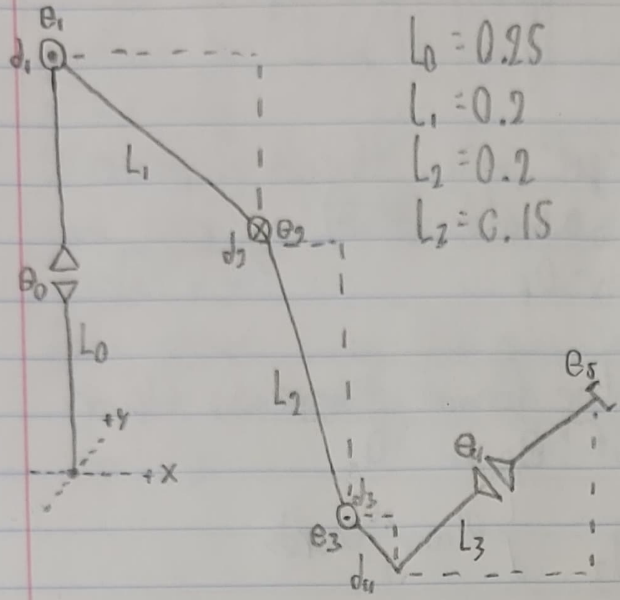
Homework 1



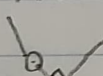
$L_0 = 0.25$ $\quad d_1 = -0.04$
$L_1 = 0.2$ $\quad d_2 = 0.04$
$L_2 = 0.2$ $\quad d_3 = -0.04$
$L_3 = 0.15$ $\quad d_4 = -0.04$

FORWARD KINEMATIS

θ_4 and θ_5 have no effect on position.
d_1 and d_2 cancel out so they can be ignored

Start with the planar slace then rotate around θ_0

$X_1 = 0$
$Y_1 = 0$
$Z_1 = L_0$



$$\begin{cases} X_4 = X_3 + d_4 \cos(\theta_1 + \theta_2 + (\theta_3 - \pi/2)) \\ Y_4 = d_3 \\ Z_4 = Z_3 - d_4 \sin(\theta_1 + \theta_2 + (\theta_3 - \pi/2)) \end{cases}$$

$X_2 = L_1 \cos(\theta_1)$
$Y_2 = 0$
$Z_2 = Z_1 - L_1 \sin(\theta_1)$

$X_5 = X_4 + L_3 \cos(\theta_1 + \theta_2 + \theta_3)$
$Y_5 = d_3$
$Z_5 = Z_4 - L_3 \cos(\theta_1 + \theta_2 + \theta_3)$

now rotate!

$X_3 = X_2 + L_2 \cos(\theta_1 + \theta_2)$
$Y_3 = 0$
$Z_3 = Z_2 - L_2 \sin(\theta_1 + \theta_2)$

$L_f = \sqrt{x_5^2 + y_5^2}$
$\theta_f = \text{atan2}(y_5, x_5)$
$x_f = L_f \cos(\theta_0 + \theta_f)$
$y_f = L_f \sin(\theta_0 + \theta_f)$
$z_f = Z_5$

INVERSE KINEMATICS

first rotate θ_0 to get into the X-Z plane

$\theta_a = \text{atan2}(Y, X)$ $\quad\theta_f = \text{asin}(d_3/\sqrt{x^2+y^2})$

$\theta_a - \theta_0 = \theta_f$ $\quad\theta_0 = \theta_a - \theta_f$

θ_f is the same as the forward kinematics. θ_f is the θ between X-Y if θ_0 is 0.

$$\left.\begin{aligned} X &= \sqrt{x^2+y^2}\cos(\theta_f) \\ Y &= \sqrt{x^2+y^2}\sin(\theta_f) = d_3 \end{aligned}\right\} \text{Planar coords}$$

we know the last link points down so

$\theta_1 + \theta_2 + \theta_3 = \pi/2$

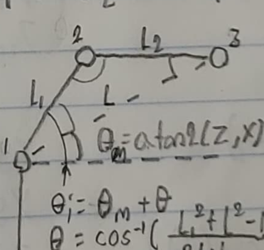
Solve the equations from forward kinematics

$x = L_1\cos(\theta_1)$

$\quad L_2\cos(\theta_1+\theta_2) \rightarrow L_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$

$\quad d_4\cos(0) \rightarrow d_4$

$\quad L_3\cos(\pi/2) \rightarrow 0$



$\theta_1' = \theta_m + \theta$

$\theta = \cos^{-1}\left(\frac{L_1^2 + L^2 - L_2^2}{2L_1L}\right)$

$\theta_2' = \cos^{-1}\left(\frac{L_1^2 + L_2^2 - L}{2L_1L_2}\right)$

$\theta_3 = \pi/2 - \theta_1 - \theta_2$

$Z = L_0$

$\quad - L_1\sin(\theta_1)$

$\quad - L_2\sin(\theta_1+\theta_2) \rightarrow L_2(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)$

$\quad - d_4\sin(0) \rightarrow 0$

$\quad - L_3\sin(\pi/2) \rightarrow L_3$

We're trying to use law of cosine so only relative positions matter. The last two components of both X and Z can be ignored. now use law of cosine. Then solve for the actual values from the triangle

$\theta_1 = -\theta_1'$ $\quad\theta_2 = \pi - \theta_2'$ $\quad\theta_3 = \pi/2 - \theta_1 - \theta_2$

To be a bit more verbose you 'ignore' the constant positions of the point by subtracting them

$x' = x - d_4$

$z' = y + L_3 - L_0$

you then solve for the other length of the triangle

$L = \sqrt{x'^2 + z'^2}$

then solve using the previous equations. θ_3 is known by solving the previous equation $\theta_1 + \theta_2 + \theta_3 = \pi/2$ for θ_3.