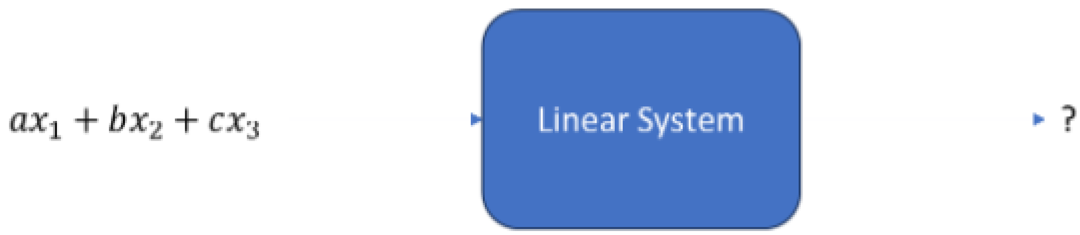
Landon Moon ID: 1001906270

CSE 3313 - Homework #2 - Discrete linear Shift-Invariant Systems

LINEARITY

1. We know that putting x_1 into a linear system results in the output y_1; putting x_2 into the system results in the output y_2 and putting x_3 into the system results in the output y_3. What is the output of that linear system with the input below?



$ay_1 + by_2 + cy_3$

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2. Test the following systems for **linearity** using the test procedure given in class and determine whether they are linear or non-linear.

a) $y[n] = 2x[n]+1$
1. $y_1[n] = 2x_1[n]+1$
   $y_2[n] = 2x_2[n]+1$
2. $x_3[n] = ax_1[n]+bx_2[n]$
3. $y_3[n] = 2x_3[n]+1$
   $y_3[n] = 2(ax_1[n]+bx_2[n])+1$
   $y_3[n] = 2ax_1[n] + 2bx_2[n] + 1$
4. $y_3[n] = ay_1[n]+by_2[n]$
   $y_3[n] = a(2x_1[n]+1)+b(2x_2[n]+1)$
   $y_3[n] = 2ax_1[n] + 2bx_2[n] + a + b$
5. $2ax_1[n] + 2bx_2[n] + a + b \neq 2ax_1[n] + 2bx_2[n] + 1$

<u>NO, the system is not LINEAR</u>

b) $y[n] = \frac{1}{2}x[n] - \frac{1}{4}x[n-1]$
1. $y_1[n] = \frac{1}{2}x_1[n] - \frac{1}{4}x_1[n-1]$
   $y_2[n] = \frac{1}{2}x_2[n] - \frac{1}{4}x_2[n-1]$
2. $x_3[n] = ax_1[n]+bx_2[n]$
3. $y_3[n] = \frac{1}{2}x_3[n] - \frac{1}{4}x_3[n-1]$
   $y_3[n] = \frac{1}{2}(ax_1[n] + bx_2[n]) - \frac{1}{4}(ax_1[n-1] + bx_2[n-1])$
4. $y_3[n] = ay_1[n] + by_2[n]$
   $y_3[n] = a\left(\frac{1}{2}x_1[n] - \frac{1}{4}x_1[n-1]\right) + b\left(\frac{1}{2}x_2[n] - \frac{1}{4}x_2[n-1]\right)$
   $y_3[n] = \frac{1}{2}(ax_1[n] + bx_2[n]) - \frac{1}{4}(ax_1[n-1] + bx_2[n-1])$
   $y_3[n] = \frac{1}{2}(ax_1[n] + bx_2[n]) - \frac{1}{4}(ax_1[n-1] + bx_2[n-1])$
5. $\frac{1}{2}(ax_1[n] + bx_2[n]) - \frac{1}{4}(ax_1[n-1] + bx_2[n-1]) == \frac{1}{2}(ax_1[n] + bx_2[n]) - \frac{1}{4}(ax_1[n-1] + bx_2[n-1])$

<u>YES, the system is LINEAR</u>

c) $y[n] = x[2n]$
1. $y_1[n] = x_1[2n]$
   $y_1[n] = x_2[2n]$
2. $x_3[n] = ax_1[n]+bx_2[n]$
3. $y_3[n] = x_3[2n]$
   $y_3[n] = ax_1[2n]+bx_2[2n]$
4. $y_3[n] = ay_1[n]+by_2[n]$
   $y_3[n] = ax_1[2n]+bx_2[2n]$
5. $ax_1[2n]+bx_2[2n] == ax_1[2n]+bx_2[2n]$

<u>YES, the system is LINEAR</u>

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SHIFT-INVARIANCE

3. We know that putting x into a shift-invarient system results in the output y. What is the output of that shift-invariant system with the input given below?

<u>The result is y[n-2].</u>

This is due to the shift invarience condition where if $x[n] \to y[n]$ then $x[n-n_0] \to y[n-n_0]$

4. Test the above systems in problems 2z, 2b, 2c for shift-invariance using the test procedure given in class and determine whether they are shift-invarient or not shift-invarient

a) $y[n] = 2x[n]+1$
   1. $y[n - n_0] = 2x[n - n_0]+1$
   2. $y_1[n - n_0] = 2x[n - n_0]+1$
   3. $2x[n - n_0]+1 == 2x[n - n_0]+1$

<u>YES, the system is SHIFT-INVARIENT</u>

b) $y[n] = \frac{1}{2}x[n] - \frac{1}{4}x[n-1]$
   1. $y[n - n_0] = \frac{1}{2}x[n - n_0] - \frac{1}{4}x[n - 1 - n_0]$
   2. $y[n - n_0] = \frac{1}{2}x[n - n_0] - \frac{1}{4}x[n - n_0 - 1]$
   3. $y[n - n_0] = \frac{1}{2}x[n - n_0] - \frac{1}{4}x[n - 1 - n_0] == y[n - n_0] = \frac{1}{2}x[n - n_0] - \frac{1}{4}x[n - n_0 - 1]$

<u>YES, the system is SHIFT-INVARIENT</u>

c) $y[n] = x[2n]$
   1. $y[n - n_0] = x[2n - n_0]$
   2. $y[n - n_0] = x[2(n-n_0)]$
      $y[n - n_0] = x[2n - 2n_0]$
   3. $x[2n - n_0] \neq x[2n - 2n_0]$

<u>NO, the system is not SHIFT-INVARIENT</u>

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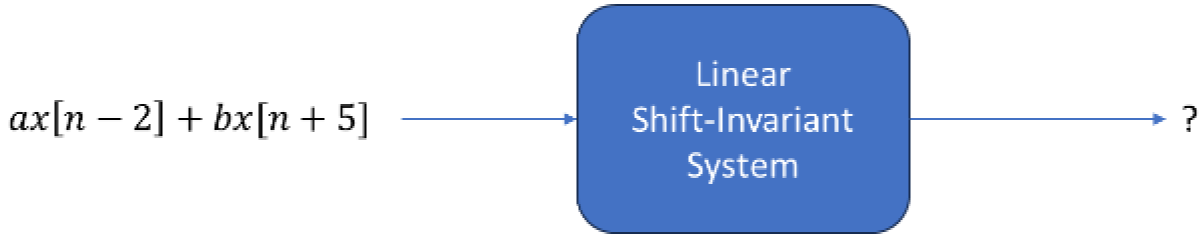
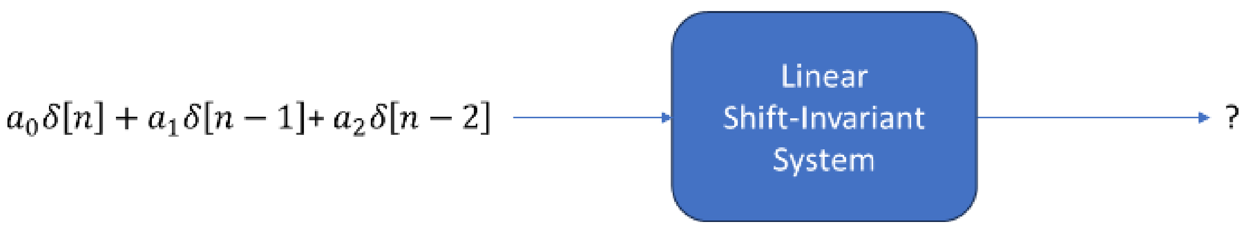
LINEAR SHIFT-INVARIENT SYSTEMS

5. If we know that x into the sytem results in an output of y, what is the output of the linear shift-invariant (LSI) system below with the given input?



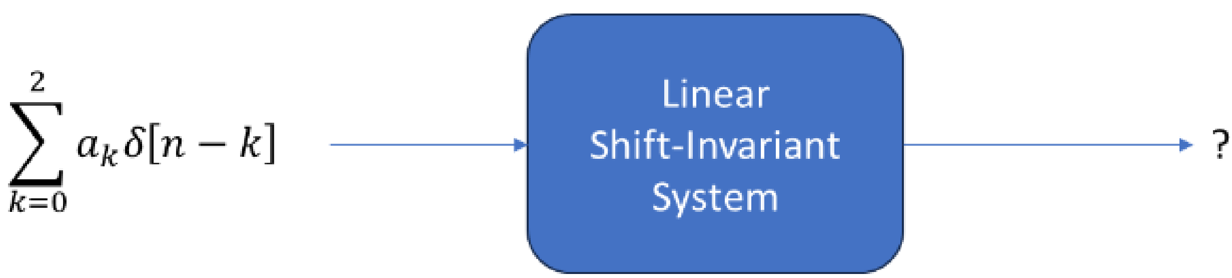
ay[n-2] + by[n+5]

6. If we know that a delta function ($\delta[n]$) as an input to an LSI system results in an output of h[n], what is the output of the following LSI system?



$a_0h[n] + a_1h[n-1] + a_2h[n-2]$

7. If we know that a delta function ($\delta[n]$) as an input to an LSI system results in an output of h[n], what is the output of the following LSI system?



$a_0h[n] + a_1h[n-1] + a_2h[n-2]$

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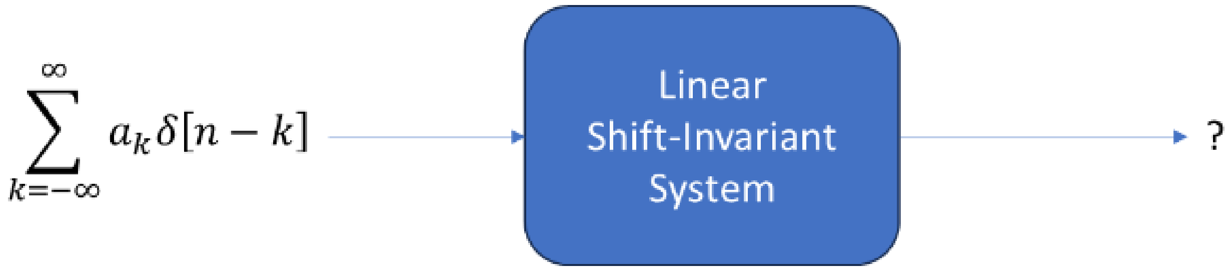
8. If we know that a delta function ($\delta[n]$) as an input to an LSI system results in an output of h[n], what is the output of the following LSI system?



$$\sum_{k=-\infty}^{\infty} a_k h[n-k]$$

9. Write the decomposition of a infinitely long general sequence x[n] into a sum of weighted and shifted delta functions ($\delta[n]$).

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Where x[k] are the individual unit weights and δ[n-k] are the shifted delta functions.

10. If we know that a delta function ($\delta[n]$) as an input to an LSI system results in an output of $h[n]$,what is the output of an LSI system with your decomposed general sequence from question 9 as the input?

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Where x[k] are the weights and h[n-k] are the outputs from δ[n-k].