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# MultiPEM Toolbox: User Manual

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## 1 Introduction

This document explains use of the **Multi-Phenomenology Explosion Monitoring** (Multi-PEM) Toolbox, a collection of R scripts for estimating the unknown device parameters of a new event with uncertainty quantification. The methodology and application used for illustration in this user manual are fully documented in a Los Alamos National Laboratory technical report<sup>1</sup> hereafter designated “WPA” for reference. Additional details on the application are found in a recent journal article<sup>2</sup>. Two assessment types are available: *rapid* and *complete*.

Rapid assessments are conducted in two stages, as described in Section 2. In the first stage, calibration data are used to estimate forward and error model parameters (WPA, §5.1) and (if relevant) errors-in-variables yield values for calibration sources (WPA, §3, Equation (3)). In the second stage, new event data are used to estimate the unknown new event device parameters (WPA, §5.2) with uncertainty quantification. Two options for treating the inferred first stage parameters in second stage Bayesian analysis are available: fixing them at their maximum likelihood estimate (default), or multiple imputation<sup>3</sup>. Multiple imputation involves utilizing several posterior samples (imputations) of the first stage parameters as fixed values in the second stage posterior sampling of the new event device parameters. Second stage sampling is conducted across imputations in parallel to improve computational efficiency. This method produces improved uncertainty quantification of the new event device parameters compared with the default treatment of the first stage parameters, at the expense of additional computation.

Complete assessments are conducted in a single stage, as described in Section 3. Calibration and (if relevant) new event data are used simultaneously to estimate all forward model, error model, and (if relevant) new event device parameters with uncertainty quantification on the latter.

As the name suggests, rapid assessments generally run substantially faster than complete

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<sup>1</sup>Williams, B.J., Picard, R.R., & Anderson, D.N. (2023). Multi-phenomenology Yield Characterization. Los Alamos National Laboratory Technical Report LA-UR-23-21950 (rev.1).

<sup>2</sup>Ford, S.R., Bulaevskaya, V., Ramirez, A., Johannesson, G., & Rodgers, A.J. (2021). Joint Bayesian inference for near-surface explosion yield and height-of-burst, *J Geophys Res Solid Earth* 126:e2020JB020968.

<sup>3</sup>Plummer, M. (2015). Cuts in Bayesian graphical models, *Stat Comput* 25:37-43.

assessments (even with multiple imputation), because the results of first stage analysis can be stored and incorporated into estimating a relatively low-dimensional space of new event device parameters whenever relevant new event data becomes available. On the other hand, complete assessments must be run on the full set of model and device parameters with calibration and new event data every time the latter becomes available.

## 1.1 Running An Application in MultiPEM Toolbox

MultiPEM Toolbox applications can be run directly in R, or through Docker. The latter is useful if a common run environment is desired for multiple users. Details are provided here for the former, followed by brief remarks on the latter in the next subsection.

Begin with the following initial steps:

- Install the latest version of R
- Install the following auxillary R packages
  - `Matrix`
  - `numDeriv`
  - `doFuture`
  - `adaptMCMC`
  - `FME`
  - `abind` (optional for Sequential Monte Carlo (SMC) sampling)
  - `ramcmc` (optional for SMC sampling)

For example, to install the `Matrix` package, run the command

```
> install.packages("Matrix")
```

inside an R session. The `FME` package requires R version 4.0 or higher (through its dependency on the `MASS` package) and its function `modMCMC` is used as the default posterior sampler in the run files associated with the rapid (`runMPEM_0.r`) and complete (`runMPEM.r`) assessments. If an older version of R is used, the `iMCMC` command in these run files should be changed from "FME" to "RAM".

- Unpack the MultiPEM Toolbox package into the directory
 

```
% ~/MultiPEM_Toolbox_Package
```

There are two MultiPEM analyses contained in the illustrative application (WPA, §6), named **3-Phen** and **4-Phen** below. The former estimates new event device parameters based on fusion of data from the *seismic*, *acoustic*, and *optical* phenomenologies, while the latter performs this same task but with addition of the *surface effects* (*crater*) phenomenology.

### 1.1.1 Rapid Assessments

The following steps may be invoked in sequential order to run the MultiPEM Toolbox on the illustrative application for rapid assessments. A similar workflow will pertain to any application. This application uses a single core for likelihood/posterior maximization and posterior sampling. Computation time can be significantly reduced by utilizing the multi-core options for these tasks as described in Section 2.

1. Create a symbolic link to the global code directory

```
% cd ./MultiPEM_Toolbox_Package/Runfiles
% # link to global code (used by all applications)
% ln -s ../Code/ Code
```

2. Create symbolic links to the application (“IYDT”) data and code directories

```
% cd IYDT
% # link to application (IYDT) data files
% ln -s ../../Applications/Data/IYDT/ Data
% # link to application code (used by IYDT application)
% ln -s ../../Applications/Code/IYDT/ Code
```

3. Run the first stage analysis for each single phenomenology

- Seismic (phenomenology 1)

```
% cd Seismic
% # link to phenomenology code (used by seismic
% # phenomenology to specify prior distribution
% # of the forward model coefficients (and its gradient))
% ln -s ../../../../Applications/Code/IYDT/Phenomenology/ Code
% cd I-SUGAR-hob-0
% R CMD BATCH runMPEM.r runMPEM.out &
% # check status
% tail runMPEM.out
% # upon completion of run (~2 hours), copy
% # .RData file for use in all relevant future
% # seismic second stage rapid assessments
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
% cp .RData .RData-s
```

- Acoustic (2)

```
% cd ../../Acoustic/I-SUGAR-hob-0
% R CMD BATCH runMPEM.r runMPEM.out &
% # check status
% tail runMPEM.out
% # upon completion of run (~35 minutes), copy
% # .RData file for use in all relevant future
```

```

% # acoustic second stage rapid assessments
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
% cp .RData .RData-a

```

- Optical (3)

```

% cd ../../Optical/I-EIV-SUGAR-hob-0
% R CMD BATCH runMPEM.r runMPEM.out &
% # check status
% tail runMPEM.out
% # upon completion of run (~1 hour), copy
% # .RData file for use in all relevant future
% # optical second stage rapid assessments
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
% cp .RData .RData-o

```
- Surface Effects/Crater (4)

```

% cd ../../Crater
% # link to phenomenology code (used by crater
% # phenomenology to specify prior distribution
% # of log-yield (and its gradient))
% ln -s ../../Applications/Code/IYDT/Phenomenology/ Code
% cd I-EIV-SUGAR-0
% R CMD BATCH runMPEM.r runMPEM.out &
% # check status
% tail runMPEM.out
% # upon completion of run (~5 minutes), copy
% # .RData file for use in all relevant future
% # crater second stage rapid assessments
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
% cp .RData .RData-c

```

The `runMPEM.r` and `runMPEM.out` files are described in Section 2.1. The status of running code is checked by issuing the following command in the run directory,

```
% tail runMPEM.out
```

Successfully completed runs show the results of the `proc.time()` command at the end of the `runMPEM.out` file. Symbolic links are created to a phenomenology-specific code directory for **Seismic** and **Crater** prior to conducting the runs (see comments in above code). The `.RData` files resulting from each completed analysis should be copied and stored for use in relevant future second stage analyses (see comments in above code). Upon completion of these runs, copy each resulting `opt.RData` file to the **Opt** directory in each MultiPEM analysis, to be used as starting values for MultiPEM log-likelihood

maximization.

```
% cd ../../3-Phen/Opt
% cp ../../Seismic/I-SUGAR-hob-0/opt.RData opt_1_0.RData
% cp ../../Acoustic/I-SUGAR-hob-0/opt.RData opt_2_0.RData
% cp ../../Optical/I-EIV-SUGAR-hob-0/opt.RData opt_3_eiv_0.RData
% cd ../../4-Phen/Opt
% cp ../../Seismic/I-SUGAR-hob-0/opt.RData opt_1_0.RData
% cp ../../Acoustic/I-SUGAR-hob-0/opt.RData opt_2_0.RData
% cp ../../Optical/I-EIV-SUGAR-hob-0/opt.RData opt_3_eiv_0.RData
% cp ../../Crater/I-EIV-SUGAR-0/opt.RData opt_4_eiv_0.RData
```

4. Run the second stage analysis for each single phenomenology, copying the `.RData` file from the first stage analysis into the run directories if necessary. For example, the two directories

```
./MultiPEM_Toolbox_Package/Runfiles/IYDT/Seismic/I-SUGAR-hob-0
./MultiPEM_Toolbox_Package/Runfiles/IYDT/Seismic/I-SUGAR-hob-pi-0
```

contain two MultiPEM analyses for the new event device parameters. The first assumes a “flat” prior on these parameters, while the second assumes an informative prior (WPA, §6.6, pp. 18-19). However, both analyses use the same first stage results. To perform the second stage run for the informative prior, the first stage `.RData` file is copied into the run directory,

```
% cd ./MultiPEM_Toolbox_Package/Runfiles/IYDT/Seismic/I-SUGAR-hob-pi-0
% cp ../I-SUGAR-hob-0/.RData-s .RData
```

and the analysis is run in the `I-EIV-SUGAR-hob-pi-0` directory as shown below.

- Seismic (phenomenology 1)

```
% # noninformative prior distribution on new event
% # device parameters
% cd ../../Seismic/I-SUGAR-hob-0
% # if necessary, change iMCMC to "RAM" in runMPEM_0.r
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM_0.out file
% # informative prior distribution on new event
% # device parameters
% cd ../I-SUGAR-hob-pi-0
% cp ../I-SUGAR-hob-0/.RData-s .RData
% # if necessary, change iMCMC to "RAM" in runMPEM_0.r
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM_0.out file
```

- Acoustic (2)

```
% # noninformative prior distribution on new event
% # device parameters
% cd ../../Acoustic/I-SUGAR-hob-0
% # if necessary, change iMCMC to "RAM" in runMPEM_0.r
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM_0.out file
% # informative prior distribution on new event
% # device parameters
% cd ../I-SUGAR-hob-pi-0
% cp ../I-SUGAR-hob-0/.RData-a .RData
% # if necessary, change iMCMC to "RAM" in runMPEM_0.r
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM_0.out file
```

- Optical (3)

```
% # noninformative prior distribution on new event
% # device parameters
% cd ../../Optical/I-EIV-SUGAR-hob-0
% # if necessary, change iMCMC to "RAM" in runMPEM_0.r
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM_0.out file
% # informative prior distribution on new event
% # device parameters
% cd ../I-EIV-SUGAR-hob-pi-0
% cp ../I-EIV-SUGAR-hob-0/.RData-o .RData
% # if necessary, change iMCMC to "RAM" in runMPEM_0.r
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM_0.out file
```

- Surface Effects/Crater (4)

```
% # noninformative prior distribution on new event
% # device parameters
% cd ../../Crater/I-EIV-SUGAR-0
% # if necessary, change iMCMC to "RAM" in runMPEM_0.r
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM_0.out file
% # informative prior distribution on new event
% # device parameters
% cd ../I-EIV-SUGAR-pi-0
% cp ../I-EIV-SUGAR-0/.RData-c .RData
```



```
% # if necessary, change iMCMC to "RAM" in runMPEM_0.r
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM_0.out file
```

The runMPEM\_0.r and runMPEM\_0.out files are described in Section 2.2.

5. Run the first stage MultiPEM analysis (illustrated here for 4-Phen)

```
% cd ../../4-Phen
% # link to phenomenology code (used by seismic
% # phenomenology to specify prior distribution
% # of the forward model coefficients (and its gradient))
% ln -s ../../Applications/Code/IYDT/Phenomenology/ Code
% cd I-EIV-SUGAR-hob-0
% R CMD BATCH runMPEM.r runMPEM.out &
% # upon completion of run (~7 hours), copy
% # .RData file for use in all relevant future
% # multiPEM second stage rapid assessments
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
% cp .RData .RData-4
```

The runMPEM.r and runMPEM.out files are described in Section 2.1. A symbolic link is created to a phenomenology-specific code directory prior to conducting the run (see comments in above code). The .RData file resulting from the completed analysis should be copied and stored for use in future second stage analyses (see comments in above code).

6. Run the second stage MultiPEM analyses, copying the .RData file from the first stage analysis into the run directories if necessary.

```
% # noninformative prior distribution on new event
% # device parameters
% # if necessary, change iMCMC to "RAM" in runMPEM_0.r
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM_0.out file
% # informative prior distribution on new event
% # device parameters
% cd ../I-EIV-SUGAR-hob-pi-0
% cp ../I-EIV-SUGAR-hob-0/.RData-4 .RData
% # if necessary, change iMCMC to "RAM" in runMPEM_0.r
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM_0.out file
```

The runMPEM\_0.r and runMPEM\_0.out files are described in Section 2.2.

### 1.1.2 Complete Assessments

The following steps may be invoked in sequential order to run the MultiPEM Toolbox on the illustrative application for complete assessments. A similar workflow will pertain to any application. This application uses a single core for likelihood/posterior maximization and posterior sampling. Computation time can be significantly reduced by utilizing the multi-core options for these tasks as described in Section 3.

1. Create a symbolic link to the global code directory

```
% cd ./MultiPEM_Toolbox_Package/Runfiles
% # link to global code (used by all applications)
% # NOT REQUIRED IF LINK CREATED PREVIOUSLY
% ln -s ../Code/ Code
```

2. Create symbolic links to the application ("IYDT") data and code directories

```
% cd IYDT
% # link to application (IYDT) data files
% # NOT REQUIRED IF LINK CREATED PREVIOUSLY
% ln -s ../../Applications/Data/IYDT/ Data
% # link to application code (used by IYDT application)
% # NOT REQUIRED IF LINK CREATED PREVIOUSLY
% ln -s ../../Applications/Code/IYDT/ Code
```

3. Run the calibration analysis for each single phenomenology

- Seismic (phenomenology 1)

```
% cd Seismic
% # link to phenomenology code (used by seismic
% # phenomenology to specify prior distribution
% # of the forward model coefficients (and its gradient))
% # NOT REQUIRED IF LINK CREATED PREVIOUSLY
% ln -s ../../../../Applications/Code/IYDT/Phenomenology/ Code
% cd I
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # check status
% tail runMPEM.out
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
```

- Acoustic (2)

```
% cd ../../Acoustic/I
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # check status
% tail runMPEM.out
```

```

% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file

• Optical (3)

% cd ../../Optical/I-EIV
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # check status
% tail runMPEM.out
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file

• Surface Effects/Crater (4)

% cd ../../Crater
% # link to phenomenology code (used by crater
% # phenomenology to specify prior distribution
% # of log-yield (and its gradient))
% # NOT REQUIRED IF LINK CREATED PREVIOUSLY
% ln -s ../../Applications/Code/IYDT/Phenomenology/ Code
% cd I-EIV
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # check status
% tail runMPEM.out
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file

```

The runMPEM.r and runMPEM.out files are described in Section 3. The status of running code is checked by issuing the following command in the run directory,

```
% tail runMPEM.out
```

Successfully completed runs show the results of the `proc.time()` command at the end of the runMPEM.out file. Symbolic links are created to a phenomenology-specific code directory for **Seismic** and **Crater** prior to conducting the runs (see comments in above code; not required to recreate links if they were created previously, e.g. for rapid assessments). Upon completion of the maximum likelihood estimation component of these runs (typically much earlier than the entire run), copy each resulting `opt.RData` file to the `Opt` directory in each MultiPEM analysis, to be used as starting values for MultiPEM log-likelihood maximization.

```

% cd ../../3-Phen/Opt
% cp ../../Seismic/I/opt.RData opt_1.RData
% cp ../../Acoustic/I/opt.RData opt_2.RData
% cp ../../Optical/I-EIV/opt.RData opt_3_eiv.RData
% cd ../../4-Phen/Opt
% cp ../../Seismic/I/opt.RData opt_1.RData

```

```
% cp ../../Acoustic/I/opt.RData opt_2.RData
% cp ../../Optical/I-EIV/opt.RData opt_3_eiv.RData
% cp ../../Crater/I-EIV/opt.RData opt_4_eiv.RData
```

#### 4. Run the complete analysis for each single phenomenology

- Seismic (phenomenology 1)

```
% # noninformative prior distribution on new event
% # device parameters
% cd ../../Seismic/I-SUGAR-hob
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
% # informative prior distribution on new event
% # device parameters
% cd ../I-SUGAR-hob-pi
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
```

- Acoustic (2)

```
% # noninformative prior distribution on new event
% # device parameters
% cd ../../Acoustic/I-SUGAR-hob
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
% # informative prior distribution on new event
% # device parameters
% cd ../I-SUGAR-hob-pi
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
```

- Optical (3)

```
% # noninformative prior distribution on new event
% # device parameters
% cd ../../Optical/I-EIV-SUGAR-hob
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # a completed run will show the results of proc.time()
```

```
% # at the end of the runMPEM.out file
% # informative prior distribution on new event
% # device parameters
% cd ../I-EIV-SUGAR-hob-pi
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
```

- Surface Effects/Crater (4)

```
% # noninformative prior distribution on new event
% # device parameters
% cd ../../Crater/I-EIV-SUGAR
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
% # informative prior distribution on new event
% # device parameters
% cd ../I-EIV-SUGAR-pi
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
```

5. Run the complete MultiPEM analysis (illustrated here for 4-Phen)

```
% cd ../../4-Phen
% # link to phenomenology code (used by seismic
% # phenomenology to specify prior distribution
% # of the forward model coefficients (and its gradient))
% # NOT REQUIRED IF LINK CREATED PREVIOUSLY
% ln -s ../../Applications/Code/IYDT/Phenomenology/ Code
% # noninformative prior distribution on new event
% # device parameters
% cd I-EIV-SUGAR-hob
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # a completed run will show the results of proc.time()
% # at the end of the runMPEM.out file
% # informative prior distribution on new event
% # device parameters
% cd ../I-EIV-SUGAR-hob-pi
% # if necessary, change iMCMC to "RAM" in runMPEM.r
% R CMD BATCH runMPEM.r runMPEM.out &
% # a completed run will show the results of proc.time()
```

% # at the end of the runMPEM.out file

The `runMPEM.r` and `runMPEM.out` files are described in Section 3. A symbolic link is created to a phenomenology-specific code directory prior to conducting the run (see comments in above code; not required to recreate links if they were created previously, e.g. for rapid assessments).

## 1.2 Running MultiPEM Toolbox Through Docker

Applications can be run in the MultiPEM Toolbox through Docker, assuming Docker has been installed on the user's system. The basic steps are stated in the following `README` file,

```
% less ./Runfiles-Docker/README
```

First, a global Docker image is built. This installs the desired version of `R` with the required supporting packages, and incorporates the global subroutines. Second, an application-specific Docker image is built on top of the global image. This incorporates all application relevant subroutines and data. Third, single phenomenology or MultiPEM analysis-specific Docker images are built on top of the application image. These incorporate all run files and (if relevant) `R` data objects containing a starting value for optimization. Finally – for each single phenomenology or MultiPEM analysis – a Docker image is built on top of the analysis image and a Docker container is started to conduct the run, for each use case. Details are provided in the following `README` files for both rapid and complete assessments,

```
% less ./Runfiles-Docker/IYDT/Seismic/README
% less ./Runfiles-Docker/IYDT/Acoustic/README
% less ./Runfiles-Docker/IYDT/Optical/README
% less ./Runfiles-Docker/IYDT/Crater/README
% less ./Runfiles-Docker/IYDT/3-Phen/README
% less ./Runfiles-Docker/IYDT/4-Phen/README
```

As with the analyses of Section 1.1, all single phenomenology runs are conducted first, and (if needed) all optimization results are copied to the MultiPEM `Opt` directories prior to conducting the subsequent MultiPEM runs.

## 2 Rapid Assessment

Rapid assessments will be illustrated by examining the run files associated with a multi-phenomenology analysis in which signals from four phenomenologies are combined to infer the log-yield and height-of-burst (HOB)/depth-of-burial (DOB) of a near-surface nuclear explosion (WPA, §6).

```
% cd ./Runfiles/IYDT/4-Phen/I-EIV-SUGAR-hob-0
```

Rapid assessments consist of two stages. In the first stage, calibration data are employed to estimate forward model parameters (e.g. regression coefficients) and error model parameters (e.g. source bias, path bias, observational error covariance), and (if relevant) errors-in-variables yield values of calibration sources. This stage may be run for one or multiple scenarios of interest upon identification of relevant historical data for each scenario, and the resulting `.RData` file(s) stored for later use in processing new event data.

In the second stage, new event data are processed to infer unknown device parameters (e.g. yield, HOB/DOB, geolocation, event time) with uncertainty quantification. In Bayesian analysis, forward and error model parameters, and (if relevant) calibration source errors-in-variables yields, may be treated in two ways:

- Fixed at values obtained from the first stage, or
- Imputed using posterior samples from the first stage<sup>4</sup>.

Either approach results in rapid assessments being executed with far less compute time than complete assessments. The first approach has the potential consequence of underestimating uncertainty in the unknown device parameters of interest for the new event, which is avoided by selecting the second approach with the expense of additional compute time.

### 2.1 First Stage

The first stage analysis is defined in the `runMPEM.r` file, provided in the first three sections of Appendix A with line numbers referred to in the ensuing discussion. Appendix A.1 provides the preprocessing component of the first stage, Appendix A.2 provides the code employed to maximize the likelihood function of the calibration data with respect to the parameters of the forward and error models, while Appendix A.3 provides the code employed to optionally sample the posterior distribution of these parameters.

The first stage analysis is run in batch mode as follows,

```
% R CMD BATCH runMPEM.r runMPEM.out &
```

The main features of the output file `runMPEM.out` are provided in Appendix A.4.

---

<sup>4</sup>Plummer, M. (2015). Cuts in Bayesian graphical models, *Stat Comput* 25:37-43.

### 2.1.1 Preprocessing

The preprocessing component of the first stage analysis in Appendix A.1 is primarily responsible for describing features of the calibration data, and the parameters of the forward and error models.

- Line 12+: Load all R packages utilized by multiple supporting subroutines, most notably log-likelihood and log-prior calculations and their associated gradients.
- Line 23: Specify directory location (relative to run directory) of all global (application independent) subroutines.
- Line 26: Read in code performing first stage preprocessing of calibration data.
- Line 29: Specify directory location (relative to run directory) of all application-specific subroutines.
- Line 32: Specify root directory (relative to run directory) containing all application-specific calibration data files.
- Lines 35-38: A scalar or vector specifying the names of calibration data files for each phenomenology, utilizing an ordering of the phenomenologies (for MultiPEM analysis) that is maintained throughout the input deck (as indicated here in Lines 41-44). Data files are text files (CSV formatted) containing all measured signatures (in the first column(s)) and input covariates (in succeeding column(s)) including all those required in forward and error model calculations. Directories specifying the exact locations of these files relative to the root data directory (Line 32) may also be included in the filenames.
- Line 47: A scalar or vector specifying the number of observed signatures for each phenomenology; in this example, 2 for each phenomenology.
- Lines 51-58: Specify the number of *common* forward model parameters for each phenomenology (WPA, §5.1, first paragraph). For a given forward model, common parameters maintain the same constant value for every log-likelihood calculation. The `pbeta` object is initialized as a null list with elements for each phenomenology in the proper order (Line 51), initialized to zero vectors of length equal to the number of observed signatures (Line 52). Subsequent lines specify the number of common forward model parameters for each signature within each phenomenology. For example, the *acoustic* forward model for each signature contains 2 common forward model parameters (Line 54).
- Lines 61-64: Specify if the forward model(s) for any phenomenology depend on event emplacement conditions (Line 61), followed by (if relevant) a vector indicating the number of distinct emplacement conditions considered for each phenomenology in the proper order (Line 63). This specification allows distinct forward model parameters to be associated with different emplacement conditions (as specified subsequently). If `Th` is `TRUE` (Line 61), a factor named `Type` must be present in the calibration (and new event) data file for each relevant phenomenology, indicating the emplacement condition pertaining to each entry. In this example, the *seismic* and *acoustic* forward



model parameters may vary for 3 distinct emplacements (“soft”, “hard”, and “wet” rock types), while the *optical* and *surface effects* forward models are independent of emplacement condition.

- Lines 68-83: Specify the number of *emplacement* dependent forward model parameters for each phenomenology if relevant (WPA, §5.1, first paragraph). For a given forward model, emplacement parameters remain constant for log-likelihood calculations with a given emplacement condition, but may be modified for each distinct emplacement. The `pbetat` object is initialized as a null list with elements for each phenomenology in the proper order (Line 69), initialized as null lists with elements for each emplacement condition (Line 71) if multiple emplacements are present. Subsequent lines specify the number of forward model parameters for each signature within each emplacement condition for each phenomenology. For example, the *seismic* forward model for each signature within each emplacement contains 5 forward model parameters (Line 76) allowed to vary across emplacements. `pbetat` must be specified if multiple emplacements are present for any phenomenology (at least one element of `Th` is greater than 1).
- Lines 87-102: Specify the location of *common* forward model parameters within the full parameter vector, for phenomenologies possessing both common and emplacement dependent parameters. The `ibetar` object is initialized as a null list with elements for each phenomenology in the proper order (Line 88), initialized as null lists with elements for each signature within each emplacement condition (Line 93) if multiple emplacements are present. Subsequent lines specify the position of common parameters in the full forward model parameter vector, for phenomenologies possessing both common and emplacement dependent forward model parameters. For example, the *acoustic* forward model parameter vector takes common parameter values in its first two positions for each signature within each emplacement condition (Line 99).
- Line 105: Indicate if errors-in-variables yield values for calibration events will be modeled (WPA, §3, Equation (3); §A.4). If `TRUE`, this allows uncertain yields for calibration events (often assumed known with certainty) to vary within user-specified guidelines.
- Lines 108-124: If relevant, specify details of errors-in-variables yield models for calibration events.
  - Line 111: Specify phenomenologies for application of errors-in-variables yield models to calibration events
  - Lines 115-116: Provide the sources subject to errors-in-variables yield models for each phenomenology. The `seiv` object is initialized as a null list with elements for each phenomenology in the proper order (Line 115), with vectors indicating the relevant sources for each relevant phenomenology (Line 116). The "ALL" designation indicates that every source in the calibration data set for the indicated phenomenology will be modeled with an errors-in-variables yield. `seiv` must be specified if `ieiv` is provided.
  - Line 119: The standard deviation of the errors-in-variables Gaussian distribution for each calibration event log-yield. For each event, the mean of this distribution

is taken to be its provided (design or measured) log-yield. In this example, a “total” error (3 standard deviations) of 10% in each provided yield is allowed. Note that this error is relative because yields are treated on a logarithmic scale. `eiv_w_sd` must be specified if `ieiv` is provided.

- Lines 128-137: Specify if level 1 (*source*) random effects (WPA, §3, Equation (2); §4; §A.5) should be included in the error model (Line 128). If so, the `pvc_1` object is initialized as a null list with elements for each phenomenology in the proper order (Line 131), initialized to zero vectors of length equal to the number of observed signatures (Line 132). Subsequent lines specify the number of level 1 random effects for each signature within each phenomenology. For example, the *seismic* error model for each signature contains a single source bias term (Line 134). If `pvc_1` is `TRUE` (Line 128), a factor named `Source` may be provided in the calibration (and new event) data file for each relevant phenomenology, identifying the source pertaining to each entry. This factor must be present if there is more than one data entry for any source. In order to include source random effects in the error model for an observed signature, the calibration data must contain more than one source, with at least one source containing more than one observation. A warning message will be printed to the log file if one of these conditions is violated.
- Lines 140-149: Specify if level 2 (*path*) random effects (WPA, §3, Equation (2); §4; §A.5) should be included in the error model (Line 140). If so, the `pvc_2` object is initialized as a null list with elements for each phenomenology in the proper order (Line 143), initialized to zero vectors of length equal to the number of observed signatures (Line 144). Subsequent lines specify the number of level 2 random effects for each signature within each phenomenology. For example, the *seismic* error model for each signature contains a single path bias term (Line 146). If `pvc_2` is `TRUE` (Line 140), a factor named `Path` must be provided in the calibration (and new event) data file for each relevant phenomenology, identifying the path (e.g. sensor network) pertaining to each entry. In order to include path random effects in the error model for an observed signature, a source random effect must also be present, the calibration data must contain more than one path for at least one source, with more than one observation for at least one of those paths. A warning message will be printed to the the log file if one of these conditions is violated.
- Line 153: Indicate if the user is providing code to compute coefficient matrices for level 1 (*source*) or level 2 (*path*) random effects (WPA, §5.1). If `FALSE`, the functions `calc_zmat.r` and `calc_zmat_0.r` located in the global code directory,

`MultiPEM_Toolbox_Package/Code`

compute default coefficient matrices for the calibration and new event data, respectively. If `TRUE`, then two user-provided functions of the same names must be placed in the application code directory; in this example,

`MultiPEM_Toolbox_Package/Applications/Code/IYDT`

The following table shows data for the first *seismic* calibration source in this example,

Y1	Y2	Source	Path	Type	lRange	W	HOB
−15.667	−9.625	HRI-1	Path_1	1	6.932	6.291	5
−15.665	−9.554	HRI-1	Path_1	1	6.932	6.291	5
−16.412	−10.591	HRI-1	Path_2	1	7.570	6.291	5
−16.468	−10.554	HRI-1	Path_2	1	7.570	6.291	5
−16.752	−10.931	HRI-1	Path_2	1	7.800	6.291	5
−17.483	−11.739	HRI-1	Path_2	1	8.371	6.291	5
−17.507	−11.711	HRI-1	Path_2	1	8.371	6.291	5

If level 1 and level 2 random effects are included in the error model, the source and path bias vectors (WPA, §5.1, p. 7) associated with this source are given by

$$\mathbf{E}_{S,11r} = \begin{pmatrix} \mathbf{Z}_{11r,1} \\ \mathbf{Z}_{11r,2} \end{pmatrix} b_{1r}^{(S)} \text{ and } \mathbf{E}_{P,11r} = \begin{pmatrix} \mathbf{E}_{P,111r} \\ \mathbf{E}_{P,112r} \end{pmatrix} = \begin{bmatrix} \mathbf{Z}_{111r} & \mathbf{0}_2 \\ \mathbf{0}_5 & \mathbf{Z}_{112r} \end{bmatrix} \begin{pmatrix} b_{1r,1}^{(P)} \\ b_{1r,2}^{(P)} \end{pmatrix},$$

where the default coefficient matrices are given by

$$\begin{aligned} \mathbf{Z}_{11r,1} &= \mathbf{1}_2 & \mathbf{Z}_{111r} &= \mathbf{1}_2 \\ \mathbf{Z}_{11r,2} &= \mathbf{1}_5 & \mathbf{Z}_{112r} &= \mathbf{1}_5 \end{aligned}$$

for  $\mathbf{1}_q$  and  $\mathbf{0}_q$  the  $q$ -vectors of ones and zeros, respectively. For each signature, this structure indicates there is a single source bias effect applied to every observation, while observations from each path are adjusted by distinct (and independently distributed) path bias effects (for this source, signatures are collected from two pathways).

- Lines 156-159: Calls the preprocessing function `prepro_cal` for the calibration data. Table 1 describes all inputs to this function with default values. Only inputs with no default values must be provided.

### 2.1.2 Maximum Likelihood Estimation

The maximum likelihood estimation component of the first stage analysis in Appendix A.2 is responsible for utilizing calibration data to estimate the parameters of the forward and error models, and possibly the yield of each calibration source for phenomenologies adopting the errors-in-variables yield model (WPA, §A.4). The resulting estimates are supplied to all relevant second stage analyses.

- Line 6: Read in code performing first stage maximum likelihood estimation of forward and error model parameters, and (if relevant) calibration source errors-in-variables yields, based on calibration data.
- Line 9: User specified seed to ensure repeatability of maximum likelihood estimation.
- Lines 13-17: Provide names of forward models for each signature by phenomenology (WPA, §6.1-6.4). The `fm` object is initialized as a null list with elements for each phenomenology in the proper order (Line 13). Subsequent lines specify the function

Table 1: Inputs to `prepro_cal` function.

Input	Default	Brief Description
<code>gdir</code>	none	directory location of global subroutines
<code>adir</code>	none	directory location of application subroutines
<code>rdir</code>	none	root directory location of data files
<code>cdir</code>	none	directory locations (if relevant) and names of calibration data files under <code>rdir</code>
<code>Rh</code>	none	vector with number of signatures for each phenomenology
<code>pbeta</code>	none	list containing empirical model common parameter counts by phenomenology
<code>izmat</code>	FALSE	user-provided code for computing variance component coefficient matrices
<code>ieiv</code>	NULL	numerical identifier of phenomenologies utilizing errors-in-variables yields in analysis of calibration data
<code>seiv</code>	NULL	list containing identifiers of calibration sources assigned errors-in-variables yields by phenomenology ( <b>ALL</b> – every source)
<code>ewsd</code>	NULL	standard deviation of errors-in-variables Gaussian likelihood
<code>Th</code>	NULL	number of emplacement conditions for each phenomenology
<code>pbetat</code>	NULL	list containing empirical model emplacement-dependent parameter counts by phenomenology
<code>ibetar</code>	NULL	list containing locations of empirical model common parameters in full parameter vector by phenomenology
<code>pvc_1</code>	NULL	list containing level 1 (source) variance component parameter counts by phenomenology
<code>pvc_2</code>	NULL	list containing level 2 (path) variance component parameter counts by phenomenology

names as vectors of strings having length equal to the number of signatures for each phenomenology (Lines 14-17). The code for all forward models from each phenomenology is concatenated into a single file named `forward.r` and placed in the application code directory; in this example,

`MultiPEM_Toolbox_Package/Applications/Code/IYDT`

Note that these forward models accept the parameters to be calibrated as their main argument. In this example, the *seismic* forward model  $f_{sr}(\cdot)$  as a function of the calibration parameters  $\beta_{sr}$  is given as follows (WPA, §6.2, p. 11),

$$\begin{aligned}
\log(\tilde{d}_{sr}(\beta_{sr})) &= \beta_{sr,1} + \beta_{sr,2} \log(\tilde{\delta}_s) + \beta_{sr,3} \text{logistic}(\beta_{sr,4} \tilde{h}_s + \beta_{sr,5}) \\
f_{sr}(\beta_{sr}) &= \log(d_{sr}(\beta_{sr}))
\end{aligned} \tag{1}$$

for

$$\text{logistic}(x) = \frac{1}{1 + \exp(-x)}.$$

The scaled signatures and covariates of this forward model are given by

$$\begin{aligned}\tilde{d}_{s1} &= d_{s1} \exp(-w/3) & \tilde{d}_{s2} &= d_{s2} \\ \tilde{\delta}_s &= \delta \exp(-w/3) & \tilde{h}_s &= h \exp(-w/3),\end{aligned}$$

where  $d_{s1}$  and  $d_{s2}$  are P-wave displacement and maximum velocity, and the covariates  $v = (w, h, \delta)$  are log-yield, HOB/DOB, and range. The function `f_s` returns a vector of forward model calculations evaluated for the supplied value of  $\beta_{sr}$ , each element corresponding to each row of a matrix of covariates (having columns  $(w, h, \delta)$ ).

- Line 20: Indicate if forward model Jacobian matrices are provided for efficient log-likelihood maximization.
- Lines 22-30: If `igrad` is `TRUE`, names of forward model Jacobian functions must be provided for each signature by phenomenology. The `gfm` object is initialized as a null list with elements for each phenomenology in the proper order (Line 25). Subsequent lines specify the Jacobian function names as vectors of strings having length equal to the number of signatures for each phenomenology (Lines 26-29). The code for all forward model Jacobian functions from each phenomenology is concatenated into a single file named `jacobian.r` and placed in the application code directory; in this example,

`MultiPEM_Toolbox_Package/Applications/Code/IYDT`

Note that these Jacobian functions accept the parameters to be calibrated as their main argument. In this example, the gradient vector of the *seismic* forward model of Equation (1) is computed from the partial derivatives of  $f_{sr}(\cdot)$  for each calibration parameter as follows,

$$\begin{aligned}\frac{\partial f_{sr}}{\partial \beta_{sr,1}} &= 1 \\ \frac{\partial f_{sr}}{\partial \beta_{sr,2}} &= \log(\tilde{\delta}_s) \\ \frac{\partial f_{sr}}{\partial \beta_{sr,3}} &= \text{logistic}(\beta_{sr,4}\tilde{h}_s + \beta_{sr,5}) \\ \frac{\partial f_{sr}}{\partial \beta_{sr,4}} &= \beta_{sr,3}\tilde{h}_s \times \text{logistic}(\beta_{sr,4}\tilde{h}_s + \beta_{sr,5}) \times \text{logistic}(-\beta_{sr,4}\tilde{h}_s - \beta_{sr,5}) \\ \frac{\partial f_{sr}}{\partial \beta_{sr,5}} &= \beta_{sr,3} \times \text{logistic}(\beta_{sr,4}\tilde{h}_s + \beta_{sr,5}) \times \text{logistic}(-\beta_{sr,4}\tilde{h}_s - \beta_{sr,5})\end{aligned}$$

The function `g_s` returns a Jacobian matrix (`jbeta_s`) of forward model gradients for the calibration parameters, evaluated at the supplied value of  $\beta_{sr}$ , with rows corresponding to the rows of a matrix of covariates (having columns  $(w, h, \delta)$ ). If `eiv` is `TRUE` (Line 105 of Appendix A.1), the partial derivative of  $f_{sr}(\cdot)$  for log-yield  $w$  is also

required,

$$\begin{aligned} \frac{\partial f_{sr}}{\partial w} = & -\frac{1}{3} \left( \beta_{sr,2} + \beta_{sr,3} \beta_{sr,4} \tilde{h}_s \times \text{logistic}(\beta_{sr,4} \tilde{h}_s + \beta_{sr,5}) \times \text{logistic}(-\beta_{sr,4} \tilde{h}_s - \beta_{sr,5}) \right) \\ & + \frac{1}{3} \delta_1(r) \end{aligned} \quad (2)$$

for  $\delta_A(x)$  the indicator function of set  $A$ . The function `g_s` will also return a Jacobian vector (`jtheta_s`) of forward model partial derivatives for log-yield, evaluated at the supplied value of  $\beta_{sr}$ , each element corresponding to each row of the same covariate matrix used in the calculation of `jbeta_s`.

- Line 35: Indicate if the same forward model function is used to compute multiple signatures, and signature-specific code within this function is required.
- Lines 37-41: If `iResponse` is `TRUE`, initialize `iResponse` to a null list with elements for each phenomenology in the proper order (Line 38). For each relevant phenomenology, subsequent lines provide vectors of length equal to the number of signatures, each element of which is a tag identifying code specific to the corresponding signature. This mechanism is utilized for the *seismic* (Line 39) and *acoustic* (Line 40) phenomenologies.
- Line 44: Indicate if fixed inputs are to be provided to the forward models for at least one phenomenology.
- Lines 46-55: If `fPars` is `TRUE`, initialize `fPars` to a null list with elements for each phenomenology in the proper order (Line 47). For each relevant phenomenology, subsequent lines provide the value(s) of all fixed inputs. For example, the *optical* forward model requires fixed values for `yield_scaling` (Line 52), `pressure_scaling` (Line 53), and `temp_scaling` (Line 54).
- Line 58: Specify the number of starting parameter vectors for the log-likelihood maximization routine.
- Line 61: Specify the number of cores to use for parallel optimization (across distinct starting values) of the calibration data log-likelihood function.
- Line 64: Specify if the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm is to be used for maximization of the log-likelihood function. If `TRUE`, functions to compute forward model Jacobian matrices analytically must be provided, or numerical gradients will be utilized (generally increasing compute time). If `FALSE`, the gradient-free Nelder-Mead simplex algorithm will be utilized for optimization, which is generally much slower computationally than BFGS with analytical gradients.
- Lines 68-71: If relevant, specifies the location (relative to the run directory) of parameter values or estimates to be used as the first starting value for log-likelihood maximization. These values are stored in a `.RData` object as a list, with elements corresponding to forward model (e.g. common parameters `beta`, emplacement-dependent parameters `tbeta`) and error model (e.g. level 1 variance components `vc_1`, level 2 variance components `vc_2`, observation error parameters `eps`) quantities of interest. If relevant, calibration source errors-in-variables yield estimates (`w_eiv`) are also provided. For

multi-phenomenology analyses, values or estimates from individual phenomenologies may be input in the proper order, and they will be concatenated appropriately.

- Line 75: If desired, name of output `.RData` file to store optimization results from this run. The elements of the list to be written are described in the previous item.
- Line 79: Indicate if phenomenology specific code is required in the postprocessing function.
- Lines 81-83: If `Phen` is `TRUE`, specifies a matrix in which the first column provides the numerical phenomenology indicator (see Lines 41-44 of the preprocessing code in Appendix A.1), and the second column provides the phenomenology name in string format. In this example, specific code is required to process results for the *seismic* phenomenology (Line 82).
- Line 86: Indicate if gradient verification is to be conducted on the log-likelihood function. If `TRUE` and `igrad` is `TRUE` (Line 20), analytical and numerical gradients at the optimal parameter value, and other randomly sampled parameter values, are compared for consistency.
- Line 89: Specify the strategy for running parallel jobs using the `future` package in R. The available options are given by starting an R session and issuing the following commands,  
  

```
% R
> require(future)
> help(plan)
```
- Lines 92-96: Calls the log-likelihood maximization function `calc_mle_cal` for the calibration data. Table 2 describes all inputs to this function with default values. Only inputs with no default values must be provided.

### 2.1.3 Bayesian Analysis

The optional Bayesian inference component of the analysis in Appendix A.3 is responsible for sampling forward and error model parameters, and calibration source errors-in-variables yields (if relevant) from their joint posterior distribution using calibration data. This Bayesian component must be run if multiple imputation of forward and error model parameters is desired in second stage Bayesian inference for new event device parameters.

- Line 6: Indicate if first stage Bayesian analysis is to be conducted. If second stage multiple imputation is desired, `iBayes` should be `TRUE`.
- Line 10: Read in code performing Bayesian analysis on forward and error model parameters, and calibration source errors-in-variables yields (if relevant), using calibration data.
- Line 14: Indicate if a log-prior density for the forward model parameters is supplied by the user (WPA, §6.5, p. 15). If `iBetaPrior` is `FALSE`, a “flat prior” (uniform on the domain) on these parameters is assumed.

Table 2: Inputs to `calc_mle_cal` function.

Input	Default	Brief Description
<code>p_cal</code>	<code>none</code>	environment storing all objects needed in log-likelihood calculations
<code>gdir</code>	<code>none</code>	directory location of global subroutines
<code>adir</code>	<code>none</code>	directory location of application subroutines
<code>f</code>	<code>none</code>	names of forward model functions for each signature by phenomenology
<code>nst</code>	<code>10</code>	number of starting values for log-likelihood maximization
<code>ncor</code>	<code>1</code>	number of cores for log-likelihood maximization
<code>igrad</code>	<code>TRUE</code>	forward model Jacobian provided
<code>bfgs</code>	<code>TRUE</code>	log-likelihood maximization uses BFGS methods
<code>igrck</code>	<code>TRUE</code>	conduct log-likelihood function gradient verification
<code>g</code>	<code>NULL</code>	names of forward model Jacobian functions for each signature by phenomenology
<code>iresp</code>	<code>NULL</code>	flags for modified calculation by signature in a common forward model for each relevant phenomenology
<code>fp_fm</code>	<code>NULL</code>	fixed inputs required by forward models
<code>fopt_in</code>	<code>NULL</code>	location of input R data file(s) providing an initial starting value for log-likelihood maximization (if multiple files, starting value created by concatenating over phenomenologies)
<code>Xst</code>	<code>NULL</code>	matrix of starting values for log-likelihood maximization if not generated by this function
<code>fopt_out</code>	<code>NULL</code>	location to write output R data file with results of log-likelihood maximization
<code>phen</code>	<code>NULL</code>	phenomenology number and type (if needed for postprocessing)
<code>pl</code>	<code>"multicore"</code>	strategy for running parallel jobs using the <code>future</code> package

- Lines 16-35: If relevant, specify details of user-provided log-prior distributions for forward model parameters. For each relevant phenomenology, the list object `lp_beta` is used for common coefficients, while the list object `lp_betat` is used for emplacement-dependent coefficients (as demonstrated below in this application).
  - Line 18: Specify location(s) of log-prior function(s). Must be provided if `iBetaPrior` is `TRUE`. In this example, a single log-prior function is provided, located at  
`../Code/lp_beta.s.r`
  - Line 20: If `igrad` is `TRUE` (Line 20 of Appendix A.2), specify location(s) of the log-prior gradient function(s). In this example, a single log-prior gradient function is provided, located at  
`../Code/glp_beta.s.r`
  - Line 25: For each relevant phenomenology, initialize a null list `lp_betat` of length



equal to the number of emplacement conditions containing distinct forward model parameters.

- Line 27: For each relevant phenomenology and emplacement condition, provide the name(s) of the log-prior function(s) for each signature. In this example, the *seismic* phenomenology utilizes a log-prior function `lp_s` for each signature within each emplacement condition.
- Line 29: If `igrad` is `TRUE` (Line 20 of Appendix A.2), then for each relevant phenomenology and emplacement condition, provide the name(s) of the log-prior gradient function(s) for each signature. In this example, the *seismic* phenomenology utilizes a log-prior gradient function `lq_s` for each signature within each emplacement condition.
- Line 39: Specify a *fixed* value for the scale parameter `A` of half-Cauchy prior distribution(s) for the level 1 (*source*) and level 2 (*path*) variance component parameters if relevant (WPA, §6.5, p. 15; §6.6, p. 17). Prior distributions for a non-empty collection of variance component parameters are taken to be mutually independent. Comment out if this parameter is to be sampled from its posterior distribution.
- Line 43: Specify a *fixed* value for the shape parameter  $\eta$  of the Lewandowski-Kurowicka-Joe (LKJ) prior distribution for the observational error model correlation parameters (WPA, §6.5, p. 15; §6.6, p. 17).
- Line 51: If `eiv` is `TRUE` (Line 105 of Appendix A.1), specify a *fixed* value for the parameter that controls the number of modes in the *flexible generalized skew-normal* (FGSN) prior distribution for the errors-in-variables yields of the calibration events (WPA, §6.5, p. 15; §6.6, p. 23).
- Line 57: Select the Markov chain Monte Carlo (MCMC) algorithm to use for posterior sampling, from one of three options: `RAM`, `FME`, and `NUTS`. `RAM` is the robust adaptive Metropolis algorithm of Vihola<sup>5</sup> implemented in the R package `adaptMCMC`. `FME` is the delayed rejection adaptive Metropolis algorithm of Haario, Laine, and Mira<sup>6</sup> implemented in the R package `FME`. `NUTS` is the No-U-Turn Sampler of Hoffman and Gelman<sup>7</sup>. The `NUTS` option requires the analytical gradient of the log-posterior density, which in turn requires `igrad` to be `TRUE` (Line 20 of Appendix A.2).
- Line 60: Specify the per core sample size of the burn-in period for MCMC sampling (pre-equilibrium stage of Markov chain). These samples are discarded prior to any inference using the posterior samples.
- Line 63: Specify the sample size of the MCMC production run. These samples are kept for posterior inference.

---

<sup>5</sup>Vihola, M. (2012). Robust adaptive Metropolis algorithm with coerced acceptance rate. *Stat Comput* 22:997-1008.

<sup>6</sup>Haario, H., Laine, M., and Mira, A. (2006). DRAM: Efficient adaptive MCMC. *Stat Comput* 16:339-354.

<sup>7</sup>Hoffman, M. D. and Gelman, A. (2014). The No-U-Turn Sampler: Adaptively setting path lengths in Hamiltonian Monte Carlo. *J Mach Learn Res* 15:1593-1623.

- Line 66: Specify the rate at which MCMC production samples are thinned for estimation of the Deviance Information Criterion<sup>8</sup> (DIC) and the Predictive Information Criterion<sup>9</sup> (PIC). In this example, the `nthin` value of 20 indicates that every 20-th production sample is kept for DIC and PIC estimation.
- Line 69: Specify the number of cores to use for parallel optimization (across distinct starting values) of the calibration data log-posterior function.
- Line 72: Specify the number of cores used to run parallel MCMC chains. The burn-in period for each chain is determined by `nburn` (Line 60), while the `nmcmc` (Line 63) production runs are split between the `ncores_mc` processors and combined at the conclusion of the runs.
- Line 75: Indicate if gradient verification is to be conducted on the log-prior function. If `TRUE` and `igrad` is `TRUE` (Line 20 of Appendix A.2), analytical and numerical gradients at the maximum *a posteriori* parameter value, and other randomly sampled parameter values, are compared for consistency.
- Line 78: Indicate if gradient verification is to be conducted on the log-posterior function. If `TRUE` and `igrad` is `TRUE` (Line 20 of Appendix A.2), analytical and numerical gradients at the maximum *a posteriori* parameter value, and other randomly sampled parameter values, are compared for consistency.
- Lines 81-88: Calls the Bayesian analysis function `calc_bayes_cal` for the calibration data. Table 3 describes all inputs to this function with default values. Only inputs with no default values must be provided.

### 2.1.4 Output

The output file `runMPEM.out` from the first stage analysis contains a summary of (if relevant) errors-in-variables yield estimates for the relevant calibration sources, forward and error model parameter estimates and (if relevant) posterior samples derived from the calibration data. The desired output is supplied by the user function `print_sumstats.r`, placed in the application code directory; in this example,

`MultiPEM_Toolbox_Package/Applications/Code/IYDT`

The output presented in Appendix A.4 contains the most pertinent information extracted from the full file.

- Lines 6-17: Output from the preprocessing function `prepro_cal`. These warning messages explain which variance component models are allowed (if any) for each signature of each phenomenology based on the structure of the calibration data. In this example, only level 1 (*source*) random effects are allowed for each *acoustic* signature (Lines 6-9), while no random effects are allowed for *optical* or *surface effects* phenomenologies

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<sup>8</sup>Spiegelhalter, D.J., Best, N.G., Carlin, B.P., & van der Linde, A. (2002). Bayesian measures of model complexity and fit (with discussion), *J R Stat Soc Ser B* 64:583-639.

<sup>9</sup>Ando, T. (2011). Predictive Bayesian model selection, *Am J Math Manag Sci* 31:13-38.

Table 3: Inputs to `calc_bayes_cal` function.

Input	Default	Brief Description
<code>p_cal</code>	<code>none</code>	environment storing all objects needed in log-posterior calculations
<code>gdir</code>	<code>none</code>	directory location of global subroutines
<code>adir</code>	<code>none</code>	directory location of application subroutines
<code>nst</code>	<code>10</code>	number of starting values for log-posterior maximization
<code>nburn</code>	<code>10000</code>	number of per core MCMC burn-in samples
<code>nmcmc</code>	<code>20000</code>	number of MCMC production samples
<code>nthin</code>	<code>20</code>	posterior sample thinning rate
<code>ncor_map</code>	<code>1</code>	number of cores for log-posterior maximization
<code>ncor_mc</code>	<code>1</code>	number of cores for generating parallel MCMC chains
<code>igrad</code>	<code>TRUE</code>	forward model Jacobian provided
<code>igrck_pr</code>	<code>TRUE</code>	conduct log-prior function gradient verification
<code>igrck_po</code>	<code>TRUE</code>	conduct log-posterior function gradient verification
<code>bfgs</code>	<code>TRUE</code>	log-posterior maximization uses BFGS methods
<code>ibpr</code>	<code>FALSE</code>	prior density function(s) provided for forward model coefficients
<code>fpr_b</code>	<code>NULL</code>	location of functions computing log-prior density for forward model coefficients
<code>fgpr_b</code>	<code>NULL</code>	location of functions computing gradients of log-prior density for forward model coefficients
<code>Xnom</code>	<code>NULL</code>	matrix of starting values for hyperparameters in log-posterior maximization if not generated by this function
<code>imcmc</code>	<code>"FME"</code>	MCMC algorithm (current options: <code>"RAM"</code> , <code>"FME"</code> , <code>"NUTS"</code> )
<code>pl</code>	<code>"multicore"</code>	strategy for running parallel jobs using the <code>future</code> package

(Lines 10-17). There are no warning messages for *seismic* signatures, indicating level 1 and level 2 (*path*) random effects are allowed.

- Lines 25-240: Output from the maximum likelihood estimation function `calc_mle_cal`:
  - Line 27: Convergence code from the R optimization function `optim`. In this example, ‘0’ indicates successful completion.
  - Line 28: Number of optimization restarts in which the relative absolute maximum log-likelihood difference is  $\leq 10^{-8}$ . The algorithm exits after 2 such restarts, which is attained in this example.
  - Lines 33-36: Maximum likelihood estimates of errors-in-variables yields for the relevant calibration sources. Source names (Lines 33 and 35) are given above yield estimates (Lines 34 and 36). Errors-in-variables yields are only estimated if `eiv` is `TRUE` (Line 105 of Appendix A.1).
  - Lines 40-62: Maximum likelihood estimates of *common* forward model parameters for each signature of each phenomenology (where present).

- Lines 66-112: Maximum likelihood estimates of *emplacement-dependent* forward model parameters for each signature of each phenomenology (where present).
- Lines 116-130: Maximum likelihood estimates of *source* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 134-140: Maximum likelihood estimates of *path* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 144-190: Maximum likelihood estimates of observational error variances for each signature, and correlations between signatures, for each phenomenology (WPA, §A.5).
- Line 192: Akaike Information Criterion<sup>10</sup> (AIC) value based on calibration data. Used for selecting among competing forward or error model specifications (WPA, §6.5, p. 15; §6.6, Tables 4 and 5, p. 18).
- Line 194: Bayesian Information Criterion<sup>11</sup> (BIC) value based on calibration data. Used for selecting among competing forward or error model specifications (WPA, §6.5, p. 15; §6.6, Tables 4 and 5, p. 18).
- Lines 199-240: Example of log-likelihood gradient verification at a single sampled parameter vector. Additional checks were deleted for brevity.
  - \* Lines 200-218: Analytic gradient calculation
  - \* Lines 220-238: Numerical gradient calculation using the R package `numDeriv`
  - \* Line 240: Largest negative (first entry) and positive (second entry) differences between the analytic and numerical gradients
- Lines 253-1283: Output from the Bayesian analysis function `calc_bayes_cal`:
  - Line 255: Convergence code from the R optimization function `optim`. In this example, ‘0’ indicates successful completion.
  - Line 256: Number of optimization restarts in which the relative absolute maximum log-posterior difference is  $\leq 10^{-8}$ . The algorithm exits after 2 such restarts, which is attained in this example.
  - Lines 261-264: Maximum *a posteriori* estimates of errors-in-variables yields for the relevant calibration sources. Source names (Lines 261 and 263) are given above yield estimates (Lines 262 and 264). Errors-in-variables yields are only estimated if `eiv` is `TRUE` (Line 105 of Appendix A.1).
  - Lines 268-290: Maximum *a posteriori* estimates of *common* forward model parameters for each signature of each phenomenology (where present).

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<sup>10</sup>Akaike, H. (1973). Information Theory and an Extension of the Maximum Likelihood Principle. In: Petrov, B.N. & Csaki, F., Eds., International Symposium on Information Theory, 267-281.

<sup>11</sup>Schwarz, G. (1978). Estimating the dimension of a model, *Ann Stat* 6:461-464.

- Lines 294-340: Maximum *a posteriori* estimates of *emplacement-dependent* forward model parameters for each signature of each phenomenology (where present).
- Lines 344-358: Maximum *a posteriori* estimates of *source* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 362-368: Maximum *a posteriori* estimates of *path* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 372-418: Maximum *a posteriori* estimates of observational error variances for each signature, and correlations between signatures, for each phenomenology (WPA, §A.5).
- Lines 422-424: Maximum *a posteriori* estimates of FGSN prior distribution parameters (WPA, §6.6, p. 23;  $\text{Alpha} = \mu$ ,  $\text{Omega} = v$  (two coefficients)).
- Lines 428-471: Example of log-prior gradient verification at a single sampled parameter vector. Additional checks were deleted for brevity.
  - \* Lines 429-448: Analytic gradient calculation
  - \* Lines 450-469: Numerical gradient calculation using the R package `numDeriv`
  - \* Line 471: Largest negative (first entry) and positive (second entry) differences between the analytic and numerical gradients
- Lines 475-518: Example of log-posterior gradient verification at a single sampled parameter vector. Additional checks were deleted for brevity.
  - \* Lines 476-495: Analytic gradient calculation
  - \* Lines 497-516: Numerical gradient calculation using the R package `numDeriv`
  - \* Line 518: Largest negative (first entry) and positive (second entry) differences between the analytic and numerical gradients
- Line 522: Acceptance rate of the Delayed Rejection Adaptive Metropolis (DRAM) posterior sampling method implemented in R package `FME`. Note that one delayed rejection step is allowed in the default implementation.
- Lines 528-580: Means and user specified quantiles of samples from the marginal posterior distributions of errors-in-variables yields for the relevant calibration sources. The ordering of calibration sources is provided with the maximum *a posteriori* estimates (Lines 261 and 263). Errors-in-variables yields are only estimated if `eiv` is `TRUE` (Line 105 of Appendix A.1).
- Lines 584-666: Means and user specified quantiles of samples from the marginal posterior distributions of *common* forward model parameters for each signature of each phenomenology (where present).

- Lines 670-872: Means and user specified quantiles of samples from the marginal posterior distributions of *emplacement-dependent* forward model parameters for each signature of each phenomenology (where present).
- Lines 876-930: Means and user specified quantiles of samples from the marginal posterior distributions of *source* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 934-960: Means and user specified quantiles of samples from the marginal posterior distributions of *path* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 964-1238: Means and user specified quantiles of samples from the marginal posterior distributions of observational error variances for each signature, and correlations between signatures, for each phenomenology (WPA, §A.5).
- Lines 1242-1279: Means and user specified quantiles of samples from the marginal posterior distributions of FGSN prior distribution parameters (WPA, §6.6, p. 23;  $\text{Alpha} = \mu$ ,  $\text{Omega} = v$  (two coefficients)).
- Line 1281: DIC value based on calibration data. Used for selecting among competing forward or error model specifications (WPA, §6.5, pp. 15-16; §6.6, Tables 4 and 5, p. 18).
- Line 1283: PIC value based on calibration data. Used for selecting among competing forward or error model specifications (WPA, §6.5, pp. 15-16; §6.6, Tables 4 and 5, p. 18).

The `p_cal` environment resulting from this run contains several elements of potential interest for additional post-processing:

- `p_cal$mle_cal`: Maximum likelihood estimate of (if relevant) calibration source errors-in-variables yields, forward and error model parameters based on calibration data
- `p_cal$map_cal`: If `iBayes` is `TRUE` (Line 6 of Appendix A.3), maximum *a posteriori* estimate of (if relevant) calibration source errors-in-variables yields, forward and error model parameters based on calibration data
- `p_cal$mpi`: If `iBayes` is `TRUE` (Line 6 of Appendix A.3), posterior samples of (if relevant) calibration source errors-in-variables yields, forward and error model parameters based on calibration data

## 2.2 Second Stage

The second stage analysis is defined in the `runMPEM.0.r` file, provided in Appendix A with line numbers referred to in the ensuing discussion. Appendix A.5 provides the preprocessing component of the second stage, Appendix A.6 provides the code employed to maximize the likelihood function of the new event data with respect to the new event device parameters, while Appendix A.7 provides the code employed to optionally sample the posterior distribution of these parameters.

The second stage analysis is run in batch mode as follows,

```
% R CMD BATCH runMPEM_0.r runMPEM_0.out &
```

This job requires the `.RData` file from the completion of the first stage run to be copied into the second stage run directory. The main features of the output file `runMPEM_0.out` are provided in Appendix A.8.

### 2.2.1 Preprocessing

The preprocessing component of the second stage analysis in Appendix A.5 is primarily responsible for describing features of the new event data and device parameters of inferential interest.

- Line 13+: Load all R packages utilized by multiple supporting subroutines, most notably log-likelihood and log-prior calculations and their associated gradients.
- Line 24: Specify directory location (relative to run directory) of all global (application independent) subroutines.
- Line 27: Read in code performing second stage preprocessing of new event data.
- Line 30: Specify directory location (relative to run directory) of all application-specific subroutines.
- Line 33: Specify root directory (relative to run directory) containing all application-specific new event data files.
- Lines 36-39: A scalar or vector specifying the names of new event data files for each phenomenology, utilizing an ordering of the phenomenologies (for MultiPEM analysis) that is consistent with first stage preprocessing and maintained throughout the input deck (as indicated here in Lines 42-45). Data files are text files (CSV formatted) containing all measured signatures (in the first column(s)) and input covariates (in succeeding column(s)) including all those required in forward and error model calculations, but excepting the new event device parameters that are unknown and subject to second stage inference. Directories specifying the exact locations of these files relative to the root data directory (Line 33) may also be included in the filenames.
- Line 48: Specify the names of the new event device parameters of inferential interest as a vector of strings. This information is utilized in postprocessing.
- Line 52: Number of first stage forward and error model parameter, and calibration source errors-in-variables yield (if relevant), posterior samples utilized in the multiple imputation algorithm for generating second stage new event device parameter posterior samples. If `nimpute` is set to 1 (default), the first stage maximum likelihood estimate of the forward and error model parameters, and the calibration source errors-in-variables yields (if relevant), is used in place of the imputation samples.
- Line 55: Specify if bounded optimization of new event device parameters is to be conducted. The default is to optimize all new event device parameters on an unbounded input space, transforming them to their input domain (specified subsequently in this

preprocessing file) as necessary for forward model calculations. If `opt_B` is `TRUE`, the new event device parameters are optimized directly on their input domain.

- Line 58: Indicate if the new event device parameters are subjected to a user-provided bijective transformation supplied to assist likelihood maximization or posterior sampling. If `itransform` is `TRUE`, the code implementing this transformation is concatenated into a single file named `transform.r` and placed in the application code directory; in this example,

`MultiPEM_Toolbox_Package/Applications/Code/IYDT`

The functions that must be provided in `transform.r` include the following:

- `tau`: Function  $\tau(\cdot)$  applied to transformed variables  $\tilde{\boldsymbol{\theta}}_0$  with the new event device parameters  $\boldsymbol{\theta}_0$  as its image,

$$\boldsymbol{\theta}_0 = \tau(\tilde{\boldsymbol{\theta}}_0)$$

- `j_tau`: Jacobian matrix of  $\tau(\cdot)$ ,

$$\mathbf{J}_\tau(\tilde{\boldsymbol{\theta}}_0) = \begin{bmatrix} \frac{\partial \tau_1(\tilde{\boldsymbol{\theta}}_0)}{\partial \tilde{\theta}_{0,1}} & \dots & \frac{\partial \tau_1(\tilde{\boldsymbol{\theta}}_0)}{\partial \tilde{\theta}_{0,q}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \tau_q(\tilde{\boldsymbol{\theta}}_0)}{\partial \tilde{\theta}_{0,1}} & \dots & \frac{\partial \tau_q(\tilde{\boldsymbol{\theta}}_0)}{\partial \tilde{\theta}_{0,q}} \end{bmatrix}$$

where  $q$  is the dimension of  $\boldsymbol{\theta}_0$ .

- `log_absdet_j_tau`: Logarithm of the absolute value of the determinant of the Jacobian matrix computed from `j_tau`,

$$\log \text{abs}(\det(\mathbf{J}_\tau(\tilde{\boldsymbol{\theta}}_0)))$$

- `dlog_absdet_j_tau`: Gradient of the log absolute Jacobian determinant with respect to  $\tilde{\boldsymbol{\theta}}_0$
- `inv_tau`: Inverse function of  $\tau(\cdot)$

In this example, the new event device parameters of inferential interest are log-yield  $w$  and height-of-burst  $h$ , that is  $\boldsymbol{\theta}_0 = (w, h)$ . The relevant forward models are functions of a scaled height-of-burst,  $\tilde{h} = h \exp(-w/3)$ , suggesting the possible utility of likelihood maximization or posterior sampling in terms of  $\tilde{\boldsymbol{\theta}}_0 = (\tilde{w}, \tilde{h})$  for  $\tilde{w} = w$ .

- Lines 61-68: If `itransform` is `TRUE`, and if `tPars` is `TRUE`, initialize `tPars` to a null list (Line 65). Subsequent lines provide the value(s) for all fixed inputs required to compute the function `tau` (see previous item). In this example, a `yield_scaling` value is required (Line 66).
- Lines 72-75: Specify lower and upper bounds for the new event device parameters if needed. By default, lower bounds are set to  $-\infty$  (Line 72) and upper bounds to



$+\infty$  (Line 74). In this example, the second parameter (height-of-burst) is restricted to the range  $(-10, 160)$  (Lines 73 and 75). Note that likelihood maximization and posterior sampling are conducted on an unbounded parameter space. If lower or upper bounds are specified for any parameter, they are applied just prior to objective function calculations using the `transform` function of the `transform.r` file located in the global code directory,

#### MultiPEM\_Toolbox\_Package/Code

- Lines 78-83: If `tsub` is `TRUE`, the forward model for at least one phenomenology depends only on a subset of the full vector  $\theta_0$  of new event device parameters. The `tsub` object is initialized to a null list with elements for each phenomenology in the proper order (Line 81). The `theta_names` vector (Line 48) describes the order of elements in  $\theta_0$ . For relevant phenomenologies, parameter subsets are specified as integer vectors identifying the extracted elements of  $\theta_0$ . The forward models of all other phenomenologies depend on the full  $\theta_0$ . In this example, the *surface effects (crater)* phenomenology only depends on log-yield (Line 82), while the other phenomenologies depend on both log-yield and height-of-burst.
- Lines 86-88: Calls the preprocessing function `prepro_0` for the new event data. Table 4 describes all inputs to this function with default values. Only inputs with no default values must be provided.

Table 4: Inputs to `prepro_0` function.

Input	Default	Brief Description
<code>p_cal</code>	<code>none</code>	environment storing all objects needed in log-likelihood and log-posterior calculations
<code>gdir</code>	<code>none</code>	directory location of global subroutines
<code>adir</code>	<code>none</code>	directory location of application subroutines
<code>rdir</code>	<code>none</code>	root directory location of data files
<code>ndir</code>	<code>none</code>	directory locations (if relevant) and names of new event data files under <code>rdir</code>
<code>tnames</code>	<code>none</code>	names of new event parameters
<code>nimp</code>	<code>1</code>	number of first stage imputation samples used in second stage new event parameter posterior sampling
<code>bopt</code>	<code>FALSE</code>	new event parameter bounds supplied to log-likelihood maximization
<code>itr</code>	<code>FALSE</code>	bijective transform of new event parameters provided
<code>fp_tr</code>	<code>NULL</code>	fixed inputs to new event parameter transform
<code>tlb</code>	<code>NULL</code>	lower bounds for new event parameters
<code>tub</code>	<code>NULL</code>	upper bounds for new event parameters
<code>tsub</code>	<code>NULL</code>	list containing index sets identifying new event parameter subsets by phenomenology if relevant

- Lines 89-95: If `opt_B` is `TRUE` (Line 55), the preprocessor function `prepro_0` returns a list (designated here as `tmp`) with objects `p_cal` and `t_cal`, which are then assigned as

follows

```
% p_cal = tmp$p_cal  
% t_cal = tmp$t_cal
```

and both are utilized for maximum likelihood estimation and Bayesian analysis. Otherwise, `p_cal` is the only object returned,

```
% p_cal = tmp$p_cal
```

and utilized in subsequent analyses.

### 2.2.2 Maximum Likelihood Estimation

The maximum likelihood estimation component of the second stage analysis in Appendix A.6 is responsible for integrating calibrated forward and error model parameter values, and calibration source errors-in-variables yields (if relevant), from the first stage with new event data to estimate new event device parameters of interest with uncertainty quantification (WPA, §A.2).

- Line 6: Read in code performing second stage maximum likelihood estimation and uncertainty quantification of new event device parameters, using calibrated forward and error model parameters, and calibration source errors-in-variables yields (if relevant), from the first stage.
- Line 9: User specified seed to ensure repeatability of maximum likelihood estimation.
- Lines 13-17: Provide names of forward models for each signature by phenomenology (WPA, §6.1-6.4). The `fm0` object is initialized as a null list with elements for each phenomenology in the proper order (Line 13). Subsequent lines specify the function names as vectors of strings having length equal to the number of signatures for each phenomenology (Lines 14-17). The code for all forward models from each phenomenology is concatenated into a single file named `forward_0.r` and placed in the application code directory; in this example,

`MultiPEM_Toolbox_Package/Applications/Code/IYDT`

Note that these forward models compute the same signatures as those used in the first stage. However, they accept only the new event device parameters of inferential interest (designated  $\theta_0$  previously) as their main argument. Forward model parameter values are passed in as fixed quantities. In this example, the *seismic* forward model  $f_{sr}^0(\cdot)$  as a function of the new event device parameters  $\theta_0 = (w, h)$  – for fixed calibration parameters  $\beta_{sr}$  – is given by Equation (1). The function `f0_s` returns a vector of forward model calculations evaluated for the supplied value of  $\theta_0$  (fixed  $\beta_{sr}$  passed in as `params$beta`), each element corresponding to each row of a matrix of covariates (in this case, a column vector of ranges  $\delta$ ).

- Line 20: Indicate if forward model Jacobian matrices are provided for efficient log-likelihood maximization.

- Lines 22-30: If `igrad` is `TRUE`, names of forward model Jacobian functions must be provided for each signature by phenomenology. The `gfm0` object is initialized as a null list with elements for each phenomenology in the proper order (Line 25). Subsequent lines specify the Jacobian function names as vectors of strings having length equal to the number of signatures for each phenomenology (Lines 26-29). The code for all forward model Jacobian functions from each phenomenology is concatenated into a single file named `jacobian_0.r` and placed in the application code directory; in this example,

`MultiPEM_Toolbox_Package/Applications/Code/IYDT`

Note that these Jacobian functions accept only the new event device parameters of inferential interest (designated  $\theta_0$  previously) as their main argument. Forward model parameter values are passed in as fixed quantities. In this example, the gradient vector of the *seismic* forward model (see description above) is computed from the partial derivatives of  $f_{sr}^0(\cdot)$  for each new event device parameter in  $\theta_0$ , for fixed calibration parameters  $\beta_{sr}$ . The partial derivative for log-yeild  $w$  is given in Equation (2). For HOB/DOB  $h$ ,

$$\frac{\partial f_{sr}^0}{\partial h} = \beta_{sr,3}\beta_{sr,4} \exp(-w/3) \times \text{logistic}(\beta_{sr,4}\tilde{h}_s + \beta_{sr,5}) \times \text{logistic}(-\beta_{sr,4}\tilde{h}_s - \beta_{sr,5})$$

The function `g0_s` returns a Jacobian matrix (`jtheta_s`) of forward model gradients for the new event device parameters, evaluated at the supplied value of  $\theta_0$  (fixed  $\beta_{sr}$  passed in as `params$beta`), with rows corresponding to the rows of a matrix of covariates (in this case, a column vector of ranges  $\delta$ ).

- Line 33: Specify the number of starting new event device parameter vectors for the log-likelihood maximization routine.
- Line 36: Specify the number of cores to use for parallel optimization (across distinct starting values) of the new event data log-likelihood function.
- Line 39: Specify if the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm is to be used for maximization of the log-likelihood function. If `TRUE`, functions to compute forward model Jacobian matrices analytically must be provided, or numerical gradients will be utilized (generally increasing compute time). If `FALSE`, the gradient-free Nelder-Mead simplex algorithm will be utilized for optimization, which is generally much slower computationally than BFGS with analytical gradients.
- Line 43: If relevant, specifies the location (relative to the run directory) of parameter values or estimates to be used as the first starting value for log-likelihood maximization. These values are stored in a `.RData` object as a list, with an element corresponding to new event device parameters (`theta0`).
- Line 47: If desired, name of output `.RData` file to store optimization results from this run. The element of the list to be written are described in the previous item.
- Lines 50-55: If `tst` is `TRUE`, specifies an initial starting value for the new event device parameters (Lines 53-54) for log-likelihood maximization. This value is superseded by

the value read in from `opt_files_in` (Line 43), if provided.

- Line 58: Specify the level of confidence intervals computed for the true values of each new event device parameter from the maximum likelihood estimate and the estimated Fisher information matrix (WPA, §A.2, Equation (19); §A.4, Equation (21)).
- Line 61: Indicate if gradient verification is to be conducted on the log-likelihood function. If `TRUE` and `igrad` is `TRUE` (Line 20), analytical and numerical gradients at the optimal parameter value, and other randomly sampled parameter values, are compared for consistency.
- Line 64: Specify the strategy for running parallel jobs using the `future` package in R. The available options are given by starting an R session and issuing the following commands,  

```
% R  
> require(future)  
> help(plan)
```
- Lines 67-71: Calls the log-likelihood maximization function `calc_mle_0` for the new event data. Table 5 describes all inputs to this function with default values. Only inputs with no default values must be provided.

### 2.2.3 Bayesian Analysis

The optional Bayesian inference component of the second stage analysis in Appendix A.7 is responsible for integrating calibrated forward and error model parameter values, and calibration source errors-in-variables yields (if relevant), from the first stage with new event data to sample new event device parameters of interest from their posterior distribution. Imputation of first stage parameters results in more complete uncertainty quantification, but is computationally more intensive than employing the maximum likelihood estimate (default). Estimates of the new event device parameters with uncertainty quantification are computed from the posterior samples.

- Line 6: Indicate if Bayesian analysis is to be conducted.
- Line 10: Read in code performing second stage Bayesian analysis on new event device parameters, using calibrated forward and error model parameters, and calibration source errors-in-variables yields (if relevant), from the first stage.
- Line 14: Select the Markov chain Monte Carlo (MCMC) algorithm to use for posterior sampling, from one of four options: `RAM`, `FME`, `NUTS`, and `SMC`. `RAM` is the robust adaptive Metropolis algorithm of Vihola<sup>12</sup> implemented in the R package `adaptMCMC`. `FME` is the delayed rejection adaptive Metropolis algorithm of Haario, Laine, and Mira<sup>13</sup> implemented in the R package `FME`. `NUTS` is the No-U-Turn Sampler of Hoffman and

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<sup>12</sup>Vihola, M. (2012). Robust adaptive Metropolis algorithm with coerced acceptance rate. *Stat Comput* 22:997-1008.

<sup>13</sup>Haario, H., Laine, M., and Mira, A. (2006). DRAM: Efficient adaptive MCMC. *Stat Comput* 16:339-354.

Table 5: Inputs to `calc_mle_0` function.

Input	Default	Brief Description
<code>p_cal</code>	none	environment storing all objects needed in log-likelihood calculations
<code>gdir</code>	none	directory location of global subroutines
<code>adir</code>	none	directory location of application subroutines
<code>f0</code>	none	names of forward model functions for each signature by phenomenology
<code>nst</code>	10	number of starting values for log-likelihood maximization
<code>ncor</code>	1	number of cores for log-likelihood maximization
<code>ci_nev</code>	0.95	confidence interval levels for new event parameter inference
<code>igrad</code>	TRUE	forward model Jacobian provided
<code>bfgs</code>	TRUE	log-likelihood maximization uses BFGS methods
<code>igrck</code>	TRUE	conduct log-likelihood function gradient verification
<code>t_cal</code>	NULL	object required if bounds supplied to log-likelihood maximization
<code>g0</code>	NULL	names of forward model Jacobian functions for each signature by phenomenology
<code>fopt_in</code>	NULL	location of input R data file providing an initial starting value for log-likelihood maximization
<code>Xst</code>	NULL	matrix of starting values for log-likelihood maximization if not generated by this function
<code>tst</code>	NULL	vector of starting values for new event parameters in log-likelihood maximization
<code>fopt_out</code>	NULL	location to write output R data file with results of log-likelihood maximization
<code>pl</code>	"multicore"	strategy for running parallel jobs using the <code>future</code> package

Gelman<sup>14</sup>. The **NUTS** option requires the analytical gradient of the log-posterior density, which in turn requires `igrad` to be **TRUE** (Line 20 of Appendix A.6). **SMC** is a Sequential Monte Carlo (SMC) method adapted for sampling challenging posterior distributions (e.g. multi-modal) of low-dimensional parameter spaces<sup>15</sup>.

- Line 17: Specify the per core sample size of the burn-in period for MCMC sampling (pre-equilibrium stage of Markov chain). These samples are discarded prior to any inference using the posterior samples.
- Line 20: Specify the sample size of the MCMC production run. These samples are kept for posterior inference.
- Line 23: Specify the rate at which MCMC production samples are thinned when mul-

<sup>14</sup>Hoffman, M. D. and Gelman, A. (2014). The No-U-Turn Sampler: Adaptively setting path lengths in Hamiltonian Monte Carlo. *J Mach Learn Res* 15:1593-1623.

<sup>15</sup>Golchi, S. and Loepky, J.L. (2016). Monte Carlo based Designs for Constrained Domains. arXiv:1512.07328v2 [stat.ME], 8 Aug. 2016.

tuple imputation of first stage parameters is invoked for improved uncertainty quantification of second stage parameters.

- Line 26: Specify the number of cores used to run multiple imputations simultaneously or parallel MCMC chains for a single imputation. The burn-in period for each chain is determined by `nburn` (Line 17), while `nmcmc` (Line 20) production runs are generated for each imputed first stage parameter value, or split between the `ncores_mc` processors and combined at the conclusion of the runs for a single imputation.
- Line 29: Indicate if a log-prior density for the new event device parameters is supplied by the user (WPA, §6.5, p. 15; §6.6, pp. 18-19). If `iTheta0Prior` is `FALSE`, a “flat prior” (uniform on the domain) on these parameters is assumed.
- Lines 31-51: If relevant, specify details of user-provided log-prior distributions for new event device parameters.
  - Line 33: Specify location of log-prior function. If `NULL`, utilize the default log-prior function contained in the file `lp_0.r` placed in the application code directory; in this example,
 

```
MultiPEM_Toolbox_Package/Applications/Code/IYDT
```
  - Line 35: If `igrad` is `TRUE` (Line 20 of Appendix A.6), specify location of the log-prior gradient function. If `NULL`, utilize the default log-prior gradient function contained in the file `glp_0.r` placed in the application code directory; in this example,
 

```
MultiPEM_Toolbox_Package/Applications/Code/IYDT
```
  - Line 39: Provide the name of the log-prior function.
  - Line 40: If `igrad` is `TRUE` (Line 20 of Appendix A.6), provide the name of the log-prior gradient function.
  - Lines 43-47: Specify all fixed quantities required for calculation of the log-prior density. In this example, the mean (Line 43) and standard deviation (Line 44) of the Gaussian prior distribution for log-yield, and the mean (Line 46) and standard deviation (Line 47) of the Gaussian prior distribution for height-of-burst, are specified.
- Line 54: Indicate if gradient verification is to be conducted on the log-prior function. If `TRUE` and `igrad` is `TRUE` (Line 20 of Appendix A.6), analytical and numerical gradients at the maximum *a posteriori* parameter value, and other randomly sampled parameter values, are compared for consistency.
- Line 57: Indicate if gradient verification is to be conducted on the log-posterior function. If `TRUE` and `igrad` is `TRUE` (Line 20 of Appendix A.6), analytical and numerical gradients at the maximum *a posteriori* parameter value, and other randomly sampled parameter values, are compared for consistency.
- Line 62: If `iMCMC` is `"SMC"` (Line 14), specify the number of cores to be used in the

inner parallelization of the Sequential Monte Carlo (SMC) code for posterior sampling of the new event device parameters. SMC is advantageous if the posterior distribution of these parameters is multi-modal.

- Line 64: Lower bounds of new event device parameters (on infinite domain) for SMC sampling. For infinite values, lower bounds are determined from the maximum likelihood estimate (MLE) of these parameters and its uncertainty.
- Line 65: Upper bounds of new event device parameters (on infinite domain) for SMC sampling. For infinite values, upper bounds are determined from the MLE of these parameters and its uncertainty.
- Lines 68-75: Calls the Bayesian analysis function `calc_bayes_0` for the new event data. Table 6 describes all inputs to this function with default values. Only inputs with no default values must be provided.

Table 6: Inputs to `calc_bayes_0` function.

Input	Default	Brief Description
<code>p_cal</code>	none	environment storing all objects needed in log-posterior calculations
<code>gdir</code>	none	directory location of global subroutines
<code>adir</code>	none	directory location of application subroutines
<code>nburn</code>	10000	number of per core MCMC burn-in samples
<code>nmcmc</code>	20000	number of MCMC production samples
<code>nthin</code>	1	posterior sample thinning rate per imputation, for multiple imputation
<code>ncor</code>	1	number of cores for multiple imputation, or for generating parallel MCMC chains if single imputation
<code>igrad</code>	TRUE	forward model Jacobian provided
<code>igrck_pr</code>	TRUE	conduct log-prior function gradient verification
<code>igrck_po</code>	TRUE	conduct log-posterior function gradient verification
<code>bfgs</code>	TRUE	log-posterior maximization uses BFGS methods
<code>itpr</code>	FALSE	prior density function provided for new event parameters
<code>fpr_t</code>	NULL	location of function computing log-prior density for new event parameters
<code>fgpr_t</code>	NULL	location of function computing gradients of log-prior density for new event parameters
<code>imcmc</code>	"FME"	MCMC algorithm (current options: "RAM", "FME", "NUTS", "SMC")
<code>pl</code>	"multicore"	strategy for running parallel jobs using the <code>future</code> package
<code>ncor_smc</code>	NULL	number of cores for inner parallelization of SMC code
<code>lb_smc</code>	NULL	lower bounds of new event parameters for SMC sampling
<code>ub_smc</code>	NULL	upper bounds of new event parameters for SMC sampling
<code>t_cal</code>	NULL	object required if bounds supplied to log-posterior maximization

## 2.2.4 Output

The output file `runMPEM_0.out` from the second stage analysis contains a summary of new event device parameter estimates and (if relevant) posterior samples derived from the new event data, based on fixed (or multiply imputed) errors-in-variables yield values for the relevant calibration sources (if relevant), and forward and error model parameter values from the first stage analysis derived from the calibration data. The desired output is supplied by the user function `print_sumstats_0.r`, placed in the application code directory; in this example,

`MultiPEM_Toolbox_Package/Applications/Code/IYDT`

The output presented in Appendix A.8 contains the most pertinent information extracted from the full file.

- Lines 7-62: Output from the maximum likelihood estimation function `calc_mle_0`:
  - Line 9: Convergence code from the R optimization function `optim`. In this example, ‘0’ indicates successful completion.
  - Line 10: Number of optimization restarts in which the relative absolute maximum log-likelihood difference is  $\leq 10^{-8}$ . The algorithm exits after 2 such restarts, which is attained in this example.
  - Line 18: Maximum likelihood estimates of the new event device parameters; in this example, log-yield (`W`) and height-of-burst (`HOB`).
  - Line 23: Standard errors of the maximum likelihood estimates of the new event device parameters, adjusted for estimation of the forward model parameters and (if relevant) calibration source errors-in-variables yields in the first stage (WPA, §A.2, Equation (19); §A.4, Equation (21)).
  - Line 28: Standard errors of the maximum likelihood estimates of the new event device parameters, assuming the forward model parameters and (if relevant) calibration source errors-in-variables yields are known with certainty (set to their first stage values) (WPA, §A.2, p. 36, calculated from  $(\mathcal{I}_{\theta_0, \theta_0}^0)^{-1}$ ).
  - Lines 32-34: Correlation matrix of the maximum likelihood estimates of the new event device parameters, adjusted for estimation of the forward model parameters and (if relevant) calibration source errors-in-variables yields in the first stage (WPA, §A.2, Equation (19); §A.4, Equation (21)).
  - Lines 38-40: Correlation matrix of the maximum likelihood estimates of the new event device parameters, assuming the forward model parameters and (if relevant) calibration source errors-in-variables yields are known with certainty (set to their first stage values) (WPA, §A.2, p. 36, calculated from  $(\mathcal{I}_{\theta_0, \theta_0}^0)^{-1}$ ).
  - Lines 45-46: 95% confidence intervals for the unknown true values of the new event device parameters, based on standard errors adjusted for estimation of the forward model parameters and (if relevant) calibration source errors-in-variables yields in the first stage (WPA, §A.2, Equation (19); §A.4, Equation (21)).



- Lines 51-52: 95% confidence intervals for the unknown true values of the new event device parameters, based on standard errors assuming the forward model parameters and (if relevant) calibration source errors-in-variables yields are known with certainty (set to their first stage values) (WPA, §A.2, p. 36, calculated from  $(\mathcal{I}_{\theta_0, \theta_0}^0)^{-1}$ ).
- Lines 57-62: Example of log-likelihood gradient verification at a single sampled parameter vector. Additional checks were deleted for brevity.
  - \* Line 58: Analytic gradient calculation
  - \* Line 60: Numerical gradient calculation using the R package `numDeriv`
  - \* Line 62: Largest negative (first entry) and positive (second entry) differences between the analytic and numerical gradients
- Lines 74-131: Output from the Bayesian analysis function `calc_bayes_0`:
  - Line 76: Convergence code from the R optimization function `optim`. In this example, ‘0’ indicates successful completion.
  - Line 77: Number of optimization restarts in which the relative absolute maximum log-posterior difference is  $\leq 10^{-8}$ . The algorithm exits after 2 such restarts, which is attained in this example.
  - Line 85: Maximum *a posteriori* estimates of the new event device parameters.
  - Lines 89-94: Example of log-prior gradient verification at a single sampled parameter vector. Additional checks were deleted for brevity.
    - \* Line 90: Analytic gradient calculation
    - \* Line 92: Numerical gradient calculation using the R package `numDeriv`
    - \* Line 94: Largest negative (first entry) and positive (second entry) differences between the analytic and numerical gradients
  - Lines 98-103: Example of log-posterior gradient verification at a single sampled parameter vector. Additional checks were deleted for brevity.
    - \* Line 99: Analytic gradient calculation
    - \* Line 101: Numerical gradient calculation using the R package `numDeriv`
    - \* Line 103: Largest negative (first entry) and positive (second entry) differences between the analytic and numerical gradients
  - Line 107: Acceptance rate of the Delayed Rejection Adaptive Metropolis (DRAM) posterior sampling method implemented in R package `FME`. Note that one delayed rejection step is allowed in the default implementation.
  - Line 113: Means of samples from the new event device parameter marginal posterior distributions.

- Line 115: Standard deviations of samples from the new event device parameter marginal posterior distributions.
- Lines 117-125: User specified quantiles of samples from the new event device parameter marginal posterior distributions.
- Lines 129-131: Correlation matrix of samples from the new event device parameter joint posterior distribution.

The `p_cal` environment resulting from this run contains several elements of potential interest for additional post-processing:

- `p_cal$mle`: Maximum likelihood estimate of unbounded new event device parameters (i.e., on scale used by the optimizer)
- `p_cal$Sigma_mle$II_nev_it`: Estimated asymptotic covariance matrix of `p_cal$mle`, adjusted for first stage estimation of quantities stated below
- `p_cal$Sigma_mle$II_nev_0_it`: Estimated asymptotic covariance matrix of `p_cal$mle`, assuming first stage estimates of quantities stated below are known with certainty
- `p_cal$tmle`: Maximum likelihood estimate of transformed new event device parameters (i.e., on correct scale)
- `p_cal$Sigma_mle$II_nev`: Estimated asymptotic covariance matrix of `p_cal$tmle`, adjusted for first stage estimation of quantities stated below
- `p_cal$Sigma_mle$II_nev_0`: Estimated asymptotic covariance matrix of `p_cal$tmle`, assuming first stage estimates of quantities stated below are known with certainty
- `p_cal$map`: If `iBayes` is TRUE (Line 6 of Appendix A.7), maximum *a posteriori* estimate of unbounded new event device parameters (i.e., on scale used by the optimizer)
- `p_cal$tmmap`: If `iBayes` is TRUE (Line 6 of Appendix A.7), maximum *a posteriori* estimate of transformed new event device parameters (i.e., on correct scale)
- `p_cal$mpi`: For multiple imputation (`nimpute` > 1; Line 52 of Appendix A.5)), first stage posterior samples of (if relevant) calibration source errors-in-variables yields, forward and error model parameters based on calibration data, used as second stage imputation values of these parameters if `iBayes` is TRUE (Line 6 of Appendix A.7)
- `p_cal$tmmpi`: If `iBayes` is TRUE (Line 6 of Appendix A.7), posterior samples of transformed new event device parameters (i.e., on correct scale)

These quantities are based on using fixed (or multiply imputed) values for the forward and error model parameters and (if relevant) the calibration source errors-in-variables yields from the first stage analysis.

### 3 Complete Assessment

Complete assessments will be illustrated by examining the run files associated with a multi-phenomenology analysis in which signals from four phenomenologies are combined to infer the log-yield and height-of-burst (HOB)/depth-of-burial (DOB) of a near-surface nuclear explosion (WPA, §6).

```
% cd ./Runfiles/IYDT/4-Phen/I-EIV-SUGAR-hob
```

Complete assessments involve combining calibration and (if relevant) new event data to simultaneously estimate forward model parameters (e.g. regression coefficients), error model parameters (e.g. source bias, path bias, observational error covariance), errors-in-variables yield values of calibration sources (if relevant), and (if relevant) new event device parameters (e.g. yield, HOB/DOB, geolocation, event time) with uncertainty quantification. Complete assessments are more computationally intensive than rapid assessments, as they require that all parameters are inferred simultaneously for each new event of interest.

The analysis is defined in the `runMPEM.r` file, provided in Appendix B with line numbers referred to in the ensuing discussion. Appendix B.1 provides the preprocessing component, Appendix B.2 provides the code employed to maximize the likelihood function of the data with respect to all of the forward model, error model, (if relevant) errors-in-variables yield, and (if relevant) new event device parameters, while Appendix B.3 provides the code employed to optionally sample the posterior distribution of these parameters.

The complete analysis is run in batch mode as follows,

```
% R CMD BATCH runMPEM.r runMPEM.out &
```

The main features of the output file `runMPEM.out` are provided in Appendix B.4.

#### 3.0.1 Preprocessing

The preprocessing component of the analysis in Appendix B.1 is primarily responsible for describing features of the calibration and (if relevant) new event data and all parameters of inferential interest.

- Line 12+: Load all R packages utilized by multiple supporting subroutines, most notably log-likelihood and log-prior calculations and their associated gradients.
- Line 23: Specify directory location (relative to run directory) of all global (application independent) subroutines.
- Line 26: Read in code performing preprocessing of calibration and (if relevant) new event data.
- Line 29: Specify directory location (relative to run directory) of all application-specific subroutines.
- Line 32: Specify root directory (relative to run directory) containing all application-specific calibration and (if relevant) new event data files.

- Lines 35-38: A scalar or vector specifying the names of calibration data files for each phenomenology, utilizing an ordering of the phenomenologies (for MultiPEM analysis) that is maintained throughout the input deck (as indicated here in Lines 41-44). Data files are text files (CSV formatted) containing all measured signatures (in the first column(s)) and input covariates (in succeeding column(s)) including all those required in forward and error model calculations. Directories specifying the exact locations of these files relative to the root data directory (Line 32) may also be included in the filenames.
- Line 47: A scalar or vector specifying the number of observed signatures for each phenomenology; in this example, 2 for each phenomenology.
- Lines 51-58: Specify the number of *common* forward model parameters for each phenomenology (WPA, §5.1, first paragraph). For a given forward model, common parameters maintain the same constant value for every log-likelihood calculation. The **pbeta** object is initialized as a null list with elements for each phenomenology in the proper order (Line 51), initialized to zero vectors of length equal to the number of observed signatures (Line 52). Subsequent lines specify the number of common forward model parameters for each signature within each phenomenology. For example, the *acoustic* forward model for each signature contains 2 common forward model parameters (Line 54).
- Lines 61-64: Specify if the forward model(s) for any phenomenology depend on event emplacement conditions (Line 61), followed by (if relevant) a vector indicating the number of distinct emplacement conditions considered for each phenomenology in the proper order (Line 63). This specification allows distinct forward model parameters to be associated with different emplacement conditions (as specified subsequently). If **Th** is **TRUE** (Line 61), a factor named **Type** must be present in the calibration and (if relevant) new event data files for each relevant phenomenology, indicating the emplacement condition pertaining to each entry. In this example, the *seismic* and *acoustic* forward model parameters may vary for 3 distinct emplacements (“soft”, “hard”, and “wet” rock types), while the *optical* and *surface effects* forward models are independent of emplacement condition.
- Lines 68-83: Specify the number of *emplacement* dependent forward model parameters for each phenomenology if relevant (WPA, §5.1, first paragraph). For a given forward model, emplacement parameters remain constant for log-likelihood calculations with a given emplacement condition, but may be modified for each distinct emplacement. The **pbetat** object is initialized as a null list with elements for each phenomenology in the proper order (Line 69), initialized as null lists with elements for each emplacement condition (Line 71) if multiple emplacements are present. Subsequent lines specify the number of forward model parameters for each signature within each emplacement condition for each phenomenology. For example, the *seismic* forward model for each signature within each emplacement contains 5 forward model parameters (Line 76) allowed to vary across emplacements. **pbetat** must be specified if multiple emplacements are present for any phenomenology (at least one element of **Th** is greater than 1).

- Lines 87-102: Specify the location of *common* forward model parameters within the full parameter vector, for phenomenologies possessing both common and emplacement dependent parameters. The `ibetar` object is initialized as a null list with elements for each phenomenology in the proper order (Line 88), initialized as null lists with elements for each signature within each emplacement condition (Line 93) if multiple emplacements are present. Subsequent lines specify the position of common parameters in the full forward model parameter vector, for phenomenologies possessing both common and emplacement dependent forward model parameters. For example, the *acoustic* forward model parameter vector takes common parameter values in its first two positions for each signature within each emplacement condition (Line 99).
- Line 105: Indicate if errors-in-variables yield values for calibration events will be modeled (WPA, §3, Equation (3); §A.4). If `TRUE`, this allows uncertain yields for calibration events (often assumed known with certainty) to vary within user-specified guidelines.
- Lines 108-124: If relevant, specify details of errors-in-variables yield models for calibration events.
  - Line 111: Specify phenomenologies for application of errors-in-variables yield models to calibration events
  - Lines 115-116: Provide the sources subject to errors-in-variables yield models for each phenomenology. The `seiv` object is initialized as a null list with elements for each phenomenology in the proper order (Line 115), with vectors indicating the relevant sources for each relevant phenomenology (Line 116). The "ALL" designation indicates that every source in the calibration data set for the indicated phenomenology will be modeled with an errors-in-variables yield. `seiv` must be specified if `ieiv` is provided.
  - Line 119: The standard deviation of the errors-in-variables Gaussian distribution for each calibration event log-yield. For each event, the mean of this distribution is taken to be its provided (design or measured) log-yield. In this example, a “total” error (3 standard deviations) of 10% in each provided yield is allowed. Note that this error is relative because yields are treated on a logarithmic scale. `eiv_w_sd` must be specified if `ieiv` is provided.
- Lines 128-137: Specify if level 1 (*source*) random effects (WPA, §3, Equation (2); §4; §A.5) should be included in the error model (Line 128). If so, the `pvc_1` object is initialized as a null list with elements for each phenomenology in the proper order (Line 131), initialized to zero vectors of length equal to the number of observed signatures (Line 132). Subsequent lines specify the number of level 1 random effects for each signature within each phenomenology. For example, the *seismic* error model for each signature contains a single source bias term (Line 134). If `pvc_1` is `TRUE` (Line 128), a factor named `Source` may be provided in the calibration and (if relevant) new event data files for each relevant phenomenology, identifying the source pertaining to each entry. This factor must be present if there is more than one data entry for any source. In order to include source random effects in the error model for an observed signature, the calibration data must contain more than one source, with at least one source

containing more than one observation. A warning message will be printed to the log file if one of these conditions is violated.

- Lines 140-149: Specify if level 2 (*path*) random effects (WPA, §3, Equation (2); §4; §A.5) should be included in the error model (Line 140). If so, the `pvc_2` object is initialized as a null list with elements for each phenomenology in the proper order (Line 143), initialized to zero vectors of length equal to the number of observed signatures (Line 144). Subsequent lines specify the number of level 2 random effects for each signature within each phenomenology. For example, the *seismic* error model for each signature contains a single path bias term (Line 146). If `pvc_2` is `TRUE` (Line 140), a factor named `Path` must be provided in the calibration and (if relevant) new event data files for each relevant phenomenology, identifying the path (e.g. sensor network) pertaining to each entry. In order to include path random effects in the error model for an observed signature, a source random effect must also be present, the calibration data must contain more than one path for at least one source, with more than one observation for at least one of those paths. A warning message will be printed to the the log file if one of these conditions is violated.
- Line 153: Indicate if the user is providing code to compute coefficient matrices for level 1 (*source*) or level 2 (*path*) random effects (WPA, §5.1). If `FALSE`, the function `calc_zmat.r` located in the global code directory,

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computes default coefficient matrices for the calibration and (if relevant) new event data. If `TRUE`, then a user-provided function of the same name must be placed in the application code directory; in this example,

`MultiPEM_Toolbox_Package/Applications/Code/IYDT`

The following table shows data for the first *seismic* calibration source in this example,

Y1	Y2	Source	Path	Type	lRange	W	HOB
-15.667	-9.625	HRI-1	Path_1	1	6.932	6.291	5
-15.665	-9.554	HRI-1	Path_1	1	6.932	6.291	5
-16.412	-10.591	HRI-1	Path_2	1	7.570	6.291	5
-16.468	-10.554	HRI-1	Path_2	1	7.570	6.291	5
-16.752	-10.931	HRI-1	Path_2	1	7.800	6.291	5
-17.483	-11.739	HRI-1	Path_2	1	8.371	6.291	5
-17.507	-11.711	HRI-1	Path_2	1	8.371	6.291	5

If level 1 and level 2 random effects are included in the error model, the source and path bias vectors (WPA, §5.1, p. 7) associated with this source are given by

$$\mathbf{E}_{S,11r} = \begin{pmatrix} \mathbf{Z}_{11r,1} \\ \mathbf{Z}_{11r,2} \end{pmatrix} b_{1r}^{(S)} \text{ and } \mathbf{E}_{P,11r} = \begin{pmatrix} \mathbf{E}_{P,111r} \\ \mathbf{E}_{P,112r} \end{pmatrix} = \begin{bmatrix} \mathbf{Z}_{111r} & \mathbf{0}_2 \\ \mathbf{0}_5 & \mathbf{Z}_{112r} \end{bmatrix} \begin{pmatrix} b_{1r,1}^{(P)} \\ b_{1r,2}^{(P)} \end{pmatrix},$$

where the default coefficient matrices are given by

$$\begin{array}{ll} \mathbf{Z}_{11r,1} = \mathbf{1}_2 & \mathbf{Z}_{111r} = \mathbf{1}_2 \\ \mathbf{Z}_{11r,2} = \mathbf{1}_5 & \mathbf{Z}_{112r} = \mathbf{1}_5 \end{array}$$

for  $\mathbf{1}_q$  and  $\mathbf{0}_q$  the  $q$ -vectors of ones and zeros, respectively. For each signature, this structure indicates there is a single source bias effect applied to every observation, while observations from each path are adjusted by distinct (and independently distributed) path bias effects (for this source, signatures are collected from two pathways).

- Line 157: Specify if bounded optimization of any unknown parameters is to be conducted. This option is currently only supported for new event device parameters. The default is to optimize all new event device parameters on an unbounded input space, transforming them to their input domain (specified subsequently in this preprocessing file) as necessary for forward model calculations. If `opt_B` is `TRUE`, the new event device parameters are optimized directly on their input domain.
- Line 160: Indicate if new event device parameters are to be estimated with uncertainty quantification simultaneously from the calibration and new event data. If `nev` is `FALSE`, only forward and error model parameters, and calibration source errors-in-variables yields (if relevant), are inferred from the calibration data.
- Lines 163-208: If relevant, specify details of new event device parameters and location(s) of new event data.
  - Lines 165-168: A scalar or vector specifying the names of new event data files for each phenomenology, utilizing an ordering of the phenomenologies (for MultiPEM analysis) that is consistent with the calibration data files and maintained throughout the input deck (as indicated here in Lines 41-44). Data files are text files (CSV formatted) containing all measured signatures (in the first column(s)) and input covariates (in succeeding column(s)) including all those required in forward and error model calculations, but excepting the new event device parameters that are unknown and subject to inference. Directories specifying the exact locations of these files relative to the root data directory (Line 32) may also be included in the filenames. Must be provided if `nev` is `TRUE` (Line 160).
  - Line 171: Specify the names of the new event device parameters of inferential interest as a vector of strings. This information is utilized in postprocessing, and must be provided if `nev` is `TRUE` (Line 160).
  - Line 174: Indicate if the new event device parameters are subjected to a user-provided bijective transformation supplied to assist likelihood maximization or posterior sampling. If `itransform` is `TRUE`, the code implementing this transformation is concatenated into a single file named `transform.r` and placed in the application code directory; in this example,

`MultiPEM_Toolbox_Package/Applications/Code/IYDT`

The functions that must be provided in `transform.r` include the following:

- \* **tau**: Function  $\tau(\cdot)$  applied to transformed variables  $\tilde{\boldsymbol{\theta}}_0$  with the new event device parameters  $\boldsymbol{\theta}_0$  as its image,

$$\boldsymbol{\theta}_0 = \tau(\tilde{\boldsymbol{\theta}}_0)$$

- \* **j\_tau**: Jacobian matrix of  $\tau(\cdot)$ ,

$$\mathbf{J}_\tau(\tilde{\boldsymbol{\theta}}_0) = \begin{bmatrix} \frac{\partial \tau_1(\tilde{\boldsymbol{\theta}}_0)}{\partial \tilde{\theta}_{0,1}} & \dots & \frac{\partial \tau_1(\tilde{\boldsymbol{\theta}}_0)}{\partial \tilde{\theta}_{0,q}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \tau_q(\tilde{\boldsymbol{\theta}}_0)}{\partial \tilde{\theta}_{0,1}} & \dots & \frac{\partial \tau_q(\tilde{\boldsymbol{\theta}}_0)}{\partial \tilde{\theta}_{0,q}} \end{bmatrix}$$

where  $q$  is the dimension of  $\boldsymbol{\theta}_0$ .

- \* **log\_absdet\_j\_tau**: Logarithm of the absolute value of the determinant of the Jacobian matrix computed from **j\_tau**,

$$\log \text{abs}(\det(\mathbf{J}_\tau(\tilde{\boldsymbol{\theta}}_0)))$$

- \* **dlog\_absdet\_j\_tau**: Gradient of the log absolute Jacobian determinant with respect to  $\tilde{\boldsymbol{\theta}}_0$

- \* **inv\_tau**: Inverse function of  $\tau(\cdot)$

In this example, the new event device parameters of inferential interest are log-yield  $w$  and height-of-burst  $h$ , that is  $\boldsymbol{\theta}_0 = (w, h)$ . The relevant forward models are functions of a scaled height-of-burst,  $\tilde{h} = h \exp(-w/3)$ , suggesting the possible utility of likelihood maximization or posterior sampling in terms of  $\tilde{\boldsymbol{\theta}}_0 = (\tilde{w}, \tilde{h})$  for  $\tilde{w} = w$ .

- Lines 178-184: If **itransform** is TRUE, and if **tPars** is TRUE, initialize **tPars** to a null list (Line 181). Subsequent lines provide the value(s) for all fixed inputs required to compute the function **tau** (see previous item). In this example, a **yield\_scaling** value is required (Line 182).
- Lines 188-191: Specify lower and upper bounds for the new event device parameters if needed. By default, lower bounds are set to  $-\infty$  (Line 188) and upper bounds to  $+\infty$  (Line 190). In this example, the second parameter (height-of-burst) is restricted to the range  $(-10, 160)$  (Lines 189 and 191). Note that likelihood maximization and posterior sampling are conducted on an unbounded parameter space. If lower or upper bounds are specified for any parameter, they are applied just prior to objective function calculations using the **transform** function of the **transform.r** file located in the global code directory,

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- Lines 194-199: If `tsub` is `TRUE`, the forward model for at least one phenomenology depends only on a subset of the full vector  $\theta_0$  of new event device parameters. The `tsub` object is initialized to a null list with elements for each phenomenology in the proper order (Line 197). The `theta_names` vector (Line 171) describes the order of elements in  $\theta_0$ . For relevant phenomenologies, parameter subsets are specified as integer vectors identifying the extracted elements of  $\theta_0$ . The forward models of all other phenomenologies depend on the full  $\theta_0$ . In this example, the *surface effects* (*crater*) phenomenology only depends on log-yield (Line 198), while the other phenomenologies depend on both log-yield and height-of-burst.
- Lines 211-215: Calls the preprocessing function `prepro` for the calibration and (if relevant) new event data. Table 7 describes all inputs to this function with default values. Only inputs with no default values must be provided.
- Lines 216-222: If `opt_B` is `TRUE` (Line 157), the preprocessor function `prepro` returns a list (designated here as `tmp`) with objects `p_cal` and `t_cal`, which are then assigned as follows

```
% p_cal = tmp$p_cal
% t_cal = tmp$t_cal
```

and both are utilized for maximum likelihood estimation and Bayesian analysis. Otherwise, `p_cal` is the only object returned,

```
% p_cal = tmp$p_cal
```

and utilized in subsequent analyses.

### 3.0.2 Maximum Likelihood Estimation

The maximum likelihood estimation component of the complete analysis in Appendix B.2 is responsible for utilizing calibration and (if relevant) new event data to simultaneously estimate the parameters of the forward and error models, possibly the yield of each calibration source for phenomenologies adopting the errors-in-variables yield model, and (if relevant) the new event device parameters (WPA, §A.1-A.2). If `nev` is `TRUE` (Line 160 of Appendix B.1), quantification of uncertainty in the new event device parameter estimates is provided, adjusting for asymptotically dependent quantities.

- Line 6: Read in code performing simultaneous maximum likelihood estimation of forward and error model parameters, calibration source errors-in-variables yields (if relevant), and new event device parameters (if relevant), based on calibration and (if relevant) new event data.
- Line 9: User specified seed to ensure repeatability of maximum likelihood estimation.
- Lines 13-17: Provide names of forward models for each signature by phenomenology (WPA, §6.1-6.4). The `fm` object is initialized as a null list with elements for each phenomenology in the proper order (Line 13). Subsequent lines specify the function names as vectors of strings having length equal to the number of signatures for each

Table 7: Inputs to **prepro** function.

Input	Default	Brief Description
<b>gdir</b>	none	directory location of global subroutines
<b>adir</b>	none	directory location of application subroutines
<b>rdir</b>	none	root directory location of data files
<b>cdir</b>	none	directory locations (if relevant) and names of calibration data files under <b>rdir</b>
<b>Rh</b>	none	vector with number of signatures for each phenomenology
<b>pbeta</b>	none	list containing empirical model common parameter counts by phenomenology
<b>bopt</b>	FALSE	parameter bounds supplied to log-likelihood maximization (currently implemented only for new event parameters)
<b>nev</b>	FALSE	analysis of new event
<b>itr</b>	FALSE	bijective transform of new event parameters provided
<b>izmat</b>	FALSE	user-provided code for computing variance component coefficient matrices
<b>ieiv</b>	NULL	numerical identifier of phenomenologies utilizing errors-in-variables yields in analysis of calibration data
<b>seiv</b>	NULL	list containing identifiers of calibration sources assigned errors-in-variables yields by phenomenology ( <b>ALL</b> – every source)
<b>ewsd</b>	NULL	standard deviation of errors-in-variables Gaussian likelihood
<b>Th</b>	NULL	number of emplacement conditions for each phenomenology
<b>pbetat</b>	NULL	list containing empirical model emplacement-dependent parameter counts by phenomenology
<b>ibetar</b>	NULL	list containing locations of empirical model common parameters in full parameter vector by phenomenology
<b>pvc.1</b>	NULL	list containing level 1 (source) variance component parameter counts by phenomenology
<b>pvc.2</b>	NULL	list containing level 2 (path) variance component parameter counts by phenomenology
<b>tnames</b>	NULL	names of new event parameters
<b>fp_tr</b>	NULL	fixed inputs to new event parameter transform
<b>tlb</b>	NULL	lower bounds for new event parameters
<b>tub</b>	NULL	upper bounds for new event parameters
<b>ndir</b>	NULL	directory locations (if relevant) and names of new event data files under <b>rdir</b>
<b>tsub</b>	NULL	list containing index sets identifying new event parameter subsets by phenomenology if relevant

phenomenology (Lines 14-17). The code for all forward models from each phenomenology is concatenated into a single file named **forward.r** and placed in the application code directory; in this example,

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Note that these forward models accept a vector of calibration and device parameters as their main argument. In this example, the *seismic* forward model  $f_{sr}(\cdot)$  as a function of the calibration parameters  $\beta_{sr}$  and device parameters log-yield ( $w$ ) and HOB/DOB ( $h$ ) is given as follows (WPA, §6.2, p. 11),

$$\begin{aligned}\log(\tilde{d}_{sr}(\beta_{sr}, (w, h))) &= \beta_{sr,1} + \beta_{sr,2} \log(\tilde{\delta}_s) + \beta_{sr,3} \text{logistic}(\beta_{sr,4} \tilde{h}_s + \beta_{sr,5}) \\ f_{sr}(\beta_{sr}, (w, h)) &= \log(d_{sr}(\beta_{sr}, (w, h)))\end{aligned}\tag{3}$$

for

$$\text{logistic}(x) = \frac{1}{1 + \exp(-x)}.$$

The scaled signatures and covariates of this forward model are given by

$$\begin{aligned}\tilde{d}_{s1} &= d_{s1} \exp(-w/3) & \tilde{d}_{s2} &= d_{s2} \\ \tilde{\delta}_s &= \delta \exp(-w/3) & \tilde{h}_s &= h \exp(-w/3),\end{aligned}$$

where  $d_{s1}$  and  $d_{s2}$  are P-wave displacement and maximum velocity, and the covariate is range  $\delta$ . The function `f_s` returns a vector of forward model calculations evaluated for the supplied value of  $(\beta_{sr}, (w, h))$ , each element corresponding to each row of a matrix of covariates (in this case, a column vector of ranges  $\delta$ ).

- Line 20: Indicate if forward model Jacobian matrices are provided for efficient log-likelihood maximization.
- Lines 22-30: If `igrad` is `TRUE`, names of forward model Jacobian functions must be provided for each signature by phenomenology. The `gfm` object is initialized as a null list with elements for each phenomenology in the proper order (Line 25). Subsequent lines specify the Jacobian function names as vectors of strings having length equal to the number of signatures for each phenomenology (Lines 26-29). The code for all forward model Jacobian functions from each phenomenology is concatenated into a single file named `jacobian.r` and placed in the application code directory; in this example,

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Note that these Jacobian functions accept a vector of calibration and device parameters as their main argument. In this example, the gradient vector of the *seismic* forward model of Equation (3) is computed from the partial derivatives of  $f_{sr}(\cdot)$  for

each calibration parameter as follows,

$$\begin{aligned}\frac{\partial f_{sr}}{\partial \beta_{sr,1}} &= 1 \\ \frac{\partial f_{sr}}{\partial \beta_{sr,2}} &= \log(\tilde{\delta}_s) \\ \frac{\partial f_{sr}}{\partial \beta_{sr,3}} &= \text{logistic}(\beta_{sr,4}\tilde{h}_s + \beta_{sr,5}) \\ \frac{\partial f_{sr}}{\partial \beta_{sr,4}} &= \beta_{sr,3}\tilde{h}_s \times \text{logistic}(\beta_{sr,4}\tilde{h}_s + \beta_{sr,5}) \times \text{logistic}(-\beta_{sr,4}\tilde{h}_s - \beta_{sr,5}) \\ \frac{\partial f_{sr}}{\partial \beta_{sr,5}} &= \beta_{sr,3} \times \text{logistic}(\beta_{sr,4}\tilde{h}_s + \beta_{sr,5}) \times \text{logistic}(-\beta_{sr,4}\tilde{h}_s - \beta_{sr,5}),\end{aligned}$$

and each device parameter as follows,

$$\begin{aligned}\frac{\partial f_{sr}}{\partial w} &= -\frac{1}{3} \left( \beta_{sr,2} + \beta_{sr,3}\beta_{sr,4}\tilde{h}_s \times \text{logistic}(\beta_{sr,4}\tilde{h}_s + \beta_{sr,5}) \times \text{logistic}(-\beta_{sr,4}\tilde{h}_s - \beta_{sr,5}) \right) \\ &\quad + \frac{1}{3} \delta_1(r) \\ \frac{\partial f_{sr}}{\partial h} &= \beta_{sr,3}\beta_{sr,4} \exp(-w/3) \times \text{logistic}(\beta_{sr,4}\tilde{h}_s + \beta_{sr,5}) \times \text{logistic}(-\beta_{sr,4}\tilde{h}_s - \beta_{sr,5})\end{aligned}$$

for  $\delta_A(x)$  the indicator function of set  $A$ . The function `g_s` returns a Jacobian matrix (`jbeta_s`) of forward model gradients for the calibration parameters, evaluated at the supplied value of  $(\beta_{sr}, (w, h))$ , with rows corresponding to the rows of a matrix of covariates (in this case, a column vector of ranges  $\delta$ ). If relevant, `g_s` will also return a Jacobian matrix (`jtheta_s`) of forward model gradients for the device parameters, evaluated at the supplied value of  $(\beta_{sr}, (w, h))$ , each element corresponding to each row of the same covariate matrix used in the calculation of `jbeta_s`.

- Line 35: Indicate if the same forward model function is used to compute multiple signatures, and signature-specific code within this function is required.
- Lines 37-41: If `iResponse` is `TRUE`, initialize `iResponse` to a null list with elements for each phenomenology in the proper order (Line 38). For each relevant phenomenology, subsequent lines provide vectors of length equal to the number of signatures, each element of which is a tag identifying code specific to the corresponding signature. This mechanism is utilized for the *seismic* (Line 39) and *acoustic* (Line 40) phenomenologies.
- Line 44: Indicate if fixed inputs are to be provided to the forward models for at least one phenomenology.
- Lines 47-54: If `fPars` is `TRUE`, initialize `fPars` to a null list with elements for each phenomenology in the proper order (Line 47). For each relevant phenomenology, subsequent lines provide the value(s) of all fixed inputs. For example, the *optical* forward model requires fixed values for `yield_scaling` (Line 52), `pressure_scaling` (Line 53), and `temp_scaling` (Line 54).

- Line 58: Specify the number of starting parameter vectors for the log-likelihood maximization routine.
- Line 61: Specify the number of cores to use for parallel optimization (across distinct starting values) of the calibration and (if relevant) new event data log-likelihood function.
- Line 64: Specify if the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm is to be used for maximization of the log-likelihood function. If **TRUE**, functions to compute forward model Jacobian matrices analytically must be provided, or numerical gradients will be utilized (generally increasing compute time). If **FALSE**, the gradient-free Nelder-Mead simplex algorithm will be utilized for optimization, which is generally much slower computationally than BFGS with analytical gradients.
- Lines 68-71: If relevant, specifies the location (relative to the run directory) of parameter values or estimates to be used as the first starting value for log-likelihood maximization. These values are stored in a **.RData** object as a list, with elements corresponding to forward model (e.g. common parameters **beta**, emplacement-dependent parameters **tbeta**) and error model (e.g. level 1 variance components **vc\_1**, level 2 variance components **vc\_2**, observation error parameters **eps**) quantities of interest. If relevant, calibration source errors-in-variables yield estimates (**w\_eiv**) are also provided. For multi-phenomenology analyses, values or estimates from individual phenomenologies may be input in the proper order, and they will be concatenated appropriately.
- Line 75: If desired, name of output **.RData** file to store optimization results from this run. The elements of the list to be written are described in the previous item.
- Lines 77-88: If **nev** is **TRUE** (Line 160 of Appendix B.1), the following options are available:
  - Lines 79-84: If **tst** is **TRUE**, specifies an initial starting value for the new event device parameters (Lines 82-83) for log-likelihood maximization. This value is superseded by the value read in from the first entry of **opt\_files\_in** (Line 68), if the **theta0** list element is provided.
  - Line 87: Specify the level of confidence intervals computed for the true values of each new event device parameter from the maximum likelihood estimate and the estimated Fisher information matrix (WPA, §A.2, Equation (19); §A.4, Equation (21)).
- Line 95: Indicate if phenomenology specific code is required in the postprocessing function.
- Lines 97-99: If **Phen** is **TRUE**, specifies a matrix in which the first column provides the numerical phenomenology indicator (see Lines 41-44 of the preprocessing code in Appendix B.1), and the second column provides the phenomenology name in string format. In this example, specific code is required to process results for the *seismic* phenomenology (Line 98).
- Line 102: Indicate if gradient verification is to be conducted on the log-likelihood

function. If `TRUE` and `igrad` is `TRUE` (Line 20), analytical and numerical gradients at the optimal parameter value, and other randomly sampled parameter values, are compared for consistency.

- Line 105: Specify the strategy for running parallel jobs using the `future` package in R. The available options are given by starting an R session and issuing the following commands,

```
% R
> require(future)
> help(plan)
```

- Lines 108-112: Calls the log-likelihood maximization function `calc_mle` for the calibration and (if relevant) new event data. Table 8 describes all inputs to this function with default values. Only inputs with no default values must be provided.

### 3.0.3 Bayesian Analysis

The optional Bayesian inference component of the analysis in Appendix B.3 is responsible for sampling forward and error model parameters, calibration source errors-in-variables yields (if relevant), and new event device parameters (if relevant) from their joint posterior distribution, using calibration and (if relevant) new event data simultaneously. If `nev` is `TRUE` (Line 160 of Appendix B.1), estimates of new event device parameters with uncertainty quantification are computed from these samples.

- Line 6: Indicate if Bayesian analysis is to be conducted.
- Line 10: Read in code performing Bayesian analysis on forward and error model parameters, calibration source errors-in-variables yields (if relevant), and new event device parameters (if relevant), using calibration and (if relevant) new event data.
- Line 14: Indicate if a log-prior density for the forward model parameters is supplied by the user (WPA, §6.5, p. 15). If `iBetaPrior` is `FALSE`, a “flat prior” (uniform on the domain) on these parameters is assumed.
- Lines 16-35: If relevant, specify details of user-provided log-prior distributions for forward model parameters. For each relevant phenomenology, the list object `lp_beta` is used for common coefficients, while the list object `lp_betat` is used for emplacement-dependent coefficients (as demonstrated below in this application).
  - Line 18: Specify location(s) of log-prior function(s). Must be provided if `iBetaPrior` is `TRUE`. In this example, a single log-prior function is provided, located at
 

```
../Code/lp_beta.s.r
```
  - Line 20: If `igrad` is `TRUE` (Line 20 of Appendix B.2), specify location(s) of the log-prior gradient function(s). In this example, a single log-prior gradient function is provided, located at
 

```
../Code/glp_beta.s.r
```

Table 8: Inputs to `calc_mle` function.

Input	Default	Brief Description
<code>p_cal</code>	<code>none</code>	environment storing all objects needed in log-likelihood calculations
<code>gdir</code>	<code>none</code>	directory location of global subroutines
<code>adir</code>	<code>none</code>	directory location of application subroutines
<code>f</code>	<code>none</code>	names of forward model functions for each signature by phenomenology
<code>nst</code>	<code>10</code>	number of starting values for log-likelihood maximization
<code>ncor</code>	<code>1</code>	number of cores for log-likelihood maximization
<code>ci_nev</code>	<code>0.95</code>	confidence interval levels for new event parameter inference
<code>igrad</code>	<code>TRUE</code>	forward model Jacobian provided
<code>bfgs</code>	<code>TRUE</code>	log-likelihood maximization uses BFGS methods
<code>igrck</code>	<code>TRUE</code>	conduct log-likelihood function gradient verification
<code>t_cal</code>	<code>NULL</code>	object required if bounds supplied to log-likelihood maximization
<code>g</code>	<code>NULL</code>	names of forward model Jacobian functions for each signature by phenomenology
<code>iresp</code>	<code>NULL</code>	flags for modified calculation by signature in a common forward model for each relevant phenomenology
<code>fp_fm</code>	<code>NULL</code>	fixed inputs required by forward models
<code>fopt_in</code>	<code>NULL</code>	location of input R data file(s) providing an initial starting value for log-likelihood maximization (if multiple files, starting value created by concatenating over phenomenologies)
<code>Xst</code>	<code>NULL</code>	matrix of starting values for log-likelihood maximization if not generated by this function
<code>tst</code>	<code>NULL</code>	vector of starting values for new event parameters in log-likelihood maximization
<code>fopt_out</code>	<code>NULL</code>	location to write output R data file with results of log-likelihood maximization
<code>phen</code>	<code>NULL</code>	phenomenology number and type (if needed for postprocessing)
<code>pl</code>	<code>"multicore"</code>	strategy for running parallel jobs using the <code>future</code> package

- Line 25: For each relevant phenomenology, initialize a null list `lp_betat` of length equal to the number of emplacement conditions containing distinct forward model parameters.
- Line 27: For each relevant phenomenology and emplacement condition, provide the name(s) of the log-prior function(s) for each signature. In this example, the *seismic* phenomenology utilizes a log-prior function `lp_s` for each signature within each emplacement condition.
- Line 29: If `igrad` is `TRUE` (Line 20 of Appendix B.2), then for each relevant phe-

nomenology and emplacement condition, provide the name(s) of the log-prior gradient function(s) for each signature. In this example, the *seismic* phenomenology utilizes a log-prior gradient function `lq_s` for each signature within each emplacement condition.

- Line 39: Specify a *fixed* value for the scale parameter `A` of half-Cauchy prior distribution(s) for the level 1 (*source*) and level 2 (*path*) variance component parameters if relevant (WPA, §6.5, p. 15; §6.6, p. 17). Prior distributions for a non-empty collection of variance component parameters are taken to be mutually independent. Comment out if this parameter is to be sampled from its posterior distribution.
- Line 43: Specify a *fixed* value for the shape parameter  $\eta$  of the Lewandowski-Kurowicka-Joe (LKJ) prior distribution for the observational error model correlation parameters (WPA, §6.5, p. 15; §6.6, p. 17).
- Line 51: If `eiv` is `TRUE` (Line 105 of Appendix B.1), specify a *fixed* value for the parameter that controls the number of modes in the *flexible generalized skew-normal* (FGSN) prior distribution for the errors-in-variables yields of the calibration events (WPA, §6.5, p. 15; §6.6, p. 23).
- Line 57: Select the Markov chain Monte Carlo (MCMC) algorithm to use for posterior sampling, from one of three options: `RAM`, `FME`, and `NUTS`. `RAM` is the robust adaptive Metropolis algorithm of Vihola<sup>16</sup> implemented in the R package `adaptMCMC`. `FME` is the delayed rejection adaptive Metropolis algorithm of Haario, Laine, and Mira<sup>17</sup> implemented in the R package `FME`. `NUTS` is the No-U-Turn Sampler of Hoffman and Gelman<sup>18</sup>. The `NUTS` option requires the analytical gradient of the log-posterior density, which in turn requires `igrad` to be `TRUE` (Line 20 of Appendix B.2).
- Line 60: Specify the per core sample size of the burn-in period for MCMC sampling (pre-equilibrium stage of Markov chain). These samples are discarded prior to any inference using the posterior samples.
- Line 63: Specify the sample size of the MCMC production run. These samples are kept for posterior inference.
- Line 66: Specify the rate at which MCMC production samples are thinned for estimation of the Deviance Information Criterion<sup>19</sup> (DIC) and the Predictive Information Criterion<sup>20</sup> (PIC). In this example, the `nthin` value of 20 indicates that every 20-th production sample is kept for DIC and PIC estimation.
- Line 69: Specify the number of cores to use for parallel optimization (across distinct

---

<sup>16</sup>Vihola, M. (2012). Robust adaptive Metropolis algorithm with coerced acceptance rate. *Stat Comput* 22:997-1008.

<sup>17</sup>Haario, H., Laine, M., and Mira, A. (2006). DRAM: Efficient adaptive MCMC. *Stat Comput* 16:339-354.

<sup>18</sup>Hoffman, M. D. and Gelman, A. (2014). The No-U-Turn Sampler: Adaptively setting path lengths in Hamiltonian Monte Carlo. *J Mach Learn Res* 15:1593-1623.

<sup>19</sup>Spiegelhalter, D.J., Best, N.G., Carlin, B.P., & van der Linde, A. (2002). Bayesian measures of model complexity and fit (with discussion), *J R Stat Soc Ser B* 64:583-639.

<sup>20</sup>Ando, T. (2011). Predictive Bayesian model selection, *Am J Math Manag Sci* 31:13-38.



starting values) of the calibration and (if relevant) new event data log-posterior function.

- Line 72: Specify the number of cores used to run parallel MCMC chains. The burn-in period for each chain is determined by `nburn` (Line 60), while the `nmcmc` (Line 63) production runs are split between the `ncores_mc` processors and combined at the conclusion of the runs.
- Line 75: Indicate if a log-prior density for the new event device parameters is supplied by the user (WPA, §6.5, p. 15; §6.6, pp. 18-19). If `iTheta0Prior` is `FALSE`, a “flat prior” (uniform on the domain) on these parameters is assumed.
- Lines 77-97: If relevant, specify details of user-provided log-prior distributions for new event device parameters.
  - Line 79: Specify location of log-prior function. If `NULL`, utilize the default log-prior function contained in the file `lp_0.r` placed in the application code directory; in this example,
 

```
MultiPEM_Toolbox_Package/Applications/Code/IYDT
```
  - Line 81: If `igrad` is `TRUE` (Line 20 of Appendix B.2), specify location of the log-prior gradient function. If `NULL`, utilize the default log-prior gradient function contained in the file `glp_0.r` placed in the application code directory; in this example,
 

```
MultiPEM_Toolbox_Package/Applications/Code/IYDT
```
  - Line 85: Provide the name of the log-prior function.
  - Line 86: If `igrad` is `TRUE` (Line 20 of Appendix B.2), provide the name of the log-prior gradient function.
  - Lines 89-93: Specify all fixed quantities required for calculation of the log-prior density. In this example, the mean (Line 89) and standard deviation (Line 90) of the Gaussian prior distribution for log-yield, and the mean (Line 92) and standard deviation (Line 93) of the Gaussian prior distribution for height-of-burst, are specified.
- Line 100: Indicate if gradient verification is to be conducted on the log-prior function. If `TRUE` and `igrad` is `TRUE` (Line 20 of Appendix B.2), analytical and numerical gradients at the maximum *a posteriori* parameter value, and other randomly sampled parameter values, are compared for consistency.
- Line 103: Indicate if gradient verification is to be conducted on the log-posterior function. If `TRUE` and `igrad` is `TRUE` (Line 20 of Appendix B.2), analytical and numerical gradients at the maximum *a posteriori* parameter value, and other randomly sampled parameter values, are compared for consistency.
- Lines 106-114: Calls the Bayesian analysis function `calc_bayes` for the new event data. Table 9 describes all inputs to this function with default values. Only inputs with no

default values must be provided.

Table 9: Inputs to `calc_bayes` function.

Input	Default	Brief Description
<code>p_cal</code>	none	environment storing all objects needed in log-posterior calculations
<code>gdir</code>	none	directory location of global subroutines
<code>adir</code>	none	directory location of application subroutines
<code>nst</code>	10	number of starting values for log-posterior maximization
<code>nburn</code>	10000	number of per core MCMC burn-in samples
<code>nmcmc</code>	20000	number of MCMC production samples
<code>nthin</code>	20	posterior sample thinning rate
<code>ncor_map</code>	1	number of cores for log-posterior maximization
<code>ncor_mc</code>	1	number of cores for generating parallel MCMC chains
<code>igrad</code>	TRUE	forward model Jacobian provided
<code>igrck_pr</code>	TRUE	conduct log-prior function gradient verification
<code>igrck_po</code>	TRUE	conduct log-posterior function gradient verification
<code>bfgs</code>	TRUE	log-posterior maximization uses BFGS methods
<code>ibpr</code>	FALSE	prior density function(s) provided for forward model coefficients
<code>itpr</code>	FALSE	prior density function provided for new event parameters
<code>fpr_b</code>	NULL	location of functions computing log-prior density for forward model coefficients
<code>fgpr_b</code>	NULL	location of functions computing gradients of log-prior density for forward model coefficients
<code>fpr_t</code>	NULL	location of function computing log-prior density for new event parameters
<code>fgpr_t</code>	NULL	location of function computing gradients of log-prior density for new event parameters
<code>Xnom</code>	NULL	matrix of starting values for hyperparameters in log-posterior maximization if not generated by this function
<code>imcmc</code>	"FME"	MCMC algorithm (current options: "RAM", "FME", "NUTS")
<code>pl</code>	"multicore"	strategy for running parallel jobs using the <code>future</code> package
<code>t_cal</code>	NULL	object required if bounds supplied to log-posterior maximization

### 3.0.4 Output

The output file `runMPEM.out` from the complete analysis contains a summary of (if relevant) errors-in-variables yield estimates for the relevant calibration sources, forward and error model parameter estimates, as well as (if relevant) new event device parameter estimates derived from the calibration and (if relevant) new event data simultaneously. The desired output is supplied by the user function `print_sumstats.r`, placed in the application code directory; in this example,

The output presented in Appendix B.4 contains the most pertinent information extracted from the full file.

- Lines 7-18: Output from the preprocessing function `prepro`. These warning messages explain which variance component models are allowed (if any) for each signature of each phenomenology based on the structure of the calibration data. In this example, only level 1 (*source*) random effects are allowed for each *acoustic* signature (Lines 7-10), while no random effects are allowed for *optical* or *surface effects* phenomenologies (Lines 11-18). There are no warning messages for *seismic* signatures, indicating level 1 and level 2 (*path*) random effects are allowed.
- Lines 26-266: Output from the maximum likelihood estimation function `calc_mle`:
  - Line 28: Convergence code from the R optimization function `optim`. In this example, '0' indicates successful completion.
  - Line 29: Number of optimization restarts in which the relative absolute maximum log-likelihood difference is  $\leq 10^{-8}$ . The algorithm exits after 2 such restarts, which is attained in this example.
  - Line 37: Maximum likelihood estimates of the new event device parameters; in this example, log-yield (`W`) and height-of-burst (`HOB`).
  - Line 42: Standard errors of the maximum likelihood estimates of the new event device parameters, adjusted for estimation of the forward model parameters and (if relevant) calibration source errors-in-variables yields (WPA, §A.2, Equation (19); §A.4, Equation (21)).
  - Lines 46-48: Correlation matrix of the maximum likelihood estimates of the new event device parameters, adjusted for estimation of the forward model parameters and (if relevant) calibration source errors-in-variables yields (WPA, §A.2, Equation (19); §A.4, Equation (21)).
  - Lines 52-54: 95% confidence intervals for the unknown true values of the new event device parameters, based on standard errors adjusted for estimation of the forward model parameters and (if relevant) calibration source errors-in-variables yields (WPA, §A.2, Equation (19); §A.4, Equation (21)).
  - Lines 59-62: Maximum likelihood estimates of errors-in-variables yields for the relevant calibration sources. Source names (Lines 59 and 61) are given above yield estimates (Lines 60 and 62). Errors-in-variables yields are only estimated if `eiv` is `TRUE` (Line 105 of Appendix B.1).
  - Lines 66-88: Maximum likelihood estimates of *common* forward model parameters for each signature of each phenomenology (where present).
  - Lines 92-138: Maximum likelihood estimates of *emplacement-dependent* forward model parameters for each signature of each phenomenology (where present).

- Lines 142-156: Maximum likelihood estimates of *source* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 160-166: Maximum likelihood estimates of *path* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 170-216: Maximum likelihood estimates of observational error variances for each signature, and correlations between signatures, for each phenomenology (WPA, §A.5).
- Line 218: Akaike Information Criterion<sup>21</sup> (AIC) value based on calibration and (if relevant) new event data. Used for selecting among competing forward or error model specifications (WPA, §6.5, p. 15; §6.6, Tables 4 and 5, p. 18).
- Line 220: Bayesian Information Criterion<sup>22</sup> (BIC) value based on calibration and (if relevant) new event data. Used for selecting among competing forward or error model specifications (WPA, §6.5, p. 15; §6.6, Tables 4 and 5, p. 18).
- Lines 225-266: Example of log-likelihood gradient verification at a single sampled parameter vector. Additional checks were deleted for brevity.
  - \* Lines 226-244: Analytic gradient calculation
  - \* Lines 246-264: Numerical gradient calculation using the R package `numDeriv`
  - \* Line 266: Largest negative (first entry) and positive (second entry) differences between the analytic and numerical gradients
- Lines 268-1341: Output from the Bayesian analysis function `calc_bayes`:
  - Line 282: Convergence code from the R optimization function `optim`. In this example, ‘0’ indicates successful completion.
  - Line 283: Number of optimization restarts in which the relative absolute maximum log-posterior difference is  $\leq 10^{-8}$ . The algorithm exits after 2 such restarts, which is attained in this example.
  - Line 291: Maximum *a posteriori* estimates of the new event device parameters.
  - Lines 296-299: Maximum *a posteriori* estimates of errors-in-variables yields for the relevant calibration sources. Source names (Lines 296 and 298) are given above yield estimates (Lines 297 and 299). Errors-in-variables yields are only estimated if `eiv` is `TRUE` (Line 105 of Appendix B.1).
  - Lines 303-325: Maximum *a posteriori* estimates of *common* forward model parameters for each signature of each phenomenology (where present).

---

<sup>21</sup>Akaike, H. (1973). Information Theory and an Extension of the Maximum Likelihood Principle. In: Petrov, B.N. & Csaki, F., Eds., International Symposium on Information Theory, 267-281.

<sup>22</sup>Schwarz, G. (1978). Estimating the dimension of a model, *Ann Stat* 6:461-464.

- Lines 329-375: Maximum *a posteriori* estimates of *emplacement-dependent* forward model parameters for each signature of each phenomenology (where present).
- Lines 379-393: Maximum *a posteriori* estimates of *source* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 397-403: Maximum *a posteriori* estimates of *path* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 407-453: Maximum *a posteriori* estimates of observational error variances for each signature, and correlations between signatures, for each phenomenology (WPA, §A.5).
- Lines 457-459: Maximum *a posteriori* estimates of FGSN prior distribution parameters (WPA, §6.6, p. 23;  $\mathbf{Alpha} = \mu$ ,  $\mathbf{Omega} = v$  (two coefficients)).
- Lines 463-506: Example of log-prior gradient verification at a single sampled parameter vector. Additional checks were deleted for brevity.
  - \* Lines 464-483: Analytic gradient calculation
  - \* Lines 485-504: Numerical gradient calculation using the R package `numDeriv`
  - \* Line 506: Largest negative (first entry) and positive (second entry) differences between the analytic and numerical gradients
- Lines 510-553: Example of log-posterior gradient verification at a single sampled parameter vector. Additional checks were deleted for brevity.
  - \* Lines 511-530: Analytic gradient calculation
  - \* Lines 532-551: Numerical gradient calculation using the R package `numDeriv`
  - \* Line 553: Largest negative (first entry) and positive (second entry) differences between the analytic and numerical gradients
- Line 557: Acceptance rate of the Delayed Rejection Adaptive Metropolis (DRAM) posterior sampling method implemented in R package `FME`. Note that one delayed rejection step is allowed in the default implementation.
- Line 563: Means of samples from the new event device parameter marginal posterior distributions.
- Line 565: Standard deviations of samples from the new event device parameter marginal posterior distributions.
- Lines 567-575: User specified quantiles of samples from the new event device parameter marginal posterior distributions.
- Lines 579-581: Correlation matrix of samples from the new event device parameter joint posterior distribution.

- Lines 585-637: Means and user specified quantiles of samples from the marginal posterior distributions of errors-in-variables yields for the relevant calibration sources. The ordering of calibration sources is provided with the maximum *a posteriori* estimates (Lines 296 and 298). Errors-in-variables yields are only estimated if `eiv` is `TRUE` (Line 105 of Appendix B.1).
- Lines 641-723: Means and user specified quantiles of samples from the marginal posterior distributions of *common* forward model parameters for each signature of each phenomenology (where present).
- Lines 727-930: Means and user specified quantiles of samples from the marginal posterior distributions of *emplacement-dependent* forward model parameters for each signature of each phenomenology (where present).
- Lines 934-988: Means and user specified quantiles of samples from the marginal posterior distributions of *source* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 992-1018: Means and user specified quantiles of samples from the marginal posterior distributions of *path* random effect (error model) variance component parameters for each signature of each phenomenology (where present).
- Lines 1022-1296: Means and user specified quantiles of samples from the marginal posterior distributions of observational error variances for each signature, and correlations between signatures, for each phenomenology (WPA, §A.5).
- Lines 1300-1337: Means and user specified quantiles of samples from the marginal posterior distributions of FGSN prior distribution parameters (WPA, §6.6, p. 23;  $\text{Alpha} = \mu$ ,  $\text{Omega} = v$  (two coefficients)).
- Line 1339: DIC value based on calibration and (if relevant) new event data. Used for selecting among competing forward or error model specifications (WPA, §6.5, pp. 15-16; §6.6, Tables 4 and 5, p. 18).
- Line 1341: PIC value based on calibration and (if relevant) new event data. Used for selecting among competing forward or error model specifications (WPA, §6.5, pp. 15-16; §6.6, Tables 4 and 5, p. 18).

The `p_cal` environment resulting from this run contains several elements of potential interest for additional post-processing:

- `p_cal$mle`: Maximum likelihood estimate of (if relevant) unbounded new event device parameters (i.e., on scale used by the optimizer), calibration source errors-in-variables yields (if relevant), and forward and error model parameters
- `p_cal$Sigma_mle$II_nev_it`: If relevant, estimated asymptotic covariance matrix of new event device parameter elements of `p_cal$mle`
- `p_cal$tmle`: If relevant, maximum likelihood estimate of transformed new event device parameters (i.e., on correct scale)

- `p_cal$Sigma_mle$II_nev`: If relevant, estimated asymptotic covariance matrix of `p_cal$tmle`
- `p_cal$map`: If `iBayes` is TRUE (Line 6 of Appendix B.3), maximum *a posteriori* estimate of (if relevant) unbounded new event device parameters (i.e., on scale used by the optimizer), calibration source errors-in-variables yields (if relevant), and forward and error model parameters
- `p_cal$tmmap`: If `iBayes` is TRUE (Line 6 of Appendix B.3), maximum *a posteriori* estimate of transformed new event device parameters (i.e., on correct scale)
- `p_cal$tm_pi_0`: If `iBayes` is TRUE (Line 6 of Appendix B.3), posterior samples of transformed new event device parameters (i.e., on correct scale)

# A Rapid Assessment Run Files

This appendix provides example run files and output files for rapid assessments.

## A.1 Calibration Data: Preprocessing

```
1 #####
2 #
3 # This file is the input deck for MultiPEM Toolbox estimation of
4 # forward and error model parameters based on calibration data.
5 #
6 #####
7
8 #
9 # REQUIRED R PACKAGES
10 #
11
12 require(Matrix)
13
14 #
15 # END REQUIRED R PACKAGES
16 #
17
18 #
19 # PREPROCESSING
20 #
21
22 # Specify directory for general subroutines
23 gen_dir = "../..../Code"
24
25 # Source supporting R function
26 source(paste(gen_dir,"/prepro_cal.r",sep=""))
27
28 # Specify directory for application subroutines
29 app_dir = "../..../Code"
30
31 # Specify root data directory
32 dat_dir = "../..../Data"
33
34 # Specify calibration data directories
35 dat_cal = c("seismic_cal.csv",
36             "acoustic_cal.csv",
37             "optical_cal.csv",
38             "crater_cal.csv")
39
```



```

40 # Phenomenologies for this analysis
41 # 1 - seismic
42 # 2 - acoustic
43 # 3 - optical
44 # 4 - crater (surface effects)
45
46 # Specify number of responses for each phenomenology
47 Rh = c(2,2,2,2)
48
49 # Empirical model parameter count: common
50 # list with elements corresponding to phenomenologies
51 pbeta = vector("list",length(Rh))
52 for( hh in 1:length(Rh) ){ pbeta[[hh]] = numeric(Rh[hh]) }
53 # phenomenology 2
54 pbeta[[2]] = c(2,2)
55 # phenomenology 3
56 pbeta[[3]] = c(2,2)
57 # phenomenology 4
58 pbeta[[4]] = c(2,2)
59
60 # Specify number of emplacement conditions for each phenomenology
61 Th = TRUE
62
63 if( Th ){ Th = c(3,3,0,0)
64 } else { Th = NULL }
65
66 # Empirical model parameter count: emplacement condition
67 # list with elements corresponding to phenomenologies
68 if( !is.null(Th) ){
69   pbetat = vector("list",length(Rh))
70   for( hh in 1:length(Rh) ){
71     if( Th[hh] > 1 ){ pbetat[[hh]] = vector("list",Th[hh]) }
72   }
73   # phenomenology 1
74   for( tt in 1:Th[1] ){
75     pbetat[[1]][[tt]] = numeric(Rh[1])
76     pbetat[[1]][[tt]] = c(5,5)
77   }
78   # phenomenology 2
79   for( tt in 1:Th[2] ){
80     pbetat[[2]][[tt]] = numeric(Rh[2])
81     pbetat[[2]][[tt]] = c(1,1)
82   }
83 } else { pbetat = NULL }
84

```

```

85 # Locations of common parameters in full parameter vector
86 # list with elements corresponding to phenomenologies
87 if( !is.null(Th) ){
88   ibetar = vector("list",length(Rh))
89   for( hh in 1:length(Rh) ){
90     if( Th[hh] > 1 ){
91       # lists with elements for each response within
92       # emplacement condition
93       ibetar[[hh]] = vector("list",Th[hh]*Rh[hh])
94     }
95   }
96   # phenomenology 2
97   for( tt in 1:Th[2] ){
98     for( rr in 1:Rh[2] ){
99       ibetar[[2]][[(tt-1)*Rh[2]+rr]] = 1:2
100     }
101   }
102 } else { ibetar = NULL }
103
104 # Indicate analysis with errors-in-variables (eiv)
105 eiv = TRUE
106
107 # Specifications for errors-in-variables
108 if( eiv ){
109   # Specify phenomenologies utilizing
110   # errors-in-variables yields
111   ieiv = 3:4
112
113   # Errors-in-variables source lists by
114   # phenomenology
115   seiv = vector("list",length(Rh))
116   for( hh in ieiv ){ seiv[[hh]] = "ALL" }
117
118   # Set standard deviation of eiv Gaussian likelihood
119   eiv_w_sd = 0.1/3
120 } else {
121   ieiv = NULL
122   seiv = NULL
123   eiv_w_sd = NULL
124 }
125
126 # Specify Error Model
127 # Level 1 variance component parameter count
128 pvc_1 = TRUE
129

```

```

130 if( pvc_1 ){
131   pvc_1 = vector("list",length(Rh))
132   for( hh in 1:length(Rh) ){ pvc_1[[hh]] = numeric(Rh[hh]) }
133   # phenomenology 1
134   pvc_1[[1]] = c(1,1)
135   # phenomenology 2
136   pvc_1[[2]] = c(1,1)
137 } else { pvc_1 = NULL }
138
139 # Level 2 variance component parameter count
140 pvc_2 = TRUE
141
142 if( pvc_2 ){
143   pvc_2 = vector("list",length(Rh))
144   for( hh in 1:length(Rh) ){ pvc_2[[hh]] = numeric(Rh[hh]) }
145   # phenomenology 1
146   pvc_2[[1]] = c(1,1)
147   # phenomenology 2
148   #pvc_2[[2]] =
149 } else { pvc_2 = NULL }
150
151 # Set flag for user-provided code to calculate variance
152 # component coefficient matrices
153 calc_Z = FALSE
154
155 # Preprocessing for statistical analysis routines
156 p_cal = prepro_cal(gen_dir,app_dir,dat_dir,dat_cal,Rh,pbeta,
157                   izmat=calc_Z,ieiv=ieiv,seiv=seiv,ewsd=eiv_w_sd,
158                   Th=Th,pbetat=pbetat,ibetar=ibetar,pvc_1=pvc_1,
159                   pvc_2=pvc_2)
160 save.image()
161
162 #
163 # END PREPROCESSING
164 #

```

## A.2 Calibration Data: Maximum Likelihood Estimation

```
1  #
2  # MAXIMUM LIKELIHOOD CALCULATION
3  #
4
5  # Source supporting R function
6  source(paste(gen_dir,"/calc_mle_cal.r",sep=""))
7
8  # Set seed for repeatability of analysis
9  set.seed(621)
10
11 # Names of forward models for each response
12 # by phenomenology
13 fm = vector("list",length(Rh))
14 fm[[1]] = c("f_s","f_s")
15 fm[[2]] = c("f_a","f_a")
16 fm[[3]] = c("f_o","f_o")
17 fm[[4]] = c("f_c","f_c")
18
19 # Indicate if forward model gradients provided
20 igrad = TRUE
21
22 if( igrad ){
23   # Names of forward model gradients for each response
24   # by phenomenology
25   gfm = vector("list",length(Rh))
26   gfm[[1]] = c("g_s","g_s")
27   gfm[[2]] = c("g_a","g_a")
28   gfm[[3]] = c("g_o","g_o")
29   gfm[[4]] = c("g_c","g_c")
30 } else { gfm = NULL }
31
32 # Specifications for forward model calculations
33 # a) flags for modified forward model calculation by
34 #   response for each relevant phenomenology
35 iResponse = TRUE
36
37 if( iResponse ){
38   iResponse = vector("list",length(Rh))
39   iResponse[[1]] = c(TRUE,FALSE)
40   iResponse[[2]] = c(TRUE,FALSE)
41 } else { iResponse = NULL }
42
43 # b) fixed quantities required by forward models
```

```

44 fPars = TRUE
45
46 if( fPars ){
47     fPars = vector("list",length(Rh))
48     fPars[[1]]$yield_scaling = 1/3
49     fPars[[2]]$yield_scaling = 1/3
50     fPars[[2]]$pressure_scaling = 1/3
51     fPars[[2]]$temp_scaling = 1/2
52     fPars[[3]]$yield_scaling = 1/3
53     fPars[[3]]$pressure_scaling = 1/3
54     fPars[[3]]$temp_scaling = 1/2
55 } else { fPars = NULL }
56
57 # Specify number of starting values for optimization
58 nstart = 10
59
60 # number of cores to use for optimization
61 ncores_mle = 1
62
63 # Indicate use of BFGS optimization methods
64 bfgs = TRUE
65
66 # Location of R data files with starting values
67 # for input to MLE optimization
68 opt_files_in = c("../Opt/opt_1_0.RData",
69                 "../Opt/opt_2_0.RData",
70                 "../Opt/opt_3_eiv_0.RData",
71                 "../Opt/opt_4_eiv_0.RData")
72
73 # Location of R data file to write the results of
74 # MLE optimization
75 opt_files_out = "../opt.RData"
76
77 # Indicate phenomenology number and type (if needed
78 # for postprocessing)
79 Phen = TRUE
80
81 if( Phen ){
82     Phen = matrix(c(1,"Seismic"),nrow=1)
83 } else { Phen = NULL }
84
85 # Indicator of MLE gradient check
86 mle_grad_ck = TRUE
87
88 # Strategy for running parallel jobs (future package)

```

```

89 parallel_plan = "multicore"
90
91 # MLE calculations
92 p_cal = calc_mle_cal(p_cal,gen_dir,app_dir,fm,nst=nstart,
93                     ncor=ncores_mle,igrad=igrad,bfgs=bfgs,
94                     igrck=mle_grad_ck,g=gfm,iresp=iResponse,
95                     fp_fm=fPars,fopt_in=opt_files_in,Xst=NULL,
96                     fopt_out=opt_files_out,phen=Phen,pl=parallel_plan)
97 save.image()
98
99 #
100 # END MAXIMUM LIKELIHOOD CALCULATION
101 #

```

### A.3 Calibration Data: Bayesian Analysis

```
1  #
2  # BAYESIAN ANALYSIS
3  #
4
5  # Specify if Bayesian analysis is to be conducted
6  iBayes = FALSE
7
8  if( iBayes ){
9    # Source supporting R function
10    source(paste(gen_dir,"/calc_bayes_cal.r",sep=""))
11
12    # Indicator of prior distribution for forward model
13    # coefficients
14    iBetaPrior = TRUE
15
16    if( iBetaPrior ){
17      # location of code for computing log-prior densities and gradients
18      prior_files_beta = "../Code/lp_beta_s.r"
19      if( igrad ){
20        gr_prior_files_beta = "../Code/glp_beta_s.r"
21      } else { gr_prior_files_beta = NULL }
22
23      # prior distribution for phenomenology 1
24      # forward model coefficients
25      p_cal$h[[1]]$lp_betat = vector("list",Th[1])
26      for( tt in 1:Th[1] ){
27        p_cal$h[[1]]$lp_betat[[tt]]$f = c("lp_s","lp_s")
28        if( igrad ){
29          p_cal$h[[1]]$lp_betat[[tt]]$g = c("lq_s","lq_s")
30        }
31      }
32    } else {
33      prior_files_beta = NULL
34      gr_prior_files_beta = NULL
35    }
36
37    # fixed scale parameters for variance component prior
38    # comment out if these parameters should vary
39    p_cal$A = 20
40
41    # eta parameter in Lewandowski-Kurowicka-Joe (LKJ) prior
42    # distribution for correlation parameters
43    p_cal$lp_corr$eta = 1
```

```

44
45 # FGSN parameters for errors-in-variables yields prior
46 # number of components
47 p_cal$K = 0
48 # total number of FGSN parameters
49 p_cal$p_fgsn = 0
50 if( eiv ){
51     p_cal$K = 2
52     p_cal$p_fgsn = p_cal$K + 2
53 }
54
55 # specify Markov chain Monte Carlo (MCMC) algorithm
56 # options: "RAM", "FME", or "NUTS"
57 iMCMC = "FME"
58
59 # burn-in
60 nburn = 10000
61
62 # production
63 nmcmc = 20000
64
65 # posterior sample thinning rate
66 nthin = 20
67
68 # number of cores to use for optimization
69 ncores_map = 1
70
71 # number of cores to use for generating parallel MCMC chains
72 ncores_mc = 1
73
74 # Indicator of prior gradient check
75 prior_grad_ck = TRUE
76
77 # Indicator of posterior gradient check
78 post_grad_ck = TRUE
79
80 # Bayesian calculations
81 p_cal = calc_bayes_cal(p_cal,gen_dir,app_dir,nst=nstart,nburn=nburn,
82                       nmcmc=nmcmc,nthin=nthin,ncor_map=ncores_map,
83                       ncor_mc=ncores_mc,igrad=igrad,
84                       igrck_pr=prior_grad_ck,igrck_po=post_grad_ck,
85                       bfgs=bfgs,ibpr=iBetaPrior,
86                       fpr_b=prior_files_beta,
87                       fgpr_b=gr_prior_files_beta,Xnom=NULL,
88                       imcmc=iMCMC,pl=parallel_plan)

```



```
89     save.image()
90 }
91
92 #
93 # END BAYESIAN ANALYSIS
94 #
```

## A.4 Calibration Data: Output File

```

1 > # Preprocessing for statistical analysis routines
2 > p_cal = prepro_cal(gen_dir,app_dir,dat_dir,dat_cal,Rh,pbeta,
3 +                   izmat=calc_Z,ieiv=ieiv,seiv=seiv,ewsd=eiv_w_sd,
4 +                   Th=Th,pbetat=pbetat,ibetar=ibetar,pvc_1=pvc_1,
5 +                   pvc_2=pvc_2)
6 [1] "Warning: Insufficient Paths for Level 2 Variance Component models with
7     Phenomenology 2 and Response 1."
8 [1] "Warning: Insufficient Paths for Level 2 Variance Component models with
9     Phenomenology 2 and Response 2."
10 [1] "Warning: Insufficient number of observations per Source for Variance
11     Component models with Phenomenology 3 and Response 1."
12 [1] "Warning: Insufficient number of observations per Source for Variance
13     Component models with Phenomenology 3 and Response 2."
14 [1] "Warning: Insufficient number of observations per Source for Variance
15     Component models with Phenomenology 4 and Response 1."
16 [1] "Warning: Insufficient number of observations per Source for Variance
17     Component models with Phenomenology 4 and Response 2."
18
19 > # MLE calculations
20 > p_cal = calc_mle_cal(p_cal,gen_dir,app_dir,fm,nst=nstart,
21 +                   ncor=ncores_mle,igrad=igrad,bfgs=bfgs,
22 +                   igrck=mle_grad_ck,g=gfm,iresp=iResponse,
23 +                   fp_fm=fPars,fopt_in=opt_files_in,Xst=NULL,
24 +                   fopt_out=opt_files_out,phen=Phen,pl=parallel_plan)
25 [1] "MLE CONVERGENCE STATUS"
26
27 [1] 0
28 [1] 2
29 [1] "MAXIMUM LIKELIHOOD SUMMARY"
30
31 [1] "ERRORS-IN-VARIABLES YIELDS"
32
33      7      8      9     10     11     13     14     16     17     20     21     22     23
34 16.29 16.21 16.51 16.61 17.00 12.21 17.57 17.27 16.52 14.50 15.75 17.59 15.23
35     24     25     28     29     30     31     33     34     35     36     37     38     39
36 15.88 16.46 14.52 12.13 17.70 23.07 23.42 17.51 21.96 22.34 16.71 21.02 18.52
37
38 [1] "COMMON COEFFICIENTS"
39
40           [1] "Phenomenology: 2; Response: 1"
41
42           [1] 6.67 -1.14
43

```

44 [1] "Phenomenology: 2; Response: 2"

45

46 [1] -5.08 0.23

47

48 [1] "Phenomenology: 3; Response: 1"

49

50 [1] -11.07 1.89

51

52 [1] "Phenomenology: 3; Response: 2"

53

54 [1] -8.64 1.72

55

56 [1] "Phenomenology: 4; Response: 1"

57

58 [1] -3.34 0.43

59

60 [1] "Phenomenology: 4; Response: 2"

61

62 [1] -2.64 0.29

63

64 [1] "EMPLACEMENT CONDITION DEPENDENT COEFFICIENTS"

65

66 [1] "Phenomenology: 1; Emplacement: 1; Response: 1"

67

68 [1] -10.07 -1.31 -1.43 3.52 0.39

69

70 [1] "Phenomenology: 1; Emplacement: 1; Response: 2"

71

72 [1] -1.52 -1.44 -1.23 2.31 0.65

73

74 [1] "Phenomenology: 1; Emplacement: 2; Response: 1"

75

76 [1] -11.48 -1.09 -3.59 4.32 0.16

77

78 [1] "Phenomenology: 1; Emplacement: 2; Response: 2"

79

80 [1] -2.33 -1.22 -7.78 1.32 -1.36

81

82 [1] "Phenomenology: 1; Emplacement: 3; Response: 1"

83

84 [1] -9.53 -1.16 -4.47 4.95 0.34

85

86 [1] "Phenomenology: 1; Emplacement: 3; Response: 2"

87

88 [1] -2.85 -0.71 -2.11 2.68 0.13

89 [1] "Phenomenology: 2; Emplacement: 1; Response: 1"

90

91

92 [1] 3.87

93

94 [1] "Phenomenology: 2; Emplacement: 1; Response: 2"

95

96 [1] -0.17

97

98 [1] "Phenomenology: 2; Emplacement: 2; Response: 1"

99

100 [1] 3.11

101

102 [1] "Phenomenology: 2; Emplacement: 2; Response: 2"

103

104 [1] -0.9

105

106 [1] "Phenomenology: 2; Emplacement: 3; Response: 1"

107

108 [1] 2.16

109

110 [1] "Phenomenology: 2; Emplacement: 3; Response: 2"

111

112 [1] -0.94

113

114 [1] "LEVEL 1 VARIANCE COMPONENTS"

115

116 [1] "Phenomenology: 1; Response: 1"

117

118 [1] 0.0011

119

120 [1] "Phenomenology: 1; Response: 2"

121

122 [1] 0.0017

123

124 [1] "Phenomenology: 2; Response: 1"

125

126 [1] 0.0038

127

128 [1] "Phenomenology: 2; Response: 2"

129

130 [1] 0.0016

131

132 [1] "LEVEL 2 VARIANCE COMPONENTS"

133

```

134         [1] "Phenomenology: 1; Response: 1"
135
136         [1] 0.001
137
138         [1] "Phenomenology: 1; Response: 2"
139
140         [1] 9e-04
141
142     [1] "OBSERVATIONAL ERROR COVARIANCE PARAMETERS"
143
144     [1] "Phenomenology 1"
145
146     [1] "Variances"
147
148     [1] 0.0021 0.0034
149
150     [1] "Correlations"
151
152         [,1] [,2]
153     [1,]    1 0.28
154     [2,]    0 1.00
155
156     [1] "Phenomenology 2"
157
158     [1] "Variances"
159
160     [1] 1e-03 3e-04
161
162     [1] "Correlations"
163
164         [,1] [,2]
165     [1,]    1 -0.25
166     [2,]    0 1.00
167
168     [1] "Phenomenology 3"
169
170     [1] "Variances"
171
172     [1] 0.0011 0.0011
173
174     [1] "Correlations"
175
176         [,1] [,2]
177     [1,]    1 0.93
178     [2,]    0 1.00

```

```

179
180 [1] "Phenomenology 4"
181
182 [1] "Variances"
183
184 [1] 0.0260 0.0077
185
186 [1] "Correlations"
187
188      [,1] [,2]
189 [1,]    1 0.73
190 [2,]    0 1.00
191
192 [1] "AIC = -5840.98"
193
194 [1] "BIC = -5511.71"
195
196 Loading required package: numDeriv
197 [1] "CHECK LOG-LIKELIHOOD GRADIENTS"
198
199 [1] "Analytic gradient"
200 [1] -8.431868e-04 -7.038053e-07 -5.595059e-05 -2.702571e-04  4.145765e-04
201 [6] -6.877652e-04 -5.414314e-05  1.630249e-04 -7.642356e-04 -2.549322e-03
202 [11] -9.760283e-04  3.702140e-04 -1.655408e-05 -2.803807e-03 -1.524976e-03
203 [16] -1.081318e-03 -2.981360e-03  4.701484e-04  7.538052e-04  2.782379e-04
204 [21]  6.314769e-04 -4.285824e-04  8.941258e-04  6.184055e-04  3.258050e-04
205 [26] -4.870028e-04 -1.503716e-03 -7.324998e-02 -2.978359e-02 -5.224518e-01
206 [31]  1.255690e-01  5.041580e-02 -1.384946e-01 -4.589536e-02 -2.084526e-03
207 [36] -3.723123e-02  4.302576e-03  8.161440e-02 -2.400975e-03 -8.966039e-03
208 [41] -2.170289e-03 -1.693288e-04  5.219861e-04 -6.309496e-04  2.186223e-03
209 [46]  2.912214e-04  2.204081e-05  3.314070e-04 -1.062880e-02 -7.688789e-02
210 [51] -6.298861e-03 -6.976100e-04  7.529508e-03  1.421339e-03  3.434872e-02
211 [56]  1.317798e-02 -3.648180e-04 -2.097170e-02  7.638015e-03  5.206311e-02
212 [61]  3.562170e-03  1.863806e-03 -1.890228e-03  1.781585e-03  1.291352e-02
213 [66]  2.920229e-03  6.897676e-04  1.963322e-04  7.009887e-04 -1.052502e-03
214 [71] -3.373052e-04  3.676417e-04 -5.854999e-04  6.978043e-04 -2.989961e-05
215 [76] -2.478198e-04 -6.438579e-05  1.975623e-04  3.884852e-04 -1.331501e-04
216 [81]  7.588398e-04 -1.283586e-03  4.477531e-03  5.487680e-04 -9.010081e-04
217 [86]  1.037655e-01  6.236496e-03 -1.749379e-04 -1.290433e-02  2.277553e-04
218 [91]  3.282681e-04  1.789334e-03
219 [1] "Numerical gradient"
220 [1] -8.431866e-04 -7.061032e-07 -5.596042e-05 -2.702460e-04  4.145769e-04
221 [6] -6.877675e-04 -5.413925e-05  1.630274e-04 -7.642502e-04 -2.549334e-03
222 [11] -9.760306e-04  3.702130e-04 -1.656036e-05 -2.803783e-03 -1.524965e-03
223 [16] -1.081294e-03 -2.981438e-03  4.701524e-04  7.538070e-04  2.782443e-04

```

```

224 [21] 6.314669e-04 -4.285834e-04 8.941207e-04 6.183993e-04 3.257979e-04
225 [26] -4.869988e-04 -1.503741e-03 -7.325015e-02 -2.978351e-02 -5.224522e-01
226 [31] 1.255690e-01 5.041530e-02 -1.384947e-01 -4.589501e-02 -2.084511e-03
227 [36] -3.723102e-02 4.302586e-03 8.161507e-02 -2.400988e-03 -8.966151e-03
228 [41] -2.171152e-03 -1.693386e-04 5.216883e-04 -6.309502e-04 2.186201e-03
229 [46] 2.909891e-04 2.202312e-05 3.314891e-04 -1.062881e-02 -7.688785e-02
230 [51] -6.298773e-03 -6.975891e-04 7.529809e-03 1.421349e-03 3.434873e-02
231 [56] 1.317800e-02 -3.647383e-04 -2.097168e-02 7.638015e-03 5.206300e-02
232 [61] 3.562154e-03 1.863808e-03 -1.890134e-03 1.781589e-03 1.291348e-02
233 [66] 2.920191e-03 6.897655e-04 1.962526e-04 7.009899e-04 -1.052988e-03
234 [71] -3.372905e-04 3.676777e-04 -5.856120e-04 6.976661e-04 -2.991824e-05
235 [76] -2.478187e-04 -6.428826e-05 1.975601e-04 3.884810e-04 -1.331631e-04
236 [81] 7.588249e-04 -1.283589e-03 4.476626e-03 5.487913e-04 -9.010005e-04
237 [86] 1.037792e-01 6.235949e-03 -1.747034e-04 -1.290695e-02 2.279298e-04
238 [91] 3.282717e-04 1.789455e-03
239 [1] "Difference"
240 [1] -1.374109e-05 2.622069e-06
241
242 + # Bayesian calculations
243 + p_cal = calc_bayes_cal(p_cal,gen_dir,app_dir,nst=nstart,nburn=nburn,
244 + nmcmc=nmcmc,nthin=nthin,ncor_map=ncores_map,
245 + ncor_mc=ncores_mc,igrad=igrad,
246 + igrck_pr=prior_grad_ck,igrck_po=post_grad_ck,
247 + bfgs=bfgs,ibpr=iBetaPrior,
248 + fpr_b=prior_files_beta,
249 + fgpr_b=gr_prior_files_beta,Xnom=NULL,
250 + imcmc=iMCMC,pl=parallel_plan)
251 + save.image()
252 + }
253 [1] "MAP CONVERGENCE STATUS"
254
255 [1] 0
256 [1] 2
257 [1] "MAXIMUM A POSTERIORI SUMMARY"
258
259 [1] "ERRORS-IN-VARIABLES YIELDS"
260
261 7 8 9 10 11 13 14 16 17 20 21 22 23
262 16.28 16.21 16.51 16.61 17.00 12.21 17.57 17.27 16.52 14.50 15.75 17.59 15.23
263 24 25 28 29 30 31 33 34 35 36 37 38 39
264 15.87 16.46 14.53 12.13 17.71 23.08 23.41 17.51 21.96 22.34 16.71 21.02 18.52
265
266 [1] "COMMON COEFFICIENTS"
267
268 [1] "Phenomenology: 2; Response: 1"

```

269 [1] 6.67 -1.14

270

271 [1] "Phenomenology: 2; Response: 2"

272

273 [1] -5.08 0.23

274

275 [1] "Phenomenology: 3; Response: 1"

276

277 [1] -11.07 1.89

278

279 [1] "Phenomenology: 3; Response: 2"

280

281 [1] -8.64 1.72

282

283 [1] "Phenomenology: 4; Response: 1"

284

285 [1] -3.35 0.43

286

287 [1] "Phenomenology: 4; Response: 2"

288

289 [1] -2.64 0.29

290

291 [1] "EMPLACEMENT CONDITION DEPENDENT COEFFICIENTS"

292

293 [1] "Phenomenology: 1; Emplacement: 1; Response: 1"

294

295 [1] -10.07 -1.31 -1.43 3.51 0.39

296

297 [1] "Phenomenology: 1; Emplacement: 1; Response: 2"

298

299 [1] -1.51 -1.44 -1.24 2.26 0.63

300

301 [1] "Phenomenology: 1; Emplacement: 2; Response: 1"

302

303 [1] -11.48 -1.09 -3.62 4.18 0.15

304

305 [1] "Phenomenology: 1; Emplacement: 2; Response: 2"

306

307 [1] -2.16 -1.22 -56.31 0.93 -3.42

308

309 [1] "Phenomenology: 1; Emplacement: 3; Response: 1"

310

311 [1] -9.53 -1.16 -4.47 4.95 0.34

312

313



314 [1] "Phenomenology: 1; Emplacement: 3; Response: 2"  
 315  
 316 [1] -2.85 -0.71 -2.17 2.60 0.08  
 317  
 318 [1] "Phenomenology: 2; Emplacement: 1; Response: 1"  
 319  
 320 [1] 3.87  
 321  
 322 [1] "Phenomenology: 2; Emplacement: 1; Response: 2"  
 323  
 324 [1] -0.17  
 325  
 326 [1] "Phenomenology: 2; Emplacement: 2; Response: 1"  
 327  
 328 [1] 3.11  
 329  
 330 [1] "Phenomenology: 2; Emplacement: 2; Response: 2"  
 331  
 332 [1] -0.9  
 333  
 334 [1] "Phenomenology: 2; Emplacement: 3; Response: 1"  
 335  
 336 [1] 2.16  
 337  
 338 [1] "Phenomenology: 2; Emplacement: 3; Response: 2"  
 339  
 340 [1] -0.94  
 341  
 342 [1] "LEVEL 1 VARIANCE COMPONENTS"  
 343  
 344 [1] "Phenomenology: 1; Response: 1"  
 345  
 346 [1] 0.0012  
 347  
 348 [1] "Phenomenology: 1; Response: 2"  
 349  
 350 [1] 0.0017  
 351  
 352 [1] "Phenomenology: 2; Response: 1"  
 353  
 354 [1] 0.0039  
 355  
 356 [1] "Phenomenology: 2; Response: 2"  
 357  
 358 [1] 0.0016

```

359
360 [1] "LEVEL 2 VARIANCE COMPONENTS"
361
362     [1] "Phenomenology: 1; Response: 1"
363
364     [1] 0.001
365
366     [1] "Phenomenology: 1; Response: 2"
367
368     [1] 0.001
369
370 [1] "OBSERVATIONAL ERROR COVARIANCE PARAMETERS"
371
372 [1] "Phenomenology 1"
373
374 [1] "Variances"
375
376 [1] 0.0021 0.0034
377
378 [1] "Correlations"
379
380     [,1] [,2]
381 [1,]    1 0.27
382 [2,]    0 1.00
383
384 [1] "Phenomenology 2"
385
386 [1] "Variances"
387
388 [1] 1e-03 3e-04
389
390 [1] "Correlations"
391
392     [,1] [,2]
393 [1,]    1 -0.25
394 [2,]    0 1.00
395
396 [1] "Phenomenology 3"
397
398 [1] "Variances"
399
400 [1] 1e-03 9e-04
401
402 [1] "Correlations"
403

```

```

404      [,1] [,2]
405 [1,]    1 0.92
406 [2,]    0 1.00
407
408 [1] "Phenomenology 4"
409
410 [1] "Variances"
411
412 [1] 0.0218 0.0059
413
414 [1] "Correlations"
415
416      [,1] [,2]
417 [1,]    1 0.59
418 [2,]    0 1.00
419
420 [1] "FGSN PRIOR PARAMETERS"
421
422 [1] "Alpha = 17.33"
423 [1] "Lambda squared = 8"
424 [1] "Omega = -1.66" "Omega = 0.56"
425
426 [1] "CHECK LOG-PRIOR GRADIENTS"
427
428 [1] "Analytic gradient"
429 [1] -1.055466e-01 -8.327724e-02 -1.787279e-01 -2.140887e-01 -3.507367e-01
430 [6]  1.997990e+00 -5.483409e-01 -4.466296e-01 -1.832292e-01  3.561637e-01
431 [11]  5.548120e-02 -5.553768e-01  1.865502e-01  1.979674e-02 -1.646971e-01
432 [16]  3.490028e-01  2.182226e+00 -5.901511e-01 -3.792248e-01 -6.430252e-01
433 [21] -5.276501e-01  3.937098e-01  2.550063e-01 -2.494903e-01  1.570684e-01
434 [26] -7.524566e-01  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
435 [31]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
436 [36]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
437 [41]  1.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
438 [46]  1.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
439 [51]  7.751886e-01  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
440 [56]  1.572709e-01  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
441 [61]  6.610821e-01  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
442 [66]  1.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
443 [71]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  4.999971e-01
444 [76]  4.999956e-01  4.999902e-01  4.999959e-01  4.999974e-01  4.999975e-01
445 [81]  0.000000e+00 -7.753342e-01 -1.413558e+01 -4.440892e-16 -8.113389e-01
446 [86]  4.223002e+01  2.220446e-16  1.518082e+00 -9.080578e+01  0.000000e+00
447 [91]  5.581077e-02 -2.313183e+01  1.965379e-02 -3.608055e-03  2.519400e-03
448 [96]  1.691543e-02

```

```

449 [1] "Numerical gradient"
450 [1] -1.055466e-01 -8.327723e-02 -1.787279e-01 -2.140887e-01 -3.507367e-01
451 [6] 1.997990e+00 -5.483409e-01 -4.466296e-01 -1.832292e-01 3.561637e-01
452 [11] 5.548120e-02 -5.553768e-01 1.865502e-01 1.979674e-02 -1.646971e-01
453 [16] 3.490028e-01 2.182226e+00 -5.901511e-01 -3.792248e-01 -6.430252e-01
454 [21] -5.276501e-01 3.937098e-01 2.550063e-01 -2.494903e-01 1.570684e-01
455 [26] -7.524566e-01 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
456 [31] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
457 [36] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
458 [41] 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
459 [46] 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
460 [51] 7.751886e-01 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
461 [56] 1.572709e-01 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
462 [61] 6.610821e-01 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
463 [66] 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
464 [71] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 4.999971e-01
465 [76] 4.999956e-01 4.999902e-01 4.999959e-01 4.999974e-01 4.999975e-01
466 [81] 0.000000e+00 -7.753342e-01 -1.413558e+01 0.000000e+00 -8.113389e-01
467 [86] 4.223002e+01 -2.776737e-20 1.518082e+00 -9.080578e+01 0.000000e+00
468 [91] 5.581077e-02 -2.313183e+01 1.965379e-02 -3.608056e-03 2.519400e-03
469 [96] 1.691543e-02
470 [1] "Difference"
471 [1] -3.718726e-08 4.307974e-07
472
473 [1] "CHECK LOG-POSTERIOR GRADIENTS"
474
475 [1] "Analytic gradient"
476 [1] -0.143725992 -0.060385357 -0.128368006 0.021998510 -0.025658369
477 [6] -0.036306696 -0.050177600 -0.085249094 0.015305390 0.103565181
478 [11] -0.201184644 0.035028210 0.057834796 0.119313668 -0.047274099
479 [16] -0.138810545 0.143130873 0.012883495 0.027928439 0.031038567
480 [21] 0.009257306 0.048872686 0.042925809 0.021272475 0.044293980
481 [26] -0.012342070 0.055484797 0.464397999 0.145253071 3.136130832
482 [31] -0.085779845 0.227551255 -0.021211292 0.153970860 -0.008353076
483 [36] -0.072768259 -0.001231599 -0.129250679 -0.066407399 0.009340489
484 [41] 0.057159667 0.001311732 -0.005017758 -0.025711262 -0.061502299
485 [46] 0.006932307 0.033351768 -0.022771865 0.151675870 -0.235282410
486 [51] -1.248225356 -0.037197888 -0.473227835 0.096449963 -0.414093631
487 [56] -0.154049649 -0.678058084 -0.964376932 -0.004970885 0.022785614
488 [61] 0.058071746 -0.001887512 0.008393102 0.006164484 0.041256854
489 [66] 0.017139968 0.011643573 -0.012383597 -0.015343286 0.063412903
490 [71] -0.035682482 -0.037305676 -0.076071754 0.021156892 0.012114242
491 [76] 0.007195484 -0.007422780 0.002099905 -0.003198358 -0.002760457
492 [81] 0.032427579 0.010586708 0.047392728 -0.038438158 -0.030227541
493 [86] -0.839483224 -0.016865095 -0.038679600 -0.002776678 0.016996863

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494 [91] 0.028550901 -0.031829322 0.019653787 -0.003608055 0.002519400
495 [96] 0.016915435
496 [1] "Numerical gradient"
497 [1] -0.143726008 -0.060385354 -0.128368009 0.021998512 -0.025658378
498 [6] -0.036306692 -0.050177604 -0.085249091 0.015305380 0.103565176
499 [11] -0.201184641 0.035028212 0.057834794 0.119313685 -0.047274109
500 [16] -0.138810540 0.143130810 0.012883498 0.027928440 0.031038559
501 [21] 0.009257302 0.048872688 0.042925807 0.021272464 0.044293981
502 [26] -0.012342073 0.055484827 0.464398287 0.145253137 3.136132448
503 [31] -0.085779761 0.227551232 -0.021211309 0.153970961 -0.008353068
504 [36] -0.072768079 -0.001231649 -0.129250734 -0.066407414 0.009340623
505 [41] 0.057159404 0.001311717 -0.005018124 -0.025711252 -0.061502296
506 [46] 0.006931771 0.033351792 -0.022771965 0.151675863 -0.235282339
507 [51] -1.248225359 -0.037197899 -0.473227186 0.096449898 -0.414093621
508 [56] -0.154049648 -0.678058194 -0.964376936 -0.004970892 0.022785708
509 [61] 0.058071708 -0.001887510 0.008392921 0.006164449 0.041256794
510 [66] 0.017139825 0.011643578 -0.012383603 -0.015343273 0.063411939
511 [71] -0.035682449 -0.037305829 -0.076071821 0.021156943 0.012114223
512 [76] 0.007195512 -0.007422785 0.002099886 -0.003198378 -0.002760494
513 [81] 0.032427562 0.010586659 0.047382834 -0.038438287 -0.030227440
514 [86] -0.839521589 -0.016865429 -0.038679788 -0.002761438 0.016996873
515 [91] 0.028550994 -0.031826601 0.019653788 -0.003608078 0.002519406
516 [96] 0.016915378
517 [1] "Difference"
518 [1] -1.524005e-05 3.836539e-05
519
520 [1] "ACCEPTANCE RATES:"
521
522 [1] "Core 1: 0.8553666666666667"
523
524 [1] "POSTERIOR SUMMARY"
525
526 [1] "ERRORS-IN-VARIABLES YIELDS"
527
528 [1] "POSTERIOR MEAN: 16.28" "POSTERIOR MEAN: 16.21" "POSTERIOR MEAN: 16.51"
529 [4] "POSTERIOR MEAN: 16.61" "POSTERIOR MEAN: 17" "POSTERIOR MEAN: 12.21"
530 [7] "POSTERIOR MEAN: 17.57" "POSTERIOR MEAN: 17.27" "POSTERIOR MEAN: 16.52"
531 [10] "POSTERIOR MEAN: 14.5" "POSTERIOR MEAN: 15.75" "POSTERIOR MEAN: 17.6"
532 [13] "POSTERIOR MEAN: 15.23" "POSTERIOR MEAN: 15.87" "POSTERIOR MEAN: 16.46"
533 [16] "POSTERIOR MEAN: 14.53" "POSTERIOR MEAN: 12.13" "POSTERIOR MEAN: 17.71"
534 [19] "POSTERIOR MEAN: 23.08" "POSTERIOR MEAN: 23.41" "POSTERIOR MEAN: 17.51"
535 [22] "POSTERIOR MEAN: 21.96" "POSTERIOR MEAN: 22.34" "POSTERIOR MEAN: 16.71"
536 [25] "POSTERIOR MEAN: 21.02" "POSTERIOR MEAN: 18.52"
537
538 [1] "LEVEL 2.5%: 16.27" "LEVEL 2.5%: 16.21" "LEVEL 2.5%: 16.49"

```

539 [4] "LEVEL 2.5%: 16.6" "LEVEL 2.5%: 16.99" "LEVEL 2.5%: 12.21"  
540 [7] "LEVEL 2.5%: 17.55" "LEVEL 2.5%: 17.26" "LEVEL 2.5%: 16.51"  
541 [10] "LEVEL 2.5%: 14.48" "LEVEL 2.5%: 15.74" "LEVEL 2.5%: 17.58"  
542 [13] "LEVEL 2.5%: 15.22" "LEVEL 2.5%: 15.86" "LEVEL 2.5%: 16.46"  
543 [16] "LEVEL 2.5%: 14.51" "LEVEL 2.5%: 12.12" "LEVEL 2.5%: 17.7"  
544 [19] "LEVEL 2.5%: 23.05" "LEVEL 2.5%: 23.4" "LEVEL 2.5%: 17.49"  
545 [22] "LEVEL 2.5%: 21.94" "LEVEL 2.5%: 22.33" "LEVEL 2.5%: 16.7"  
546 [25] "LEVEL 2.5%: 21.01" "LEVEL 2.5%: 18.51"  
547  
548 [1] "LEVEL 5%: 16.28" "LEVEL 5%: 16.21" "LEVEL 5%: 16.49" "LEVEL 5%: 16.6"  
549 [5] "LEVEL 5%: 16.99" "LEVEL 5%: 12.21" "LEVEL 5%: 17.56" "LEVEL 5%: 17.26"  
550 [9] "LEVEL 5%: 16.51" "LEVEL 5%: 14.48" "LEVEL 5%: 15.74" "LEVEL 5%: 17.59"  
551 [13] "LEVEL 5%: 15.23" "LEVEL 5%: 15.86" "LEVEL 5%: 16.46" "LEVEL 5%: 14.51"  
552 [17] "LEVEL 5%: 12.12" "LEVEL 5%: 17.7" "LEVEL 5%: 23.06" "LEVEL 5%: 23.4"  
553 [21] "LEVEL 5%: 17.49" "LEVEL 5%: 21.94" "LEVEL 5%: 22.33" "LEVEL 5%: 16.7"  
554 [25] "LEVEL 5%: 21.01" "LEVEL 5%: 18.51"  
555  
556 [1] "LEVEL 50%: 16.28" "LEVEL 50%: 16.21" "LEVEL 50%: 16.51" "LEVEL 50%: 16.61"  
557 [5] "LEVEL 50%: 17" "LEVEL 50%: 12.21" "LEVEL 50%: 17.57" "LEVEL 50%: 17.27"  
558 [9] "LEVEL 50%: 16.52" "LEVEL 50%: 14.5" "LEVEL 50%: 15.75" "LEVEL 50%: 17.59"  
559 [13] "LEVEL 50%: 15.24" "LEVEL 50%: 15.87" "LEVEL 50%: 16.46" "LEVEL 50%: 14.53"  
560 [17] "LEVEL 50%: 12.13" "LEVEL 50%: 17.71" "LEVEL 50%: 23.08" "LEVEL 50%: 23.41"  
561 [21] "LEVEL 50%: 17.51" "LEVEL 50%: 21.96" "LEVEL 50%: 22.34" "LEVEL 50%: 16.71"  
562 [25] "LEVEL 50%: 21.02" "LEVEL 50%: 18.52"  
563  
564 [1] "LEVEL 95%: 16.29" "LEVEL 95%: 16.22" "LEVEL 95%: 16.52" "LEVEL 95%: 16.62"  
565 [5] "LEVEL 95%: 17.01" "LEVEL 95%: 12.22" "LEVEL 95%: 17.59" "LEVEL 95%: 17.28"  
566 [9] "LEVEL 95%: 16.53" "LEVEL 95%: 14.51" "LEVEL 95%: 15.75" "LEVEL 95%: 17.6"  
567 [13] "LEVEL 95%: 15.24" "LEVEL 95%: 15.89" "LEVEL 95%: 16.47" "LEVEL 95%: 14.54"  
568 [17] "LEVEL 95%: 12.14" "LEVEL 95%: 17.72" "LEVEL 95%: 23.09" "LEVEL 95%: 23.43"  
569 [21] "LEVEL 95%: 17.52" "LEVEL 95%: 21.97" "LEVEL 95%: 22.35" "LEVEL 95%: 16.72"  
570 [25] "LEVEL 95%: 21.03" "LEVEL 95%: 18.52"  
571  
572 [1] "LEVEL 97.5%: 16.3" "LEVEL 97.5%: 16.22" "LEVEL 97.5%: 16.52"  
573 [4] "LEVEL 97.5%: 16.62" "LEVEL 97.5%: 17.01" "LEVEL 97.5%: 12.22"  
574 [7] "LEVEL 97.5%: 17.59" "LEVEL 97.5%: 17.28" "LEVEL 97.5%: 16.53"  
575 [10] "LEVEL 97.5%: 14.52" "LEVEL 97.5%: 15.76" "LEVEL 97.5%: 17.61"  
576 [13] "LEVEL 97.5%: 15.24" "LEVEL 97.5%: 15.89" "LEVEL 97.5%: 16.47"  
577 [16] "LEVEL 97.5%: 14.54" "LEVEL 97.5%: 12.15" "LEVEL 97.5%: 17.72"  
578 [19] "LEVEL 97.5%: 23.1" "LEVEL 97.5%: 23.43" "LEVEL 97.5%: 17.53"  
579 [22] "LEVEL 97.5%: 21.98" "LEVEL 97.5%: 22.35" "LEVEL 97.5%: 16.72"  
580 [25] "LEVEL 97.5%: 21.03" "LEVEL 97.5%: 18.52"  
581  
582 [1] "COMMON COEFFICIENTS"  
583

584 [1] "Phenomenology: 2; Response: 1"

585

586 [1] "POSTERIOR MEAN: 6.67" "POSTERIOR MEAN: -1.14"

587

588 [1] "LEVEL 2.5%: 6.67" "LEVEL 2.5%: -1.14"

589

590 [1] "LEVEL 5%: 6.67" "LEVEL 5%: -1.14"

591

592 [1] "LEVEL 50%: 6.67" "LEVEL 50%: -1.14"

593

594 [1] "LEVEL 95%: 6.68" "LEVEL 95%: -1.14"

595

596 [1] "LEVEL 97.5%: 6.68" "LEVEL 97.5%: -1.14"

597

598 [1] "Phenomenology: 2; Response: 2"

599

600 [1] "POSTERIOR MEAN: -5.08" "POSTERIOR MEAN: 0.23"

601

602 [1] "LEVEL 2.5%: -5.08" "LEVEL 2.5%: 0.23"

603

604 [1] "LEVEL 5%: -5.08" "LEVEL 5%: 0.23"

605

606 [1] "LEVEL 50%: -5.08" "LEVEL 50%: 0.23"

607

608 [1] "LEVEL 95%: -5.08" "LEVEL 95%: 0.23"

609

610 [1] "LEVEL 97.5%: -5.08" "LEVEL 97.5%: 0.23"

611

612 [1] "Phenomenology: 3; Response: 1"

613

614 [1] "POSTERIOR MEAN: -11.07" "POSTERIOR MEAN: 1.89"

615

616 [1] "LEVEL 2.5%: -11.09" "LEVEL 2.5%: 1.88"

617

618 [1] "LEVEL 5%: -11.08" "LEVEL 5%: 1.88"

619

620 [1] "LEVEL 50%: -11.07" "LEVEL 50%: 1.89"

621

622 [1] "LEVEL 95%: -11.06" "LEVEL 95%: 1.91"

623

624 [1] "LEVEL 97.5%: -11.06" "LEVEL 97.5%: 1.91"

625

626 [1] "Phenomenology: 3; Response: 2"

627

628 [1] "POSTERIOR MEAN: -8.64" "POSTERIOR MEAN: 1.72"

629 [1] "LEVEL 2.5%: -8.65" "LEVEL 2.5%: 1.7"

630

631

632 [1] "LEVEL 5%: -8.65" "LEVEL 5%: 1.7"

633

634 [1] "LEVEL 50%: -8.64" "LEVEL 50%: 1.72"

635

636 [1] "LEVEL 95%: -8.63" "LEVEL 95%: 1.73"

637

638 [1] "LEVEL 97.5%: -8.63" "LEVEL 97.5%: 1.73"

639

640 [1] "Phenomenology: 4; Response: 1"

641

642 [1] "POSTERIOR MEAN: -3.34" "POSTERIOR MEAN: 0.43"

643

644 [1] "LEVEL 2.5%: -3.53" "LEVEL 2.5%: 0.42"

645

646 [1] "LEVEL 5%: -3.5" "LEVEL 5%: 0.42"

647

648 [1] "LEVEL 50%: -3.35" "LEVEL 50%: 0.43"

649

650 [1] "LEVEL 95%: -3.16" "LEVEL 95%: 0.44"

651

652 [1] "LEVEL 97.5%: -3.12" "LEVEL 97.5%: 0.44"

653

654 [1] "Phenomenology: 4; Response: 2"

655

656 [1] "POSTERIOR MEAN: -2.63" "POSTERIOR MEAN: 0.29"

657

658 [1] "LEVEL 2.5%: -2.77" "LEVEL 2.5%: 0.28"

659

660 [1] "LEVEL 5%: -2.74" "LEVEL 5%: 0.28"

661

662 [1] "LEVEL 50%: -2.63" "LEVEL 50%: 0.29"

663

664 [1] "LEVEL 95%: -2.52" "LEVEL 95%: 0.3"

665

666 [1] "LEVEL 97.5%: -2.5" "LEVEL 97.5%: 0.3"

667

668 [1] "EMPLACEMENT CONDITION DEPENDENT COEFFICIENTS"

669

670 [1] "Phenomenology: 1; Emplacement: 1; Response: 1"

671

672 [1] "POSTERIOR MEAN: -10.07" "POSTERIOR MEAN: -1.31" "POSTERIOR MEAN: -1.43"

673 [4] "POSTERIOR MEAN: 3.51" "POSTERIOR MEAN: 0.38"



674  
675 [1] "LEVEL 2.5%: -10.1" "LEVEL 2.5%: -1.31" "LEVEL 2.5%: -1.46"  
676 [4] "LEVEL 2.5%: 3.39" "LEVEL 2.5%: 0.35"  
677  
678 [1] "LEVEL 5%: -10.09" "LEVEL 5%: -1.31" "LEVEL 5%: -1.46" "LEVEL 5%: 3.42"  
679 [5] "LEVEL 5%: 0.36"  
680  
681 [1] "LEVEL 50%: -10.07" "LEVEL 50%: -1.31" "LEVEL 50%: -1.43"  
682 [4] "LEVEL 50%: 3.51" "LEVEL 50%: 0.38"  
683  
684 [1] "LEVEL 95%: -10.05" "LEVEL 95%: -1.31" "LEVEL 95%: -1.41"  
685 [4] "LEVEL 95%: 3.6" "LEVEL 95%: 0.41"  
686  
687 [1] "LEVEL 97.5%: -10.04" "LEVEL 97.5%: -1.31" "LEVEL 97.5%: -1.41"  
688 [4] "LEVEL 97.5%: 3.62" "LEVEL 97.5%: 0.42"  
689  
690 [1] "Phenomenology: 1; Emplacement: 1; Response: 2"  
691  
692 [1] "POSTERIOR MEAN: -1.52" "POSTERIOR MEAN: -1.44" "POSTERIOR MEAN: -1.24"  
693 [4] "POSTERIOR MEAN: 2.26" "POSTERIOR MEAN: 0.63"  
694  
695 [1] "LEVEL 2.5%: -1.56" "LEVEL 2.5%: -1.45" "LEVEL 2.5%: -1.28"  
696 [4] "LEVEL 2.5%: 2.13" "LEVEL 2.5%: 0.59"  
697  
698 [1] "LEVEL 5%: -1.55" "LEVEL 5%: -1.45" "LEVEL 5%: -1.27" "LEVEL 5%: 2.15"  
699 [5] "LEVEL 5%: 0.6"  
700  
701 [1] "LEVEL 50%: -1.51" "LEVEL 50%: -1.44" "LEVEL 50%: -1.24" "LEVEL 50%: 2.25"  
702 [5] "LEVEL 50%: 0.63"  
703  
704 [1] "LEVEL 95%: -1.48" "LEVEL 95%: -1.44" "LEVEL 95%: -1.21" "LEVEL 95%: 2.38"  
705 [5] "LEVEL 95%: 0.67"  
706  
707 [1] "LEVEL 97.5%: -1.48" "LEVEL 97.5%: -1.44" "LEVEL 97.5%: -1.2"  
708 [4] "LEVEL 97.5%: 2.4" "LEVEL 97.5%: 0.68"  
709  
710 [1] "Phenomenology: 1; Emplacement: 2; Response: 1"  
711  
712 [1] "POSTERIOR MEAN: -11.48" "POSTERIOR MEAN: -1.09" "POSTERIOR MEAN: -3.62"  
713 [4] "POSTERIOR MEAN: 4.18" "POSTERIOR MEAN: 0.15"  
714  
715 [1] "LEVEL 2.5%: -11.5" "LEVEL 2.5%: -1.1" "LEVEL 2.5%: -3.8"  
716 [4] "LEVEL 2.5%: 3.5" "LEVEL 2.5%: 0.06"  
717  
718 [1] "LEVEL 5%: -11.5" "LEVEL 5%: -1.1" "LEVEL 5%: -3.77" "LEVEL 5%: 3.59"

719 [5] "LEVEL 5%: 0.08"

720

721 [1] "LEVEL 50%: -11.48" "LEVEL 50%: -1.09" "LEVEL 50%: -3.62"

722 [4] "LEVEL 50%: 4.17" "LEVEL 50%: 0.15"

723

724 [1] "LEVEL 95%: -11.46" "LEVEL 95%: -1.09" "LEVEL 95%: -3.48"

725 [4] "LEVEL 95%: 4.79" "LEVEL 95%: 0.22"

726

727 [1] "LEVEL 97.5%: -11.45" "LEVEL 97.5%: -1.09" "LEVEL 97.5%: -3.45"

728 [4] "LEVEL 97.5%: 4.89" "LEVEL 97.5%: 0.23"

729

730 [1] "Phenomenology: 1; Emplacement: 2; Response: 2"

731

732 [1] "POSTERIOR MEAN: -2.16" "POSTERIOR MEAN: -1.22" "POSTERIOR MEAN: -55.75"

733 [4] "POSTERIOR MEAN: 0.93" "POSTERIOR MEAN: -3.41"

734

735 [1] "LEVEL 2.5%: -2.28" "LEVEL 2.5%: -1.23" "LEVEL 2.5%: -67.22"

736 [4] "LEVEL 2.5%: 0.87" "LEVEL 2.5%: -3.66"

737

738 [1] "LEVEL 5%: -2.26" "LEVEL 5%: -1.23" "LEVEL 5%: -65.77" "LEVEL 5%: 0.88"

739 [5] "LEVEL 5%: -3.62"

740

741 [1] "LEVEL 50%: -2.16" "LEVEL 50%: -1.22" "LEVEL 50%: -55.44"

742 [4] "LEVEL 50%: 0.93" "LEVEL 50%: -3.41"

743

744 [1] "LEVEL 95%: -2.07" "LEVEL 95%: -1.21" "LEVEL 95%: -47.11"

745 [4] "LEVEL 95%: 0.99" "LEVEL 95%: -3.22"

746

747 [1] "LEVEL 97.5%: -2.05" "LEVEL 97.5%: -1.21" "LEVEL 97.5%: -46.03"

748 [4] "LEVEL 97.5%: 1" "LEVEL 97.5%: -3.2"

749

750 [1] "Phenomenology: 1; Emplacement: 3; Response: 1"

751

752 [1] "POSTERIOR MEAN: -9.53" "POSTERIOR MEAN: -1.16" "POSTERIOR MEAN: -4.47"

753 [4] "POSTERIOR MEAN: 4.96" "POSTERIOR MEAN: 0.34"

754

755 [1] "LEVEL 2.5%: -9.56" "LEVEL 2.5%: -1.16" "LEVEL 2.5%: -4.49"

756 [4] "LEVEL 2.5%: 4.89" "LEVEL 2.5%: 0.33"

757

758 [1] "LEVEL 5%: -9.55" "LEVEL 5%: -1.16" "LEVEL 5%: -4.49" "LEVEL 5%: 4.9"

759 [5] "LEVEL 5%: 0.33"

760

761 [1] "LEVEL 50%: -9.53" "LEVEL 50%: -1.16" "LEVEL 50%: -4.47" "LEVEL 50%: 4.96"

762 [5] "LEVEL 50%: 0.34"

763

764 [1] "LEVEL 95%: -9.5" "LEVEL 95%: -1.15" "LEVEL 95%: -4.45" "LEVEL 95%: 5.01"  
765 [5] "LEVEL 95%: 0.35"  
766  
767 [1] "LEVEL 97.5%: -9.5" "LEVEL 97.5%: -1.15" "LEVEL 97.5%: -4.44"  
768 [4] "LEVEL 97.5%: 5.02" "LEVEL 97.5%: 0.35"  
769  
770 [1] "Phenomenology: 1; Emplacement: 3; Response: 2"  
771  
772 [1] "POSTERIOR MEAN: -2.85" "POSTERIOR MEAN: -0.71" "POSTERIOR MEAN: -2.16"  
773 [4] "POSTERIOR MEAN: 2.61" "POSTERIOR MEAN: 0.09"  
774  
775 [1] "LEVEL 2.5%: -2.89" "LEVEL 2.5%: -0.71" "LEVEL 2.5%: -2.36"  
776 [4] "LEVEL 2.5%: 2.34" "LEVEL 2.5%: -0.05"  
777  
778 [1] "LEVEL 5%: -2.88" "LEVEL 5%: -0.71" "LEVEL 5%: -2.33" "LEVEL 5%: 2.38"  
779 [5] "LEVEL 5%: -0.03"  
780  
781 [1] "LEVEL 50%: -2.85" "LEVEL 50%: -0.71" "LEVEL 50%: -2.16" "LEVEL 50%: 2.61"  
782 [5] "LEVEL 50%: 0.09"  
783  
784 [1] "LEVEL 95%: -2.81" "LEVEL 95%: -0.7" "LEVEL 95%: -2.01" "LEVEL 95%: 2.84"  
785 [5] "LEVEL 95%: 0.2"  
786  
787 [1] "LEVEL 97.5%: -2.8" "LEVEL 97.5%: -0.7" "LEVEL 97.5%: -1.99"  
788 [4] "LEVEL 97.5%: 2.88" "LEVEL 97.5%: 0.21"  
789  
790 [1] "Phenomenology: 2; Emplacement: 1; Response: 1"  
791  
792 [1] "POSTERIOR MEAN: 3.87"  
793  
794 [1] "LEVEL 2.5%: 3.86"  
795  
796 [1] "LEVEL 5%: 3.86"  
797  
798 [1] "LEVEL 50%: 3.87"  
799  
800 [1] "LEVEL 95%: 3.88"  
801  
802 [1] "LEVEL 97.5%: 3.88"  
803  
804 [1] "Phenomenology: 2; Emplacement: 1; Response: 2"  
805  
806 [1] "POSTERIOR MEAN: -0.17"  
807  
808 [1] "LEVEL 2.5%: -0.19"

809 [1] "LEVEL 5%: -0.19"

810

811 [1] "LEVEL 50%: -0.17"

812

813 [1] "LEVEL 95%: -0.15"

814

815 [1] "LEVEL 97.5%: -0.15"

816

817 [1] "Phenomenology: 2; Emplacement: 2; Response: 1"

818

819 [1] "POSTERIOR MEAN: 3.11"

820

821 [1] "LEVEL 2.5%: 3.09"

822

823 [1] "LEVEL 5%: 3.09"

824

825 [1] "LEVEL 50%: 3.11"

826

827 [1] "LEVEL 95%: 3.14"

828

829 [1] "LEVEL 97.5%: 3.14"

830

831 [1] "Phenomenology: 2; Emplacement: 2; Response: 2"

832

833 [1] "POSTERIOR MEAN: -0.9"

834

835 [1] "LEVEL 2.5%: -0.96"

836

837 [1] "LEVEL 5%: -0.95"

838

839 [1] "LEVEL 50%: -0.9"

840

841 [1] "LEVEL 95%: -0.86"

842

843 [1] "LEVEL 97.5%: -0.85"

844

845 [1] "Phenomenology: 2; Emplacement: 3; Response: 1"

846

847 [1] "POSTERIOR MEAN: 2.16"

848

849 [1] "LEVEL 2.5%: 2.14"

850

851 [1] "LEVEL 5%: 2.14"

852

853

854 [1] "LEVEL 50%: 2.16"

855

856 [1] "LEVEL 95%: 2.17"

857

858 [1] "LEVEL 97.5%: 2.18"

859

860 [1] "Phenomenology: 2; Emplacement: 3; Response: 2"

861

862 [1] "POSTERIOR MEAN: -0.94"

863

864 [1] "LEVEL 2.5%: -0.99"

865

866 [1] "LEVEL 5%: -0.98"

867

868 [1] "LEVEL 50%: -0.94"

869

870 [1] "LEVEL 95%: -0.9"

871

872 [1] "LEVEL 97.5%: -0.9"

873

874 [1] "LEVEL 1 VARIANCE COMPONENTS"

875

876 [1] "Phenomenology: 1; Response: 1"

877

878 [1] "POSTERIOR MEAN: 0.0012"

879

880 [1] "LEVEL 2.5%: 9e-04"

881

882 [1] "LEVEL 5%: 9e-04"

883

884 [1] "LEVEL 50%: 0.0012"

885

886 [1] "LEVEL 95%: 0.0015"

887

888 [1] "LEVEL 97.5%: 0.0015"

889

890 [1] "Phenomenology: 1; Response: 2"

891

892 [1] "POSTERIOR MEAN: 0.0017"

893

894 [1] "LEVEL 2.5%: 0.0016"

895

896 [1] "LEVEL 5%: 0.0017"

897

898 [1] "LEVEL 50%: 0.0017"

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899
900      [1] "LEVEL 95%: 0.0019"
901
902      [1] "LEVEL 97.5%: 0.0019"
903
904      [1] "Phenomenology: 2; Response: 1"
905
906      [1] "POSTERIOR MEAN: 0.0039"
907
908      [1] "LEVEL 2.5%: 0.0035"
909
910      [1] "LEVEL 5%: 0.0036"
911
912      [1] "LEVEL 50%: 0.0039"
913
914      [1] "LEVEL 95%: 0.0043"
915
916      [1] "LEVEL 97.5%: 0.0044"
917
918      [1] "Phenomenology: 2; Response: 2"
919
920      [1] "POSTERIOR MEAN: 0.0016"
921
922      [1] "LEVEL 2.5%: 0.0015"
923
924      [1] "LEVEL 5%: 0.0015"
925
926      [1] "LEVEL 50%: 0.0016"
927
928      [1] "LEVEL 95%: 0.0018"
929
930      [1] "LEVEL 97.5%: 0.0018"
931
932      [1] "LEVEL 2 VARIANCE COMPONENTS"
933
934      [1] "Phenomenology: 1; Response: 1"
935
936      [1] "POSTERIOR MEAN: 0.001"
937
938      [1] "LEVEL 2.5%: 9e-04"
939
940      [1] "LEVEL 5%: 9e-04"
941
942      [1] "LEVEL 50%: 0.001"
943

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944         [1] "LEVEL 95%: 0.0012"
945
946         [1] "LEVEL 97.5%: 0.0013"
947
948         [1] "Phenomenology: 1; Response: 2"
949
950         [1] "POSTERIOR MEAN: 0.001"
951
952         [1] "LEVEL 2.5%: 8e-04"
953
954         [1] "LEVEL 5%: 8e-04"
955
956         [1] "LEVEL 50%: 0.001"
957
958         [1] "LEVEL 95%: 0.0012"
959
960         [1] "LEVEL 97.5%: 0.0012"
961
962     [1] "OBSERVATIONAL ERROR COVARIANCE PARAMETERS"
963
964     [1] "Phenomenology 1"
965
966     [1] "POSTERIOR MEAN:"
967
968     [1] "Variances"
969
970     [1] 0.0021 0.0034
971
972     [1] "Correlations"
973
974         [,1] [,2]
975     [1,]    1 0.27
976     [2,]    0 1.00
977
978     [1] "Variances"
979
980         [1] "LEVEL 2.5%: 0.002" "LEVEL 2.5%: 0.0032"
981
982     [1] "Correlations"
983
984         [1] "LEVEL 2.5%:"
985             [,1] [,2]
986     [1,]    1 0.26
987     [2,]    0 1.00
988

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```

989 [1] "Variances"
990
991 [1] "LEVEL 5%: 0.002" "LEVEL 5%: 0.0032"
992
993 [1] "Correlations"
994
995 [1] "LEVEL 5%:"
996 [1] [1] [2]
997 [1,] 1 0.26
998 [2,] 0 1.00
999
1000 [1] "Variances"
1001
1002 [1] "LEVEL 50%: 0.0021" "LEVEL 50%: 0.0034"
1003
1004 [1] "Correlations"
1005
1006 [1] "LEVEL 50%:"
1007 [1] [1] [2]
1008 [1,] 1 0.27
1009 [2,] 0 1.00
1010
1011 [1] "Variances"
1012
1013 [1] "LEVEL 95%: 0.0022" "LEVEL 95%: 0.0035"
1014
1015 [1] "Correlations"
1016
1017 [1] "LEVEL 95%:"
1018 [1] [1] [2]
1019 [1,] 1 0.28
1020 [2,] 0 1.00
1021
1022 [1] "Variances"
1023
1024 [1] "LEVEL 97.5%: 0.0022" "LEVEL 97.5%: 0.0036"
1025
1026 [1] "Correlations"
1027
1028 [1] "LEVEL 97.5%:"
1029 [1] [1] [2]
1030 [1,] 1 0.28
1031 [2,] 0 1.00
1032
1033 [1] "Phenomenology 2"

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1034
1035 [1] "POSTERIOR MEAN:"
1036
1037 [1] "Variances"
1038
1039 [1] 1e-03 3e-04
1040
1041 [1] "Correlations"
1042
1043      [,1] [,2]
1044 [1,]    1 -0.25
1045 [2,]    0  1.00
1046
1047 [1] "Variances"
1048
1049      [1] "LEVEL 2.5%: 0.001" "LEVEL 2.5%: 3e-04"
1050
1051 [1] "Correlations"
1052
1053      [1] "LEVEL 2.5%:"
1054      [,1] [,2]
1055 [1,]    1 -0.3
1056 [2,]    0  1.0
1057
1058 [1] "Variances"
1059
1060      [1] "LEVEL 5%: 0.001" "LEVEL 5%: 3e-04"
1061
1062 [1] "Correlations"
1063
1064      [1] "LEVEL 5%:"
1065      [,1] [,2]
1066 [1,]    1 -0.29
1067 [2,]    0  1.00
1068
1069 [1] "Variances"
1070
1071      [1] "LEVEL 50%: 0.001" "LEVEL 50%: 3e-04"
1072
1073 [1] "Correlations"
1074
1075      [1] "LEVEL 50%:"
1076      [,1] [,2]
1077 [1,]    1 -0.25
1078 [2,]    0  1.00

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1079
1080 [1] "Variances"
1081
1082           [1] "LEVEL 95%: 0.0011" "LEVEL 95%: 3e-04"
1083
1084 [1] "Correlations"
1085
1086           [1] "LEVEL 95%:"
1087           [,1] [,2]
1088 [1,]      1 -0.21
1089 [2,]      0  1.00
1090
1091 [1] "Variances"
1092
1093           [1] "LEVEL 97.5%: 0.0011" "LEVEL 97.5%: 3e-04"
1094
1095 [1] "Correlations"
1096
1097           [1] "LEVEL 97.5%:"
1098           [,1] [,2]
1099 [1,]      1 -0.21
1100 [2,]      0  1.00
1101
1102 [1] "Phenomenology 3"
1103
1104 [1] "POSTERIOR MEAN:"
1105
1106 [1] "Variances"
1107
1108 [1] 1e-03 9e-04
1109
1110 [1] "Correlations"
1111
1112           [,1] [,2]
1113 [1,]      1 0.92
1114 [2,]      0 1.00
1115
1116 [1] "Variances"
1117
1118           [1] "LEVEL 2.5%: 8e-04" "LEVEL 2.5%: 8e-04"
1119
1120 [1] "Correlations"
1121
1122           [1] "LEVEL 2.5%:"
1123           [,1] [,2]

```

```

1124 [1,]    1  0.9
1125 [2,]    0  1.0
1126
1127 [1] "Variances"
1128
1129      [1] "LEVEL 5%: 9e-04" "LEVEL 5%: 8e-04"
1130
1131 [1] "Correlations"
1132
1133      [1] "LEVEL 5%:"
1134      [,1] [,2]
1135 [1,]    1  0.9
1136 [2,]    0  1.0
1137
1138 [1] "Variances"
1139
1140      [1] "LEVEL 50%: 0.001" "LEVEL 50%: 9e-04"
1141
1142 [1] "Correlations"
1143
1144      [1] "LEVEL 50%:"
1145      [,1] [,2]
1146 [1,]    1 0.92
1147 [2,]    0 1.00
1148
1149 [1] "Variances"
1150
1151      [1] "LEVEL 95%: 0.0011" "LEVEL 95%: 0.0011"
1152
1153 [1] "Correlations"
1154
1155      [1] "LEVEL 95%:"
1156      [,1] [,2]
1157 [1,]    1 0.93
1158 [2,]    0 1.00
1159
1160 [1] "Variances"
1161
1162      [1] "LEVEL 97.5%: 0.0012" "LEVEL 97.5%: 0.0011"
1163
1164 [1] "Correlations"
1165
1166      [1] "LEVEL 97.5%:"
1167      [,1] [,2]
1168 [1,]    1 0.93

```

```

1169 [2,]    0 1.00
1170
1171 [1] "Phenomenology 4"
1172
1173 [1] "POSTERIOR MEAN:"
1174
1175 [1] "Variances"
1176
1177 [1] 0.0223 0.0061
1178
1179 [1] "Correlations"
1180
1181      [,1] [,2]
1182 [1,]    1 0.59
1183 [2,]    0 1.00
1184
1185 [1] "Variances"
1186
1187      [1] "LEVEL 2.5%: 0.0163" "LEVEL 2.5%: 0.0045"
1188
1189 [1] "Correlations"
1190
1191      [1] "LEVEL 2.5%:"
1192      [,1] [,2]
1193 [1,]    1 0.45
1194 [2,]    0 1.00
1195
1196 [1] "Variances"
1197
1198      [1] "LEVEL 5%: 0.0169" "LEVEL 5%: 0.0046"
1199
1200 [1] "Correlations"
1201
1202      [1] "LEVEL 5%:"
1203      [,1] [,2]
1204 [1,]    1 0.47
1205 [2,]    0 1.00
1206
1207 [1] "Variances"
1208
1209      [1] "LEVEL 50%: 0.022" "LEVEL 50%: 0.006"
1210
1211 [1] "Correlations"
1212
1213      [1] "LEVEL 50%:"

```

```

1214           [,1] [,2]
1215 [1,]      1 0.6
1216 [2,]      0 1.0
1217
1218 [1] "Variances"
1219
1220           [1] "LEVEL 95%: 0.029" "LEVEL 95%: 0.0077"
1221
1222 [1] "Correlations"
1223
1224           [1] "LEVEL 95%:"
1225           [,1] [,2]
1226 [1,]      1 0.7
1227 [2,]      0 1.0
1228
1229 [1] "Variances"
1230
1231           [1] "LEVEL 97.5%: 0.0307" "LEVEL 97.5%: 0.008"
1232
1233 [1] "Correlations"
1234
1235           [1] "LEVEL 97.5%:"
1236           [,1] [,2]
1237 [1,]      1 0.71
1238 [2,]      0 1.00
1239
1240 [1] "FGSN PRIOR PARAMETERS"
1241
1242 [1] "POSTERIOR MEAN:"
1243
1244 [1] "Alpha = 17.33"
1245 [1] "Lambda squared = 8"
1246 [1] "Omega = -1.67" "Omega = 0.56"
1247
1248 [1] "Alpha:"
1249 [1] "LEVEL 2.5%: 17.13"
1250
1251 [1] "LEVEL 5%: 17.16"
1252
1253 [1] "LEVEL 50%: 17.33"
1254
1255 [1] "LEVEL 95%: 17.48"
1256
1257 [1] "LEVEL 97.5%: 17.51"
1258

```

```

1259 [1] "Lambda squared:"
1260 [1] "LEVEL 2.5%: 7.83"
1261
1262 [1] "LEVEL 5%: 7.85"
1263
1264 [1] "LEVEL 50%: 8"
1265
1266 [1] "LEVEL 95%: 8.13"
1267
1268 [1] "LEVEL 97.5%: 8.15"
1269
1270 [1] "Omega:"
1271 [1] "LEVEL 2.5%: -1.94" "LEVEL 2.5%: 0.49"
1272
1273 [1] "LEVEL 5%: -1.9" "LEVEL 5%: 0.5"
1274
1275 [1] "LEVEL 50%: -1.67" "LEVEL 50%: 0.56"
1276
1277 [1] "LEVEL 95%: -1.42" "LEVEL 95%: 0.62"
1278
1279 [1] "LEVEL 97.5%: -1.37" "LEVEL 97.5%: 0.63"
1280
1281 [1] "DIC = -6011.41"
1282
1283 [1] "PIC = -6005.05"

```

## A.5 New Event Data: Preprocessing

```
1 #####
2 #
3 # This file is the input deck for MultiPEM Toolbox rapid post-
4 # detonation analysis, based on using fixed values of the forward and
5 # error model parameters obtained from calibration data.
6 #
7 #####
8
9 #
10 # REQUIRED R PACKAGES
11 #
12
13 require(Matrix)
14
15 #
16 # END REQUIRED R PACKAGES
17 #
18
19 #
20 # PREPROCESSING
21 #
22
23 # Specify directory for general subroutines
24 gen_dir = "../..../Code"
25
26 # Source supporting R function
27 source(paste(gen_dir,"/prepro_0.r",sep=""))
28
29 # Specify directory for application subroutines
30 app_dir = "../..../Code"
31
32 # Specify root data directory
33 dat_dir = "../..../Data"
34
35 # Specify new event data directories
36 dat_new = c("seismic_new.csv",
37             "acoustic_new.csv",
38             "optical_new.csv",
39             "crater_new.csv")
40
41 # Phenomenologies for this analysis
42 # 1 - seismic
43 # 2 - acoustic
```

```

44 # 3 - optical
45 # 4 - crater (surface effects)
46
47 # Names of new event inference parameters
48 theta_names = c("W","HOB")
49
50 # Number of calibration parameter imputations utilized in
51 # Markov chain Monte Carlo (MCMC) for new event parameters
52 nimpute = 1
53
54 # Set flag for bounded optimization
55 opt_B = FALSE
56
57 # Indicate nev parameter transform
58 itransform = FALSE
59
60 # Specify fixed parameters for nev parameter transform
61 if( itransform ){
62     tPars = TRUE
63
64     if( tPars ){
65         tPars = vector("list",0)
66         tPars$yield_scaling = 1/3
67     } else { tPars = NULL }
68 } else { tPars = NULL }
69
70 # Set up parameter constraints
71 # lower and upper bounds (use -Inf and Inf if unbounded)
72 lb_theta0 = rep(-Inf,length(theta_names))
73 lb_theta0[2] = -10
74 ub_theta0 = rep(Inf,length(theta_names))
75 ub_theta0[2] = 160
76
77 # Set up parameter subsets by phenomenology
78 tsub = TRUE
79
80 if( tsub ) {
81     tsub = vector("list",length(Rh))
82     tsub[[4]] = 1 # only log-yield for crater
83 } else { tsub = NULL }
84
85 # Preprocessing for statistical analysis routines
86 tmp = prepro_0(p_cal,gen_dir,app_dir,dat_dir,dat_new,theta_names,
87               nimp=nimpute,bopt=opt_B,itr=itransform,fp_tr=tPars,
88               tlb=lb_theta0,tub=ub_theta0,tsub=tsub)

```



```
89  if( opt_B ){
90    p_cal = tmp$p_cal
91    t_cal = tmp$t_cal
92  } else {
93    p_cal = tmp
94    t_cal = NULL
95  }
96  rm(tmp)
97  save.image()
98
99  #
100 # END PREPROCESSING
101 #
```

## A.6 New Event Data: Maximum Likelihood Estimation

```
1  #
2  # MAXIMUM LIKELIHOOD CALCULATION
3  #
4
5  # Source supporting R function
6  source(paste(gen_dir,"/calc_mle_0.r",sep=""))
7
8  # Set seed for repeatability of analysis
9  set.seed(631)
10
11 # Names of forward models for each response
12 # by phenomenology
13 fm0 = vector("list",length(Rh))
14 fm0[[1]] = c("f0_s","f0_s")
15 fm0[[2]] = c("f0_a","f0_a")
16 fm0[[3]] = c("f0_o","f0_o")
17 fm0[[4]] = c("f0_c","f0_c")
18
19 # Indicate if forward model gradients provided
20 igrad = TRUE
21
22 if( igrad ){
23   # Names of forward model gradients for each response
24   # by phenomenology
25   gfm0 = vector("list",length(Rh))
26   gfm0[[1]] = c("g0_s","g0_s")
27   gfm0[[2]] = c("g0_a","g0_a")
28   gfm0[[3]] = c("g0_o","g0_o")
29   gfm0[[4]] = c("g0_c","g0_c")
30 } else { gfm0 = NULL }
31
32 # Specify number of starting values for optimization
33 nstart = 10
34
35 # number of cores to use for optimization
36 ncores_mle = 1
37
38 # Indicate use of BFGS optimization methods
39 bfgs = TRUE
40
41 # Location of R data files with starting values
42 # for input to MLE optimization
43 opt_files_in = NULL
```

```

44
45 # Location of R data file to write the results of
46 # MLE optimization
47 opt_files_out = "./opt_nev.RData"
48
49 # Initial start value for theta0
50 tst = TRUE
51
52 if( tst ){
53     tst = numeric(p_cal$ntheta0)
54     tst[2] = runif(1,-1.e-6,1.e-6)
55 } else { tst = NULL }
56
57 # Confidence interval levels for new event parameter inference
58 ci_nev = 0.95
59
60 # Indicator of MLE gradient check
61 mle_grad_ck = TRUE
62
63 # Strategy for running parallel jobs (future package)
64 parallel_plan = "multicore"
65
66 # MLE calculations
67 p_cal = calc_mle_0(p_cal,gen_dir,app_dir,fm0,nst=nstart,ncor=ncores_mle,
68                   ci_nev=ci_nev,igrad=igrad,bfgs=bfgs,
69                   igrck=mle_grad_ck,t_cal=t_cal,g0=gfm0,
70                   fopt_in=opt_files_in,Xst=NULL,tst=tst,
71                   fopt_out=opt_files_out,pl=parallel_plan)
72 save.image()
73
74 #
75 # END MAXIMUM LIKELIHOOD CALCULATION
76 #

```

## A.7 New Event Data: Bayesian Analysis

```
1  #
2  # BAYESIAN ANALYSIS
3  #
4
5  # Specify if Bayesian analysis is to be conducted
6  iBayes = TRUE
7
8  if( iBayes ){
9    # Source supporting R function
10    source(paste(gen_dir,"/calc_bayes_0.r",sep=""))
11
12    # specify MCMC algorithm
13    # options: "RAM", "FME", "NUTS", or "SMC"
14    iMCMC = "FME"
15
16    # burn-in
17    nburn = 10000
18
19    # production
20    nmcmc = 20000
21
22    # posterior sample thinning rate (for multiple imputation)
23    nthin = 1
24
25    # number of cores to use for generating parallel MCMC chains
26    ncores_mc = 1
27
28    # Indicator of prior distribution for theta0
29    iTheta0Prior = FALSE
30
31    if( iTheta0Prior ){
32      # location of code for computing log-prior densities and gradients
33      prior_files_theta0 = NULL
34      if( igrad ){
35        gr_prior_files_theta0 = NULL
36      } else { gr_prior_files_theta0 = NULL }
37
38      # prior distribution for new event parameters (theta0)
39      p_cal$lp_theta0$f = "lp_0"
40      if( igrad ){ p_cal$lp_theta0$g = "lq_0" }
41
42      # parameters for log yield parameter prior (Gaussian)
43      p_cal$pi_w_mu = (log(10)+log(10000000))/2
```

```

44     p_cal$pi_w_sd = (log(100000000)-log(10))/6
45     # parameters for HOB/DOB parameter prior (Gaussian)
46     p_cal$pi_h_mu = 0
47     p_cal$pi_h_sd = 160/3
48 } else {
49     prior_files_theta0 = NULL
50     gr_prior_files_theta0 = NULL
51 }
52
53 # Indicator of prior gradient check
54 prior_grad_ck = TRUE
55
56 # Indicator of posterior gradient check
57 post_grad_ck = TRUE
58
59 # Options for Sequential Monte Carlo (SMC) sampling
60 # (iMCMC = "SMC")
61 # number of cores to use for parallelization within SMC algorithm
62 ncores_smc = NULL
63 # new event parameter ranges for initial SMC sample
64 lb_smc = rep(-Inf,length(theta_names))
65 ub_smc = rep(Inf,length(theta_names))
66
67 # Bayesian calculations
68 p_cal = calc_bayes_0(p_cal,gen_dir,app_dir,nburn=nburn,nmcmc=nmcmc,
69                     nthin=ntthin,ncor=ncores_mc,igrad=igrad,
70                     igrck_pr=prior_grad_ck,igrck_po=post_grad_ck,
71                     bfgs=bfgs,itpr=iTheta0Prior,
72                     fpr_t=prior_files_theta0,
73                     fgpr_t=gr_prior_files_theta0,imcmc=iMCMC,
74                     pl=parallel_plan,ncor_smc=ncores_smc,
75                     lb_smc=lb_smc,ub_smc=ub_smc,t_cal=t_cal)
76 save.image()
77 }
78
79 #
80 # END BAYESIAN ANALYSIS
81 #

```

## A.8 New Event Data: Output File

```
1 > # MLE calculations
2 > p_cal = calc_mle_0(p_cal,gen_dir,app_dir,fm0,nst=nstart,ncor=ncores_mle,
3 +                   ci_nev=ci_nev,igrad=igrad,bfgs=bfgs,
4 +                   igrck=mle_grad_ck,t_cal=t_cal,g0=gfm0,
5 +                   fopt_in=opt_files_in,Xst=NULL,tst=tst,
6 +                   fopt_out=opt_files_out,pl=parallel_plan)
7 [1] "MLE CONVERGENCE STATUS"
8
9 [1] 0
10 [1] 2
11 [1] "MAXIMUM LIKELIHOOD SUMMARY"
12
13 [1] "NEW EVENT INFERENCE PARAMETERS"
14
15 [1] "ESTIMATE: "
16
17      W    HOB
18 14.02 -2.70
19
20 [1] "STANDARD DEVIATION: "
21
22      W    HOB
23 0.05 1.97
24
25 [1] "STANDARD DEVIATION FIXED MODEL PARAMETERS: "
26
27      W    HOB
28 0.05 1.79
29
30 [1] "CORRELATION MATRIX: "
31
32      W    HOB
33 W      1.00 -0.43
34 HOB -0.43  1.00
35
36 [1] "CORRELATION MATRIX FIXED MODEL PARAMETERS: "
37
38      W    HOB
39 W      1.00 -0.36
40 HOB -0.36  1.00
41
42 [1] "95%: CONFIDENCE INTERVAL:"
43
```

```

44      W    HOB
45 lb 13.92 -5.46
46 ub 14.13  3.30
47
48 [1] "95%: CONFIDENCE INTERVAL FIXED MODEL PARAMETERS:"
49
50      W    HOB
51 lb_0 13.93 -5.28
52 ub_0 14.11  2.52
53
54 Loading required package: numDeriv
55 [1] "CHECK LOG-LIKELIHOOD GRADIENTS"
56
57 [1] "Analytic gradient"
58 [1] 3.612240e-06 1.545468e-07
59 [1] "Numerical gradient"
60 [1] 3.611856e-06 1.526450e-07
61 [1] "Difference"
62 [1] 3.836284e-10 1.901811e-09
63
64 + # Bayesian calculations
65 + p_cal = calc_bayes_0(p_cal,gen_dir,app_dir,nburn=nburn,nmcmc=nmcmc,
66 +                      nthin=nthin,ncor=ncores_mc,igrad=igrad,
67 +                      igrck_pr=prior_grad_ck,igrck_po=post_grad_ck,
68 +                      bfgs=bfgs,itpr=iTheta0Prior,
69 +                      fpr_t=prior_files_theta0,
70 +                      fgpr_t=gr_prior_files_theta0,imcmc=iMCMC,
71 +                      pl=parallel_plan,t_cal=t_cal)
72 + save.image()
73 + }
74 [1] "MAP CONVERGENCE STATUS"
75
76 [1] 0
77 [1] 2
78 [1] "MAXIMUM A POSTERIORI SUMMARY"
79
80 [1] "NEW EVENT INFERENCE PARAMETERS"
81
82 [1] "ESTIMATE: "
83
84      W    HOB
85 14.02 -2.11
86
87 [1] "CHECK LOG-PRIOR GRADIENTS"
88

```

```

89 [1] "Analytic gradient"
90 [1] 0.000000 0.682004
91 [1] "Numerical gradient"
92 [1] 0.000000 0.682004
93 [1] "Difference"
94 [1] 0.000000e+00 7.597145e-12
95
96 [1] "CHECK LOG-POSTERIOR GRADIENTS"
97
98 [1] "Analytic gradient"
99 [1] 4.523644e-06 3.942643e-06
100 [1] "Numerical gradient"
101 [1] 4.524224e-06 3.942255e-06
102 [1] "Difference"
103 [1] -5.799625e-10 3.878053e-10
104
105 [1] "ACCEPTANCE RATES:"
106
107 [1] "Core 1: 0.8163666666666667"
108
109 [1] "POSTERIOR SUMMARY"
110
111 [1] "NEW EVENT INFERENCE PARAMETERS"
112
113 [1] "POSTERIOR MEAN: 14.03" "POSTERIOR MEAN: -0.77"
114
115 [1] "POSTERIOR SD: 0.05" "POSTERIOR SD: 3.04"
116
117 [1] "LEVEL 2.5%: 13.94" "LEVEL 2.5%: -5.54"
118
119 [1] "LEVEL 5%: 13.95" "LEVEL 5%: -5.03"
120
121 [1] "LEVEL 50%: 14.03" "LEVEL 50%: -1.42"
122
123 [1] "LEVEL 95%: 14.1" "LEVEL 95%: 4.48"
124
125 [1] "LEVEL 97.5%: 14.12" "LEVEL 97.5%: 5.25"
126
127 [1] "CORRELATION MATRIX:"
128
129      W HOB
130 W    1    0
131 HOB  0    1

```



## B Complete Assessment Run Files

This appendix provides an example run file and output file for complete assessments.

### B.1 Preprocessing

```
1 #####
2 #
3 # This file is the input deck for MultiPEM Toolbox complete post-
4 # detonation analysis.
5 #
6 #####
7
8 #
9 # REQUIRED R PACKAGES
10 #
11
12 require(Matrix)
13
14 #
15 # END REQUIRED R PACKAGES
16 #
17
18 #
19 # PREPROCESSING
20 #
21
22 # Specify directory for general subroutines
23 gen_dir = "../..../Code"
24
25 # Source supporting R function
26 source(paste(gen_dir,"/prepro.r",sep=""))
27
28 # Specify directory for application subroutines
29 app_dir = "../..../Code"
30
31 # Specify root data directory
32 dat_dir = "../..../Data"
33
34 # Specify calibration data directories
35 dat_cal = c("seismic_cal.csv",
36             "acoustic_cal.csv",
37             "optical_cal.csv",
38             "crater_cal.csv")
39
```

```

40 # Phenomenologies for this analysis
41 # 1 - seismic
42 # 2 - acoustic
43 # 3 - optical
44 # 4 - crater (surface effects)
45
46 # Specify number of responses for each phenomenology
47 Rh = c(2,2,2,2)
48
49 # Empirical model parameter count: common
50 # list with elements corresponding to phenomenologies
51 pbeta = vector("list",length(Rh))
52 for( hh in 1:length(Rh) ){ pbeta[[hh]] = numeric(Rh[hh]) }
53 # phenomenology 2
54 pbeta[[2]] = c(2,2)
55 # phenomenology 3
56 pbeta[[3]] = c(2,2)
57 # phenomenology 4
58 pbeta[[4]] = c(2,2)
59
60 # Specify number of emplacement conditions for each phenomenology
61 Th = TRUE
62
63 if( Th ){ Th = c(3,3,0,0)
64 } else { Th = NULL }
65
66 # Empirical model parameter count: emplacement condition
67 # list with elements corresponding to phenomenologies
68 if( !is.null(Th) ){
69   pbetat = vector("list",length(Rh))
70   for( hh in 1:length(Rh) ){
71     if( Th[hh] > 1 ){ pbetat[[hh]] = vector("list",Th[hh]) }
72   }
73   # phenomenology 1
74   for( tt in 1:Th[1] ){
75     pbetat[[1]][[tt]] = numeric(Rh[1])
76     pbetat[[1]][[tt]] = c(5,5)
77   }
78   # phenomenology 2
79   for( tt in 1:Th[2] ){
80     pbetat[[2]][[tt]] = numeric(Rh[2])
81     pbetat[[2]][[tt]] = c(1,1)
82   }
83 } else { pbetat = NULL }
84

```

```

85 # Locations of common parameters in full parameter vector
86 # list with elements corresponding to phenomenologies
87 if( !is.null(Th) ){
88   ibetar = vector("list",length(Rh))
89   for( hh in 1:length(Rh) ){
90     if( Th[hh] > 1 ){
91       # lists with elements for each response within
92       # emplacement condition
93       ibetar[[hh]] = vector("list",Th[hh]*Rh[hh])
94     }
95   }
96   # phenomenology 2
97   for( tt in 1:Th[2] ){
98     for( rr in 1:Rh[2] ){
99       ibetar[[2]][[(tt-1)*Rh[2]+rr]] = 1:2
100     }
101   }
102 } else { ibetar = NULL }
103
104 # Indicate analysis with errors-in-variables (eiv)
105 eiv = TRUE
106
107 # Specifications for errors-in-variables
108 if( eiv ){
109   # Specify phenomenologies utilizing
110   # errors-in-variables yields
111   ieiv = 3:4
112
113   # Errors-in-variables source lists by
114   # phenomenology
115   seiv = vector("list",length(Rh))
116   for( hh in ieiv ){ seiv[[hh]] = "ALL" }
117
118   # Set standard deviation of eiv Gaussian likelihood
119   eiv_w_sd = 0.1/3
120 } else {
121   ieiv = NULL
122   seiv = NULL
123   eiv_w_sd = NULL
124 }
125
126 # Specify Error Model
127 # Level 1 variance component parameter count
128 pvc_1 = TRUE
129

```

```

130 if( pvc_1 ){
131   pvc_1 = vector("list",length(Rh))
132   for( hh in 1:length(Rh) ){ pvc_1[[hh]] = numeric(Rh[hh]) }
133   # phenomenology 1
134   pvc_1[[1]] = c(1,1)
135   # phenomenology 2
136   pvc_1[[2]] = c(1,1)
137 } else { pvc_1 = NULL }
138
139 # Level 2 variance component parameter count
140 pvc_2 = TRUE
141
142 if( pvc_2 ){
143   pvc_2 = vector("list",length(Rh))
144   for( hh in 1:length(Rh) ){ pvc_2[[hh]] = numeric(Rh[hh]) }
145   # phenomenology 1
146   pvc_2[[1]] = c(1,1)
147   # phenomenology 2
148   #pvc_2[[2]] =
149 } else { pvc_2 = NULL }
150
151 # Set flag for user-provided code to calculate variance
152 # component coefficient matrices
153 calc_Z = FALSE
154
155 # Set flag for bounded optimization
156 # currently only supported for new event parameters
157 opt_B = FALSE
158
159 # Indicate analysis of new event (nev)
160 nev = TRUE
161
162 # Specifications for new event
163 if( nev ){
164   # Specify new event data directories
165   dat_new = c("seismic_new.csv",
166               "acoustic_new.csv",
167               "optical_new.csv",
168               "crater_new.csv")
169
170   # Names of new event inference parameters
171   theta_names = c("W","HOB")
172
173   # Indicate nev parameter transform
174   itransform = FALSE

```

```

175
176 # Specify fixed parameters for nev parameter transform
177 if( itransform ){
178     tPars = TRUE
179
180     if( tPars ){
181         tPars = vector("list",0)
182         tPars$yield_scaling = 1/3
183     } else { tPars = NULL }
184 } else { tPars = NULL }
185
186 # Set up parameter constraints
187 # lower and upper bounds (use -Inf and Inf if unbounded)
188 lb_theta0 = rep(-Inf,length(theta_names))
189 lb_theta0[2] = -10
190 ub_theta0 = rep(Inf,length(theta_names))
191 ub_theta0[2] = 160
192
193 # Set up parameter subsets by phenomenology
194 tsub = TRUE
195
196 if( tsub ){
197     tsub = vector("list",length(Rh))
198     tsub[[4]] = 1 # only log-yield for crater
199 } else { tsub = NULL }
200 } else {
201     dat_new = NULL
202     theta_names = NULL
203     itransform = FALSE
204     tPars = NULL
205     lb_theta0 = NULL
206     ub_theta0 = NULL
207     tsub = NULL
208 }
209
210 # Preprocessing for statistical analysis routines
211 tmp = prepro(gen_dir,app_dir,dat_dir,dat_cal,Rh,pbeta,bopt=opt_B,
212             nev=nev,itr=itransform,izmat=calc_Z,ieiv=ieiv,seiv=seiv,
213             ewsd=eiv_w_sd,Th=Th,pbetat=pbetat,ibetar=ibetar,
214             pvc_1=pvc_1,pvc_2=pvc_2,tnames=theta_names,fp_tr=tPars,
215             tlb=lb_theta0,tub=ub_theta0,ndir=dat_new,tsub=tsub)
216 if( opt_B ){
217     p_cal = tmp$p_cal
218     t_cal = tmp$t_cal
219 } else {

```

```
220     p_cal = tmp
221     t_cal = NULL
222 }
223 rm(tmp)
224 save.image()
225
226 #
227 # END PREPROCESSING
228 #
```

## B.2 Maximum Likelihood Estimation

```
1  #
2  # MAXIMUM LIKELIHOOD CALCULATION
3  #
4
5  # Source supporting R function
6  source(paste(gen_dir,"/calc_mle.r",sep=""))
7
8  # Set seed for repeatability of analysis
9  set.seed(611)
10
11 # Names of forward models for each response
12 # by phenomenology
13 fm = vector("list",length(Rh))
14 fm[[1]] = c("f_s","f_s")
15 fm[[2]] = c("f_a","f_a")
16 fm[[3]] = c("f_o","f_o")
17 fm[[4]] = c("f_c","f_c")
18
19 # Indicate if forward model gradients provided
20 igrad = TRUE
21
22 if( igrad ){
23   # Names of forward model gradients for each response
24   # by phenomenology
25   gfm = vector("list",length(Rh))
26   gfm[[1]] = c("g_s","g_s")
27   gfm[[2]] = c("g_a","g_a")
28   gfm[[3]] = c("g_o","g_o")
29   gfm[[4]] = c("g_c","g_c")
30 } else { gfm = NULL }
31
32 # Specifications for forward model calculations
33 # a) flags for modified forward model calculation by
34 #   response for each relevant phenomenology
35 iResponse = TRUE
36
37 if( iResponse ){
38   iResponse = vector("list",length(Rh))
39   iResponse[[1]] = c(TRUE,FALSE)
40   iResponse[[2]] = c(TRUE,FALSE)
41 } else { iResponse = NULL }
42
43 # b) fixed quantities required by forward models
```

```

44 fPars = TRUE
45
46 if( fPars ){
47   fPars = vector("list",length(Rh))
48   fPars[[1]]$yield_scaling = 1/3
49   fPars[[2]]$yield_scaling = 1/3
50   fPars[[2]]$pressure_scaling = 1/3
51   fPars[[2]]$temp_scaling = 1/2
52   fPars[[3]]$yield_scaling = 1/3
53   fPars[[3]]$pressure_scaling = 1/3
54   fPars[[3]]$temp_scaling = 1/2
55 } else { fPars = NULL }
56
57 # Specify number of starting values for optimization
58 nstart = 10
59
60 # number of cores to use for optimization
61 ncores_mle = 1
62
63 # Indicate use of BFGS optimization methods
64 bfgs = TRUE
65
66 # Location of R data files with starting values
67 # for input to MLE optimization
68 opt_files_in = c("../Opt/opt_1.RData",
69                 "../Opt/opt_2.RData",
70                 "../Opt/opt_3_eiv.RData",
71                 "../Opt/opt_4_eiv.RData")
72
73 # Location of R data file to write the results of
74 # MLE optimization
75 opt_files_out = "../opt.RData"
76
77 if( nev ){
78   # Initial start value for theta0
79   tst = TRUE
80
81   if( tst ){
82     tst = numeric(p_cal$ntheta0)
83     tst[2] = runif(1,-1.e-6,1.e-6)
84   } else { tst = NULL }
85
86   # Confidence interval levels for new event parameter inference
87   ci_nev = 0.95
88 } else {

```



```

89     tst = NULL
90     ci_nev = NULL
91 }
92
93 # Indicate phenomenology number and type (if needed
94 # for postprocessing)
95 Phen = TRUE
96
97 if( Phen ){
98     Phen = matrix(c(1,"Seismic"),nrow=1)
99 } else { Phen = NULL }
100
101 # Indicator of MLE gradient check
102 mle_grad_ck = TRUE
103
104 # Strategy for running parallel jobs (future package)
105 parallel_plan = "multicore"
106
107 # MLE calculations
108 p_cal = calc_mle(p_cal,gen_dir,app_dir,fm,nst=nstart,ncor=ncores_mle,
109                 ci_nev=ci_nev,igrad=igrad,bfgs=bfgs,igrck=mle_grad_ck,
110                 t_cal=t_cal,g=gfm,iresp=iResponse,fp_fm=fPars,
111                 fopt_in=opt_files_in,Xst=NULL,tst=tst,
112                 fopt_out=opt_files_out,phen=Phen,pl=parallel_plan)
113 save.image()
114
115 #
116 # END MAXIMUM LIKELIHOOD CALCULATION
117 #

```

## B.3 Bayesian Analysis

```
1  #
2  # BAYESIAN ANALYSIS
3  #
4
5  # Specify if Bayesian analysis is to be conducted
6  iBayes = TRUE
7
8  if( iBayes ){
9    # Source supporting R function
10   source(paste(gen_dir,"/calc_bayes.r",sep=""))
11
12   # Indicator of prior distribution for forward model
13   # coefficients
14   iBetaPrior = TRUE
15
16   if( iBetaPrior ){
17     # location of code for computing log-prior densities and gradients
18     prior_files_beta = "../Code/lp_beta_s.r"
19     if( igrad ){
20       gr_prior_files_beta = "../Code/glp_beta_s.r"
21     } else { gr_prior_files_beta = NULL }
22
23     # prior distribution for phenomenology 1
24     # forward model coefficients
25     p_cal$h[[1]]$lp_betat = vector("list",Th[1])
26     for( tt in 1:Th[1] ){
27       p_cal$h[[1]]$lp_betat[[tt]]$f = c("lp_s","lp_s")
28       if( igrad ){
29         p_cal$h[[1]]$lp_betat[[tt]]$g = c("lq_s","lq_s")
30       }
31     }
32   } else {
33     prior_files_beta = NULL
34     gr_prior_files_beta = NULL
35   }
36
37   # fixed scale parameters for variance component prior
38   # comment out if these parameters should vary
39   p_cal$A = 20
40
41   # eta parameter in Lewandowski-Kurowicka-Joe (LKJ) prior
42   # distribution for correlation parameters
43   p_cal$lp_corr$eta = 1
```

```

44
45 # FGSN parameters for errors-in-variables yields prior
46 # number of components
47 p_cal$K = 0
48 # total number of FGSN parameters
49 p_cal$p_fgsn = 0
50 if( eiv ){
51     p_cal$K = 2
52     p_cal$p_fgsn = p_cal$K + 2
53 }
54
55 # specify Markov chain Monte Carlo (MCMC) algorithm
56 # options: "RAM", "FME", or "NUTS"
57 iMCMC = "FME"
58
59 # burn-in
60 nburn = 10000
61
62 # production
63 nmcmc = 20000
64
65 # posterior sample thinning rate
66 nthin = 20
67
68 # number of cores to use for optimization
69 ncores_map = 1
70
71 # number of cores to use for generating parallel MCMC chains
72 ncores_mc = 1
73
74 # Indicator of prior distribution for theta0
75 iTheta0Prior = FALSE
76
77 if( nev && iTheta0Prior ){
78     # location of code for computing log-prior densities and gradients
79     prior_files_theta0 = NULL
80     if( igrad ){
81         gr_prior_files_theta0 = NULL
82     } else { gr_prior_files_theta0 = NULL }
83
84     # prior distribution for new event parameters (theta0)
85     p_cal$lp_theta0$f = "lp_0"
86     if( igrad ){ p_cal$lp_theta0$g = "lq_0" }
87
88     # parameters for log yield parameter prior (Gaussian)

```

```

89     p_cal$pi_w_mu = (log(10)+log(10000000))/2
90     p_cal$pi_w_sd = (log(10000000)-log(10))/6
91     # parameters for HOB/DOB parameter prior (Gaussian)
92     p_cal$pi_h_mu = 0
93     p_cal$pi_h_sd = 160/3
94 } else {
95     prior_files_theta0 = NULL
96     gr_prior_files_theta0 = NULL
97 }
98
99 # Indicator of prior gradient check
100 prior_grad_ck = TRUE
101
102 # Indicator of posterior gradient check
103 post_grad_ck = TRUE
104
105 # Bayesian calculations
106 p_cal = calc_bayes(p_cal,gen_dir,app_dir,nst=nstart,nburn=nburn,
107                   nmcmlc=nmcmlc,nthin=nthin,ncor_map=ncores_map,
108                   ncor_mc=ncores_mc,igrad=igrad,
109                   igrck_pr=prior_grad_ck,igrck_po=post_grad_ck,
110                   bfgs=bfgs,ibpr=iBetaPrior,itpr=iTheta0Prior,
111                   fpr_b=prior_files_beta,fgpr_b=gr_prior_files_beta,
112                   fpr_t=prior_files_theta0,
113                   fgpr_t=gr_prior_files_theta0,Xnom=NULL,imcmc=iMCMC,
114                   pl=parallel_plan,t_cal=t_cal)
115 save.image()
116 }
117
118 #
119 # END BAYESIAN ANALYSIS
120 #

```

## B.4 Output File

```
1 > # Preprocessing for statistical analysis routines
2 > tmp = prepro(gen_dir,app_dir,dat_dir,dat_cal,Rh,pbeta,bopt=opt_B,
3 +             nev=nev,itr=itransform,izmat=calc_Z,ieiv=ieiv,seiv=seiv,
4 +             ewsd=eiv_w_sd,Th=Th,pbetat=pbetat,ibetar=ibetar,
5 +             pvc_1=pvc_1,pvc_2=pvc_2,tnames=theta_names,fp_tr=tPars,
6 +             tlb=lb_theta0,tub=ub_theta0,ndir=dat_new,tsub=tsub)
7 [1] "Warning: Insufficient Paths for Level 2 Variance Component models with
8     Phenomenology 2 and Response 1."
9 [1] "Warning: Insufficient Paths for Level 2 Variance Component models with
10    Phenomenology 2 and Response 2."
11 [1] "Warning: Insufficient number of observations per Source for Variance
12    Component models with Phenomenology 3 and Response 1."
13 [1] "Warning: Insufficient number of observations per Source for Variance
14    Component models with Phenomenology 3 and Response 2."
15 [1] "Warning: Insufficient number of observations per Source for Variance
16    Component models with Phenomenology 4 and Response 1."
17 [1] "Warning: Insufficient number of observations per Source for Variance
18    Component models with Phenomenology 4 and Response 2."
19
20 > # MLE calculations
21 > p_cal = calc_mle(p_cal,gen_dir,app_dir,fm,nst=nstart,ncor=ncores_mle,
22 +                 ci_nev=ci_nev,igrad=igrad,bfgs=bfgs,igrck=mle_grad_ck,
23 +                 t_cal=t_cal,g=gfm,iresp=iResponse,fp_fm=fPars,
24 +                 fopt_in=opt_files_in,Xst=NULL,tst=tst,
25 +                 fopt_out=opt_files_out,phen=Phen,pl=parallel_plan)
26 [1] "MLE CONVERGENCE STATUS"
27
28 [1] 0
29 [1] 2
30 [1] "MAXIMUM LIKELIHOOD SUMMARY"
31
32 [1] "NEW EVENT INFERENCE PARAMETERS"
33
34 [1] "ESTIMATE: "
35
36     W    HOB
37 14.02 -2.62
38
39 [1] "STANDARD DEVIATION: "
40
41     W    HOB
42 0.05 1.93
43
```

```

44 [1] "CORRELATION MATRIX: "
45
46      W    HOB
47 W    1.00 -0.44
48 HOB -0.44  1.00
49
50 [1] "95%: CONFIDENCE INTERVAL:"
51
52      W    HOB
53 lb 13.92 -5.35
54 ub 14.12  3.18
55
56
57 [1] "ERRORS-IN-VARIABLES YIELDS"
58
59      7      8      9      10      11      13      14      16      17      20      21      22      23
60 16.29 16.21 16.51 16.61 17.00 12.21 17.57 17.27 16.52 14.50 15.75 17.59 15.23
61      24      25      28      29      30      31      33      34      35      36      37      38      39
62 15.88 16.46 14.52 12.13 17.71 23.08 23.42 17.51 21.96 22.34 16.71 21.02 18.52
63
64 [1] "COMMON COEFFICIENTS"
65
66      [1] "Phenomenology: 2; Response: 1"
67
68      [1] 6.67 -1.14
69
70      [1] "Phenomenology: 2; Response: 2"
71
72      [1] -5.08  0.23
73
74      [1] "Phenomenology: 3; Response: 1"
75
76      [1] -11.07  1.89
77
78      [1] "Phenomenology: 3; Response: 2"
79
80      [1] -8.64  1.71
81
82      [1] "Phenomenology: 4; Response: 1"
83
84      [1] -3.31  0.43
85
86      [1] "Phenomenology: 4; Response: 2"
87
88      [1] -2.38  0.28

```

```

89
90 [1] "EMPLACEMENT CONDITION DEPENDENT COEFFICIENTS"
91
92 [1] "Phenomenology: 1; Emplacement: 1; Response: 1"
93
94 [1] -10.07 -1.31 -1.43 3.53 0.40
95
96 [1] "Phenomenology: 1; Emplacement: 1; Response: 2"
97
98 [1] -1.52 -1.44 -1.23 2.29 0.63
99
100 [1] "Phenomenology: 1; Emplacement: 2; Response: 1"
101
102 [1] -11.48 -1.09 -3.59 4.32 0.16
103
104 [1] "Phenomenology: 1; Emplacement: 2; Response: 2"
105
106 [1] -2.19 -1.22 -26.48 0.99 -2.65
107
108 [1] "Phenomenology: 1; Emplacement: 3; Response: 1"
109
110 [1] -9.53 -1.16 -4.47 4.95 0.34
111
112 [1] "Phenomenology: 1; Emplacement: 3; Response: 2"
113
114 [1] -2.85 -0.71 -2.10 2.68 0.13
115
116 [1] "Phenomenology: 2; Emplacement: 1; Response: 1"
117
118 [1] 3.87
119
120 [1] "Phenomenology: 2; Emplacement: 1; Response: 2"
121
122 [1] -0.17
123
124 [1] "Phenomenology: 2; Emplacement: 2; Response: 1"
125
126 [1] 3.12
127
128 [1] "Phenomenology: 2; Emplacement: 2; Response: 2"
129
130 [1] -0.9
131
132 [1] "Phenomenology: 2; Emplacement: 3; Response: 1"
133

```

```

134         [1] 2.16
135
136         [1] "Phenomenology: 2; Emplacement: 3; Response: 2"
137
138         [1] -0.94
139
140 [1] "LEVEL 1 VARIANCE COMPONENTS"
141
142         [1] "Phenomenology: 1; Response: 1"
143
144         [1] 0.0011
145
146         [1] "Phenomenology: 1; Response: 2"
147
148         [1] 0.0016
149
150         [1] "Phenomenology: 2; Response: 1"
151
152         [1] 0.0037
153
154         [1] "Phenomenology: 2; Response: 2"
155
156         [1] 0.0015
157
158 [1] "LEVEL 2 VARIANCE COMPONENTS"
159
160         [1] "Phenomenology: 1; Response: 1"
161
162         [1] 0.001
163
164         [1] "Phenomenology: 1; Response: 2"
165
166         [1] 9e-04
167
168 [1] "OBSERVATIONAL ERROR COVARIANCE PARAMETERS"
169
170 [1] "Phenomenology 1"
171
172 [1] "Variances"
173
174 [1] 0.0021 0.0034
175
176 [1] "Correlations"
177
178         [,1] [,2]

```



```

179 [1,]    1 0.27
180 [2,]    0 1.00
181
182 [1] "Phenomenology 2"
183
184 [1] "Variances"
185
186 [1] 1e-03 3e-04
187
188 [1] "Correlations"
189
190      [,1] [,2]
191 [1,]    1 -0.25
192 [2,]    0  1.00
193
194 [1] "Phenomenology 3"
195
196 [1] "Variances"
197
198 [1] 0.0011 0.0010
199
200 [1] "Correlations"
201
202      [,1] [,2]
203 [1,]    1 0.93
204 [2,]    0 1.00
205
206 [1] "Phenomenology 4"
207
208 [1] "Variances"
209
210 [1] 0.0222 0.0087
211
212 [1] "Correlations"
213
214      [,1] [,2]
215 [1,]    1 0.65
216 [2,]    0 1.00
217
218 [1] "AIC = -5913.11"
219
220 [1] "BIC = -5573.35"
221
222 Loading required package: numDeriv
223 [1] "CHECK LOG-LIKELIHOOD GRADIENTS"

```

```

224
225 [1] "Analytic gradient"
226 [1] 5.112159e-04 -2.542508e-05 -1.863792e-03 6.228479e-04 -1.625812e-03
227 [6] 1.907117e-03 2.310639e-04 8.065429e-04 -8.404374e-05 -9.984542e-04
228 [11] -4.226071e-04 -1.989370e-03 -1.157463e-03 1.812303e-03 3.537858e-03
229 [16] -2.784636e-03 7.371172e-04 7.547859e-04 -1.494877e-03 4.223703e-03
230 [21] 1.534175e-03 -2.357543e-03 1.895850e-03 -3.989834e-04 1.732983e-03
231 [26] 9.354533e-04 -4.440143e-05 9.331977e-06 -2.694365e-03 -4.299476e-02
232 [31] -1.507931e-02 -2.445686e-01 -3.765500e-02 -7.708215e-04 2.986042e-02
233 [36] 6.118387e-03 -6.676220e-04 -1.555101e-02 1.448662e-03 2.788605e-02
234 [41] 9.125824e-04 2.695089e-04 -2.401801e-03 -1.526986e-04 -4.231445e-04
235 [46] -1.511128e-03 1.417248e-03 1.029923e-03 -1.364681e-04 3.342011e-04
236 [51] -1.656650e-02 -1.223063e-01 4.508649e-02 3.934503e-03 -7.615629e-03
237 [56] 1.978269e-03 1.821235e-03 4.439611e-03 1.006111e-02 4.886488e-03
238 [61] -3.438471e-04 2.571739e-03 1.720799e-03 2.864180e-04 -7.508282e-04
239 [66] 9.688080e-04 2.185358e-03 -1.105926e-03 1.041878e-04 -4.157820e-04
240 [71] 5.369795e-04 -4.609019e-04 2.385899e-04 -6.059969e-05 3.177182e-04
241 [76] 2.788174e-04 5.402526e-04 2.458910e-04 -3.436638e-05 -1.133472e-04
242 [81] -3.018574e-04 -1.905102e-04 1.069394e-04 5.970366e-04 -5.390348e-04
243 [86] 6.469861e-04 2.939823e-05 1.613974e-01 8.178509e-04 6.931910e-04
244 [91] 2.285890e-02 -1.018627e-04 4.539666e-05 2.014041e-04
245 [1] "Numerical gradient"
246 [1] 5.112092e-04 -2.539405e-05 -1.863777e-03 6.228507e-04 -1.625809e-03
247 [6] 1.907111e-03 2.310578e-04 8.065436e-04 -8.404145e-05 -9.984549e-04
248 [11] -4.225894e-04 -1.989392e-03 -1.157464e-03 1.812304e-03 3.537854e-03
249 [16] -2.784620e-03 7.370990e-04 7.547591e-04 -1.494772e-03 4.223713e-03
250 [21] 1.534173e-03 -2.357541e-03 1.895848e-03 -3.989830e-04 1.732981e-03
251 [26] 9.354541e-04 -4.440477e-05 9.337693e-06 -2.694363e-03 -4.299474e-02
252 [31] -1.507930e-02 -2.445664e-01 -3.765495e-02 -7.709241e-04 2.986035e-02
253 [36] 6.117932e-03 -6.675813e-04 -1.555099e-02 1.448742e-03 2.788585e-02
254 [41] 9.125534e-04 2.693702e-04 -2.401588e-03 -1.526865e-04 -4.228457e-04
255 [46] -1.511103e-03 1.417144e-03 1.029829e-03 -1.364287e-04 3.343325e-04
256 [51] -1.656650e-02 -1.223064e-01 4.508646e-02 3.934508e-03 -7.616756e-03
257 [56] 1.978321e-03 1.821328e-03 4.439639e-03 1.006101e-02 4.886500e-03
258 [61] -3.438479e-04 2.571653e-03 1.720924e-03 2.864399e-04 -7.508463e-04
259 [66] 9.688664e-04 2.185367e-03 -1.105973e-03 1.041493e-04 -4.163252e-04
260 [71] 5.369532e-04 -4.615999e-04 2.386148e-04 -6.041131e-05 3.177212e-04
261 [76] 2.789338e-04 5.402541e-04 2.458948e-04 -3.428431e-05 -1.133899e-04
262 [81] -3.018444e-04 -1.905103e-04 1.069402e-04 5.970440e-04 -5.336226e-04
263 [86] 6.470927e-04 2.945024e-05 1.614883e-01 8.171414e-04 6.932402e-04
264 [91] 2.286268e-02 -1.016709e-04 4.562640e-05 2.005773e-04
265 [1] "Difference"
266 [1] -9.084942e-05 1.126289e-06
267
268 + # Bayesian calculations

```

```

269 + p_cal = calc_bayes(p_cal,gen_dir,app_dir,nst=nstart,nburn=nburn,
270 +                    nmcnc=nmcmc,nthin=nthin,ncor_map=ncores_map,
271 +                    ncor_mc=ncores_mc,igrad=igrad,
272 +                    igrck_pr=prior_grad_ck,igrck_po=post_grad_ck,
273 +                    bfgs=bfgs,ibpr=iBetaPrior,itpr=iTheta0Prior,
274 +                    fpr_b=prior_files_beta,fgpr_b=gr_prior_files_beta,
275 +                    fpr_t=prior_files_theta0,
276 +                    fgpr_t=gr_prior_files_theta0,Xnom=NULL,imcmc=iMCMC,
277 +                    pl=parallel_plan,t_cal=t_cal)
278 + save.image()
279 + }
280 [1] "MAP CONVERGENCE STATUS"
281
282 [1] 0
283 [1] 2
284 [1] "MAXIMUM A POSTERIORI SUMMARY"
285
286 [1] "NEW EVENT INFERENCE PARAMETERS"
287
288 [1] "ESTIMATE: "
289
290      W      HOB
291 14.01  2.30
292
293
294 [1] "ERRORS-IN-VARIABLES YIELDS"
295
296      7      8      9     10     11     13     14     16     17     20     21     22     23
297 16.28 16.21 16.50 16.61 17.00 12.21 17.57 17.27 16.52 14.50 15.75 17.60 15.24
298     24     25     28     29     30     31     33     34     35     36     37     38     39
299 15.87 16.46 14.53 12.13 17.71 23.08 23.41 17.51 21.96 22.34 16.71 21.02 18.52
300
301 [1] "COMMON COEFFICIENTS"
302
303      [1] "Phenomenology: 2; Response: 1"
304
305      [1] 6.67 -1.14
306
307      [1] "Phenomenology: 2; Response: 2"
308
309      [1] -5.08 0.23
310
311      [1] "Phenomenology: 3; Response: 1"
312
313      [1] -11.07 1.89

```

314  
315 [1] "Phenomenology: 3; Response: 2"  
316  
317 [1] -8.64 1.71  
318  
319 [1] "Phenomenology: 4; Response: 1"  
320  
321 [1] -3.31 0.43  
322  
323 [1] "Phenomenology: 4; Response: 2"  
324  
325 [1] -2.38 0.28  
326  
327 [1] "EMPLACEMENT CONDITION DEPENDENT COEFFICIENTS"  
328  
329 [1] "Phenomenology: 1; Emplacement: 1; Response: 1"  
330  
331 [1] -10.06 -1.31 -1.44 3.49 0.38  
332  
333 [1] "Phenomenology: 1; Emplacement: 1; Response: 2"  
334  
335 [1] -1.50 -1.44 -1.25 2.24 0.61  
336  
337 [1] "Phenomenology: 1; Emplacement: 2; Response: 1"  
338  
339 [1] -11.48 -1.09 -3.60 4.25 0.16  
340  
341 [1] "Phenomenology: 1; Emplacement: 2; Response: 2"  
342  
343 [1] -2.13 -1.22 -159.29 0.90 -4.47  
344  
345 [1] "Phenomenology: 1; Emplacement: 3; Response: 1"  
346  
347 [1] -9.53 -1.16 -4.47 4.95 0.34  
348  
349 [1] "Phenomenology: 1; Emplacement: 3; Response: 2"  
350  
351 [1] -2.85 -0.71 -2.15 2.61 0.10  
352  
353 [1] "Phenomenology: 2; Emplacement: 1; Response: 1"  
354  
355 [1] 3.87  
356  
357 [1] "Phenomenology: 2; Emplacement: 1; Response: 2"  
358

359 [1] -0.17

360

361 [1] "Phenomenology: 2; Emplacement: 2; Response: 1"

362

363 [1] 3.11

364

365 [1] "Phenomenology: 2; Emplacement: 2; Response: 2"

366

367 [1] -0.91

368

369 [1] "Phenomenology: 2; Emplacement: 3; Response: 1"

370

371 [1] 2.16

372

373 [1] "Phenomenology: 2; Emplacement: 3; Response: 2"

374

375 [1] -0.94

376

377 [1] "LEVEL 1 VARIANCE COMPONENTS"

378

379 [1] "Phenomenology: 1; Response: 1"

380

381 [1] 0.0012

382

383 [1] "Phenomenology: 1; Response: 2"

384

385 [1] 0.0017

386

387 [1] "Phenomenology: 2; Response: 1"

388

389 [1] 0.0039

390

391 [1] "Phenomenology: 2; Response: 2"

392

393 [1] 0.0016

394

395 [1] "LEVEL 2 VARIANCE COMPONENTS"

396

397 [1] "Phenomenology: 1; Response: 1"

398

399 [1] 0.001

400

401 [1] "Phenomenology: 1; Response: 2"

402

403 [1] 0.001

```

404
405 [1] "OBSERVATIONAL ERROR COVARIANCE PARAMETERS"
406
407 [1] "Phenomenology 1"
408
409 [1] "Variances"
410
411 [1] 0.0021 0.0034
412
413 [1] "Correlations"
414
415      [,1] [,2]
416 [1,]    1 0.27
417 [2,]    0 1.00
418
419 [1] "Phenomenology 2"
420
421 [1] "Variances"
422
423 [1] 1e-03 3e-04
424
425 [1] "Correlations"
426
427      [,1] [,2]
428 [1,]    1 -0.25
429 [2,]    0 1.00
430
431 [1] "Phenomenology 3"
432
433 [1] "Variances"
434
435 [1] 9e-04 9e-04
436
437 [1] "Correlations"
438
439      [,1] [,2]
440 [1,]    1 0.92
441 [2,]    0 1.00
442
443 [1] "Phenomenology 4"
444
445 [1] "Variances"
446
447 [1] 0.0196 0.0071
448

```

```

449 [1] "Correlations"
450
451      [,1] [,2]
452 [1,]    1 0.52
453 [2,]    0 1.00
454
455 [1] "FGSN PRIOR PARAMETERS"
456
457 [1] "Alpha = 17.33"
458 [1] "Lambda squared = 8.05"
459 [1] "Omega = -1.65" "Omega = 0.56"
460
461 [1] "CHECK LOG-PRIOR GRADIENTS"
462
463 [1] "Analytic gradient"
464 [1] 0.000000e+00 7.980369e-01 -1.043263e-01 -8.247890e-02 -1.766663e-01
465 [6] -2.122515e-01 -3.470393e-01 1.996855e+00 -5.420832e-01 -4.415773e-01
466 [11] -1.814653e-01 3.548269e-01 5.536912e-02 -5.491970e-01 1.851314e-01
467 [16] 1.968395e-02 -1.633027e-01 3.474537e-01 2.182895e+00 -5.838604e-01
468 [21] -3.733141e-01 -6.350369e-01 -5.217498e-01 3.876847e-01 2.520478e-01
469 [26] -2.463583e-01 1.526028e-01 -7.433990e-01 0.000000e+00 0.000000e+00
470 [31] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
471 [36] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
472 [41] 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00
473 [46] 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00
474 [51] 0.000000e+00 0.000000e+00 7.782174e-01 0.000000e+00 0.000000e+00
475 [56] 0.000000e+00 0.000000e+00 9.276835e-02 0.000000e+00 0.000000e+00
476 [61] 0.000000e+00 0.000000e+00 6.610552e-01 0.000000e+00 0.000000e+00
477 [66] 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00
478 [71] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
479 [76] 0.000000e+00 4.999971e-01 4.999957e-01 4.999904e-01 4.999960e-01
480 [81] 4.999974e-01 4.999976e-01 -2.220446e-16 -7.766602e-01 -1.404975e+01
481 [86] 0.000000e+00 -8.150896e-01 4.186579e+01 -4.440892e-16 1.514024e+00
482 [91] -9.310054e+01 -4.440892e-16 -1.766204e-01 -1.871582e+01 -3.044490e-02
483 [96] -2.429394e-02 -7.026312e-02 -1.405367e-01
484 [1] "Numerical gradient"
485 [1] 0.000000e+00 7.980369e-01 -1.043263e-01 -8.247890e-02 -1.766663e-01
486 [6] -2.122515e-01 -3.470393e-01 1.996855e+00 -5.420832e-01 -4.415773e-01
487 [11] -1.814653e-01 3.548269e-01 5.536912e-02 -5.491970e-01 1.851314e-01
488 [16] 1.968395e-02 -1.633027e-01 3.474538e-01 2.182895e+00 -5.838604e-01
489 [21] -3.733141e-01 -6.350369e-01 -5.217498e-01 3.876847e-01 2.520478e-01
490 [26] -2.463583e-01 1.526028e-01 -7.433990e-01 0.000000e+00 0.000000e+00
491 [31] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
492 [36] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
493 [41] 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00

```

```

494 [46] 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00
495 [51] 0.000000e+00 0.000000e+00 7.782174e-01 0.000000e+00 0.000000e+00
496 [56] 0.000000e+00 0.000000e+00 9.276835e-02 0.000000e+00 0.000000e+00
497 [61] 0.000000e+00 0.000000e+00 6.610552e-01 0.000000e+00 0.000000e+00
498 [66] 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00
499 [71] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
500 [76] 0.000000e+00 4.999971e-01 4.999957e-01 4.999904e-01 4.999960e-01
501 [81] 4.999974e-01 4.999976e-01 0.000000e+00 -7.766602e-01 -1.404975e+01
502 [86] 0.000000e+00 -8.150896e-01 4.186579e+01 2.753625e-20 1.514024e+00
503 [91] -9.310054e+01 1.333140e-16 -1.766204e-01 -1.871582e+01 -3.044490e-02
504 [96] -2.429394e-02 -7.026312e-02 -1.405367e-01
505 [1] "Difference"
506 [1] -5.909749e-08 7.107675e-07
507
508 [1] "CHECK LOG-POSTERIOR GRADIENTS"
509
510 [1] "Analytic gradient"
511 [1] 0.0652554710 0.2809128777 0.2776747288 0.1913483992 0.1640543443
512 [6] 0.4110787063 0.3053493374 0.2732904664 0.0689460969 0.0034688791
513 [11] -0.3275711254 -0.5398117514 0.1307987365 0.3776565007 0.1278625292
514 [16] -1.0163032877 -0.3003047036 -0.5183880602 -0.4898692700 0.3585217007
515 [21] 0.1201606000 -0.0253855445 0.3277713035 -0.0943793983 0.1286572477
516 [26] 0.0709557394 -0.2406661285 -0.1030212436 0.2253706895 0.0874951338
517 [31] -0.0918057138 0.7026903265 -0.1706238390 1.2547660142 -0.0055983437
518 [36] 0.8590244663 -0.0192460560 0.4013458092 -0.0654405703 -0.5828461460
519 [41] 0.1246405443 -0.0227052968 -0.0380441080 0.0041646414 0.0634785826
520 [46] -0.0197653236 0.0154455135 0.1004165465 -0.0645952411 0.0333200130
521 [51] -0.0217611223 0.1961083586 0.3975783892 0.0062887012 -0.0368259828
522 [56] 0.0234689167 -0.1709389850 0.0385639350 -0.0001195893 -0.2933159601
523 [61] -0.0091858079 0.0182733107 0.0555022779 0.0462591964 -0.0944073914
524 [66] -0.0296077888 0.1329141126 0.2394165121 0.0738754511 -0.0903148613
525 [71] 0.0482421120 -0.3823240575 -0.0177985898 0.0577506753 -0.1026839723
526 [76] 0.0120394755 0.0211923436 0.0150026326 -0.0237693383 -0.0250745269
527 [81] 0.0140009614 0.0128335916 0.0898737506 0.0054301676 0.0055953664
528 [86] 0.0285555250 -0.0256819332 0.6947476606 0.3170921643 0.3694779435
529 [91] -0.1589376117 -0.0032056661 0.0286115155 1.1214529943 -0.0304449029
530 [96] -0.0242939414 -0.0702631185 -0.1405366559
531 [1] "Numerical gradient"
532 [1] 0.0652554649 0.2809128894 0.2776747326 0.1913483933 0.1640543491
533 [6] 0.4110787091 0.3053493421 0.2732904738 0.0689461035 0.0034688910
534 [11] -0.3275711180 -0.5398117334 0.1307987382 0.3776564972 0.1278625377
535 [16] -1.0163032755 -0.3003047094 -0.5183880474 -0.4898692439 0.3585217045
536 [21] 0.1201605998 -0.0253855458 0.3277713020 -0.0943793913 0.1286572464
537 [26] 0.0709557483 -0.2406661286 -0.1030212465 0.2253706584 0.0874950308
538 [31] -0.0918056653 0.7026919885 -0.1706238980 1.2547656219 -0.0055983545

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539 [36] 0.8590243042 -0.0192460637 0.4013454211 -0.0654405498 -0.5828455748
540 [41] 0.1246405538 -0.0227052931 -0.0380437658 0.0041647257 0.0634790314
541 [46] -0.0197652454 0.0154455414 0.1004166175 -0.0645952838 0.0333197483
542 [51] -0.0217611143 0.1961083798 0.3975782937 0.0062887030 -0.0368266050
543 [56] 0.0234689285 -0.1709390734 0.0385639399 -0.0001195863 -0.2933159334
544 [61] -0.0091857966 0.0182732944 0.0555020405 0.0462592100 -0.0944077022
545 [66] -0.0296077430 0.1329142502 0.2394164763 0.0738755689 -0.0903175582
546 [71] 0.0482421097 -0.3823248767 -0.0177985940 0.0577507259 -0.1026839206
547 [76] 0.0120396687 0.0211923580 0.0150026541 -0.0237692800 -0.0250745786
548 [81] 0.0140009232 0.0128335996 0.0898737813 0.0054301772 0.0055855654
549 [86] 0.0285555874 -0.0256819816 0.6947811056 0.3170917052 0.3694781757
550 [91] -0.1589561195 -0.0032054142 0.0286116001 1.1214522097 -0.0304449021
551 [96] -0.0242939549 -0.0702630914 -0.1405367301
552 [1] "Difference"
553 [1] -3.344501e-05 1.850774e-05
554
555 [1] "ACCEPTANCE RATES:"
556
557 [1] "Core 1: 0.8525666666666667"
558
559 [1] "POSTERIOR SUMMARY"
560
561 [1] "NEW EVENT INFERENCE PARAMETERS"
562
563 [1] "POSTERIOR MEAN: 14.01" "POSTERIOR MEAN: 2.26"
564
565 [1] "POSTERIOR SD: 0.01" "POSTERIOR SD: 0.69"
566
567 [1] "LEVEL 2.5%: 13.98" "LEVEL 2.5%: 0.99"
568
569 [1] "LEVEL 5%: 13.99" "LEVEL 5%: 1.18"
570
571 [1] "LEVEL 50%: 14.01" "LEVEL 50%: 2.23"
572
573 [1] "LEVEL 95%: 14.03" "LEVEL 95%: 3.47"
574
575 [1] "LEVEL 97.5%: 14.04" "LEVEL 97.5%: 3.69"
576
577 [1] "CORRELATION MATRIX:"
578
579      W  HOB
580 W    1.00 0.89
581 HOB 0.89 1.00
582
583 [1] "ERRORS-IN-VARIABLES YIELDS"

```

584

585 [1] "POSTERIOR MEAN: 16.28" "POSTERIOR MEAN: 16.21" "POSTERIOR MEAN: 16.51"

586 [4] "POSTERIOR MEAN: 16.61" "POSTERIOR MEAN: 17" "POSTERIOR MEAN: 12.21"

587 [7] "POSTERIOR MEAN: 17.57" "POSTERIOR MEAN: 17.27" "POSTERIOR MEAN: 16.52"

588 [10] "POSTERIOR MEAN: 14.5" "POSTERIOR MEAN: 15.75" "POSTERIOR MEAN: 17.6"

589 [13] "POSTERIOR MEAN: 15.24" "POSTERIOR MEAN: 15.87" "POSTERIOR MEAN: 16.46"

590 [16] "POSTERIOR MEAN: 14.53" "POSTERIOR MEAN: 12.13" "POSTERIOR MEAN: 17.71"

591 [19] "POSTERIOR MEAN: 23.08" "POSTERIOR MEAN: 23.41" "POSTERIOR MEAN: 17.51"

592 [22] "POSTERIOR MEAN: 21.96" "POSTERIOR MEAN: 22.34" "POSTERIOR MEAN: 16.71"

593 [25] "POSTERIOR MEAN: 21.02" "POSTERIOR MEAN: 18.52"

594

595 [1] "LEVEL 2.5%: 16.27" "LEVEL 2.5%: 16.2" "LEVEL 2.5%: 16.49"

596 [4] "LEVEL 2.5%: 16.59" "LEVEL 2.5%: 16.97" "LEVEL 2.5%: 12.2"

597 [7] "LEVEL 2.5%: 17.56" "LEVEL 2.5%: 17.25" "LEVEL 2.5%: 16.5"

598 [10] "LEVEL 2.5%: 14.48" "LEVEL 2.5%: 15.74" "LEVEL 2.5%: 17.58"

599 [13] "LEVEL 2.5%: 15.22" "LEVEL 2.5%: 15.86" "LEVEL 2.5%: 16.45"

600 [16] "LEVEL 2.5%: 14.51" "LEVEL 2.5%: 12.12" "LEVEL 2.5%: 17.7"

601 [19] "LEVEL 2.5%: 23.06" "LEVEL 2.5%: 23.4" "LEVEL 2.5%: 17.5"

602 [22] "LEVEL 2.5%: 21.94" "LEVEL 2.5%: 22.32" "LEVEL 2.5%: 16.69"

603 [25] "LEVEL 2.5%: 21" "LEVEL 2.5%: 18.5"

604

605 [1] "LEVEL 5%: 16.27" "LEVEL 5%: 16.2" "LEVEL 5%: 16.49" "LEVEL 5%: 16.59"

606 [5] "LEVEL 5%: 16.98" "LEVEL 5%: 12.2" "LEVEL 5%: 17.56" "LEVEL 5%: 17.25"

607 [9] "LEVEL 5%: 16.51" "LEVEL 5%: 14.49" "LEVEL 5%: 15.74" "LEVEL 5%: 17.59"

608 [13] "LEVEL 5%: 15.22" "LEVEL 5%: 15.86" "LEVEL 5%: 16.45" "LEVEL 5%: 14.52"

609 [17] "LEVEL 5%: 12.12" "LEVEL 5%: 17.7" "LEVEL 5%: 23.07" "LEVEL 5%: 23.4"

610 [21] "LEVEL 5%: 17.5" "LEVEL 5%: 21.95" "LEVEL 5%: 22.33" "LEVEL 5%: 16.69"

611 [25] "LEVEL 5%: 21.01" "LEVEL 5%: 18.5"

612

613 [1] "LEVEL 50%: 16.28" "LEVEL 50%: 16.21" "LEVEL 50%: 16.51" "LEVEL 50%: 16.61"

614 [5] "LEVEL 50%: 17" "LEVEL 50%: 12.21" "LEVEL 50%: 17.57" "LEVEL 50%: 17.27"

615 [9] "LEVEL 50%: 16.52" "LEVEL 50%: 14.5" "LEVEL 50%: 15.74" "LEVEL 50%: 17.6"

616 [13] "LEVEL 50%: 15.24" "LEVEL 50%: 15.87" "LEVEL 50%: 16.46" "LEVEL 50%: 14.53"

617 [17] "LEVEL 50%: 12.13" "LEVEL 50%: 17.71" "LEVEL 50%: 23.08" "LEVEL 50%: 23.41"

618 [21] "LEVEL 50%: 17.51" "LEVEL 50%: 21.96" "LEVEL 50%: 22.34" "LEVEL 50%: 16.71"

619 [25] "LEVEL 50%: 21.02" "LEVEL 50%: 18.52"

620

621 [1] "LEVEL 95%: 16.29" "LEVEL 95%: 16.22" "LEVEL 95%: 16.52" "LEVEL 95%: 16.62"

622 [5] "LEVEL 95%: 17.01" "LEVEL 95%: 12.23" "LEVEL 95%: 17.58" "LEVEL 95%: 17.28"

623 [9] "LEVEL 95%: 16.53" "LEVEL 95%: 14.51" "LEVEL 95%: 15.75" "LEVEL 95%: 17.61"

624 [13] "LEVEL 95%: 15.25" "LEVEL 95%: 15.88" "LEVEL 95%: 16.47" "LEVEL 95%: 14.54"

625 [17] "LEVEL 95%: 12.14" "LEVEL 95%: 17.72" "LEVEL 95%: 23.09" "LEVEL 95%: 23.43"

626 [21] "LEVEL 95%: 17.52" "LEVEL 95%: 21.97" "LEVEL 95%: 22.35" "LEVEL 95%: 16.72"

627 [25] "LEVEL 95%: 21.03" "LEVEL 95%: 18.53"

628

629 [1] "LEVEL 97.5%: 16.3" "LEVEL 97.5%: 16.22" "LEVEL 97.5%: 16.52"  
 630 [4] "LEVEL 97.5%: 16.63" "LEVEL 97.5%: 17.02" "LEVEL 97.5%: 12.23"  
 631 [7] "LEVEL 97.5%: 17.59" "LEVEL 97.5%: 17.29" "LEVEL 97.5%: 16.53"  
 632 [10] "LEVEL 97.5%: 14.51" "LEVEL 97.5%: 15.76" "LEVEL 97.5%: 17.61"  
 633 [13] "LEVEL 97.5%: 15.25" "LEVEL 97.5%: 15.89" "LEVEL 97.5%: 16.48"  
 634 [16] "LEVEL 97.5%: 14.54" "LEVEL 97.5%: 12.14" "LEVEL 97.5%: 17.72"  
 635 [19] "LEVEL 97.5%: 23.09" "LEVEL 97.5%: 23.43" "LEVEL 97.5%: 17.52"  
 636 [22] "LEVEL 97.5%: 21.98" "LEVEL 97.5%: 22.35" "LEVEL 97.5%: 16.73"  
 637 [25] "LEVEL 97.5%: 21.03" "LEVEL 97.5%: 18.53"  
 638  
 639 [1] "COMMON COEFFICIENTS"  
 640  
 641 [1] "Phenomenology: 2; Response: 1"  
 642  
 643 [1] "POSTERIOR MEAN: 6.67" "POSTERIOR MEAN: -1.14"  
 644  
 645 [1] "LEVEL 2.5%: 6.66" "LEVEL 2.5%: -1.14"  
 646  
 647 [1] "LEVEL 5%: 6.66" "LEVEL 5%: -1.14"  
 648  
 649 [1] "LEVEL 50%: 6.67" "LEVEL 50%: -1.14"  
 650  
 651 [1] "LEVEL 95%: 6.68" "LEVEL 95%: -1.14"  
 652  
 653 [1] "LEVEL 97.5%: 6.68" "LEVEL 97.5%: -1.14"  
 654  
 655 [1] "Phenomenology: 2; Response: 2"  
 656  
 657 [1] "POSTERIOR MEAN: -5.08" "POSTERIOR MEAN: 0.23"  
 658  
 659 [1] "LEVEL 2.5%: -5.08" "LEVEL 2.5%: 0.23"  
 660  
 661 [1] "LEVEL 5%: -5.08" "LEVEL 5%: 0.23"  
 662  
 663 [1] "LEVEL 50%: -5.08" "LEVEL 50%: 0.23"  
 664  
 665 [1] "LEVEL 95%: -5.08" "LEVEL 95%: 0.23"  
 666  
 667 [1] "LEVEL 97.5%: -5.08" "LEVEL 97.5%: 0.23"  
 668  
 669 [1] "Phenomenology: 3; Response: 1"  
 670  
 671 [1] "POSTERIOR MEAN: -11.07" "POSTERIOR MEAN: 1.89"  
 672  
 673 [1] "LEVEL 2.5%: -11.08" "LEVEL 2.5%: 1.87"

674 [1] "LEVEL 5%: -11.08" "LEVEL 5%: 1.88"

675

676 [1] "LEVEL 50%: -11.07" "LEVEL 50%: 1.89"

677

678 [1] "LEVEL 95%: -11.06" "LEVEL 95%: 1.91"

679

680 [1] "LEVEL 97.5%: -11.06" "LEVEL 97.5%: 1.91"

681

682 [1] "Phenomenology: 3; Response: 2"

683

684 [1] "POSTERIOR MEAN: -8.64" "POSTERIOR MEAN: 1.71"

685

686 [1] "LEVEL 2.5%: -8.64" "LEVEL 2.5%: 1.7"

687

688 [1] "LEVEL 5%: -8.64" "LEVEL 5%: 1.7"

689

690 [1] "LEVEL 50%: -8.64" "LEVEL 50%: 1.71"

691

692 [1] "LEVEL 95%: -8.63" "LEVEL 95%: 1.72"

693

694 [1] "LEVEL 97.5%: -8.63" "LEVEL 97.5%: 1.72"

695

696 [1] "Phenomenology: 4; Response: 1"

697

698 [1] "POSTERIOR MEAN: -3.31" "POSTERIOR MEAN: 0.43"

699

700 [1] "LEVEL 2.5%: -3.39" "LEVEL 2.5%: 0.43"

701

702 [1] "LEVEL 5%: -3.38" "LEVEL 5%: 0.43"

703

704 [1] "LEVEL 50%: -3.31" "LEVEL 50%: 0.43"

705

706 [1] "LEVEL 95%: -3.24" "LEVEL 95%: 0.43"

707

708 [1] "LEVEL 97.5%: -3.23" "LEVEL 97.5%: 0.43"

709

710 [1] "Phenomenology: 4; Response: 2"

711

712 [1] "POSTERIOR MEAN: -2.37" "POSTERIOR MEAN: 0.28"

713

714 [1] "LEVEL 2.5%: -2.45" "LEVEL 2.5%: 0.27"

715

716 [1] "LEVEL 5%: -2.43" "LEVEL 5%: 0.27"

717

718

719 [1] "LEVEL 50%: -2.37" "LEVEL 50%: 0.28"

720

721 [1] "LEVEL 95%: -2.31" "LEVEL 95%: 0.28"

722

723 [1] "LEVEL 97.5%: -2.3" "LEVEL 97.5%: 0.28"

724

725 [1] "EMPLACEMENT CONDITION DEPENDENT COEFFICIENTS"

726

727 [1] "Phenomenology: 1; Emplacement: 1; Response: 1"

728

729 [1] "POSTERIOR MEAN: -10.06" "POSTERIOR MEAN: -1.31" "POSTERIOR MEAN: -1.44"

730 [4] "POSTERIOR MEAN: 3.5" "POSTERIOR MEAN: 0.38"

731

732 [1] "LEVEL 2.5%: -10.07" "LEVEL 2.5%: -1.31" "LEVEL 2.5%: -1.47"

733 [4] "LEVEL 2.5%: 3.29" "LEVEL 2.5%: 0.32"

734

735 [1] "LEVEL 5%: -10.07" "LEVEL 5%: -1.31" "LEVEL 5%: -1.46" "LEVEL 5%: 3.32"

736 [5] "LEVEL 5%: 0.33"

737

738 [1] "LEVEL 50%: -10.06" "LEVEL 50%: -1.31" "LEVEL 50%: -1.44"

739 [4] "LEVEL 50%: 3.5" "LEVEL 50%: 0.38"

740

741 [1] "LEVEL 95%: -10.05" "LEVEL 95%: -1.31" "LEVEL 95%: -1.41"

742 [4] "LEVEL 95%: 3.68" "LEVEL 95%: 0.43"

743

744 [1] "LEVEL 97.5%: -10.05" "LEVEL 97.5%: -1.31" "LEVEL 97.5%: -1.4"

745 [4] "LEVEL 97.5%: 3.72" "LEVEL 97.5%: 0.44"

746

747 [1] "Phenomenology: 1; Emplacement: 1; Response: 2"

748

749 [1] "POSTERIOR MEAN: -1.5" "POSTERIOR MEAN: -1.44" "POSTERIOR MEAN: -1.25"

750 [4] "POSTERIOR MEAN: 2.24" "POSTERIOR MEAN: 0.6"

751

752 [1] "LEVEL 2.5%: -1.52" "LEVEL 2.5%: -1.45" "LEVEL 2.5%: -1.28"

753 [4] "LEVEL 2.5%: 2.18" "LEVEL 2.5%: 0.53"

754

755 [1] "LEVEL 5%: -1.52" "LEVEL 5%: -1.45" "LEVEL 5%: -1.27" "LEVEL 5%: 2.19"

756 [5] "LEVEL 5%: 0.55"

757

758 [1] "LEVEL 50%: -1.5" "LEVEL 50%: -1.44" "LEVEL 50%: -1.25" "LEVEL 50%: 2.24"

759 [5] "LEVEL 50%: 0.6"

760

761 [1] "LEVEL 95%: -1.49" "LEVEL 95%: -1.44" "LEVEL 95%: -1.23" "LEVEL 95%: 2.28"

762 [5] "LEVEL 95%: 0.67"

763

764 [1] "LEVEL 97.5%: -1.49" "LEVEL 97.5%: -1.44" "LEVEL 97.5%: -1.22"  
765 [4] "LEVEL 97.5%: 2.29" "LEVEL 97.5%: 0.68"  
766  
767 [1] "Phenomenology: 1; Emplacement: 2; Response: 1"  
768  
769 [1] "POSTERIOR MEAN: -11.48" "POSTERIOR MEAN: -1.09" "POSTERIOR MEAN: -3.61"  
770 [4] "POSTERIOR MEAN: 4.23" "POSTERIOR MEAN: 0.15"  
771  
772 [1] "LEVEL 2.5%: -11.51" "LEVEL 2.5%: -1.1" "LEVEL 2.5%: -3.76"  
773 [4] "LEVEL 2.5%: 3.65" "LEVEL 2.5%: 0.09"  
774  
775 [1] "LEVEL 5%: -11.5" "LEVEL 5%: -1.1" "LEVEL 5%: -3.74" "LEVEL 5%: 3.73"  
776 [5] "LEVEL 5%: 0.1"  
777  
778 [1] "LEVEL 50%: -11.48" "LEVEL 50%: -1.09" "LEVEL 50%: -3.6"  
779 [4] "LEVEL 50%: 4.23" "LEVEL 50%: 0.15"  
780  
781 [1] "LEVEL 95%: -11.45" "LEVEL 95%: -1.09" "LEVEL 95%: -3.49"  
782 [4] "LEVEL 95%: 4.7" "LEVEL 95%: 0.21"  
783  
784 [1] "LEVEL 97.5%: -11.45" "LEVEL 97.5%: -1.09" "LEVEL 97.5%: -3.46"  
785 [4] "LEVEL 97.5%: 4.79" "LEVEL 97.5%: 0.22"  
786  
787 [1] "Phenomenology: 1; Emplacement: 2; Response: 2"  
788  
789 [1] "POSTERIOR MEAN: -2.13" "POSTERIOR MEAN: -1.22"  
790 [3] "POSTERIOR MEAN: -165.12" "POSTERIOR MEAN: 0.9"  
791 [5] "POSTERIOR MEAN: -4.5"  
792  
793 [1] "LEVEL 2.5%: -2.18" "LEVEL 2.5%: -1.23" "LEVEL 2.5%: -205.6"  
794 [4] "LEVEL 2.5%: 0.85" "LEVEL 2.5%: -4.74"  
795  
796 [1] "LEVEL 5%: -2.17" "LEVEL 5%: -1.22" "LEVEL 5%: -200.61"  
797 [4] "LEVEL 5%: 0.85" "LEVEL 5%: -4.71"  
798  
799 [1] "LEVEL 50%: -2.13" "LEVEL 50%: -1.22" "LEVEL 50%: -164.39"  
800 [4] "LEVEL 50%: 0.9" "LEVEL 50%: -4.5"  
801  
802 [1] "LEVEL 95%: -2.1" "LEVEL 95%: -1.22" "LEVEL 95%: -131.19"  
803 [4] "LEVEL 95%: 0.94" "LEVEL 95%: -4.29"  
804  
805 [1] "LEVEL 97.5%: -2.09" "LEVEL 97.5%: -1.21" "LEVEL 97.5%: -127.28"  
806 [4] "LEVEL 97.5%: 0.94" "LEVEL 97.5%: -4.27"  
807  
808 [1] "Phenomenology: 1; Emplacement: 3; Response: 1"

809  
810 [1] "POSTERIOR MEAN: -9.53" "POSTERIOR MEAN: -1.16" "POSTERIOR MEAN: -4.47"  
811 [4] "POSTERIOR MEAN: 4.95" "POSTERIOR MEAN: 0.34"  
812  
813 [1] "LEVEL 2.5%: -9.54" "LEVEL 2.5%: -1.16" "LEVEL 2.5%: -4.51"  
814 [4] "LEVEL 2.5%: 4.9" "LEVEL 2.5%: 0.31"  
815  
816 [1] "LEVEL 5%: -9.54" "LEVEL 5%: -1.16" "LEVEL 5%: -4.5" "LEVEL 5%: 4.91"  
817 [5] "LEVEL 5%: 0.32"  
818  
819 [1] "LEVEL 50%: -9.53" "LEVEL 50%: -1.16" "LEVEL 50%: -4.47" "LEVEL 50%: 4.95"  
820 [5] "LEVEL 50%: 0.34"  
821  
822 [1] "LEVEL 95%: -9.51" "LEVEL 95%: -1.16" "LEVEL 95%: -4.43" "LEVEL 95%: 5"  
823 [5] "LEVEL 95%: 0.36"  
824  
825 [1] "LEVEL 97.5%: -9.51" "LEVEL 97.5%: -1.15" "LEVEL 97.5%: -4.42"  
826 [4] "LEVEL 97.5%: 5.01" "LEVEL 97.5%: 0.36"  
827  
828 [1] "Phenomenology: 1; Emplacement: 3; Response: 2"  
829  
830 [1] "POSTERIOR MEAN: -2.85" "POSTERIOR MEAN: -0.71" "POSTERIOR MEAN: -2.17"  
831 [4] "POSTERIOR MEAN: 2.6" "POSTERIOR MEAN: 0.09"  
832  
833 [1] "LEVEL 2.5%: -2.88" "LEVEL 2.5%: -0.71" "LEVEL 2.5%: -2.38"  
834 [4] "LEVEL 2.5%: 2.43" "LEVEL 2.5%: -0.11"  
835  
836 [1] "LEVEL 5%: -2.87" "LEVEL 5%: -0.71" "LEVEL 5%: -2.34" "LEVEL 5%: 2.45"  
837 [5] "LEVEL 5%: -0.07"  
838  
839 [1] "LEVEL 50%: -2.84" "LEVEL 50%: -0.71" "LEVEL 50%: -2.16" "LEVEL 50%: 2.61"  
840 [5] "LEVEL 50%: 0.09"  
841  
842 [1] "LEVEL 95%: -2.82" "LEVEL 95%: -0.7" "LEVEL 95%: -2" "LEVEL 95%: 2.75"  
843 [5] "LEVEL 95%: 0.25"  
844  
845 [1] "LEVEL 97.5%: -2.81" "LEVEL 97.5%: -0.7" "LEVEL 97.5%: -1.97"  
846 [4] "LEVEL 97.5%: 2.78" "LEVEL 97.5%: 0.27"  
847  
848 [1] "Phenomenology: 2; Emplacement: 1; Response: 1"  
849  
850 [1] "POSTERIOR MEAN: 3.87"  
851  
852 [1] "LEVEL 2.5%: 3.85"  
853

854 [1] "LEVEL 5%: 3.85"

855

856 [1] "LEVEL 50%: 3.87"

857

858 [1] "LEVEL 95%: 3.89"

859

860 [1] "LEVEL 97.5%: 3.89"

861

862 [1] "Phenomenology: 2; Emplacement: 1; Response: 2"

863

864 [1] "POSTERIOR MEAN: -0.17"

865

866 [1] "LEVEL 2.5%: -0.19"

867

868 [1] "LEVEL 5%: -0.18"

869

870 [1] "LEVEL 50%: -0.17"

871

872 [1] "LEVEL 95%: -0.15"

873

874 [1] "LEVEL 97.5%: -0.14"

875

876 [1] "Phenomenology: 2; Emplacement: 2; Response: 1"

877

878 [1] "POSTERIOR MEAN: 3.11"

879

880 [1] "LEVEL 2.5%: 3.11"

881

882 [1] "LEVEL 5%: 3.11"

883

884 [1] "LEVEL 50%: 3.11"

885

886 [1] "LEVEL 95%: 3.12"

887

888 [1] "LEVEL 97.5%: 3.12"

889

890 [1] "Phenomenology: 2; Emplacement: 2; Response: 2"

891

892 [1] "POSTERIOR MEAN: -0.9"

893

894 [1] "LEVEL 2.5%: -0.95"

895

896 [1] "LEVEL 5%: -0.94"

897

898 [1] "LEVEL 50%: -0.9"



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899
900      [1] "LEVEL 95%: -0.86"
901
902      [1] "LEVEL 97.5%: -0.85"
903
904      [1] "Phenomenology: 2; Emplacement: 3; Response: 1"
905
906      [1] "POSTERIOR MEAN: 2.16"
907
908      [1] "LEVEL 2.5%: 2.14"
909
910      [1] "LEVEL 5%: 2.14"
911
912      [1] "LEVEL 50%: 2.16"
913
914      [1] "LEVEL 95%: 2.17"
915
916      [1] "LEVEL 97.5%: 2.17"
917
918      [1] "Phenomenology: 2; Emplacement: 3; Response: 2"
919
920      [1] "POSTERIOR MEAN: -0.94"
921
922      [1] "LEVEL 2.5%: -0.99"
923
924      [1] "LEVEL 5%: -0.98"
925
926      [1] "LEVEL 50%: -0.94"
927
928      [1] "LEVEL 95%: -0.91"
929
930      [1] "LEVEL 97.5%: -0.9"
931
932      [1] "LEVEL 1 VARIANCE COMPONENTS"
933
934      [1] "Phenomenology: 1; Response: 1"
935
936      [1] "POSTERIOR MEAN: 0.0012"
937
938      [1] "LEVEL 2.5%: 0.001"
939
940      [1] "LEVEL 5%: 0.001"
941
942      [1] "LEVEL 50%: 0.0012"
943

```

944 [1] "LEVEL 95%: 0.0014"

945

946 [1] "LEVEL 97.5%: 0.0014"

947

948 [1] "Phenomenology: 1; Response: 2"

949

950 [1] "POSTERIOR MEAN: 0.0017"

951

952 [1] "LEVEL 2.5%: 0.0014"

953

954 [1] "LEVEL 5%: 0.0014"

955

956 [1] "LEVEL 50%: 0.0017"

957

958 [1] "LEVEL 95%: 0.002"

959

960 [1] "LEVEL 97.5%: 0.0021"

961

962 [1] "Phenomenology: 2; Response: 1"

963

964 [1] "POSTERIOR MEAN: 0.0038"

965

966 [1] "LEVEL 2.5%: 0.0034"

967

968 [1] "LEVEL 5%: 0.0034"

969

970 [1] "LEVEL 50%: 0.0038"

971

972 [1] "LEVEL 95%: 0.0043"

973

974 [1] "LEVEL 97.5%: 0.0044"

975

976 [1] "Phenomenology: 2; Response: 2"

977

978 [1] "POSTERIOR MEAN: 0.0016"

979

980 [1] "LEVEL 2.5%: 0.0014"

981

982 [1] "LEVEL 5%: 0.0014"

983

984 [1] "LEVEL 50%: 0.0016"

985

986 [1] "LEVEL 95%: 0.0018"

987

988 [1] "LEVEL 97.5%: 0.0019"

```

989
990 [1] "LEVEL 2 VARIANCE COMPONENTS"
991
992     [1] "Phenomenology: 1; Response: 1"
993
994     [1] "POSTERIOR MEAN: 0.001"
995
996     [1] "LEVEL 2.5%: 9e-04"
997
998     [1] "LEVEL 5%: 9e-04"
999
1000    [1] "LEVEL 50%: 0.001"
1001
1002    [1] "LEVEL 95%: 0.0011"
1003
1004    [1] "LEVEL 97.5%: 0.0012"
1005
1006    [1] "Phenomenology: 1; Response: 2"
1007
1008    [1] "POSTERIOR MEAN: 0.001"
1009
1010    [1] "LEVEL 2.5%: 9e-04"
1011
1012    [1] "LEVEL 5%: 9e-04"
1013
1014    [1] "LEVEL 50%: 0.001"
1015
1016    [1] "LEVEL 95%: 0.001"
1017
1018    [1] "LEVEL 97.5%: 0.001"
1019
1020 [1] "OBSERVATIONAL ERROR COVARIANCE PARAMETERS"
1021
1022 [1] "Phenomenology 1"
1023
1024 [1] "POSTERIOR MEAN:"
1025
1026 [1] "Variances"
1027
1028 [1] 0.0021 0.0034
1029
1030 [1] "Correlations"
1031
1032     [,1] [,2]
1033 [1,]    1 0.27

```

```

1034 [2,]    0 1.00
1035
1036 [1] "Variances"
1037
1038      [1] "LEVEL 2.5%: 0.002" "LEVEL 2.5%: 0.0033"
1039
1040 [1] "Correlations"
1041
1042      [1] "LEVEL 2.5%:"
1043          [,1] [,2]
1044 [1,]    1 0.26
1045 [2,]    0 1.00
1046
1047 [1] "Variances"
1048
1049      [1] "LEVEL 5%: 0.002" "LEVEL 5%: 0.0033"
1050
1051 [1] "Correlations"
1052
1053      [1] "LEVEL 5%:"
1054          [,1] [,2]
1055 [1,]    1 0.26
1056 [2,]    0 1.00
1057
1058 [1] "Variances"
1059
1060      [1] "LEVEL 50%: 0.0021" "LEVEL 50%: 0.0034"
1061
1062 [1] "Correlations"
1063
1064      [1] "LEVEL 50%:"
1065          [,1] [,2]
1066 [1,]    1 0.27
1067 [2,]    0 1.00
1068
1069 [1] "Variances"
1070
1071      [1] "LEVEL 95%: 0.0021" "LEVEL 95%: 0.0035"
1072
1073 [1] "Correlations"
1074
1075      [1] "LEVEL 95%:"
1076          [,1] [,2]
1077 [1,]    1 0.29
1078 [2,]    0 1.00

```

```

1079
1080 [1] "Variances"
1081
1082           [1] "LEVEL 97.5%: 0.0021" "LEVEL 97.5%: 0.0035"
1083
1084 [1] "Correlations"
1085
1086           [1] "LEVEL 97.5%:"
1087           [,1] [,2]
1088 [1,]      1 0.29
1089 [2,]      0 1.00
1090
1091 [1] "Phenomenology 2"
1092
1093 [1] "POSTERIOR MEAN:"
1094
1095 [1] "Variances"
1096
1097 [1] 1e-03 3e-04
1098
1099 [1] "Correlations"
1100
1101           [,1] [,2]
1102 [1,]      1 -0.25
1103 [2,]      0 1.00
1104
1105 [1] "Variances"
1106
1107           [1] "LEVEL 2.5%: 0.001" "LEVEL 2.5%: 3e-04"
1108
1109 [1] "Correlations"
1110
1111           [1] "LEVEL 2.5%:"
1112           [,1] [,2]
1113 [1,]      1 -0.29
1114 [2,]      0 1.00
1115
1116 [1] "Variances"
1117
1118           [1] "LEVEL 5%: 0.001" "LEVEL 5%: 3e-04"
1119
1120 [1] "Correlations"
1121
1122           [1] "LEVEL 5%:"
1123           [,1] [,2]

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1124 [1,]    1 -0.28
1125 [2,]    0  1.00
1126
1127 [1] "Variances"
1128
1129      [1] "LEVEL 50%: 0.001" "LEVEL 50%: 3e-04"
1130
1131 [1] "Correlations"
1132
1133      [1] "LEVEL 50%:"
1134      [,1] [,2]
1135 [1,]    1 -0.26
1136 [2,]    0  1.00
1137
1138 [1] "Variances"
1139
1140      [1] "LEVEL 95%: 0.0011" "LEVEL 95%: 3e-04"
1141
1142 [1] "Correlations"
1143
1144      [1] "LEVEL 95%:"
1145      [,1] [,2]
1146 [1,]    1 -0.22
1147 [2,]    0  1.00
1148
1149 [1] "Variances"
1150
1151      [1] "LEVEL 97.5%: 0.0011" "LEVEL 97.5%: 3e-04"
1152
1153 [1] "Correlations"
1154
1155      [1] "LEVEL 97.5%:"
1156      [,1] [,2]
1157 [1,]    1 -0.22
1158 [2,]    0  1.00
1159
1160 [1] "Phenomenology 3"
1161
1162 [1] "POSTERIOR MEAN:"
1163
1164 [1] "Variances"
1165
1166 [1] 9e-04 9e-04
1167
1168 [1] "Correlations"

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1169
1170      [,1] [,2]
1171 [1,]    1 0.91
1172 [2,]    0 1.00
1173
1174 [1] "Variances"
1175
1176      [1] "LEVEL 2.5%: 8e-04" "LEVEL 2.5%: 7e-04"
1177
1178 [1] "Correlations"
1179
1180      [1] "LEVEL 2.5%:"
1181      [,1] [,2]
1182 [1,]    1 0.89
1183 [2,]    0 1.00
1184
1185 [1] "Variances"
1186
1187      [1] "LEVEL 5%: 8e-04" "LEVEL 5%: 8e-04"
1188
1189 [1] "Correlations"
1190
1191      [1] "LEVEL 5%:"
1192      [,1] [,2]
1193 [1,]    1 0.89
1194 [2,]    0 1.00
1195
1196 [1] "Variances"
1197
1198      [1] "LEVEL 50%: 9e-04" "LEVEL 50%: 9e-04"
1199
1200 [1] "Correlations"
1201
1202      [1] "LEVEL 50%:"
1203      [,1] [,2]
1204 [1,]    1 0.92
1205 [2,]    0 1.00
1206
1207 [1] "Variances"
1208
1209      [1] "LEVEL 95%: 0.0011" "LEVEL 95%: 0.001"
1210
1211 [1] "Correlations"
1212
1213      [1] "LEVEL 95%:"

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1214          [,1] [,2]
1215 [1,]      1 0.93
1216 [2,]      0 1.00
1217
1218 [1] "Variances"
1219
1220          [1] "LEVEL 97.5%: 0.0011" "LEVEL 97.5%: 0.001"
1221
1222 [1] "Correlations"
1223
1224          [1] "LEVEL 97.5%:"
1225          [,1] [,2]
1226 [1,]      1 0.93
1227 [2,]      0 1.00
1228
1229 [1] "Phenomenology 4"
1230
1231 [1] "POSTERIOR MEAN:"
1232
1233 [1] "Variances"
1234
1235 [1] 0.0201 0.0071
1236
1237 [1] "Correlations"
1238
1239          [,1] [,2]
1240 [1,]      1 0.53
1241 [2,]      0 1.00
1242
1243 [1] "Variances"
1244
1245          [1] "LEVEL 2.5%: 0.0156" "LEVEL 2.5%: 0.0056"
1246
1247 [1] "Correlations"
1248
1249          [1] "LEVEL 2.5%:"
1250          [,1] [,2]
1251 [1,]      1 0.4
1252 [2,]      0 1.0
1253
1254 [1] "Variances"
1255
1256          [1] "LEVEL 5%: 0.0162" "LEVEL 5%: 0.0058"
1257
1258 [1] "Correlations"

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1259
1260         [1] "LEVEL 5%:"
1261             [,1] [,2]
1262 [1,]      1 0.42
1263 [2,]      0 1.00
1264
1265 [1] "Variances"
1266
1267         [1] "LEVEL 50%: 0.02" "LEVEL 50%: 0.0071"
1268
1269 [1] "Correlations"
1270
1271         [1] "LEVEL 50%:"
1272             [,1] [,2]
1273 [1,]      1 0.53
1274 [2,]      0 1.00
1275
1276 [1] "Variances"
1277
1278         [1] "LEVEL 95%: 0.0244" "LEVEL 95%: 0.0086"
1279
1280 [1] "Correlations"
1281
1282         [1] "LEVEL 95%:"
1283             [,1] [,2]
1284 [1,]      1 0.63
1285 [2,]      0 1.00
1286
1287 [1] "Variances"
1288
1289         [1] "LEVEL 97.5%: 0.0253" "LEVEL 97.5%: 0.0088"
1290
1291 [1] "Correlations"
1292
1293         [1] "LEVEL 97.5%:"
1294             [,1] [,2]
1295 [1,]      1 0.65
1296 [2,]      0 1.00
1297
1298 [1] "FGSN PRIOR PARAMETERS"
1299
1300 [1] "POSTERIOR MEAN:"
1301
1302 [1] "Alpha = 17.32"
1303 [1] "Lambda squared = 8.03"

```

```

1304 [1] "Omega = -1.68" "Omega = 0.57"
1305
1306 [1] "Alpha:"
1307 [1] "LEVEL 2.5%: 17.25"
1308
1309 [1] "LEVEL 5%: 17.26"
1310
1311 [1] "LEVEL 50%: 17.32"
1312
1313 [1] "LEVEL 95%: 17.38"
1314
1315 [1] "LEVEL 97.5%: 17.39"
1316
1317 [1] "Lambda squared:"
1318 [1] "LEVEL 2.5%: 7.4"
1319
1320 [1] "LEVEL 5%: 7.51"
1321
1322 [1] "LEVEL 50%: 8.01"
1323
1324 [1] "LEVEL 95%: 8.57"
1325
1326 [1] "LEVEL 97.5%: 8.68"
1327
1328 [1] "Omega:"
1329 [1] "LEVEL 2.5%: -1.93" "LEVEL 2.5%: 0.48"
1330
1331 [1] "LEVEL 5%: -1.9" "LEVEL 5%: 0.49"
1332
1333 [1] "LEVEL 50%: -1.68" "LEVEL 50%: 0.57"
1334
1335 [1] "LEVEL 95%: -1.46" "LEVEL 95%: 0.65"
1336
1337 [1] "LEVEL 97.5%: -1.41" "LEVEL 97.5%: 0.66"
1338
1339 [1] "DIC = -6086.65"
1340
1341 [1] "PIC = -6080.32"

```