Determining Velocity and Pressure Values on Surface of Cylinder In Flow Field

Landon Knipp

ABSTRACT

When a fluid flows around an object, the velocity of the flow changes when it comes in contact with the body. Based on energy conservation, this causes a non-uniform pressure distribution on the surface of the object. Understanding this pressure distribution gives us insight into understanding what external forces may be acting on the object. For example, the pressure distribution on the wing of a plane is not uniform because in order for the plane to fly, there needs to be a greater pressure on the underside of the wing than the topside. However, calculating the pressure along the surface can be extremely complex. Computer programming makes this nearly impossible task possible, to where we can calculate the conditions of the flow around nearly any type of geometry.

1 Problem Description

The goal is to determine the coefficient of pressure at a linear distribution of points along the surface of a cylinder that is in an augmented velocity field. This will be done for two different cases: no lift and lift. We want to see how the pressure changes when there is lift and when there is no lift. We can enforce a lift or non-lift situation by dictating where the fluid leaves the body on the backside; this is known as the Kutta Condition. For the non-lift case, the Kutta Condition states that there is no circulation on the trending edge of the cylinder. For the lifting case (specifically for the models computed in this project), the Kutta Condition will be placed 30° below the trending edge. For clarification, the backside of the cylinder is the opposite side to where the flow comes in (assume fluid flow is from left to right). As we will see later, we can assume that the pressure distribution will not be symmetric for the lifting case, since lift is a result of a greater pressure distribution on the bottom-side of a body.

2 Formulation

In order to be able to create a body in a coding language such as MATLAB, there needs to be a governing equation for the fluid flow. We can simulate an object surrounded by a uniform flow by placing a dipole in the flow. The dipole acts as a source and a sink, forming a circular boundary (where fluid does not cross) around its center. This is how we form the cylinder. Since a cylinder is such a simple object, we can use an analytical solution to determine the coefficient of pressure along the surface of the cylinder. This is helpful because we can later compare our results from the panel method to the analytical solution to see how accurate it is. First, we need to determine the velocity at any point. We know that there is no normal component of velocity to the surface of the cylinder, so the velocity is simply:

$$\mathbf{U}_\theta = 1 - (2sin\theta + \frac{\Gamma}{2\pi aU})^2 \tag{1}$$

Here, Γ is the amount of circulation the dipole holds. It is more of a theoretical/abstract concept than a physical attribute when utilizing it as a solid object. The circulation is what creates lift on the body. Therefore, for the no lift case, $\Gamma = 0$. However, we want a general expression for the flow that does not depend on the velocity of the flow, the radius of the cylinder, or the circulation of the dipole (by doing so we will get a general solution that can be applied to any set of conditions by scaling the results appropriately). Therefore, we make the following substitution: $\frac{\Gamma}{2\pi aU} = \star$. By doing this, we can solve for whatever $\star$ needs to be for our specific lifting case. We are told to place the Kutta Condition of the lifting case at $-150°$ (equivalent to the location of $-30°$ on the unit circle). With the equation for the coefficient of pressure:

$$Cp = 1 - U^2 \tag{2}$$

and the specific points of the Kutta Condition, we arrive at the following equations:

$$Cp = 1 - 4sin^2\theta \tag{3}$$
$$Cp = 1 - (2sin\theta + 1)^2 \tag{4}$$

Equation (3) is used for the analytical solution for the non-lifting case and Equation (4) is used for the lifting case. For the computational method, we use the theory for vortex sheets to determine the velocity along the surface of the cylinder. We first start by making multiple lines that connect point to point to make a rough outline of a circle. Then we find the circulation values on each panel by dictating that the normal component of the velocity must equal to zero (since no fluid can penetrate the body). From this, we know that:

$$\sum_{j=1}^{m}(C_{n1ij}\gamma_j + C_{n2ij}\gamma_{j+1}) = sin(\theta_i) \tag{5}$$

After solving for the circulation values (γ) from Equation (5), we can use the following equation in order to calculate the total velocity at each point on the surface of the cylinder:

$$U_i = cos(\theta_i) + \sum_{j=1}^{m}(C_{t1ij}\gamma_i + C_{t2ij}\gamma_{j+1}) \tag{6}$$

Note that $C_{n1ij}, C_{n2ij}, C_{t1ij}$, and C_{t2ij} are simply constants determined by the orientation of each panel that form the cylinder. The velocities determined from Equation (6) are then used to calculate the coefficient of pressure, using Equation (2), at each control point (control points are the points where the velocity is being analyzed, and are deliberately placed on the mid-point of each panel).

Once we have the velocities, we can use Thwaites Method to determine where the fluid begins to separate from the surface of the cylinder; these are known as the separation points. This gives us insight into the size and location of the wake region that forms in real life scenarios.

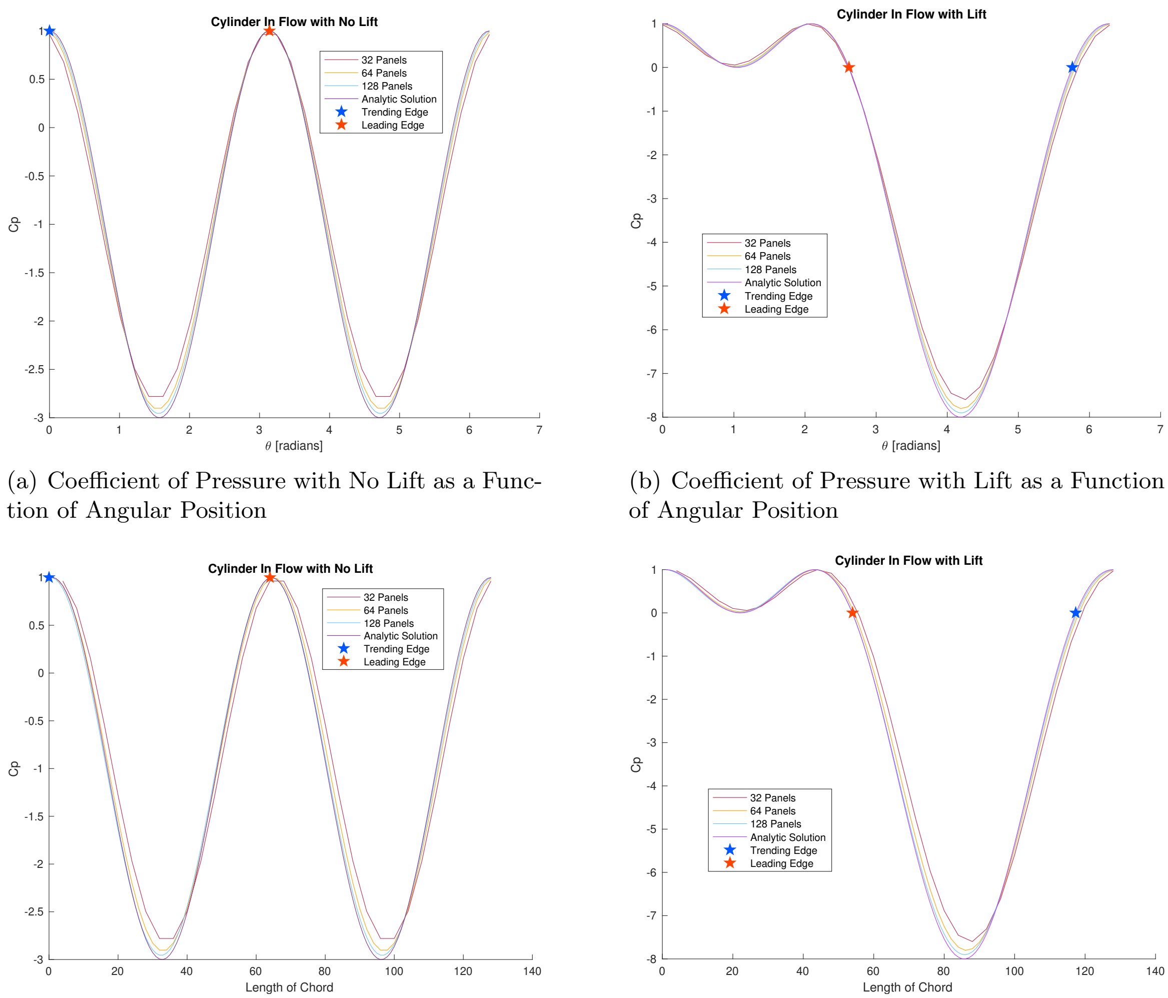
3 Code Implementation

The only input necessary to run the code is to specify the number of panels desired. The higher number of panels, the higher the resolution for the graphs (since there are more data points). In order to perform the calculations, matrices were constructed to form the coefficients from Equations (5) (Matrix A_n) and (6) (Matrix A_t), as well as for $sin(\theta_i)$ and $cos(\theta_i)$. To calculate the circulation of each panel, the inverse of Matrix A_n was multiplied with the array for $sin(\theta_i)$ (using concepts from Linear Algebra).

After finding the circulation values, I used Matrix A_t and multiplied each column by its appropriate circulation value (γ_j). I then summed up each row, and added this resultant array with the $cos(\theta_i)$ array. The result is a vertical array containing the velocities at each control point. Equation (2) was used again to determine the coefficient of pressure at each control point.

One of the most difficult tasks for this project coding-wise was manipulating the velocity data into separate arrays, $U_{e,upper}$ and $U_{e,lower}$ and determining the distance travelled along the surface of the body. This was necessary to perform Thwaites Method. I did so by locating the leading edge of the cylinder by how the velocity changes from negative to positive, relative to the orientation of the panels (which are formed clockwise). I then reallocated the data from this point, and flipped the velocity values element-wise for the underside of the cylinder, since I needed them in the opposite order.

4 Results



(a) Coefficient of Pressure with No Lift as a Function of Angular Position

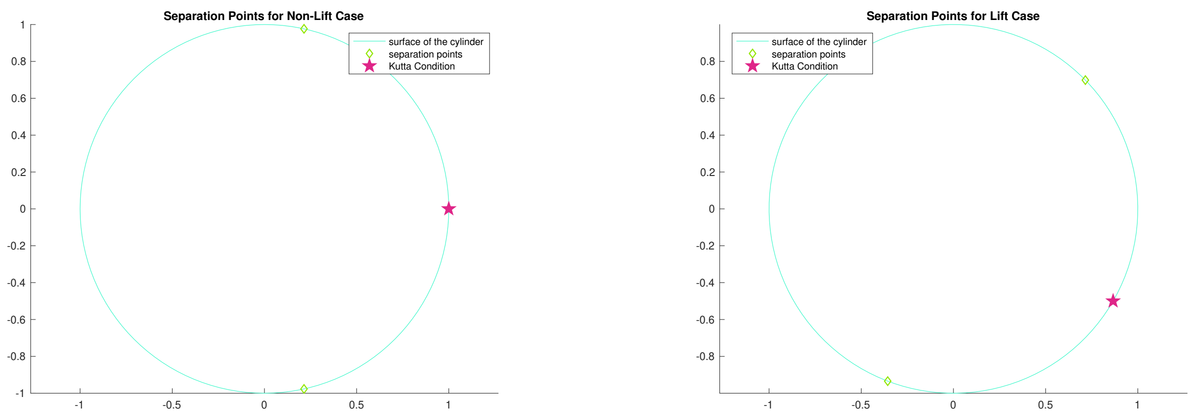
(b) Coefficient of Pressure with Lift as a Function of Angular Position

(c) Coefficient of Pressure with No Lift as a Function of Chord Length

(d) Coefficient of Pressure with Lift as a Function of Chord Length

Figure 1: Values for the Coefficient of Pressure on the Surface of the Cylinder

For a lower number of panels, the curves are less continuous. This is because there are a finite number of panels, resulting in fewer control (data) points. For the non-lifting case, the curve is symmetric. This is because the fluid flows around the cylinder the same on the top as on the bottom. However, it is not symmetric for the lifting case because the stagnation points are now on the underside of the cylinder. This is what causes the higher pressure coefficient values on the left side of the graphs. The graphs are represented as a function of angular position as well as a chord length on the surface of the cylinder. Note: $Length\ of\ Chord$ is measured from where the Kutta Condition is placed and increases positively clockwise around the cylinder. The lift case is offset from the non-lift case due to where the Kutta Condition is placed (enforcing a stagnation point on the lower side of the cylinder); thus, the leading and trending edge locations are marked on the graph for reference. The data from the panel method mimics the curve of the analytical solution surprisingly well, especially for an increasing number of panels. This is crucial because we now know that this is an accurate way of modelling the flow conditions, and may now use this method for any arbitrary geometry (such as an air foil).



(a) Points of Separation with No Lift

(b) Points of Separation with Lift

Figure 2: Separation Points Calculated Using Thwaites Method

The equation used for Thwaities Method is:

$$K(x_s) = \frac{0.45}{U_e^6(x_s)}\left(\frac{dU_e(x_s)}{dx}\right)\int_0^{x_s} U_e^5(\xi)d\xi = -0.09 \tag{7}$$

Thwaites is a difficult method to use because the equation that governs it is a non-linear differential equation of the velocity. x_s is the distance along the surface of the cylinder from the leading edge to the point where it separates from the surface of the cylinder. Equation (7) is used for both the top and bottom portions of the body, since the fluid has a separation point on each side. For the non-lifting case, the separation points are at the same distance from the leading edge. This is a result of the flow being symmetrical on the top and bottom for this case. On the other hand, for the lifting case, the upper separation point is closer to the trending edge, and the lower separation point is closer to the leading edge. This is due to the Kutta Condition creating a stagnation point on the lower back side of the cylinder instead of right on the trending edge. It appears as if the separation points are still an equal distance on each side of where the Kutta Condition is located (which is simply the location of the stagnation point on the backside of the object).