London Moon

Assignment 5

1)

$$[\vec{X}]_B = \underset{B\leftarrow A}{P}\,[\vec{X}]_A \qquad B=\left\{\begin{bmatrix}0\\-1\\3\end{bmatrix},\begin{bmatrix}4\\5\\-4\end{bmatrix},\begin{bmatrix}3\\3\\-6\end{bmatrix}\right\}$$

$$[\vec{X}]_A = (\underset{B\leftarrow A}{P})^{-1}[\vec{X}]_B \qquad \underset{B\leftarrow A}{P}=\begin{bmatrix}4 & -9 & 5\\-3 & -1 & 6\\9 & -2 & -6\end{bmatrix}$$

$$A = B\underset{B\leftarrow A}{P} = \begin{bmatrix}15 & -10 & 6\\8 & -2 & 7\\-30 & -11 & 27\end{bmatrix} \quad \cancel{\underset{A\leftarrow B}{P} = (\underset{B\leftarrow A}{P})^{-1} = \begin{bmatrix}-6/59 & 64/177 & 49/177\\-17/59 & 23/59 & 13/59\\-8/59 & 13/177 & 31/177\end{bmatrix}}$$

~~WHY!! The numbers are so ugly. The question should have P be the change of coordinates from B to A instead of the other way around.~~

5)

$$V_1=\begin{bmatrix}3\\5\\2\end{bmatrix} \qquad V_2=\begin{bmatrix}0\\2\\-5\end{bmatrix} \qquad X=\begin{bmatrix}3\\-4\\1\end{bmatrix}$$

$$V_1\cdot V_2 = 3\cdot 0 + 5\cdot 2 + 2\cdot -5$$
$$= 0 + 10 + -10$$
$$= 0$$

orthoginal

$$\hat{y} = \frac{X\cdot V_1}{V_1\cdot V_1}V_1 + \frac{X\cdot V_2}{V_2\cdot V_2}V_2$$
$$= \frac{-9}{38}V_1 + \frac{-13}{29}V_2$$

$C_1 = \frac{-9}{38}$

$C_2 = \frac{-13}{29}$

$$= \frac{-9}{38}\begin{bmatrix}3\\5\\2\end{bmatrix} + \frac{-13}{29}\begin{bmatrix}0\\2\\-5\end{bmatrix}$$

$$= \begin{bmatrix}-27/38\\-2293/1102\\974/551\end{bmatrix}$$

6)

$$v_1 = \begin{bmatrix} 3 \\ -1 \\ 2 \\ -4 \end{bmatrix} \quad v_2 = \begin{bmatrix} 2 \\ -5 \\ -2 \\ -5 \end{bmatrix} \quad z = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$$

$$c_1 = \frac{-4}{15}$$

$$\hat{z} = \frac{z \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{z \cdot v_2}{v_2 \cdot v_2} v_2$$

$$c_2 = \frac{-25}{58}$$

closest point to z

$$= \frac{-8}{30} v_1 + \frac{-25}{58} v_2$$

$$= \frac{-8}{30} \begin{bmatrix} 3 \\ -1 \\ 2 \\ -4 \end{bmatrix} + \frac{-25}{58} \begin{bmatrix} 2 \\ -5 \\ -2 \\ -5 \end{bmatrix} = \begin{bmatrix} -241/145 \\ 2107/870 \\ 143/435 \\ 2203/870 \end{bmatrix}$$

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7)

$$u_1 = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} \quad u_2 = \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix} \quad y = \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}$$

$u_1 \cdot u_2 = -12 + 12 = 0$

orthoginal

$$\hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} u_1 + \frac{y \cdot u_2}{u_2 \cdot u_2} u_2$$

$$= \frac{24}{25} u_1 + \frac{-7}{25} u_2$$

duh

just flatten the x_3 component to 0

$$\frac{24}{25} \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} + \frac{-7}{25} \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$$