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CSE 3313 - Homework #5 – z-transforms 1

Find the z-transform polynomial ratio and ROC for the following unit sample sequences:

$$\sum_{n=-\infty}^{\infty} h[n]z^{-n}, \ \ z = re^{j\omega}$$

1. $h[n] = (\frac{4}{5})^n u[n]$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=0}^{\infty} (\frac{4}{5})^n z^{-n} = \sum_{n=0}^{\infty} (\frac{4}{5z})^n = \frac{1}{1-\frac{4}{5z}} = \frac{z}{z-\frac{4}{5}}$$

$ROC: |z| > \frac{4}{5}$

2. $h[n] = 4^n u[-n-1]$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=0}^{\infty} 4^n u[-n-1]z^{-n} = \sum_{n=-1}^{-\infty} 4^n z^{-n} = \sum_{n=1}^{\infty} (\frac{z}{4})^n = \sum_{n=0}^{\infty} (\frac{z}{4})^{n+1}$$

$$= \left(\frac{z}{4}\right) \sum_{n=0}^{\infty} \left(\frac{z}{4}\right)^n = (\frac{z}{4}) \frac{1}{1-\frac{z}{4}} = \frac{z}{4-z}$$

$ROC: |z| < 4$

3. $h[n] = (\frac{1}{5})^n u[-n]$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=0}^{-\infty} (\frac{1}{5})^n z^{-n} = \sum_{n=0}^{\infty} (5z)^n = \frac{1}{1-5z}$$

$ROC: |z| < \frac{1}{5}$

4. $h[n] = 2^n u[n-1]$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=1}^{\infty} (\frac{2}{z})^n = \sum_{n=0}^{\infty} (\frac{2}{z})^{n+1} = \left(\frac{2}{z}\right) \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n = \left(\frac{2}{z}\right) \frac{1}{1-\frac{2}{z}} = \frac{2}{z-2}$$

$ROC: |z| > 2$

5. Which of the systems above has a Fourier Transform that exists?

A Fourier Transform exists when the z-transform converges at $|z|=1$. This means that the Fourier Transform exists for 1, and 2 but not 3 or 4.

Find the inverse z-transform of the following z-transforms:

$$\frac{z}{z-a} = \frac{1}{1-az^{-1}} = \sum_{n=0}^{\infty} a^n z^{-n}$$

6. $H(z) = \frac{z}{z-\frac{1}{3}}, |z| > \frac{1}{3}$

   $pole$: $z = \frac{1}{3}$, ROC is right sided. Done by inspection.

   $$\frac{z}{z-\frac{1}{3}} = \frac{1}{1-\frac{1}{3}z^{-1}} \to h[n] = (\frac{1}{3})^n u[n]$$

7. $H(z) = \frac{z}{z-2}, |z| < 2$

   $pole$: $z = 2$, ROC is left sided. Done by inspection.

   $$\frac{z}{z-2} = \frac{1}{1-2z^{-1}} \to h[n] = 2^n u[-n-1]$$

8. $H(z) = \frac{1}{1-2z^{-1}}, |z| < 2$

   $Same\ as\ above\ question$

   $h[n] = 2^n u[-n-1]$

9. $H(z) = \frac{1}{1-\frac{2}{3}z^{-1}}, |z| > \frac{2}{3}$

   $pole$: $z = \frac{2}{3}$, ROC is right sided. Done by inspection.

   $$\frac{1}{1-\frac{2}{3}z^{-1}} \to h[n] = (\frac{2}{3})^n u[n]$$

10. Which of the systems above has a Fourier Transform that exists?

    A Fourier Transform exists when the series converges at $|z| = 1$. This means that all the questions have a Fourier transform that exists.

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Find the poles and zeros of the following z-transform polynomial ratios:

11. $H(z) = \frac{z(2z-6)}{(z-2)(z-4)}$
    $zeros$: z=0, z=3
    $poles$: z=2, z=4
12. $H(z) = \frac{2-\frac{5}{6}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$
    $zeros: z = \dfrac{5}{12}$
    $poles: z = \dfrac{1}{2}, z = \dfrac{1}{3}$

Find the z-transform and ROC (region of convergence) of the following unit sample sequences:

$$\sum_{n=-\infty}^{\infty} h[n]z^{-n}, \ \ z = re^{j\omega}$$

13. $h[n] = (\frac{3}{4})^n u[n] + (\frac{1}{4})^n u[n]$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=-\infty}^{\infty} \left(\frac{3}{4}\right)^n u[n]z^{-n} + \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u[n]z^{-n}$$

$$= \sum_{n=0}^{\infty} (\frac{3}{4z})^n + \sum_{n=0}^{\infty} (\frac{1}{4z})^n = \frac{1}{1-\frac{3}{4}z^{-1}} + \frac{1}{1-\frac{1}{4}z^{-1}} = \frac{z}{z-\frac{3}{4}} + \frac{z}{z-\frac{1}{4}}$$

$$ROC: |z| > \frac{3}{4} \ \& \ |z| > \frac{1}{4} \ \ \rightarrow \ \ |z| > \frac{3}{4}$$

14. $h[n] = 2^n u[n] + 4^n u[n]$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=-\infty}^{\infty} 2^n u[n]z^{-n} + \sum_{n=-\infty}^{\infty} 4^n u[n]z^{-n}$$

$$= \sum_{n=0}^{\infty} (\frac{2}{z})^n + \sum_{n=0}^{\infty} (\frac{4}{z})^n = \frac{1}{1-2z^{-1}} + \frac{1}{1-4z^{-1}} = \frac{z}{z-2} + \frac{z}{z-4}$$

$$ROC: |z| > 2 \ \& \ |z| > 4 \ \ \rightarrow \ \ |z| > 4$$

15. $h[n] = (\frac{3}{4})^n u[n] - 2^n u[-n-1]$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=-\infty}^{\infty} (\frac{3}{4})^n u[n]z^{-n} - \sum_{n=1}^{\infty} (\frac{z}{2})^n$$

$$= \sum_{n=0}^{\infty} (\frac{3}{4z})^n - \left(\frac{z}{2}\right) \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n = \frac{z}{z-\frac{3}{4}} - \frac{z}{2-z}$$

$$ROC: |z| > \frac{3}{4} \ \& \ |z| < 2 \ \ \rightarrow \ \ \frac{3}{4} < |z| < 2$$

16. $h[n] = -2^n u[-n-1] - 5^n u[-n-1]$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = -\sum_{n=1}^{\infty}(\frac{z}{2})^n - \sum_{n=1}^{\infty}(\frac{z}{5})^n$$

$$= -\left(\frac{z}{2}\right)\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n - \left(\frac{z}{5}\right)\sum_{n=0}^{\infty}\left(\frac{z}{5}\right)^n = -\frac{z}{2-z} - \frac{z}{5-z}$$

ROC: $|z| < 2$ & $|z| < 5 \quad \rightarrow \quad |z| < 2$

17. Which of the systems above has a Fourier Transform that exists?

A Fourier Transform exists when the z-transform converges at $|z|$=1. This means that the Fourier Transform exists for 13, 15, and 16.