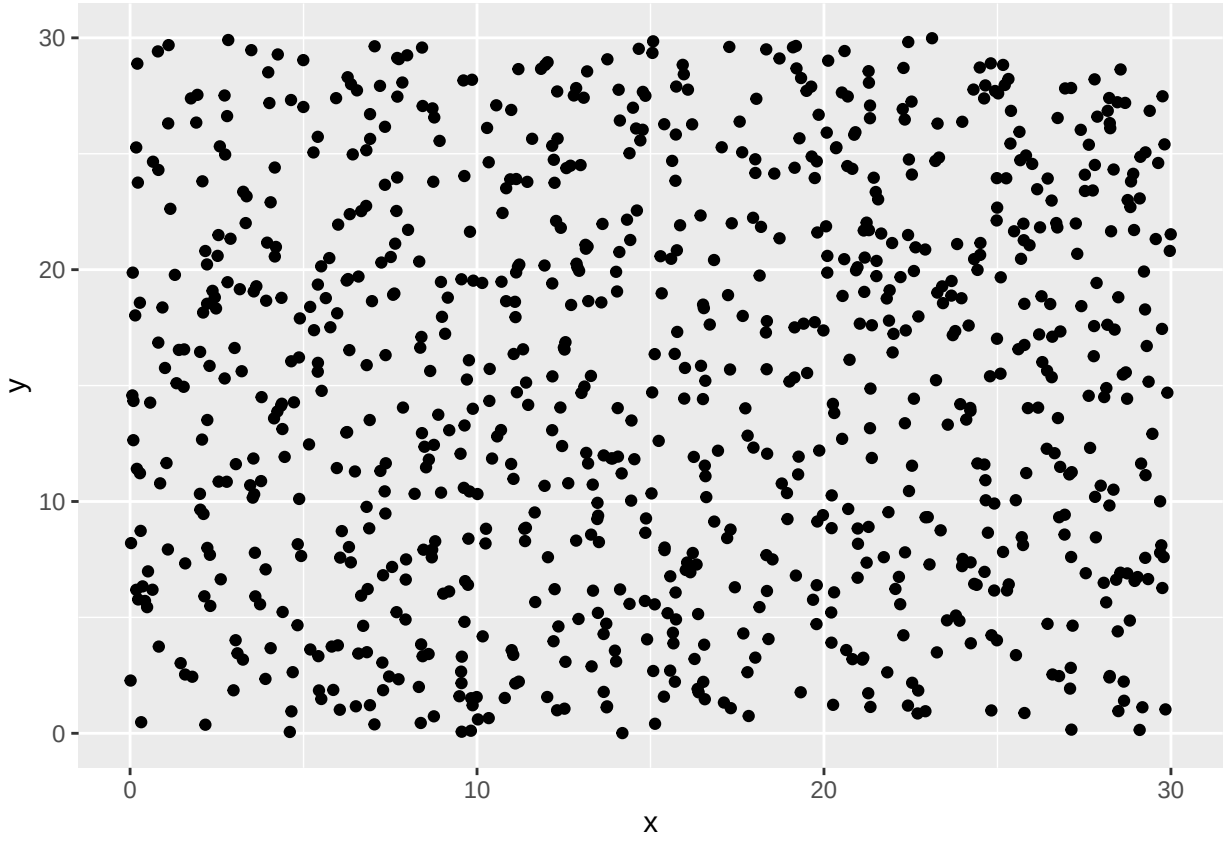
# Spatial Covariates

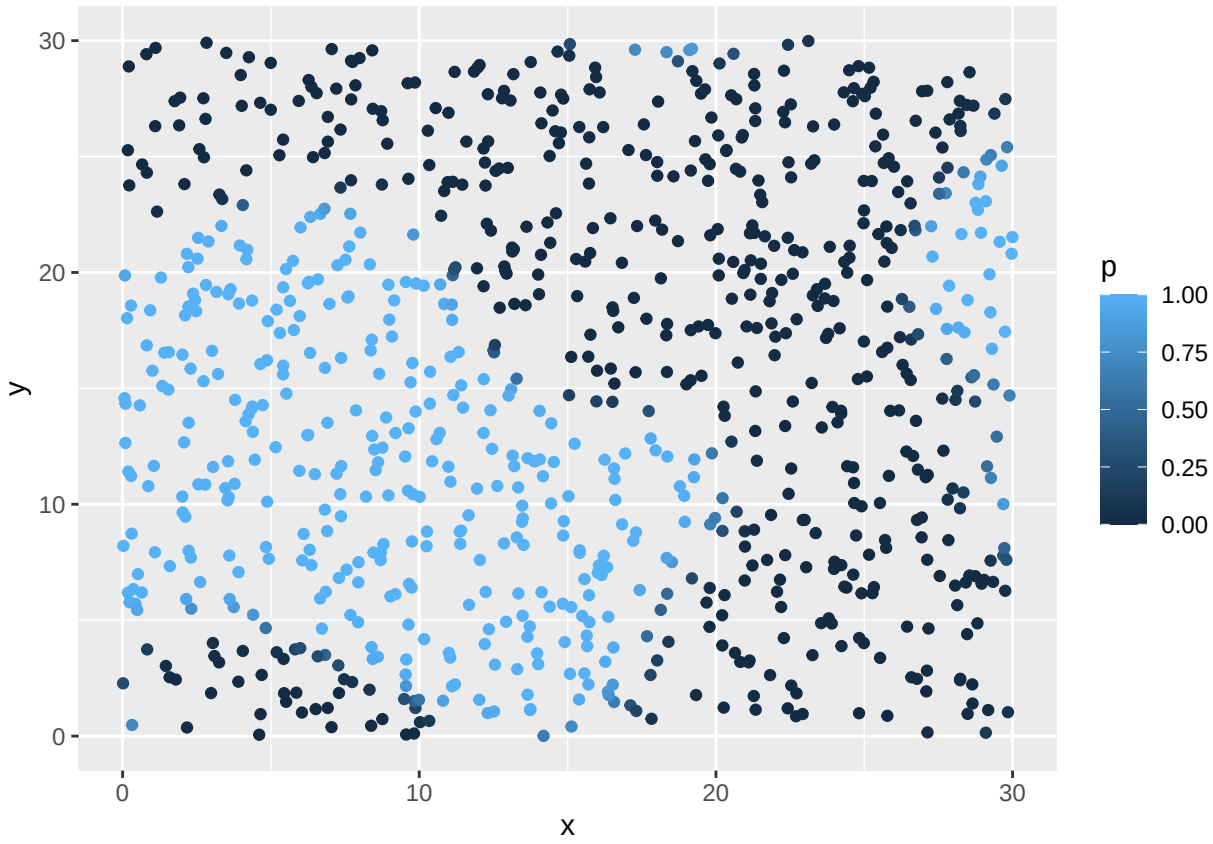
2023-04-17

## The understory fuels generation algorithm.

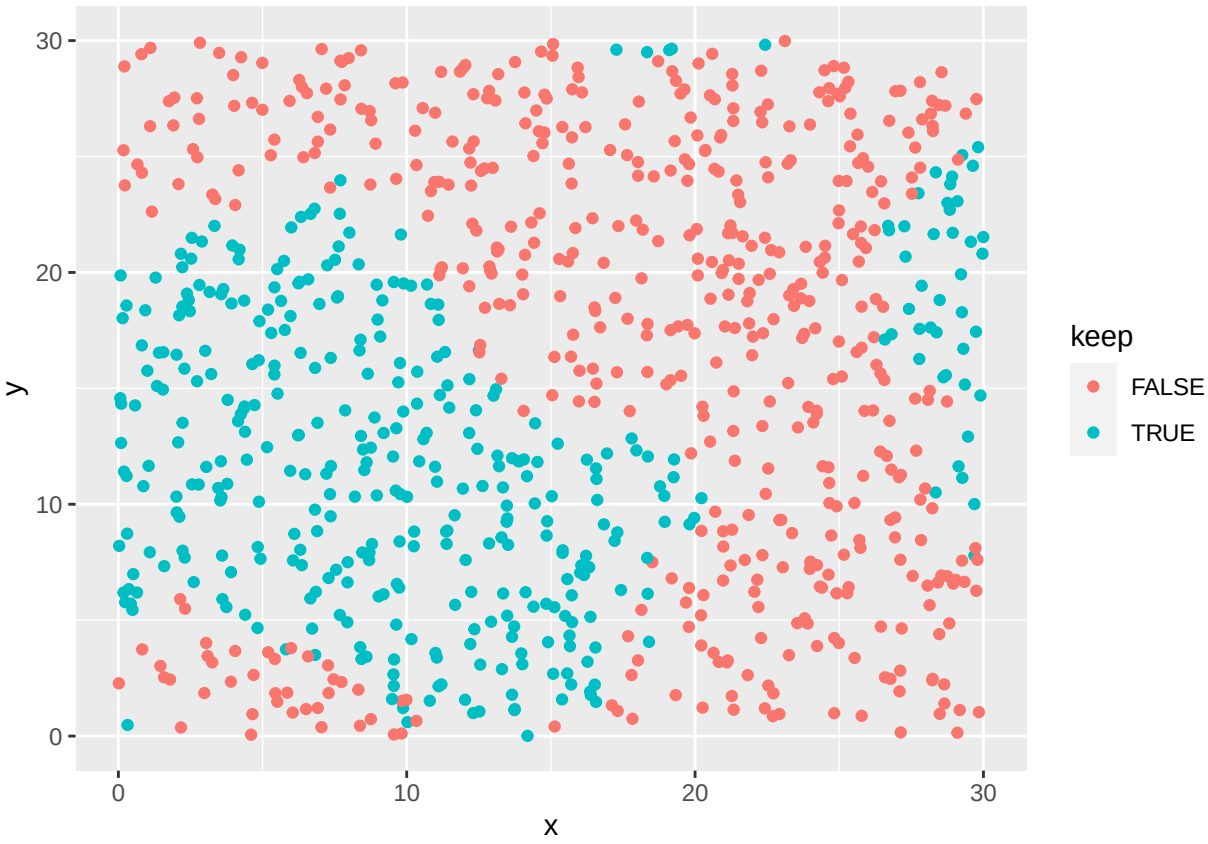
1. Draw a number of fuel elements $n \sim Pois(\lambda * X * Y)$ where $\lambda$ is the relative density parameter for a domain $[0, X] \times [0, Y]$
2. Randomly sample $x, y$ locations $[\boldsymbol{x}|\boldsymbol{y}] \sim Unif([0, X] \times [0, Y])$




3. Sample from a mean-zero intensity function (Gaussian Process) with heterogeneity parameter $\rho$ and variance $\sigma^2$ at these locations $\tilde{G}(\boldsymbol{x}, \boldsymbol{y}|\rho, \sigma)$.
4. Determine the probability of keeping a shrub at each location $\boldsymbol{p} = plogis(\tilde{G}(\boldsymbol{x}, \boldsymbol{y}|\rho, \sigma))$




5. Keep shrubs at locations where $\boldsymbol{u} < \boldsymbol{p}$ ($\boldsymbol{u} \sim Unif(0, 1)$)




## Incorporate spatial covariates

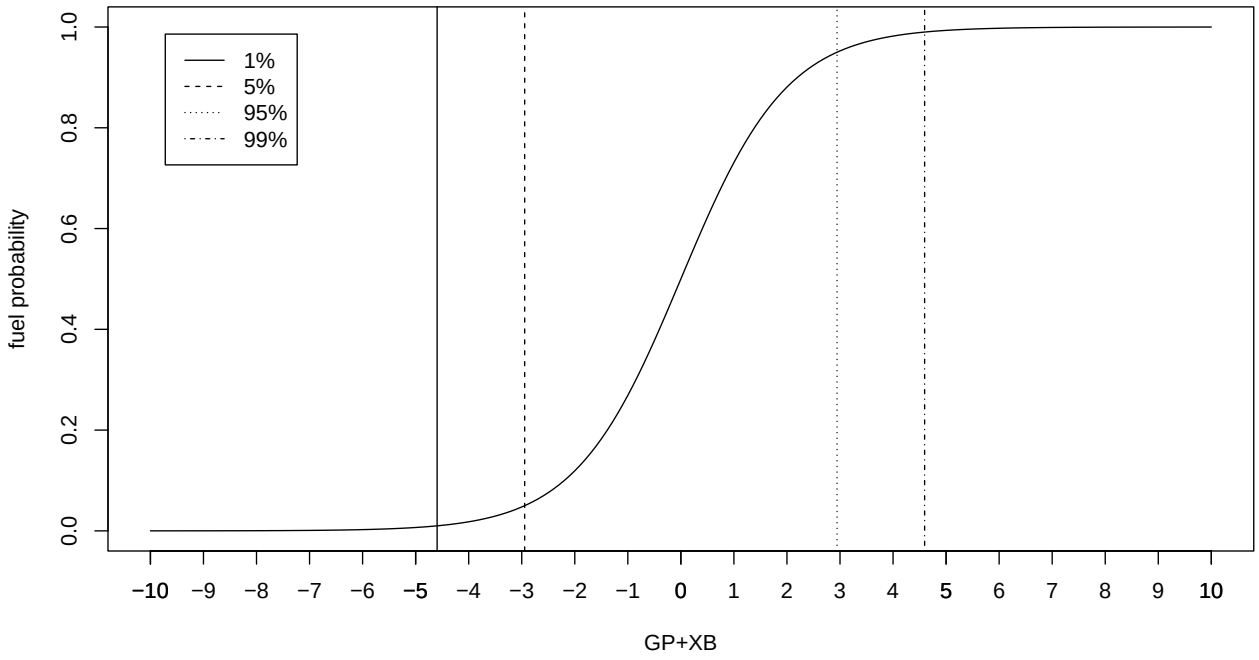
If we wish to include spatial covariates $\boldsymbol{X}$ with effect $\boldsymbol{\beta}$ we add them to the GP realization before downsampling with the logistic.

4. Determine the probability of keeping a shrub at each location $\boldsymbol{p} = plogis(\tilde{G}(\boldsymbol{x}, \boldsymbol{y}|\rho, \sigma) + \boldsymbol{X\beta})$

At this moment, the code is technically usable with any spatial covariate, however we have designed it so that $X$ is scaled between $[-1, 1]$ and $\beta \in \mathbb{R}^+$. This is so a single X can have both a positive and negative effect of shrub probability. If $X \in \mathbb{R}^+$ then X will always have a positive effect, which may not be the desired effect, especially since a mean zero Intensity function gives an average probability of 50% to each location, and we may want to significantly reduce that probability with a covariate.

### How to choose $\sigma^2$ and $\boldsymbol{\beta}$?

This plot may be useful in determining a balance between the GP standard deviation (heterogeneity scale) and the spatial covariate parameters $\boldsymbol{\beta}$. Below we show the probability of fuel placement based on the value of $GP + X\boldsymbol{\beta}$ at any given location. We can see that at $-5, 5$ the probability of fuel placement is less than 1% and greater than 99% respectively. Desired trade-offs between the two terms may be achievable with *heterogeneity scale* $\in [0, 5/3]$ and $\boldsymbol{\beta} \leq 5$. These are reasonable starting points to consider for these parameters but there may be circumstances where values of greater magnitude help to achieve a desired effect in fuel placement.


1%
5%
95%
99%
fuel probability
0.0
0.2
0.4
0.6
0.8
1.0
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10
GP+XB