CSE 3313 - Homework #4 – System Frequency Response HW

System Frequency Response

Find the system frequency response or DTFT, if it exists

1. $h[n] = (\frac{1}{2})^n u[n]$
   a. $H(e^{jw}) = \frac{1}{1-\frac{1}{2}e^{-jw}}$

$$\alpha = \frac{1}{2}$$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jwn} = \sum_{n=-\infty}^{\infty} \alpha^n u[n]e^{-jwn} = \sum_{n=0}^{\infty} (\alpha e^{-jw})^n$$
$$= \frac{1}{1-\alpha e^{-jw}} = \frac{1}{1-\frac{1}{2}e^{-jw}}$$

2. $h[n] = 3^n u[n]$
   a. Does not exist.

$$\alpha = 3$$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jwn} = \sum_{n=-\infty}^{\infty} \alpha^n u[n]e^{-jwn} = \sum_{n=0}^{\infty} (\alpha e^{-jw})^n$$
$$\rightarrow unstable\ \&\ grows\ to\ \infty$$

3. $h[n] = (\frac{2}{3})^n u[n]$
   a. $H(e^{jw}) = \frac{1}{1-\frac{2}{3}e^{-jw}}$

$$\alpha = \frac{2}{3}$$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jwn} = \sum_{n=-\infty}^{\infty} \alpha^n u[n]e^{-jwn} = \sum_{n=0}^{\infty} (\alpha e^{-jw})^n$$
$$= \frac{1}{1-\alpha e^{-jw}} = \frac{1}{1-\frac{2}{3}e^{-jw}}$$

4. $x[n] = \delta[n-1] + \delta[n+1]$

   Assuming you mean h[n] instead of x[n]. Otherwise the question doesn't make sense for this kind of problem.

   a. $H(e^{jw}) = 2cosw$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jwn} = \sum_{n=-\infty}^{\infty} (\delta[n-1] + \delta[n+1])e^{-jwn} =$$
$$= e^{jw} + e^{-jw} = 2cosw$$

5. $h[n] = 2^n u[-n-1]$

   a. $H(e^{jw}) = \frac{1}{\alpha - e^{jw}}$

$$\alpha = 2$$
$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jwn} = \sum_{n=-\infty}^{\infty} \alpha^n u[-n-1]e^{-jwn} = \sum_{n=0}^{\infty} \frac{(\alpha e^{-jw})^{-n}}{\alpha}$$
$$= \frac{\sum_{n=0}^{\infty}(\alpha^{-1}e^{jw})^n}{\alpha} = \frac{1}{\alpha(1-\alpha^{-1}e^{jw})} = \frac{1}{\alpha - e^{jw}}$$

6. $h[n] = (\frac{2}{3})^n u[-n]$

   a. Does not exist.

$$\alpha = \frac{2}{3}$$
$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jwn} = \sum_{n=-\infty}^{\infty} \alpha^n u[-n]e^{-jwn} = \sum_{n=0}^{\infty} (\alpha e^{-jw})^{-n}$$
$$\rightarrow \ unstable \ \& \ grows \ to \ \infty$$

For the difference equation below

$$y[n] = x[n] + \alpha y[n-1]$$

7. What range of values for α will result in a system frequence response that exists?
   a. $|\alpha| < 1$
8. Fing h[n] from the difference equation.
   a. $h[n] = \alpha^n u[n]$

$$h[n] = \delta[n] + \alpha\delta[\mathrm{n}-1] + \alpha^2\delta[n-2] \ldots = \sum_{k=0}^{\infty} \alpha^k \delta[n-k] = \alpha^n u[n]$$

9. Find $H(e^{j\omega})$ for the system.
   a. $H(e^{jw}) = \frac{1}{1-\alpha e^{-jw}}$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jwn} = \sum_{n=-\infty}^{\infty} \alpha^n u[n]e^{-jwn} = \sum_{n=0}^{\infty} (\alpha e^{-jw})^n$$
$$= \frac{1}{1-\alpha e^{-jw}}$$

10. Find the magnitude-squared system frequency response $\left|H(e^{j\omega})\right|^2$ for the system in terms of trigonometric functions and plot the response from w=0 to w=2π.
    a. $\left|H(e^{j\omega})\right|^2 = \frac{1}{1+\alpha^2-2\alpha\cos w}$

    To find magnitude squared of a complex frequency, you multiply by the complex conjugate.

$$\left|H(e^{j\omega})\right|^2 = \frac{1}{1-\alpha e^{-jw}} \cdot \frac{1}{1-\alpha e^{jw}} = \frac{1}{(1-\alpha e^{-jw})(1-\alpha e^{jw})}$$
$$= \frac{1}{1+(\alpha e^{-jw})(\alpha e^{jw}) - \alpha e^{-jw} - \alpha e^{jw}}$$
$$= \frac{1}{1+\alpha^2 - 2\alpha(\frac{e^{-jw}+\alpha e^{jw}}{2})} = \frac{1}{1+\alpha^2-2\alpha\cos w}$$

As a final note, the equation $y[n] + \alpha\mathrm{y}[\mathrm{n}-1] = \mathrm{x}[\mathrm{n}] \rightarrow \mathrm{h}[\mathrm{n}] = \alpha^n u[n]$ in the frequency response notes is incorrect. The sign is wrong. It should be $[n] - \alpha\mathrm{y}[\mathrm{n}-1] = \mathrm{x}[\mathrm{n}] \rightarrow \mathrm{h}[\mathrm{n}] = \alpha^n u[n]$ as shown in the difference equation notes.