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CSE 3313 - Homework #3 – LSI Systems HW

Causality & Stability

Test the following linear-shift-invariant systems for causality and stability

$$h[n] = 0 \; for \; n < 0$$

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

a. $h[n] = (\frac{1}{4})^n u[n]$
   1) Causal. All n<0 values equal 0
   2) Stable. Total sums to 4/3
b. $h[n] = \delta[n] + 2\delta[n-1]$
   1) Causal. All n<0 values equal 0
   2) Stable. Total sums to 3
c. $h[n] = 2^n u[-n-1]$
   1) Non-Causal. Non-zero n<0 values exist due to '-n'
   2) Stable. Total sums to 1
d. $h[n] = \delta[n+1] + 2\delta[n] + \delta[n-1]$
   1) Non-Causal. Non-zero n<0 values exist due to 'n+1'
   2) Stable. Total sums to 4
e. $h[n] = 3^n u[-n]$
   1) Non-Causal. Non-zero n<0 values exist due to '-n'
   2) Stable. Total sums to 3
f. $h[n] = (\frac{1}{2})^n u[-n]$
   1) Non-Causal. Non-zero n<0 values exist due to '-n'
   2) Non-Stable. Total sums to ∞ due to '$(\frac{1}{2})^n$' as n → -∞

Sum totals were approximated with custom made desmos code. This was done so I could visualize each equation.

Convolution

Calculate y[n] analytically using convolution. Remember, $x[k] * h[k] = h[k] * x[k]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

a. $x[n] = (\frac{1}{2})^n u[n],\ h[n] = \delta[n-1]$

δ [n] is only non-zero when its input is 0, so using the second equation above, the only non-zero value is when k=1 due to h[n].

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = h[1]x[n-1]$$

$$= (\frac{1}{2})^{n-1} u[n-1]$$

b. $x[n] = 3^n u[-n],\ h[n] = \delta[n+2]$

δ [n] is only non-zero when its input is 0, so using the second equation above, the only non-zero value is when k=-2 due to h[n].

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = h[-2]x[n+2]$$

$$= 3^{n+2} u[-(n+2)]$$

c. $x[n] = \delta[n-1] + \delta[n+1],\ h[n] = (\frac{1}{3})^n u[n]$

δ [n] is only non-zero when its input is 0, so using the first equation above, the only non-zero values are when k=-1 and k=1 due to x[n].

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[-1]h[n+1] + x[1]h[n-1]$$

$$= (\frac{1}{3})^{n+1} u[n+1] + (\frac{1}{3})^{n-1} u[n-1]$$

d. $x[n] = \sin(2\pi f n),\ h[n] = \delta[n+5]$

δ [n] is only non-zero when its input is 0, so using the second equation above, the only non-zero value is when k=-5 due to h[n].

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = h[-5]x[n+5]$$

$$= \sin(2\pi f(n+5))$$