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CSE 3313 - Homework #6 – z-transforms 2

For the following unit sample sequences, find the z-transform polynomial ratio, the poles, zeros, ROC of the z-transform, and the difference equation:

1. $h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[n]$

   POLYNOMIAL RATIO

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n = \frac{z}{z-\frac{1}{2}} + \frac{z}{z-\frac{1}{3}}$$

$$= \frac{z\left(z-\frac{1}{2}+z-\frac{1}{3}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} = \frac{2-\frac{5}{6}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)} = \frac{z\left(2z-\frac{5}{6}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$

$$= \frac{2-\frac{5}{6}z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$$

   ZEROS, POLES, ROC

$$Zeros: z = \frac{5}{12}, z = 0$$

$$Poles: z = \frac{1}{2}, z = \frac{1}{3}$$

$$ROC: |z| > \frac{1}{2} \;\&\; |z| > \frac{1}{3} \quad \rightarrow \quad |z| > \frac{1}{2}$$

   DIFFERENCE EQUATION

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2-\frac{5}{6}z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} \rightarrow$$

$$Y(z)\left(1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}\right) = X(z)\left(2-\frac{5}{6}z^{-1}\right) \rightarrow$$

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 2x[n] - \frac{5}{6}x[n-1]$$

2. $h[n] = \left(\frac{1}{3}\right)^n u[n] - 5^n u[-n-1]$

$POLYNOMIAL\ RATIO$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n - \left(\frac{z}{5}\right)\sum_{n=0}^{\infty}\left(\frac{z}{5}\right)^n = \frac{z}{z-\frac{1}{3}} - \frac{z}{5-z}$$

$$= \frac{z\left(5-z-z+\frac{1}{3}\right)}{\left(z-\frac{1}{3}\right)(5-z)} = \frac{-2+\frac{16}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)(5z^{-1}-1)} = \frac{z\left(-2z+\frac{16}{3}\right)}{\left(z-\frac{1}{3}\right)(5-z)}$$

$$= \frac{-2+\frac{16}{3}z^{-1}}{-1+\frac{16}{3}z^{-1}+\frac{5}{3}z^{-2}}$$

$ZEROS, POLES, ROC$

$$Zeros: z = \frac{8}{3}, z = 0$$

$$Poles: z = \frac{1}{3}, z = 5$$

$$ROC: |z| > \frac{1}{3}\ \&\ |z| < 5 \quad \rightarrow \quad \frac{1}{3} < |z| < 5$$

$DIFFERENCE\ EQUATION$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2-\frac{5}{6}z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} \rightarrow$$

$$Y(z)\left(-1+\frac{16}{3}z^{-1}+\frac{5}{3}z^{-2}\right) = X(z)\left(-2+\frac{16}{3}z^{-1}\right) \rightarrow$$

$$y[n] + \frac{16}{3}y[n-1] + \frac{5}{3}y[n-2] = -2x[n] + \frac{16}{3}x[n-1]$$

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For the following causal difference equation, find the z-transform, ROC, and the unit sample sequence:

3. $y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 2x[n] - \frac{5}{6}x[n-1]$

$Z - TRANSFORM$

$$Y(z) - \frac{5}{6}z^{-1}Y(z) + \frac{1}{6}z^{-2}Y(z) = 2X(z) - \frac{5}{6}z^{-1}X(z) \rightarrow$$

$$Y(z)\left(1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}\right) = X(z)\left(2 - \frac{5}{6}z^{-1}\right) \rightarrow$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - \frac{5}{6}z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} = \frac{2 - \frac{5}{6}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{2}z^{-1})} = \frac{z(2z - \frac{5}{6})}{(z - \frac{1}{3})(z - \frac{1}{2})}$$

ROC

$$Zeros: z = \frac{5}{3}, z = 0$$

$$Poles: z = \frac{1}{3}, z = \frac{1}{2}$$

$$ROC: |z| > \frac{1}{2} \;\&\; |z| > \frac{1}{3} \quad \rightarrow \quad |z| > \frac{1}{2}$$

$UNIT\ SAMPLE\ SEQUENCE$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[n]$$

For the following z-transform and ROC, find the difference equation and unit sample sequence:

4. $H(z) = \frac{2-5z^{-1}}{1-5z^{-1}+6z^{-2}}, ROC: |z| < 3$

$DIFFERENCE\ EQUATION$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2-5z^{-1}}{1-5z^{-1}+6z^{-2}} \rightarrow \quad Y(z)(1-5z^{-1}+6z^{-2}) = X(z)(2-5z^{-1}) \rightarrow$$

$$y[n] - 5y[n-1] + 6y[n-2] = 2x[n] - 5x[n-1]$$

$UNIT\ SAMPLE\ SEQUENCE$

$$H(z) = \frac{2-5z^{-1}}{1-5z^{-1}+6z^{-2}} = \frac{2-5z^{-1}}{(1-3z^{-1})(1-2z^{-1})}$$

$$h[n] = 3^n u[-n-1] + 2^n u[-n-1]$$