Human Powered Quad-Rotor Hovercraft Blade Design

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Abstract

Ideas behind creating vertical take off and landing (VTOL) air crafts, otherwise known as rotor crafts, are traced back all the way to 400 BC China, beginning with spinning aerial toys (Leishman, *"Principles of Helicopter Aerodynamics"*). It was not until later when people began hypothesizing about such apparatuses able to carry a human, such as Da Vinci's famous "aerial-screw" (1483 AD), based off Archimedes' water screw. The main issues with developments over the years was due to a lack of an energy source able to maintain rotation of the rotor(s), as well as vibrations experienced from external forces. Eventually, inventions like gas powered engines and specific designs to help reduce the net torque on the craft dramatically increased advancements. Specifically the design of quad rotor crafts and coaxial rotors where opposite rotors or blades rotate in opposite directions of each other to cancel the net torque.

1 Introduction

Knowledge in how specific geometries behave under specific flow conditions have also improved rotor craft technology. It is crucial to design blades specifically based on maximizing thrust and minimizing the torque necessary to maintain a constant rotation of the rotor.

2 Physics & Code Implementation

The analysis of how the geometry of a blade translates over to the amount of thrust and torque experienced goes back to the lift produced when an object moves through a liquid at a specific velocity (refer to Project 3 for further explanation and analysis). It begins with non-dimensionalyzing terms for the thrust and torque (the same is done for other terms, which refers to the coefficients of pressure, drag, and lift). These non-dimensional terms are the coefficients of forces normal to and tangential to the disk that the rotor sweeps out. The normal coefficient of force describes the thrust and the tangential coefficient of force describes the moments on the rotor. Recall that drag and lift are in a reference plane relative to the orientation of the airfoil; however, since rotor crafts have their theoretical "disk" orientated with the horizontal, these coefficients can be correlated to the coefficients used for thrust and torque:

$$C_n = \frac{T}{qA_{blade}} \Rightarrow C_n \approxeq C_l \cos(\phi) \tag{1}$$

$$C_t = \frac{Q}{qA_{blade}r} \Rightarrow C_t \approxeq C_l \sin(\phi) \tag{2}$$

ϕ is the angle the flow makes with the horizontal, and can be described as $\beta = \alpha + \phi$; β is the pitch of the blade, and α is the perceived angle of attack (angle of blade relative to angle of the flow). Since the magnitude and angle of the flow changes radially, we can discretize these values:

$$\Delta T = \frac{1}{2}\rho v_r^2(r) C_l(r) \cos(\phi(r)) c(r) \Delta r \tag{3}$$

$$\Delta Q = \frac{1}{2}\rho v_r^2(r)C_l(r)\sin(\phi(r))c(r)r\Delta r \tag{4}$$

Summing up each discretized portion results in the overall thrust and torque for a single blade. This torque is specifically the torque caused by the lifting forces acting on the blade; there is also a torque experienced due to viscous forces. These viscous effects c cause boundary layers to form over the blade. The coefficient of drag due to viscous effects is related to the Reynolds Number, a non-dimensional term that describes the behavior of the flow. If we assume the blade to be thin enough, we can approximate the drag as the same drag over a flat plate:

$$C_{t,\nu}(r) = \frac{0.664\cos(\beta)}{\sqrt{Re}} = \frac{0.664\cos(\beta)}{\sqrt{\frac{v_r(r)c(r)}{\nu}}} \tag{5}$$

Determining the coefficient of drag comes from solutions to boundary layer theory. Boundary layer theory was developed by solving the Navier-Stokes equation incorporating the viscous components of the fluid (simply put, the Navier-Stokes equation is a partial differential equation that describes the behavior of fluid i.e. mass and momentum fluxes). The boundary layer is considered to by any fluid in the region near an interface where the velicty is 99% of the free stream velocity. The drag coefficient comes from the shear stress caused by the difference in velocity within the boundary layer: $\tau = \eta\frac{du}{dy}$, and the drag force equation comes from the same convention as the other coefficient forces.

$$F_{t,\nu}(r) = \frac{1}{2}C_{t,\nu}(r)\rho v_r^2(r)c(r)\Delta r \tag{6}$$

Therefore, the total torque is the sum of the moments using Equation (6) and the summation of Equation (4). For the specific quad rotor design being analyzed, there are four rotors and two blades per rotor, thus multiplying by these values is the net amount the craft experiences. Note that since this is for a flat plate, multiply Equation (6) by two to account for the upper and lower surfaces of the blade.

3 Design Process

I began my design with identifying what the total amount of thrust a single blade needed to provide in order to counteract the weight of the human powered craft and remain level in the air. Since the craft has a total mass of $155kg$, there are 4 rotors, and two blades per rotor, a single blade needs to provide $141.0188N$ of thrust. Furthermore, the sum of all the discretized portions of thrust over the entire blade must be this value: $\sum_{i=1}^{m}\Delta T_i = T_{blade} = \frac{mg}{N_{rotors}N_{blades}}$.

I wanted to see if there was a direct correlation between thrust and torque so that I could minimize the amount of torque while seeing how my necessary amount of thrust affected it. Dividing Equation (4) by Equation (3) and isolating ΔQ yields:

$$\Delta Q = \Delta T r\tan(\phi(r)) \tag{7}$$

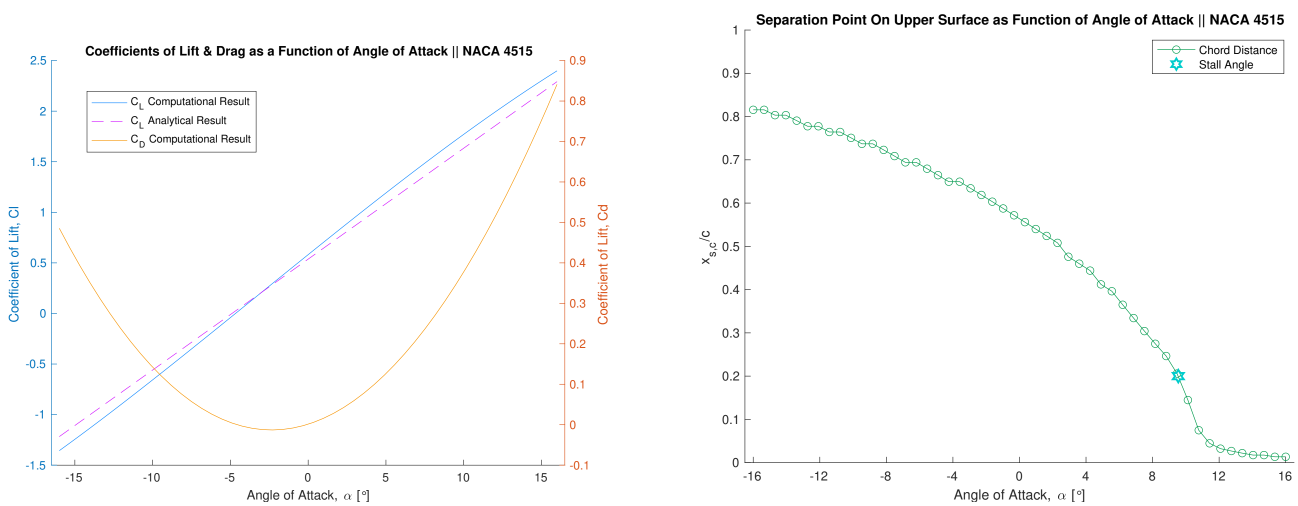
Knowing that r increases linearly towards the tip of the blade, and $\tan(\phi)$ decreases quickly, its desirable to have more thrust towards the root than the tip.

By analyzing Equation (3), the variables that can be manipulated are the chord length and the coefficient of lift (recall that $C_l \approxeq 2\pi\alpha_{absolute}$). Therefore, I wanted to maximize the total area of the blade while having more of its area located at the root. Since the maximum blade to disk ratio was $\sigma = 0.1$, the maximum area for a single blade was limited to $15.7080m^2$. After trying different boundary conditions (the chord lengths at each end) for linear and polynomial functions, a blade with a square root profile was perfect (total area of a single blade is $15.6968m^2$:

$$c(r) = 0.8090\sqrt{10.5501 - r} \qquad 1 \leqq r \leqq 10 \tag{8}$$

Since there is less torque for the same amount of thrust closer to the rotor (based on Equation (7)), I decided to have a constant coefficient of lift of $C_l = 1.008$ for the first third of the blade. The last third I determined $C_l = 0.01$ to be sufficient (r gains a value approaching 10 further out so lower coefficient in this region reduces the overall torque). After determining the coefficients of lift for the first and third portions of the blade, I decided to go to use a linear profile for the coefficient of lift for the middle portion. I did this originally for simplicity, but you can see in Figure (4) that a curve that decreases faster at first would not require enough thrust necessary to maintain a constant elevation (since the total thrust and torque is the area under the curves in Figure (1.a)). However, having a larger coefficient of lift at the first portion of the blade to balance out this loss of area if I were to have a different profile for the middle section would also increase the torque exponentially (you can see that the slope gets steeper and steeper at this section). I also tried out several different values for the pairs of coefficients at the root and tip of the blade, and there seemed to be a lot less torque for a higher coefficient of lift near the root rather than at the tip. Using these discretized values gives the angle of attack: $\alpha = \frac{C_l}{2\pi} + \alpha_{L_0}$. $\alpha_{L_0} = -4.8980°$, and was determined using the results from the vortex panel method. The pitch of the blade can now be determined: $\beta = \alpha + \phi$. Furthermore, to ensure my design supplied enough thrust for the rotor craft, I continuously checked each updated design attempt by invoking $\sum_{i=1}^{m} \Delta T_i = T_{blade}$

4 Results

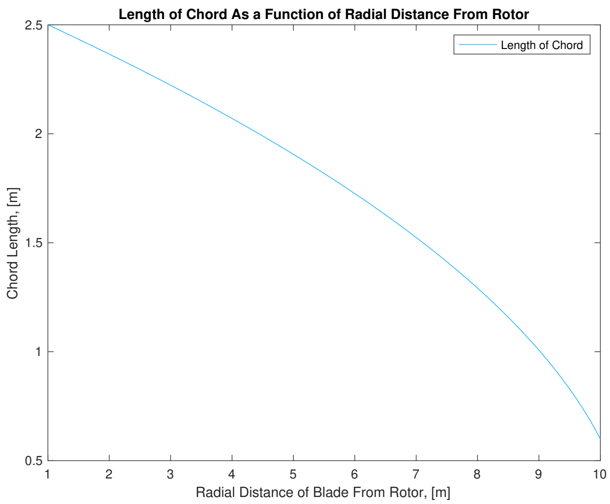


(a) Coefficients of Lift and Drag from Vortex Panel Method

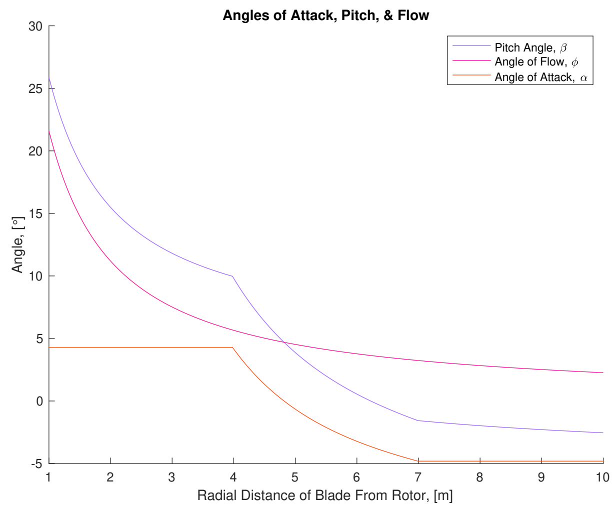
(b) Thwaites

Figure 1: Results From Vortex Panel Method

Figure (1.a) is the coefficient of lift for the NACA 4515 airfoil that was used for the blade design. I used α_{L_0} to form the curve: *'C_L Analytical Result'* to show how it deviates slightly from the coefficients of lift from the vortex panel method. Figure (1.b) shows the output values of Thwaites Method, as in Project 3. The stall angle is approximately $\alpha_{stall} = 9.5950°$. Thus, based on the parameters for the blade design, the highest value allowed for an angle of attack is $\alpha_{max} = 8.5950°$.



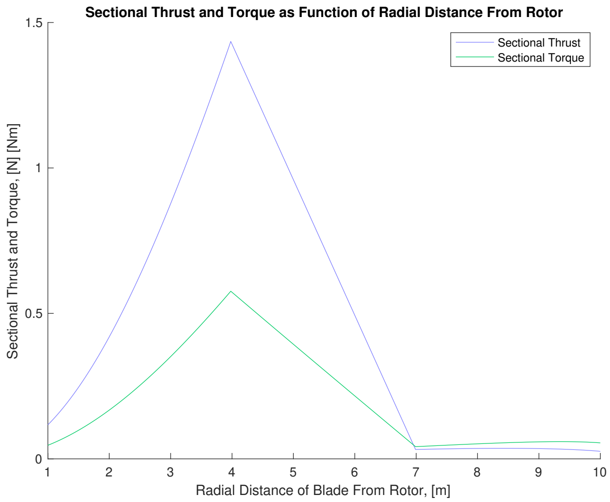
(a) Chord Variation



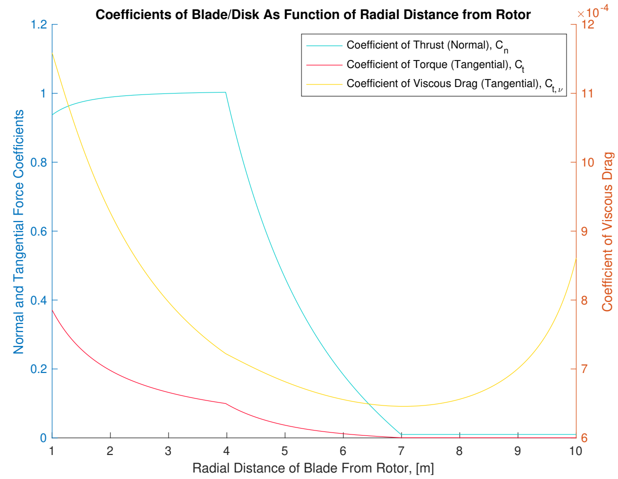
(b) Angles of Attack, Pitch, and Flow

Figure 2: Blade and Flow Characteristics

Figure (4.a) is a visible representation of the chord length along the blade. Notice how it is much thicker near the root and becomes shorter towards the tip. Figure (4.b) shows the angles of attack, pitch, and flow. The angle of the flow decreases asymptotically because the horizontal component becomes larger and larger further from the center of rotation of the rotor while the down-flow velocity remains constant. The angle of attack is constant at the first and third portions of the blade because of how I assigned the coefficients of lift. These two aspects are what give rise to the strange behavior of the pitch angle. Remember that the angle of attack is the angle of the blade profile (NACA 4515) relative to the fluid flow, so it is not necessarily the angle of the profile with the horizontal. In contrast, the pitch and flow angles are measured relative to the horizontal.



(a) Thrust and Torque



(b) Coefficients

Figure 3: Forces and Torques

Figure (2.a) shows the discretized values of thrust and torque along the blade's length. The two curves appear rather similar, and this makes since referring to Equation (7). This highlights an issue with designing rotor crafts: there will always be some finite amount of power required to maintain hovering conditions, and it will be proportional to the amount of required (even for well designed models). Figure (2.b) shows the coefficients of normal, tangential, and viscous forces along the blade. These are the coefficients used for calculating the total thrust and torque experienced by the blade. The coefficient of viscous drag is tremendously smaller than the normal and tangential force coefficients. This is because the fluid medium is air, which has a relatively low dynamic viscosity. Most of the torque required to maintain a constant rotation of the rotor comes from Equation (4).

c_{root} $[m]$	c_{tip} $[m]$	σ	α_{min} $[°]$	α_{max} $[°]$	T $[N]$	Q $[Nm]$	P $[hp]$
2.5	0.6	0.0999	$-4.8068°$	$4.2939°$	143.8176	504.3737	1.0620

The values in the above table are characteristics of the rotor and blade design, as well as the performance values. c_{root} and c_{tip} are the chord lengths at the root and tip of the blade. σ is the total blade area to disk area ratio of each rotor. α_{min} is the smallest relative angle of attack the blade makes with the flow and α_{max} in this case is the maximum relative angle of attack the blade makes with the flow. T is the total amount of thrust produced by the design (accounting for the total number of blades). Q is the total amount of torque (including viscous effects) required to maintain a constant angular velocity for all every rotor. P is the total amount of power required to have this specific rotor-craft remain in a hovering position (units are horsepower [$746\ Watts \approx 1\ horsepower$]). Relatively speaking, this is the amount of power an average human can produce for an extended amount of time.

Personal Power Output:

I have an Apple Watch and track most of my exercises. My recent average mile pace is $7 : 30\ min/mile$, and I typically run $3.25\ miles$. Therefore, the total amount of time run is approximately $24.375\ = 1462.5\ sec$. I typically burn around $550\ calories = 2301200\ J$. The definition of power is the amount of energy exerted per second; therefore, my average power output is $1573.47\ W$. Converting to horsepower: $2.109\ hp$. This is practically double the amount of power required to maintain a hovering position for this rotor-craft design.

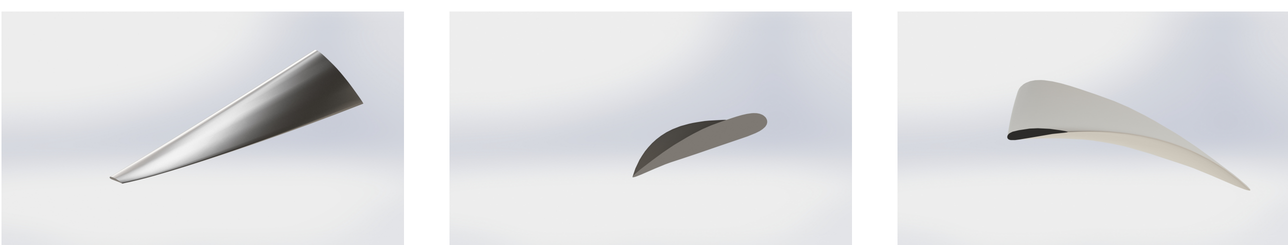
4.1 Bending & Twisting Moments

When an object is moving through a fluid, the forces exerted on it will typically produce moments relative to a point. Moments with respect to the leading edge of the profile (this would be the cross section of the blade, so the leading edge of the NACA 4515 geometry) can be calculated by knowing the velocity component at every position of the airfoil. These values are then used to determine the coefficient of pressure, and then actual pressures everywhere on the surface. A way to simplify determining the net moment relative to the leading edge is to assume a thin airfoil (Keuthe, *'Foundations of Aerodynamics'*). Using pressure differences between the top and bottom surfaces gives a net pressure-force: $F(x)$, where x is a position along the chord, and F is the net force at that position. Therefore, the net moment relative to the leading edge is then:

$$M_{LE} = \int_{x=0}^{x=c} F(x)dx \approxeq \sum_{i=1}^{m} F_i(x_i)x_i \tag{9}$$

Where the summation formulation can be used in the code with the number of discretized values recorded (m). In addition, there will also be a moment the rotor experiences by a single blade due to the thrust: $\int_{r=1}^{r=10} T(r)dr$. Since there are two blades opposite of each other on each rotor, the moment caused by each blade will cancel each other out, and there will only be a net force upward experienced by the rotor (which is simply the total thrust per rotor). However, the thrust will cause shearing stresses locally throughout the blade, and should be considered when deciding on what material to use. The material would need to be able to withstand these shearing forces so they do break nor deform. Deformations would change the profile and in turn the performance of the blade.

5 3D Model Renderings



(a) Isometric View From Tip (b) Profile From Root (c) Profile From Tip

Figure 4: Blade Model

Figure (4.a-c) is a rendering of a 3D model of the blade using SolidWorks. The isometric view clearly shows the chord profile from root to tip, while the head on views highlight how the pitch angles vary along the blade's length.

References

[1] Kuethe, A. M. and Chow, C-Y. *Foundations of aerodynamics*. 5th edition. Wiley 1998.

[2] Leishman, G *Principles of Helicopter Aerodynamics, Cambridge*. 2000