Math 122 **Name:** _____
Spring 2024
Exam 3
4/11/2024
Time Limit: 75 Minutes Signature: _____

This exam has 7 questions, for a total of 70 points and 0 bonus points. Unless otherwise specified, there is no form of technology allowed. Further, final solutions must be written in the prescribed boxes, and all work must be shown. There is paper provided in the front of the class for scratch work. Any numerical values given for a final answer must be precise.

Grade Table (for teacher use only)

Question	Points	Bonus Points	Score
1	10	0	
2	10	0	
3	10	0	
4	10	0	
5	10	0	
6	10	0	
7	10	0	
Total:	70	0	

1. (10 points) If a is a positive constant, find all critical points of $f(x) = x^3 - ax$. Find the value of a so that f has local extrema at $x = \pm 2$.

$f'(x) = 3x^2 - a$ (poly)

$f'(x) = 0 \Rightarrow 0 = 3x^2 - a \Rightarrow \frac{a}{3} = x^2 \Rightarrow \pm\sqrt{\frac{a}{3}} = x$

$\therefore \pm\sqrt{\frac{a}{3}} = x$ are critical values

$\pm\sqrt{\frac{a}{3}} = \pm 2 \Rightarrow$ (there are cases)

1. $\sqrt{\frac{a}{3}} = \pm 2 \Rightarrow a = 8$ (only solution)

2. $-\sqrt{\frac{a}{3}} = \pm 2 \Rightarrow$ $a = 8$ (only solution)

2. (10 points) Find all critical points of the function $g(x) = x^5 - 5x^4 + 35$. Determine which of these are local minimums, local maximums, inflection points, or neither.

$g'(x) = 5(x-4)x^3$

$g''(x) = 20(x-3)x^2$

$g'(x) = 0 \Rightarrow x = 0, 4 \Rightarrow$ 0, 4 crit. values

$g''(4) > 0 \Rightarrow$ local min at 4 (2nd der. test)

$g'(-1) = 5(-5)(-1)^3 = -25(-1) = 25 > 0$

$g(1) = 5(1-4)1^3 = -15 < 0$

$\Rightarrow$ local max at 0 (1st der test)

3. (10 points) Find all global maximums and minimums of the function $f(t) = \frac{t}{1+t^2}$.

$f'(t) = \frac{1-t^2}{(t^2+1)^2}$

$f'(t) = 0 \Rightarrow t = \pm 1$

$f(-1) = \frac{-1}{1+1} = -\frac{1}{2}$

$f(1) = \frac{1}{1+1} = \frac{1}{2}$

$\Rightarrow$ gl. min at -1, gl. max at 1

4. (10 points) Compute $\int (t^3 - \frac{t^2}{2} - t)dt$.

$\int (t^3 - \frac{t^2}{2} - t)dt = \int t^3 dt - \int \frac{t^2}{2} dt - \int t dt$

$= \frac{1}{4}t^4 - \frac{1}{6}t^3 - \frac{1}{2}t^2 + C$

5. (10 points) Compute $\int(\sqrt{z})^3 dz$.

$$\int(\sqrt{z})^3 dz = \int z^{\frac{3}{2}} dz = \frac{1}{\frac{3}{2}+1} z^{\frac{3}{2}+1} + C$$

$$= \frac{2}{5} z^{\frac{5}{2}} + C$$

6. (10 points) Compute $\int(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3})dx$.

$$\int\left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}\right)dx = \int x^{-1} dx + \int x^{-2} dx + \int x^{-3} dx$$

$$= \ln|x| - x^{-1} - \frac{1}{2} x^{-2} + C$$

7. (10 points) Compute $\int_1^4 (\sqrt{x})^{-1} dx$.

$$\int_1^4 (\sqrt{x})^{-1}\,dx = \int_1^4 x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}}\Big|_{x=1}^{x=4}$$

$$2\sqrt{4} - 2\sqrt{1} = 4 - 2 = \boxed{2}$$