Landon moon

due: 3/23/22

Assignment 4

Requirements for basis

1. span(cols) = $\mathbb{R}^n$
2. linearly independent

1) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ doesn't span $\mathbb{R}^3$ X

not linearly independent X

Not a Basis

2) $\begin{bmatrix} -2 & 1 & 2 & -5 \\ 0 & -3 & 3 & -3 \\ -11 & 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -3/2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \end{bmatrix}$ spans $\mathbb{R}^3$ ✓

not linearly independent X

free

Not a Basis

3) $\left[\begin{array}{ccc|c} -2 & 1 & 0 & 0 \\ 0 & -3 & -2 & 0 \\ 0 & -3 & 5 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

No free variables so the parametric form is a zero vector, a zero vector is not linearly independent so no basis exists

4) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis for A

5) $\left[\begin{array}{ccc|c} -6 & -3 & 5 & 0 \\ -4 & -2 & 2 & 0 \\ -8 & -4 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Parametric form:

$$0 = x_2 \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix}$$

6)
$$\begin{bmatrix} 2 & 1 & -1 \\ 10 & 5 & -5 \\ -8 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & -1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

using theorem 5, the pivot vectors form a basis for col A

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

---

7)

$T(0+0) = T(0) = b$ but $T(0)+T(0) = \boxed{2b}$ $\boxed{b \neq 2b}$. A affine transformation where $b \neq 0$ is not a linear transformation under addition; $b \neq 2b$

---

8)

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(30) & -\sin(30) & 0 \\ 0 & \sin(30) & \cos(30) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a: $\boxed{M = T_1 R_x T_2}$

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\sin 30 = 1/2$

b:

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ \sqrt{3}/2 \\ 1/2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ \sqrt{3}/2 \\ 1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2+\frac{\sqrt{3}}{2} \\ 3/2 \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} X &= -2 \\ Y &= 2+\frac{\sqrt{3}}{2} \\ Z &= 3/2 \end{aligned}}$$

just -30° instead of 30°

9) $R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 & 0 \\ 0 & -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad T_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

a: $\boxed{M = T_1 R_x T_2}$ $\quad T_2 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

b: $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 2+\sqrt{3}/2 \\ 3/2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ \sqrt{3}/2 \\ 1/2 \\ 1 \end{bmatrix}$ $\quad x=-3$, $y=\sqrt{3}/2$, $z=1/2$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 & 0 \\ 0 & -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ \sqrt{3}/2 \\ 1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

Propperly reverse the rotation

$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \\ 1 \end{bmatrix}$ $\quad x=-2$, $y=3$, $z=1$

10) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 4 \\ 1 & 6 & 0 \\ 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 + x_3 \\ -x_2 + 4x_3 \\ x_1 + 6x_2 \\ 4x_1 \end{bmatrix}$

$$\boxed{T = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 4 \\ 1 & 6 & 0 \\ 4 & 0 & 0 \end{bmatrix}}$$

1) a) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} k\cos\theta & k\sin\theta \\ -k\sin\theta & k\cos\theta \end{bmatrix}$ $AB=BA$

$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} k\cos\theta & k\sin\theta \\ -k\sin\theta & k\cos\theta \end{bmatrix}$

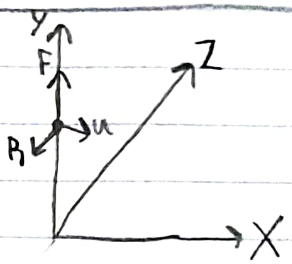
b) Rotating then shifting $\neq$ shifting then Rotating $AB \neq BA$
because by shifting you change the distance the rotation moves.

c) Scaling then shifting $\neq$ shifting then scaling $AB \neq BA$
because by scaling 2nd you also scale the shift, while if you shift 2nd then the shift is not scaled

---

2) a) $\begin{array}{c} \\ \\ \\ \\ \end{array}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{array}{l} \text{Right} \\ \text{Up} \\ \text{forward} \\ \text{coords} \end{array}$

x-1



$\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = -1$ forward

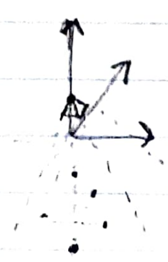
Right can be any value on X-Z plane

$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0$ Right

src = {0, 1, 0}
to = {0, 0, 0}

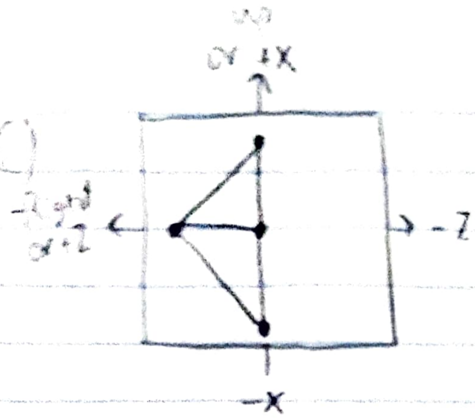
$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0$ up

b) $\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ \begin{bmatrix} 0 \\ 1/5 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} -1/5 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 1/5 \\ 0 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow X \\ \leftarrow Z \end{array}$



dist = 5 4 5 6 5

C)



point {0,5,0} is occluded by point {0,-3,0}

13) a) $\begin{bmatrix} 1 & 0 & 0 & 119/159 \\ 0 & 1 & 0 & -4/53 \\ 0 & 0 & 1 & 28/53 \end{bmatrix}$ Solutions given by program

b) $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ -6 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ -7 \\ 2 \end{bmatrix}$

C) $X_1 = 119/159$
$X_2 = -4/53$
$X_3 = 28/53$

D) Null(A) = [ ]

14)

$B[X]_B = \begin{bmatrix} -18 \\ 2 \\ 5 \end{bmatrix}$ Solution given by program

15)

a) dimentionality of A is $\mathbb{R}^{4\times 3}$ Solution given by program

b) basis of col A is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$