -.\overline{3} & .\overline{6} & .\overline{6} \\ -.29 & -.74 & .59 \\ .\overline{6} & 0 & .45 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ -2 & 3 & -2 \\ -2 & 0 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 1.\overline{33} \\ 0 & -2.23 & 3.87 \\ 0 & 0 & 6.26 \end{bmatrix}$

4) 5) 6) completed as program