

Yield Assessment Using the MultiPEM Toolbox

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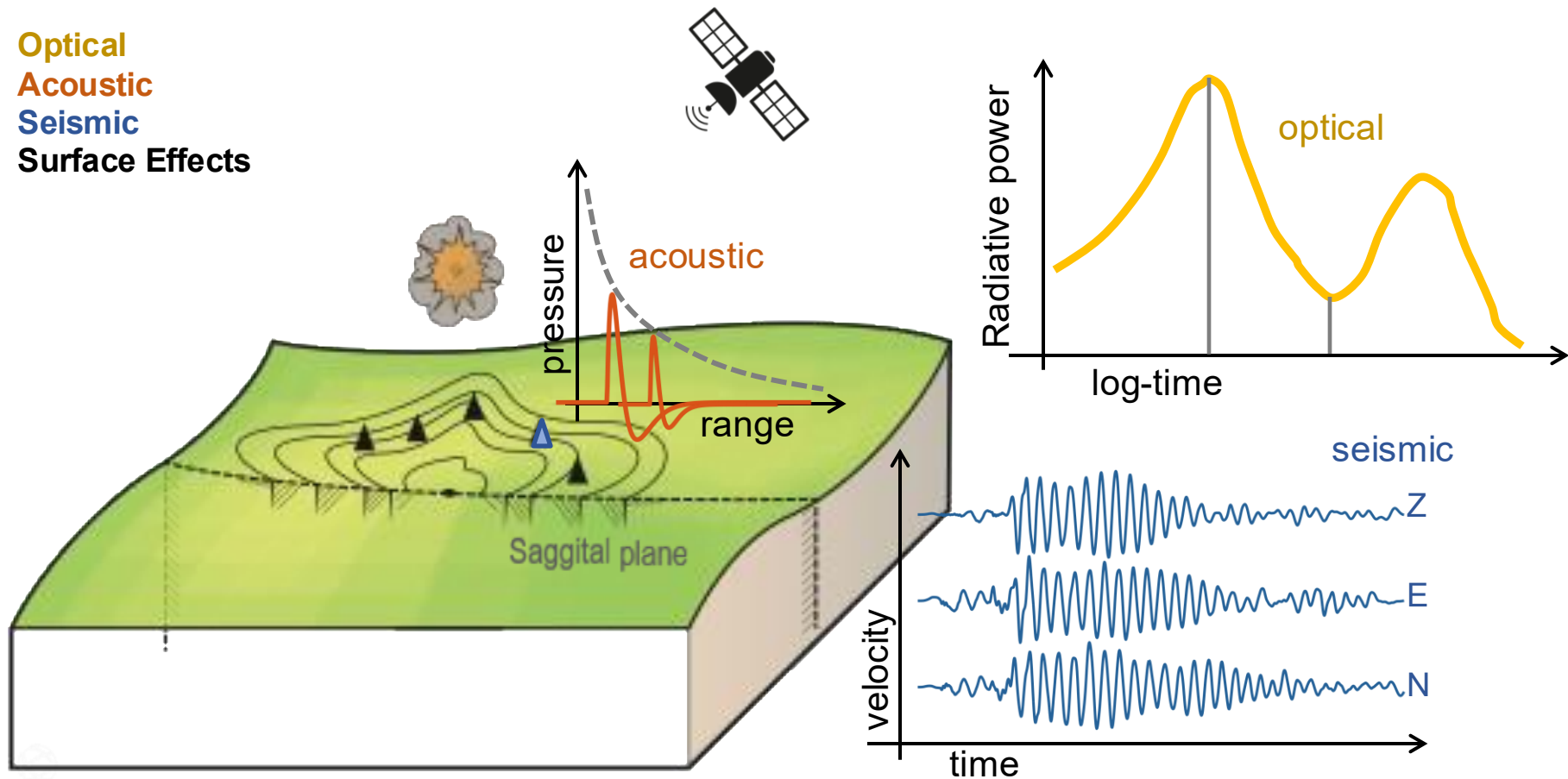
LYNM
***Low Yield Nuclear
Monitoring***

Abstract

This presentation demonstrates multi-physics (multivariate) yield estimation using disparate physical sensor signatures. Importantly, this methodology consistently aligns error propagation for yield estimation with first-order physical basis from explosion source to sensor propagation pathways. This implies that signature weighting in a yield calculation is done properly, including the ability to account for error correlation between disparate signatures. The advanced error budget includes error terms for source, propagation, and sensor site models — errors that masquerade as other uncertainties in models that do not directly incorporate these effects. These components of error in our advanced error budget, expressed as statistical variances/standard deviations, provide metrics on the technical validation of ensemble model components (e.g., source, propagation, and sensor). The approach is illustrated using simulated data from acoustic, seismic, optical, and surface effect sensor signatures.

Signatures of an Aboveground Event: *Assemble Data*

Optical
Acoustic
Seismic
Surface Effects



Introduction

- **Multi-Phenomenology Explosion Monitoring (MultiPEM) Toolbox** provides a capability for post-detonation characterization focusing on yield estimation
 - Synthesize information from multiple sensor modalities
 - Leverage historical databases to inform statistical models
 - Two-stage implementation for rapid new event device parameter characterization
 - Implemented in R code
- **References**
 - Williams, B.J., Brug, W.P., Casleton, E.M., Syracuse, E.M., Blom, P.S., Meierbachtol, C.S., Stead, R.J., MacLeod, G.A., Bauer, A.L., Shao, X.-M. & Anderson, D.N. (2021). Multiphenomenology explosion monitoring (MultiPEM): A general framework for data interpretation and yield estimation. *Geophysical Journal International*, 226, 14-32.
 - Ford, S.R., Bulaevskaya, V., Ramirez, A., Johannesson, G., & Rodgers, A.J. (2021). Joint Bayesian inference for near-surface explosion yield and height-of-burst. *Journal of Geophysical Research: Solid Earth*, 126, e2020JB020968

MultiPEM

The diagram illustrates the MultiPEM model, showing a central source (yellow starburst) emitting seismic waves. Red arrows represent the propagation of errors from the source to various stations (blue triangles). The stations are labeled with error terms: ϵ_{21} , ϵ_{22} , ϵ_{23} , ϵ_{11} , ϵ_{12} , ϵ_{32} , ϵ_{31} , ϵ_{42} , ϵ_{41} , and ϵ_{43} . A black arrow points from the source to the station labeled ϵ_{jk} .

Three probability density functions (PDFs) are shown, representing the distribution of errors:

- Source error (E_S):** A narrow, symmetric distribution centered at 0.
- Path error (E_P):** A wider, symmetric distribution centered at 0.
- Observation error (E_{jk}):** A very wide, symmetric distribution centered at 0.

Three bullet points describe the error types:

- Source model error affects all networks/stations equally
- Path model error varies by path
- Observation error varies by sensor

A large blue arrow points from the PDFs towards the bottom right corner.

Yield estimate PDFs

- Seismic (G)
- Acoustic (B)
- Unified (R)

The diagram shows three overlapping PDFs (red, green, and blue) representing the yield estimate distributions for Seismic (G), Acoustic (B), and Unified (R) methods. The x-axis is labeled "Yield" and the y-axis is labeled "PDF".

Reference: Williams, B.

- Reference:** Williams, B.J., Picard, R.R., & Anderson, D.N. (2023). **Multi-Phenomenology Yield Characterization**. Los Alamos National Laboratory Technical Report LA-UR-23-21950 (rev.4).

Basic Elements of MultiPEM Characterization

- Identify scenario of interest
 - Underground
 - **Near-surface (0 – 100 m)**
 - Free-air (> 100 m)
- Identify the **unknowns** of inferential interest for a new event
 - E.g. event time, location, **yield, HOB/DOB**
- For each scenario, identify relevant sensor modalities (**phenomenologies**)
 - **Seismic, Acoustic/Infrasound, Optical, Surface Effects**
- For each phenomenology, identify **signatures** (raw or processed waveforms)
 - Collect all relevant historical data

Basic Elements of MultiPEM Characterization

- For each signature, select a **forward model** and a statistical **error model**
 - Forward model relates signature to device parameters
 - Error model adjusts signature for unmodeled source and path effects
- Calibrate model parameters (e.g. regression coefficients) and variance components simultaneously
 - Use benchmark historical data having known device parameters
- New event characterization
 - Assimilate new event data into statistical framework
 - Infer unknown device parameters with rigorous UQ

Example: Validate Characterization of Near-Surface Nuclear Test

Data and Model Summaries (IYDT)

- Calibration and “new” event data provided by Sean Ford
- Predict the yield and HOB of the near-surface nuclear explosion **Sugar**
- Calibration data, error model, and model parameter count
 - Forward model parameters may depend on emplacement condition
 - Seismic, acoustic tests conducted in soft, hard, and wet rock
 - Chemical to nuclear equivalency: 2-1

Type	Explosion	Total Sources / Sample Size per Signature	Error Model	Total Model Parameters
Seismic	Chemical	34/392	Source + Path	37
Acoustic	Chemical	30/411	Source + Path	17
Optical	Nuclear	25/25	Observational	36 (EIV) / 11 (fixed)
Surface Effects	Nuclear	6/6	Observational	13 (EIV) / 7 (fixed)

[illegible]

**DATA NOT
AVAILABLE FOR NEW
EVENTS**

TYPE: Indicator for
e.g. emplacement condition

Acoustic IYDT Data

Y1	Y2	Source	Path	Station	Type	logTempSc	logPressureSc	lRange	W	C2N	HOB
2.076	-2.767	HRI-1	HR-103	BRDW	1	0.055	-0.196	7.809	6.291	0.693	5
3.095	-2.659	HRI-1	HR-48	E1	1	0.055	-0.196	6.714	6.291	0.693	5
1.990	-2.670	HRI-2	HR-103	BRDW	1	0.039	-0.188	7.813	6.291	0.693	3
4.195	-2.896	HRI-2	HR-12	LV1	1	0.039	-0.188	5.622	6.291	0.693	3
3.119	-2.700	HRI-2	HR-48	E1	1	0.039	-0.188	6.696	6.291	0.693	3
4.134	-2.903	HRI-3	HR-12	LV1	1	0.056	-0.192	5.608	6.291	0.693	1.5
3.068	-2.799	HRI-3	HR-48	E1	1	0.056	-0.192	6.716	6.291	0.693	1.5
3.151	-3.268	HRI-4	HR-12	LV1	1	0.028	-0.180	5.613	6.291	0.693	-1.5
2.204	-2.873	HRI-4	HR-48	E1	1	0.028	-0.180	6.707	6.291	0.693	-1.5
1.983	-2.747	HRI-5	HR-103	BRDW	1	0.042	-0.184	7.818	6.291	0.693	0.5
4.069	-2.994	HRI-5	HR-12	LV1	1	0.042	-0.184	5.651	6.291	0.693	0.5
3.069	-2.687	HRI-5	HR-48	E1	1	0.042	-0.184	6.697	6.291	0.693	0.5
1.755	-2.963	HRI-6	HR-103	BRDW	1	0.036	-0.188	7.810	6.291	0.693	-0.5
3.905	-3.099	HRI-6	HR-12	LV1	1	0.036	-0.188	5.559	6.291	0.693	-0.5
2.751	-2.852	HRI-6	HR-48	E1	1	0.036	-0.188	6.727	6.291	0.693	-0.5
-0.567	-2.856	HRI-7	HR-103	BRDW	1	0.049	-0.188	7.817	6.291	0.693	-5
1.494	-2.896	HRI-7	HR-12	LV1	1	0.049	-0.188	5.661	6.291	0.693	-5
0.573	-2.548	HRI-7	HR-48	E1	1	0.049	-0.188	6.686	6.291	0.693	-5

MultiPEM Statistical Model

$$\underbrace{y_{hijr}}_{\text{signature}} = \underbrace{f_{hijr}(\beta_{hr}, v_{hij})}_{\text{forward model}} + \underbrace{Z_{hir,j} b_{hr}^{(S)} + Z_{hijr} b_{hr}^{(P)}}_{\substack{\text{source (S) and path (P) bias} \\ \text{linear Gaussian random effects}}} + \underbrace{\epsilon_{hijr}}_{\substack{\text{observational} \\ \text{error}}}$$

error model

- Signature indexed by sensor type (h), source (i), path (j), & measurement (r)
 - New event ($i = 0$) and calibration events ($i > 0$)
 - New event QOIs θ_0 have known values in calibration data: $v_{h0j} = (\theta_0, v'_{h0j})$
- Forward model is a wrapper for source and path physics models
- Source (S) and path (P) bias terms modeled as random effects
 - Framework is therefore transportable to new test sites and sensors
- Observational errors modeled as Gaussian

MultiPEM Toolbox: Miscellaneous Notes

- Uncertainty in the yields of benchmark events can be modeled in the MultiPEM Toolbox

– *Errors-in-variables*: True unknown log-yield for source s

Given log-yield: Design, radiochemical, etc. → $\widetilde{W}_s \sim w_s + \epsilon_s, \quad \epsilon_s \sim \mathcal{N}(0, \sigma_s^2)$

Uncertainty in given log-yield:
e.g. $\sigma_s = 0.1$ specifies a 1-SD relative error in given yield of 10%

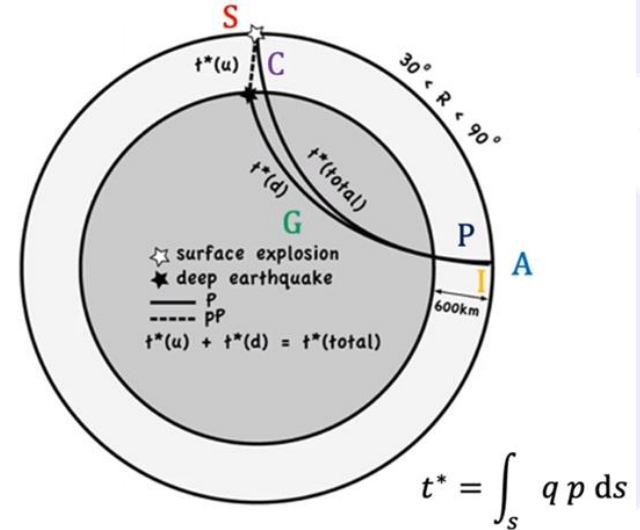
- Calibration parameters can be estimated by the MultiPEM Toolbox during benchmarking
 - Chemical-to-nuclear equivalency
- Likelihood and Bayes inference methods available in the MultiPEM Toolbox
 - Likelihood Principle*: All experimental information relevant to a statistical model's parameters is contained within the likelihood function
 - Bayes allows analysts to incorporate prior information about these parameters

Forward Models

Forward Models: Additive in Log-Spectral Space

$$\log A = \log I + \log P + \log S + \log C + \log G$$

↑
 Amplitude at sensor
 ↑
 Instrument correction
 ↑
 Near-site (i.e., station) crustal correction
 ↑
 Source term
 ↑
 Near-source crustal correction
 ↑
 Path term



Ichinose et al. (2014), Figure 1

[Ichinose et al., 2014]: Ichinose, G., Woods, M., & Dwyer, J. (2014). Mantle Attenuation Estimated from Regional and Teleseismic P-waves of Deep Earthquakes and Surface Explosions. *Pure and Applied Geophysics*, 171(3–5), 485–506. <https://doi.org/10.1007/s00024-012-0632-z>

Forward Models and Data

- For relevant (scenario, phenomenology) combinations ...
 - Identify one or more reduced order models (ROMs) connecting signatures to unknowns and controlled inputs, e.g.

$$f_{hij}(\beta_h, \mathbf{v}_{hij}) = \underbrace{\log S_{hi}(\beta_h^{(S)}, \mathbf{v}_{hi}^{(S)}) + \log P_{hij}(\beta_h^{(P)}, \mathbf{v}_{hij}^{(P)})}_{\text{forward model}}$$

h = phenomenology, i = source, j = path

- Collect data sets tailored to benchmarking unknowns (regression parameters) in each ROM
 - Chemical or nuclear explosions
- ROMs can be benchmarked to chemical and nuclear data simultaneously
 - Chemical to nuclear equivalency

Seismic Forward Model

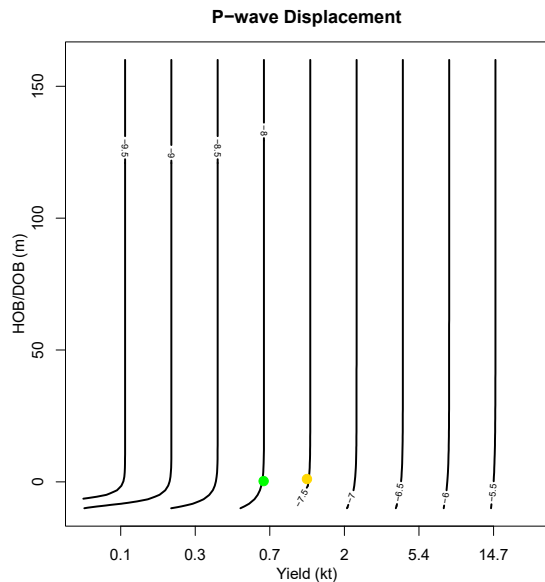
$$\log d_{sijr} = \beta_{sr,1} + \beta_{sr,2} \log \tilde{r}_{sij} + \beta_{sr,3} \text{logistic} \left(\beta_{sr,4} \tilde{h}_{si} + \beta_{sr,5} \right)$$

$r = 1$. **scaled** P-wave displacement
 $r = 2$. peak velocity

scaled range

Yield = y
 Scale factor = $y^{1/3}$

scaled height-of-burst



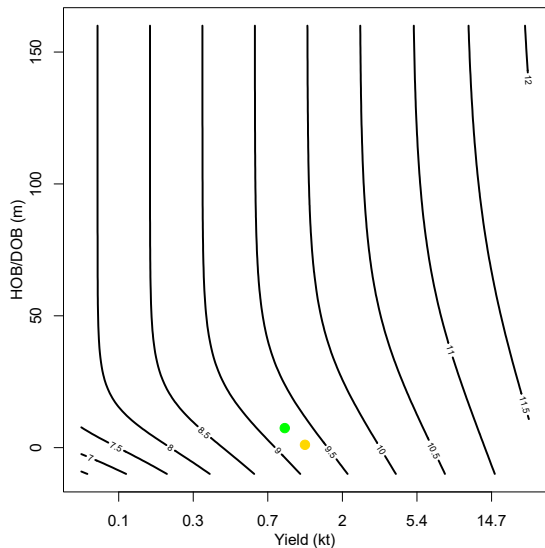
Acoustic Forward Model

$$\log d_{aijr} = \beta_{ar,1} + \beta_{ar,2} \log \tilde{r}_{aij} + \beta_{ar,3} \tilde{h}_{ai} - \log \left(1 + \exp \left(\beta_{ar,3} \tilde{h}_{ai} \right) \right)$$

$r=1$. **scaled** acoustic impulse

$r=2$. **scaled** positive phase duration

Acoustic Impulse



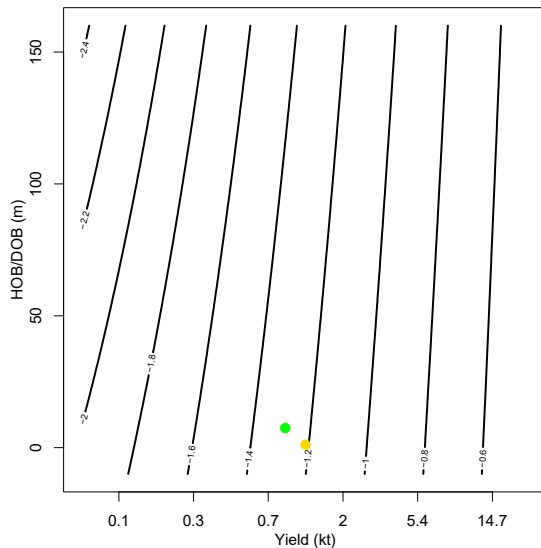
Impulse and Duration

Scale factor =

$$y^{1/3} (P/P_0)^p (T/T_0)^{1/2}$$

$$p = (1) -2/3, (2) -1/3$$

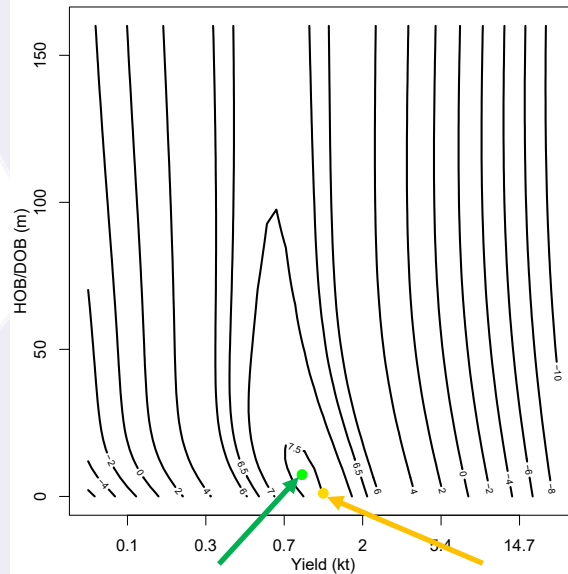
Positive Phase Duration



Range and HOB/DOB

$$\text{Scale factor} = y^{1/3} (P/P_0)^{1/3}$$

Profile Log-Likelihood



Optical Forward Model

Ford (F)

$$\log(d_{oijr}) = \beta_{or,1} + \beta_{or,2} \exp(-|\tilde{h}_{oi}|) + 100 (\tilde{h}_{oi} + 0.1) - \log \left(1 + \exp \left(100 (\tilde{h}_{oi} + 0.1) \right) \right)$$

r = 1. **scaled** first minimum of irradiance
r = 2. **scaled** second maximum of irradiance

Min and Second Max
Scale factor =
 $y^{1/3} (P/P_0)^{-1/3} (T/T_0)^{1/2}$

HOB/DOB
Scale factor = $y^{1/3} (P/P_0)^{1/3}$

Whitaker-Symbolisty (WS)

$$\log(d_{oijr}) = \beta_{or,1} + \beta_{or,2} \log y_{oi} + \log \left(1 + \beta_{or,3} \exp \left(-(\tilde{h}_{oi}/\beta_{or,4})^2 \right) \right)$$

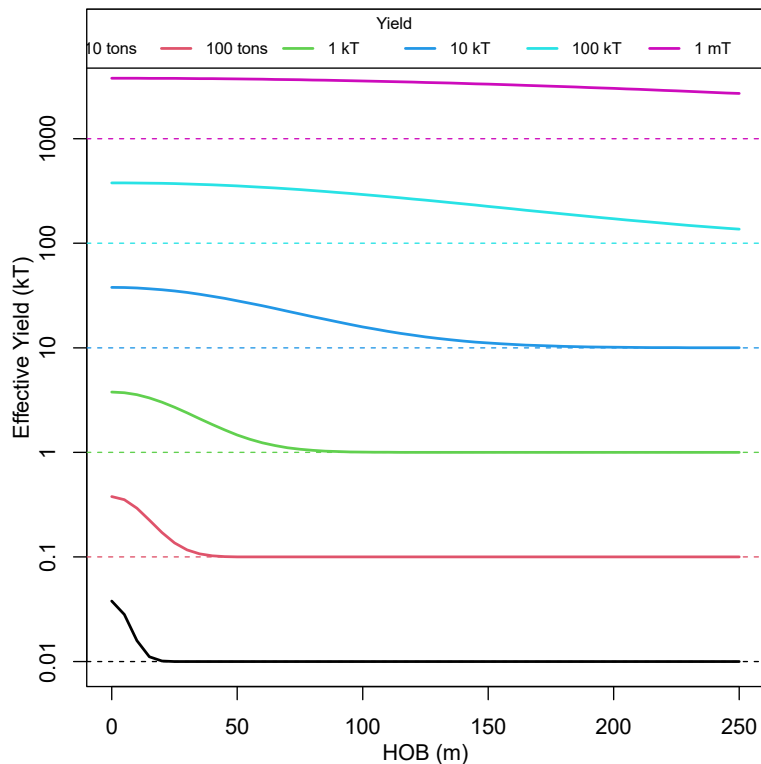
r = 1. first minimum of irradiance
r = 2. second maximum of irradiance

HOB
Scale factor = $y^{1/3}$

Information Criteria

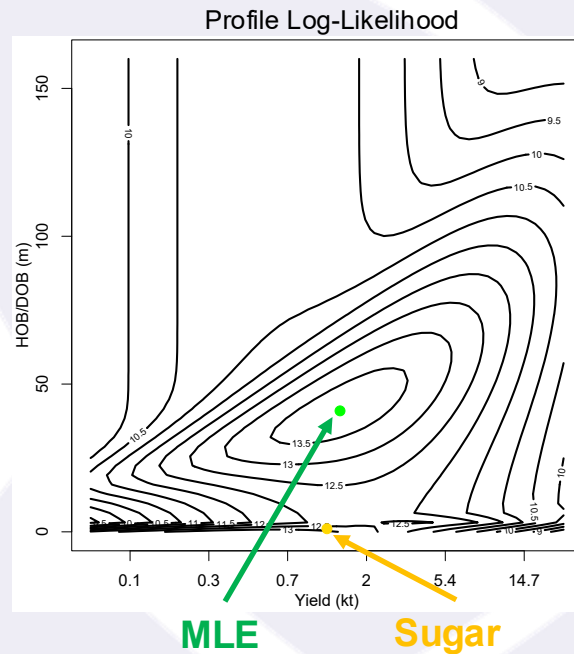
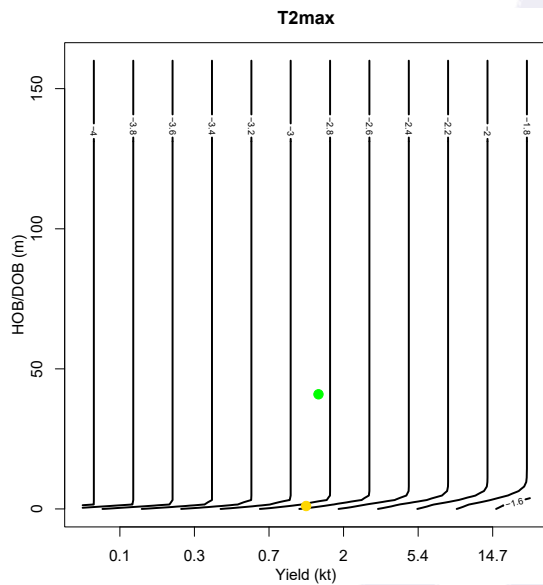
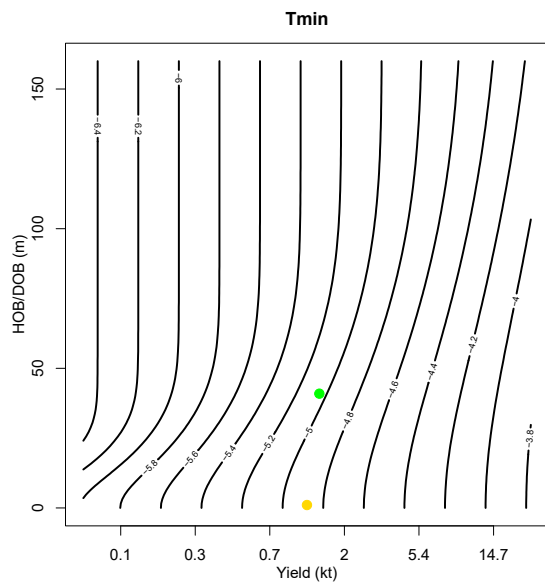
Analysis	AIC		BIC		DIC		PIC	
Model	WS	F	WS	F	WS	F	WS	F
EIV=0.3	92.98	108.21	137.22	148	97.81	110.46	133.95	142.9
Fixed	-2.57	12.53	10.94	21.23	-0.22	13.47	4.01	20.49

Whitaker-Symbolisty Optical Forward Model



- Whitaker-Symbolisty (2009) Surface to Free Air Optical Scaling Law
 - LA-UR-09-00514
- Relationship between t_{\min} and yield described by a power law for free air events
- Apparent yield of surface interacting events greater than free air yield
- Scaling law adjusts free air t_{\min} upwards for surface interacting events by a multiplicative factor depending on scaled HOB

Whitaker-Symbolisty Optical Forward Model

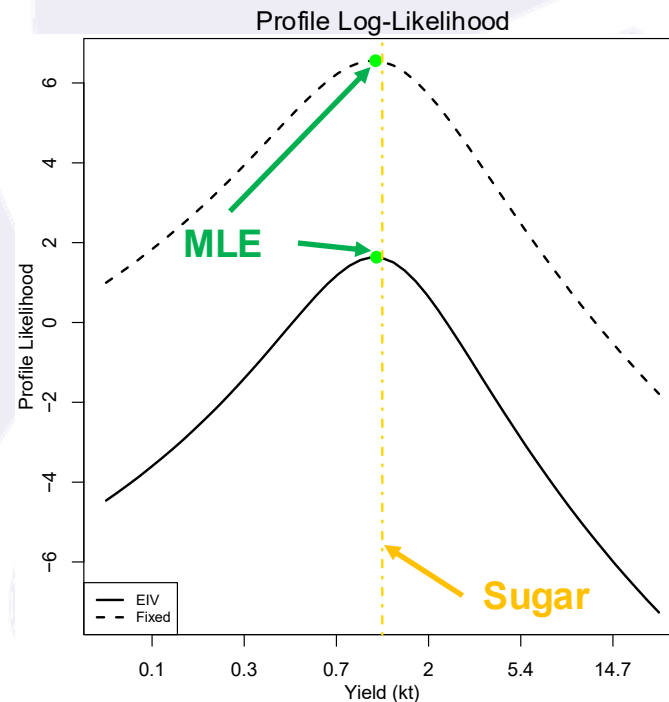
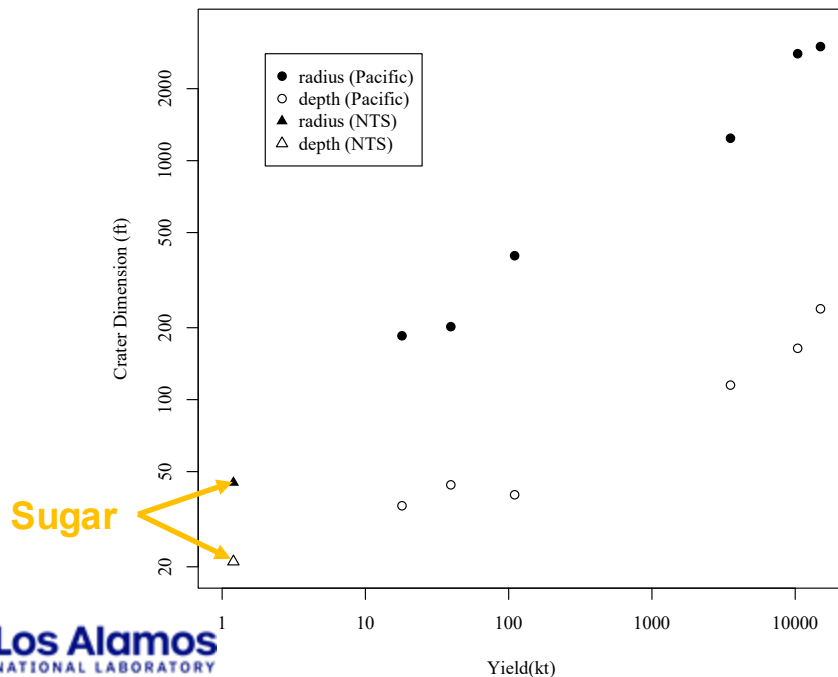


Surface Effects Forward Model

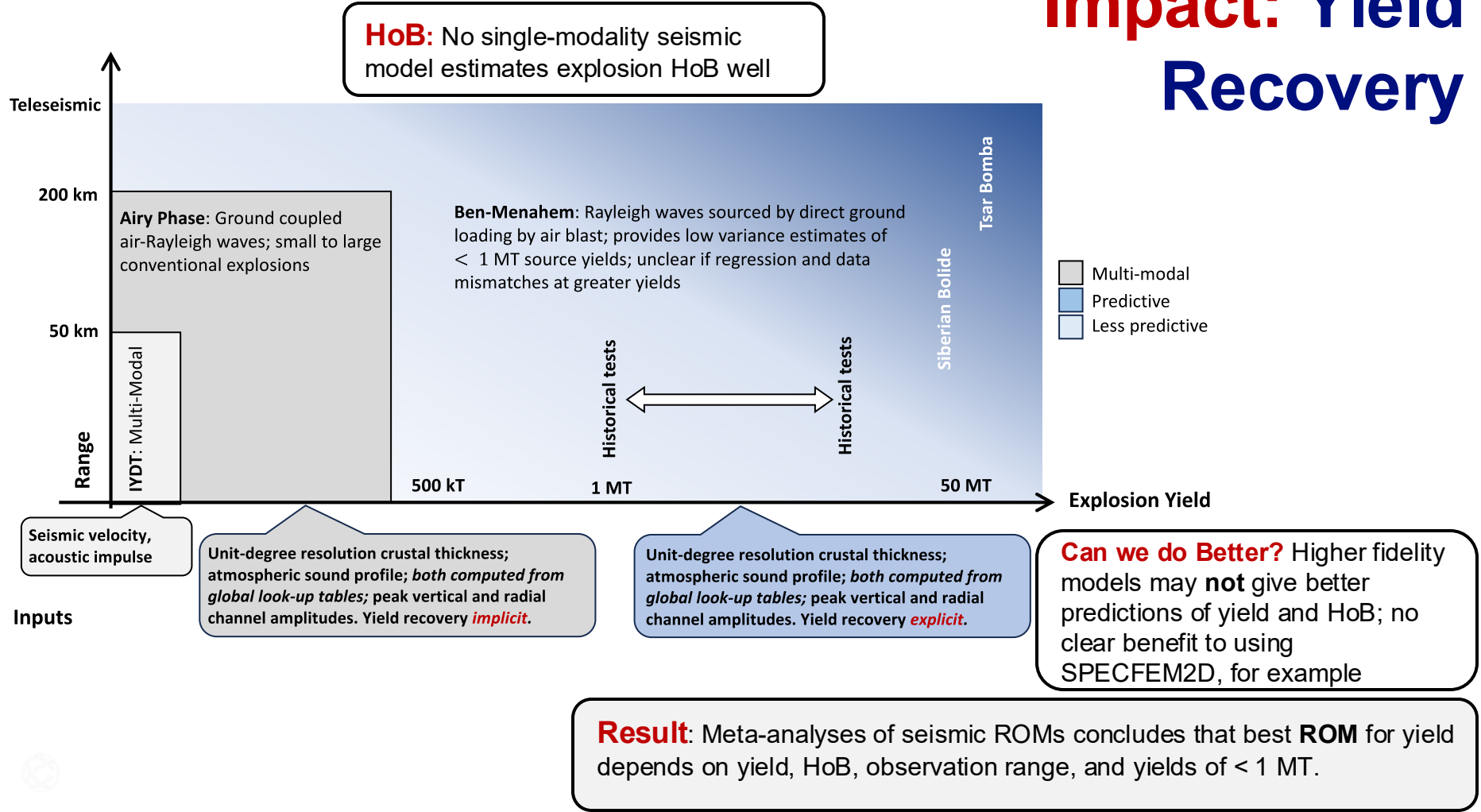
$$\log d_{cir} = \beta_{cr,1} + \beta_{cr,2} \log y_i$$

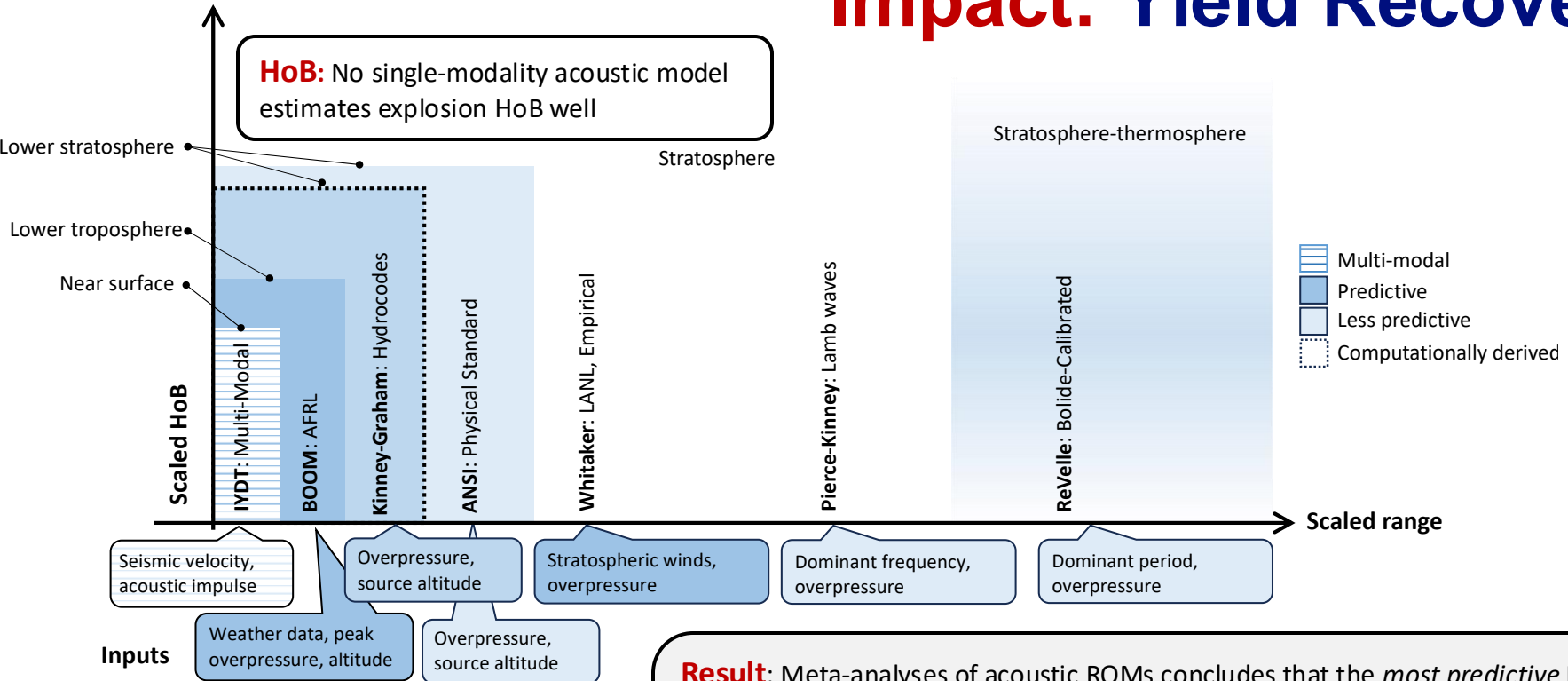
$r = 1$. crater radius

$r = 2$. crater depth



Development of Advanced Forward Models





Can we do Better? Higher fidelity models give better predictions of yield and HoB; these incur computational cost and SME intervention

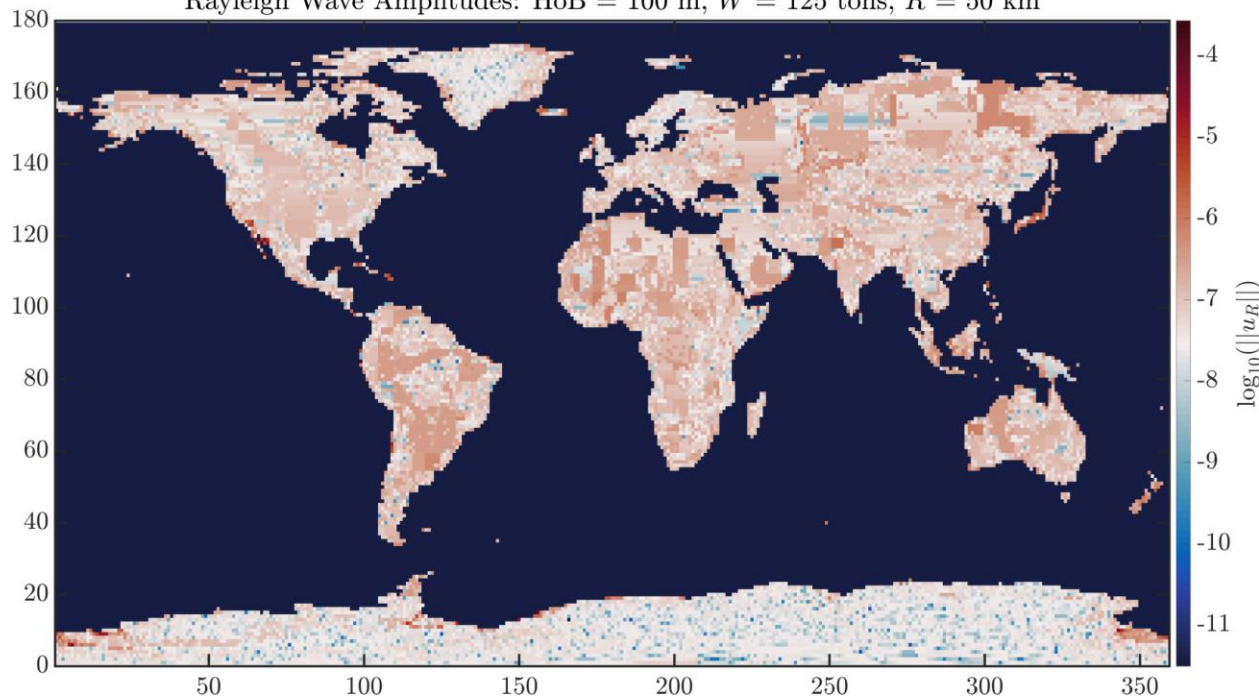
Result: Meta-analyses of acoustic ROMs concludes that the *most predictive* ROMs for yield (W) that use acoustic data are:

- Near-source distances: **IYDT**
- Local-source distances: **BOOM**
- Far-local to teleseismic distances: **Whitaker/LANL**
- Teleseismic distances: **Recommend a combined Pierce-Kinney and Revelle model**

Reduced Order Models for Seismic Rayleigh Waveforms (1/3)

Absolute Waveform Amplitudes used in ROMs

Rayleigh Wave Amplitudes: HoB = 100 m, $W = 125$ tons, $R = 50$ km



Warm colors show large amplitudes predicted from 0.125 kT explosions detonated 100m aboveground at 50km observation distances. Cool colors show lower than expected amplitudes. Some locations in Russia (for example) produce small amplitudes.

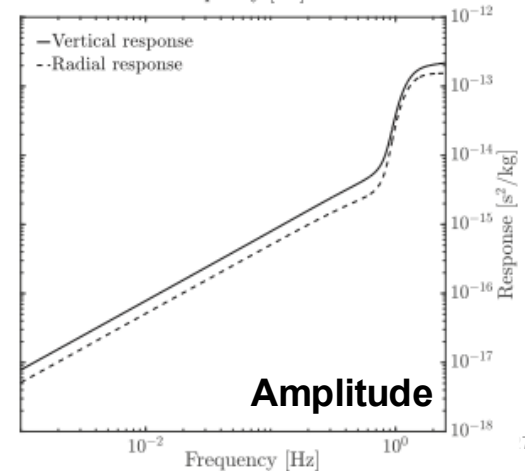
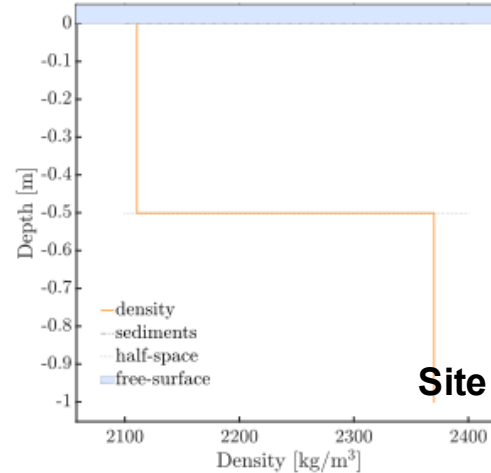
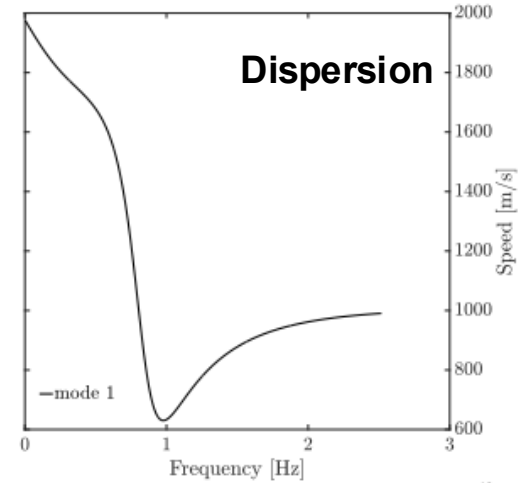
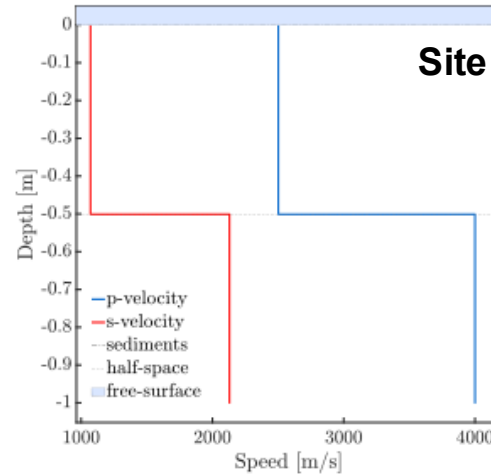
Rayleigh ROM Parameters

- Compute elastic parameters for Rayleigh wave, reduced order models every 1° over global landmasses.
- Create reduced order source models for aboveground explosions that couple acoustic and seismic waveforms.
- Compute the predicted peak displacement from these models.
- **Impact:** “Lookup tables” for Rayleigh waves at landmass locations on the globe that populate ROMs with yield and HoB parameters.

Reduced Order Models for Seismic Rayleigh Waveforms (2/3)

A Detailed Example

1. Interpolate CRUST1.0 at **site** location (Eastern Europe, here).
2. Estimate Rayleigh **dispersion curves** from the crustal structure
3. Compute peak over-pressures and positive phase durations for the air blast that loads the ground; use analytic fits of Kinney and Graham to hydrocode output (**source**).
4. Radially integrate the source over the loading disc via Murphy (1981, 1988).
5. Estimate the **site response function** from the structure

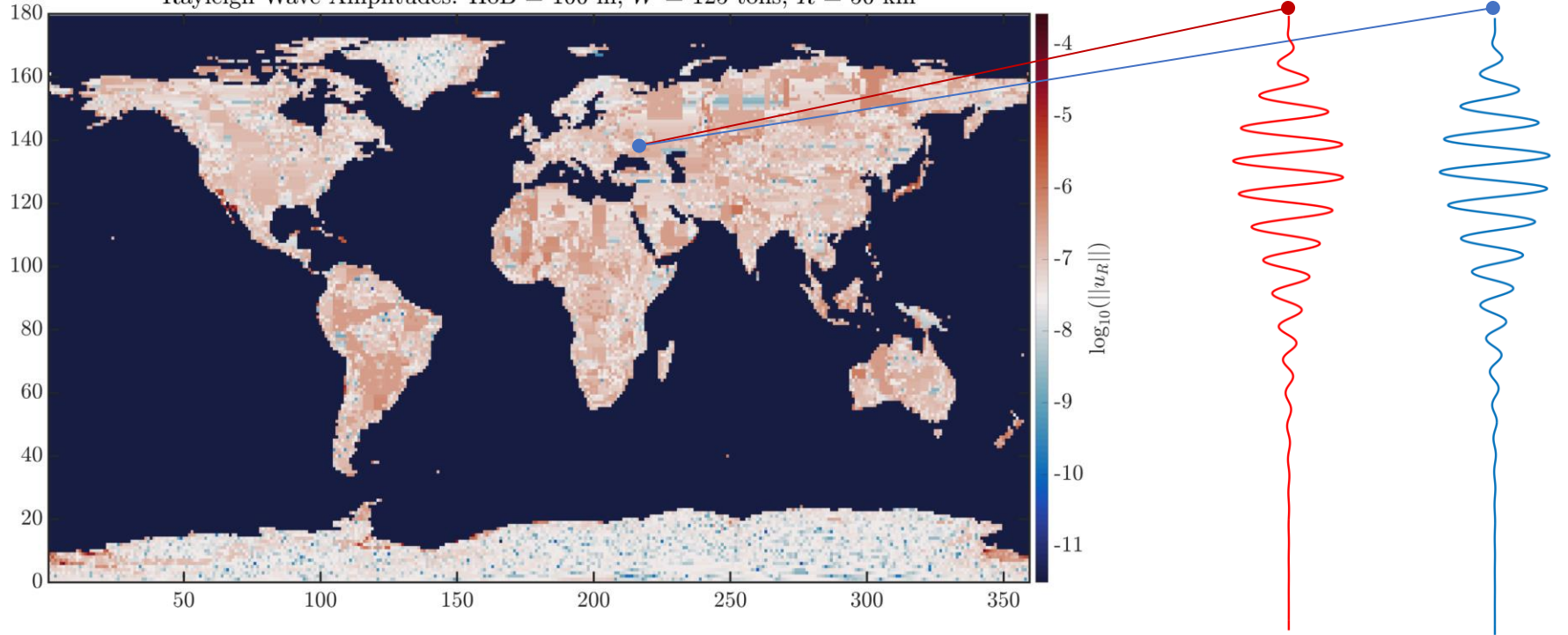


$$A(\omega_0) = S(\omega_0)A_R(\omega_0) \sqrt{\frac{c(\omega_0)}{\omega_0} \left(\frac{2U(\omega_0)^2}{\dot{U}(\omega_0)} \right)^{\frac{1}{3}}}$$

Reduced Order Models for Seismic Rayleigh Waveforms (3/3)

Absolute Waveform Amplitudes used in ROMs

Rayleigh Wave Amplitudes: HoB = 100 m, $W = 125$ tons, $R = 50$ km



Warm colors: large amplitudes predicted from 0.125 kT explosions detonated 100m aboveground at 50km observation distances. Cool colors: lower than expected amplitudes. Some locations (e.g., Russia) produce small amplitudes.

Yield, HoB, and location determine envelope shape, amplitude, and waveform frequency

Error Models

Error Model: Structure of Source and Path Bias

Y1	Y2	Source	Path	Type	lRange	W	HOB
-15.091	-9.252	HRI-1	Path_1	1	6.932	6.291	5
-15.089	-9.180	HRI-1	Path_1	1	6.932	6.291	5
-15.836	-10.218	HRI-1	Path_2	1	7.570	6.291	5
-15.892	-10.180	HRI-1	Path_2	1	7.570	6.291	5
-16.176	-10.557	HRI-1	Path_2	1	7.800	6.291	5
-16.907	-11.366	HRI-1	Path_2	1	8.371	6.291	5
-16.931	-11.338	HRI-1	Path_2	1	8.371	6.291	5
-14.835	-9.199	HRI-2	Path_1	1	6.930	6.291	3
-14.860	-9.184	HRI-2	Path_1	1	6.930	6.291	3
-15.674	-10.089	HRI-2	Path_1	1	7.568	6.291	3
-15.754	-10.197	HRI-2	Path_1	1	7.568	6.291	3
-16.002	-10.530	HRI-2	Path_2	1	7.802	6.291	3
-16.060	-10.605	HRI-2	Path_2	1	7.802	6.291	3
-16.534	-11.115	HRI-2	Path_2	1	8.239	6.291	3
-16.741	-11.230	HRI-2	Path_3	1	8.373	6.291	3
-16.737	-11.288	HRI-2	Path_3	1	8.373	6.291	3
-17.208	-11.656	HRI-2	Path_3	1	8.738	6.291	3

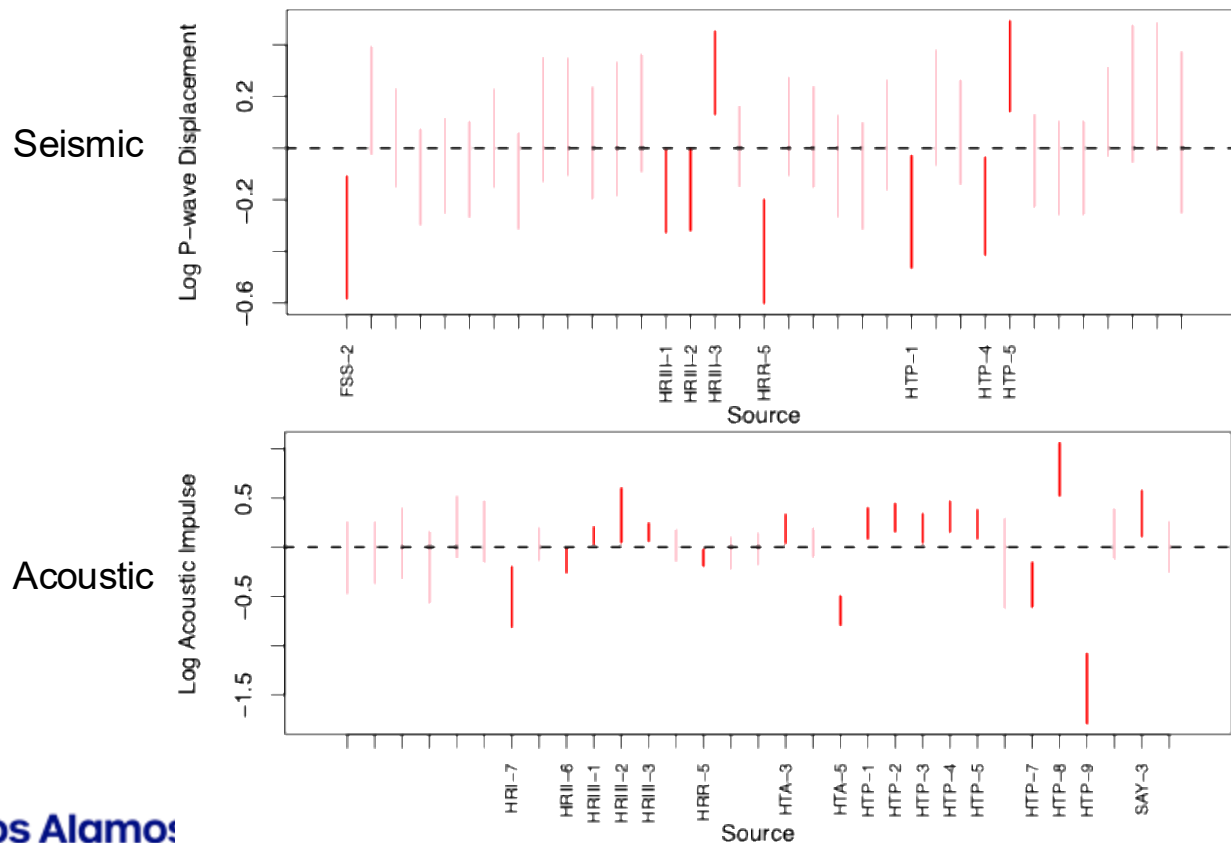
Source Bias

$$\mathbf{Z}_{1\{1,2\}r,\{1,2,3\}} \mathbf{b}_{hr}^{(S)} \begin{matrix} \text{S1P1} \\ \text{S1P2} \\ \text{S2P1} \\ \text{S2P2} \\ \text{S2P3} \\ \text{S1} & \text{S2} \end{matrix} \begin{bmatrix} 1_2 & 0_2 \\ 1_5 & 0_5 \\ 0_4 & 1_4 \\ 0_3 & 1_3 \\ 0_3 & 1_3 \end{bmatrix} \begin{pmatrix} b_{1r,1}^{(S)} \\ b_{1r,2}^{(S)} \end{pmatrix}$$

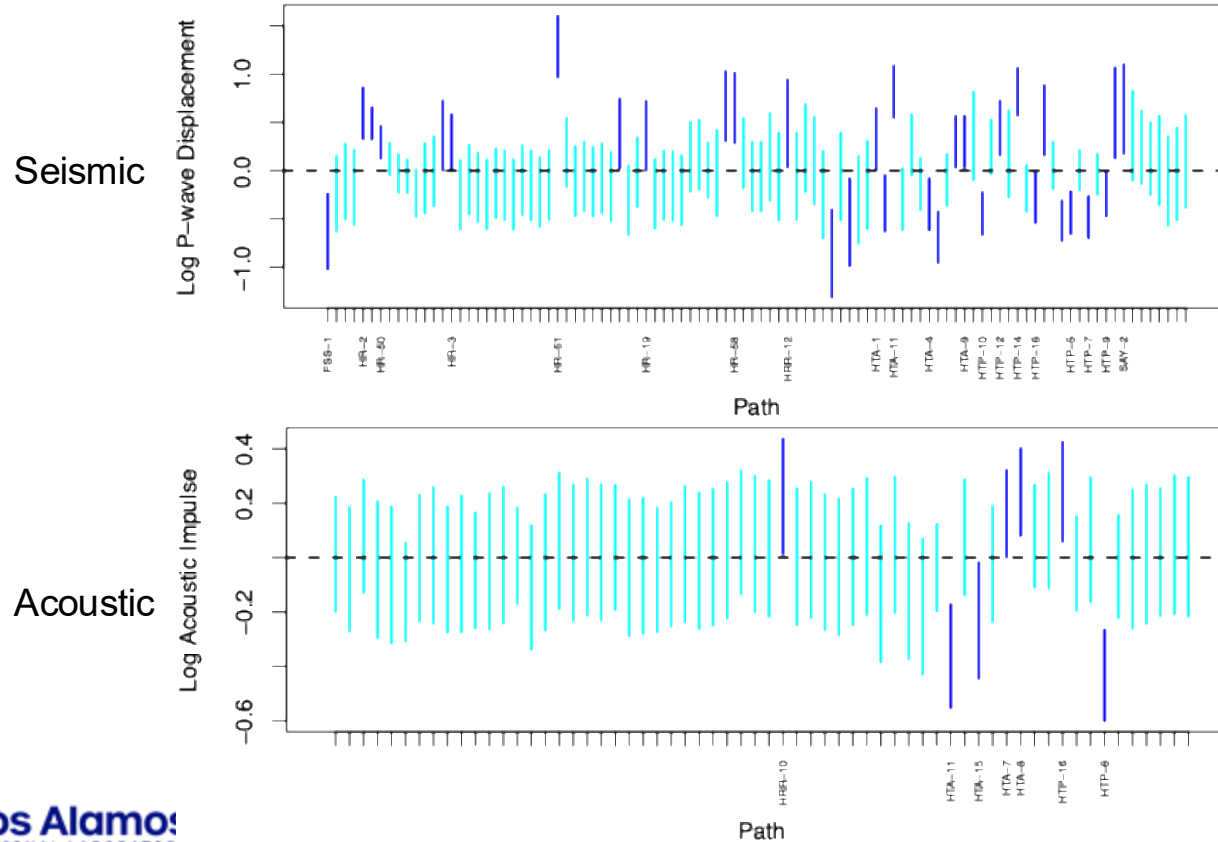
Path bias

$$\mathbf{Z}_{1\{1,2\}\{1,2,3\}r} \mathbf{b}_{hr}^{(P)} \begin{matrix} \text{S1P1} \\ \text{S1P2} \\ \text{S2P1} \\ \text{S2P2} \\ \text{S2P3} \\ \text{P1} & \text{P2} & \text{P3} \end{matrix} \begin{bmatrix} 1_2 & 0_2 & 0_2 \\ 0_5 & 1_5 & 0_5 \\ 1_4 & 0_4 & 0_4 \\ 0_3 & 1_3 & 0_3 \\ 0_3 & 0_3 & 1_3 \end{bmatrix} \begin{pmatrix} b_{1r,\{1,2\}1}^{(P)} \\ b_{1r,\{1,2\}2}^{(P)} \\ b_{1r,\{1,2\}3}^{(P)} \end{pmatrix}$$

Example: Source Random Effects

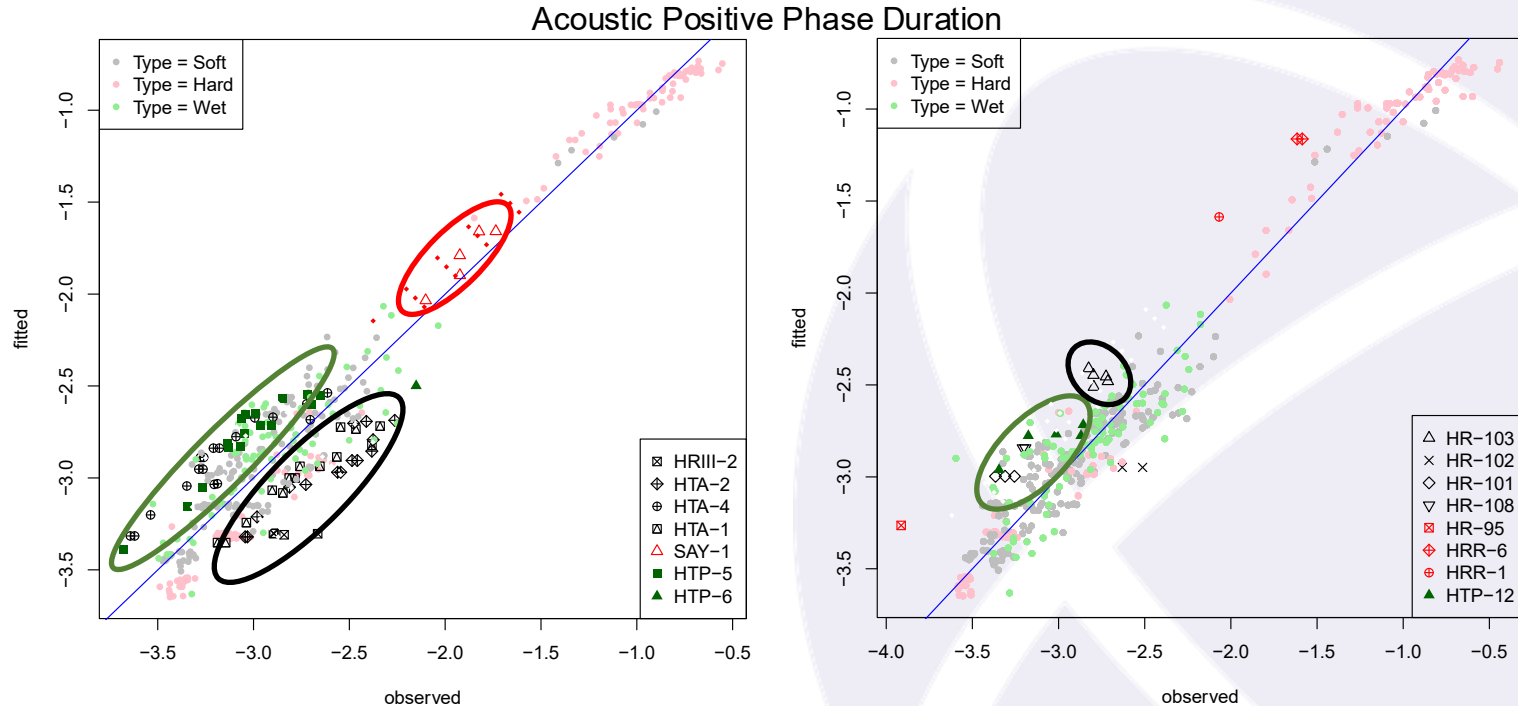


Example: Path Random Effects



95% credible intervals
for path random
effects (bias)

Example: Source and Path Bias



Adjusting observed signatures for clustering by source and path
reduces bias in forward model parameter estimates

Example: Results

Error Model Selection: Seismic

Source Dynamic Bias		Path Dynamic Bias		Information Criteria (IC)			
f_{s1}	f_{s2}	f_{s1}	f_{s2}	AIC	BIC	DIC	PIC
✓	✓	C	C	1053	1163	1064	1076
✓	✓	N	N	1315	1438	1325	1336
✓	✓	C		1161	1276	1163	1187
✓	✓	N		1345	1465	1336	1340
✓	✓		C	1214	1328	1201	1196
✓	✓		N	1314	1434	1310	1319
✓	✓			1347	1464	1348	1352
		C	C	1147	1275	1126	1141
✓		C		1194	1336	1175	1188
✓		N		1378	1522	1373	1397
✓				1380	1520	1359	1370
		C		1231	1372	1211	1225
	✓		C	1254	1396	1252	1278
	✓		N	1356	1500	1335	1342
	✓			1389	1529	1383	1392
			C	1314	1455	1296	1311
				1426	1572	1405	1417

C: "Crossed" Path Effects

Source-to-sensor propagation path effects are correlated across sources for (nearly) collocated sources and/or sensors

N: "Nested" Path Effects

Source-to-sensor propagation path effects are nested within source and independent across (nearly) collocated sources and/or sensors

Error Model Selection: Acoustic

Source Dynamic Bias		Path Dynamic Bias		Information Criteria (IC)			
f_{a1}	f_{a2}	f_{a1}	f_{a2}	AIC	BIC	DIC	PIC
✓	✓	C	C	17.58	59.26	17.9	34.38
✓	✓	N	N	60.48	103.6	60.54	76.42
✓	✓	C		80.7	121.4	81.59	97.21
✓	✓	N		111	152.3	110.7	125.5
✓	✓		C	48.4	88.3	48.73	64.11
✓	✓		N	59.6	100.3	60.8	76.4
✓	✓			109.3	148	111.4	126.4
		C	C	370.2	436.7	371.4	386.8
✓		C		251	313.2	250.5	264.9
✓		N		281.4	343.8	281	295.3
✓				279.7	337.9	280.6	294.6
		C		400.4	466	399.9	413.5
	✓		C	164.6	226.8	165.8	181
	✓		N	174.8	236.8	175	189.7
	✓			227.3	285.7	228.2	242
			C	381.2	445.3	381.2	395.1
				410.5	473.1	410.9	424

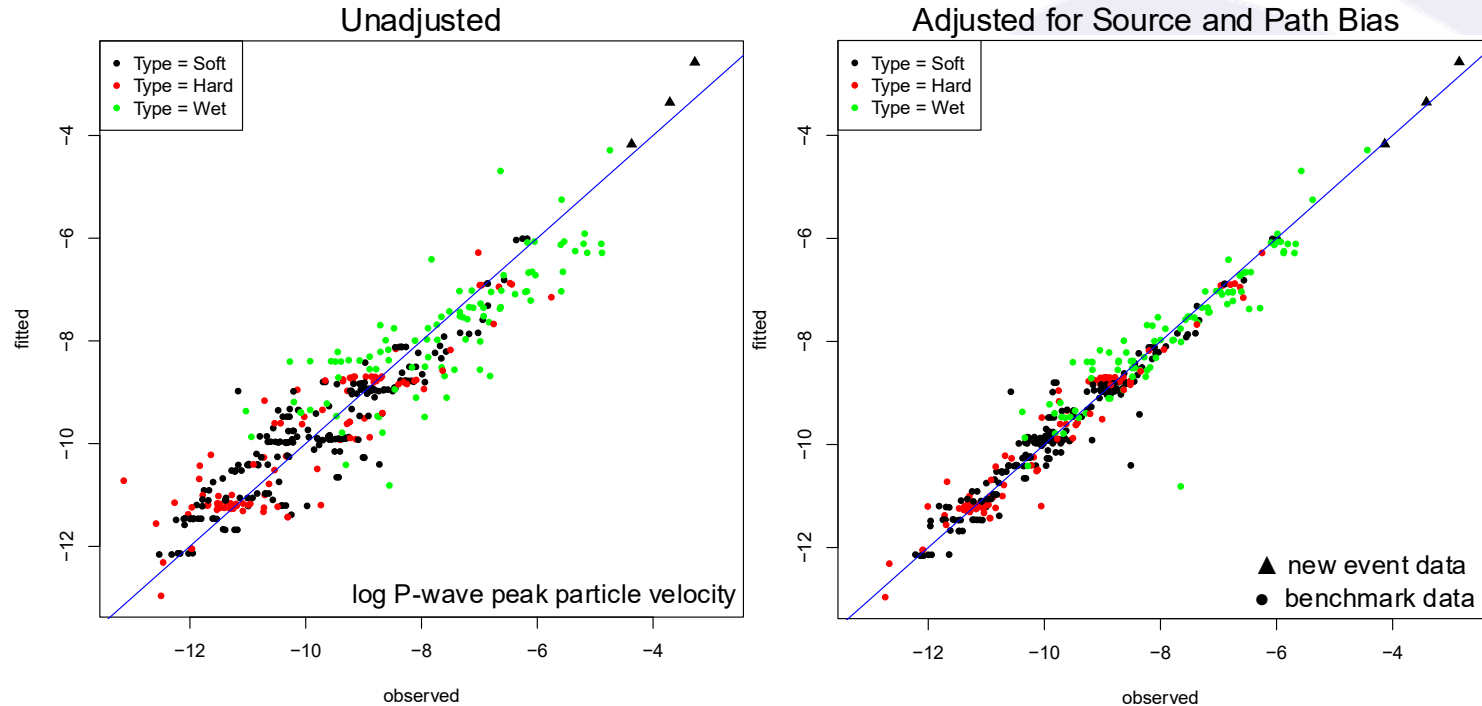
C: "Crossed" Path Effects

Source-to-sensor propagation path effects are correlated across sources for (nearly) collocated sources and/or sensors

N: "Nested" Path Effects

Source-to-sensor propagation path effects are nested within source and independent across (nearly) collocated sources and/or sensors

Model Diagnostics: Seismic



Fitted (maximum likelihood) vs. simulated seismic P-wave peak particle velocity by emplacement (rock type)

Maximum Likelihood Results

Method	Sensor Type	Yield [kt] (RE)	HOB/DOB [m]	Correlation
ML	Seismic	0.67 (620%)	0.3 (390)	0.84
ML	Acoustic	0.91 (300%)	7.4 (65)	-0.70
ML	Optical	1.42 (890%)	40 (35)	0.74
ML	Crater	1.12 (130%)		

Sugar:

1.2 kt

1.1 m

Method	Sensor Type	Yield [kt] (RE)	HOB/DOB [m]	Correlation
ML	MultiPEM	0.94 (70%)	1.1 (1.2)	0.22
ML	MultiPEM EIV	0.96 (66%)	1.1 (1.2)	0.20

EIV = “Errors in variables” allows for 30% “total uncertainty” in benchmark nuclear yields

MultiPEM: Significant uncertainty reduction compared to single sensor

MultiPEM 95% Confidence Intervals

Yield (kt): (0.56, 1.58)
HOB/DOB (m): (0.44, 5.5)

Bayesian Results

Method	Sensor Type	Yield [kt] (RE)	HOB/DOB [m]	Correlation
Bayes	Seismic	0.95 (260%)	75 (95)	0.62
Bayes	Acoustic	0.74 (220%)	65 (90)	-0.12
Bayes	Optical	1.72 (1000%)	65 (85)	0.09
Bayes	Crater	1.14 (310%)		

Sugar:

1.2 kt

1.1 m

Method	Sensor Type	Yield [kt] (RE)	HOB/DOB [m]	Correlation
Bayes	MultiPEM	0.92 (100%)	45 (70)	0.00
Bayes	MultiPEM EIV	0.95 (75%)	40 (45)	0.08

EIV = “Errors in variables” allows for 30% “total uncertainty” in benchmark nuclear yields

MultiPEM: Significant uncertainty reduction compared to single sensor

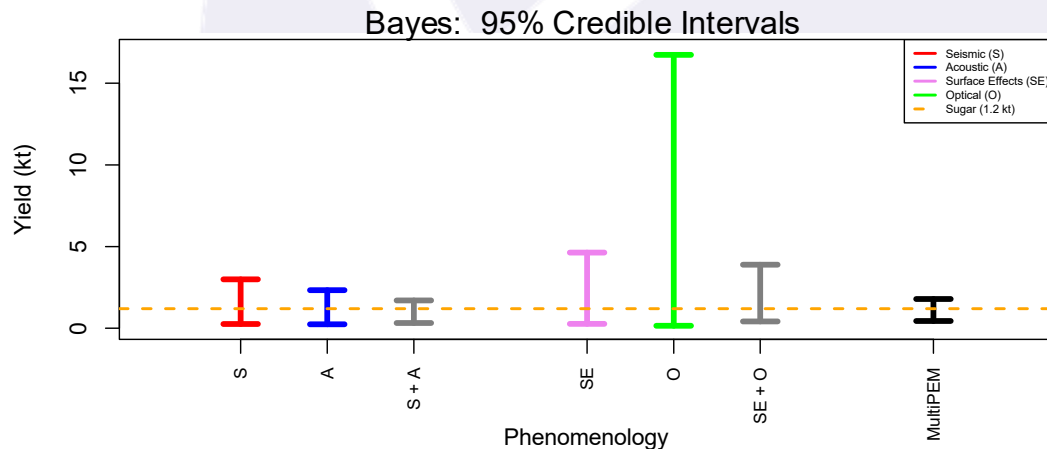
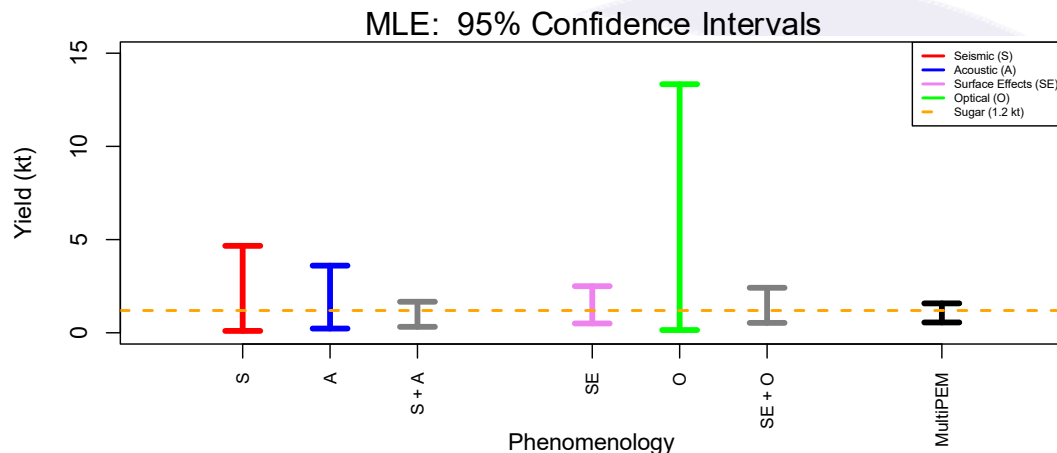
MultiPEM 95% Confidence Intervals

Yield (kt): (0.45, 1.79)
HOB/DOB (m): (1.8, 145)

Summary of Yield Characterization for Sugar

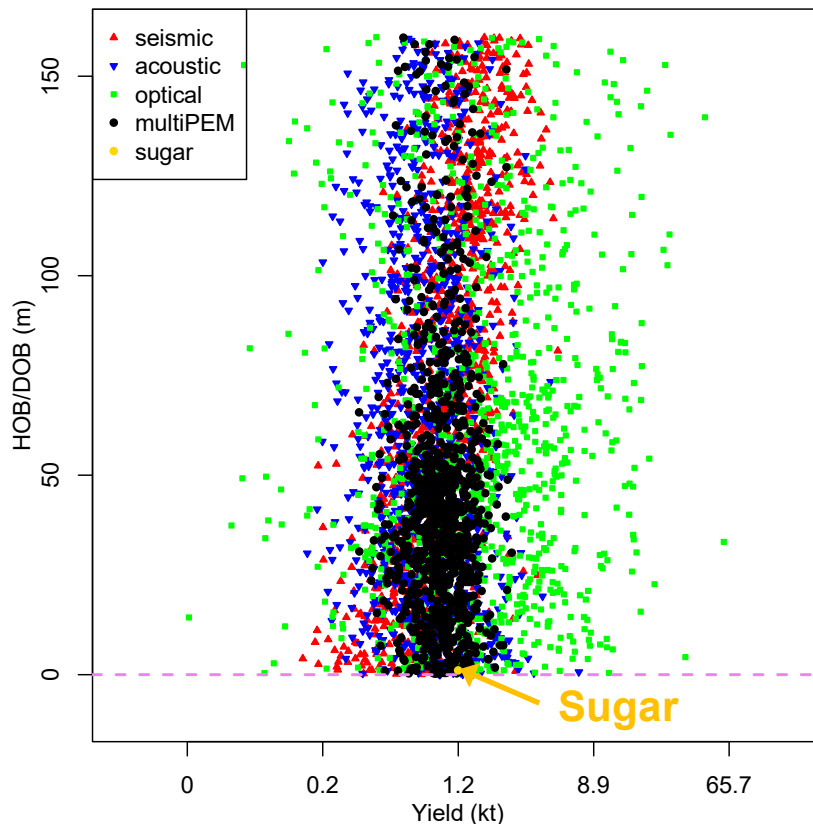
Error	MLE	95% CI	
		LB	UB
(S)+(P)	0.67	0.24	1.9
(S)	0.42	0.17	1.03

Accounting for path bias in error model
results in bias reduction of seismic
yield estimate for **SUGAR**
at true HOB = 3.5 ft



Bayesian Results: Joint Posterior

Prior Distribution
log Yield \sim Improper Uniform
HOB/DOB \sim Uniform[0, 160]



Posterior sampling
conducted via
Markov chain Monte
Carlo (MCMC):
Delayed Rejection
Adaptive Metropolis
(DRAM)

Forward/error models

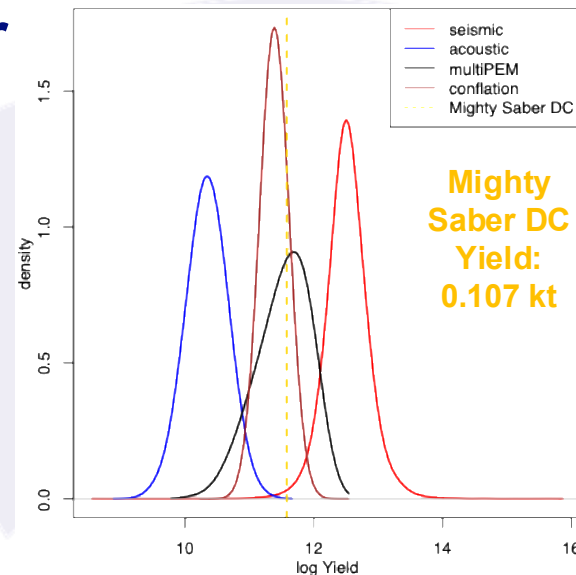
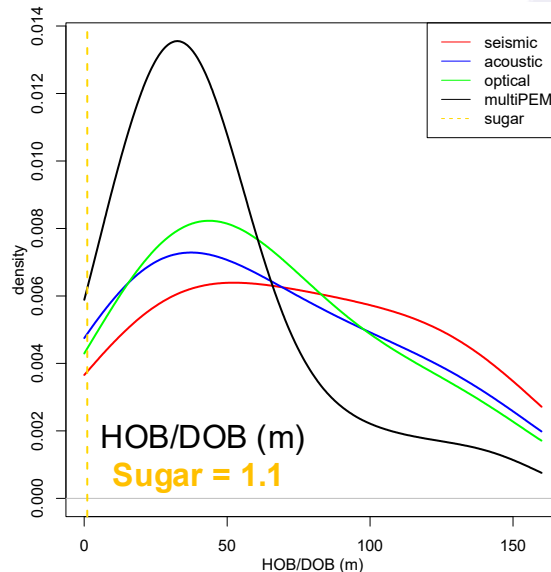
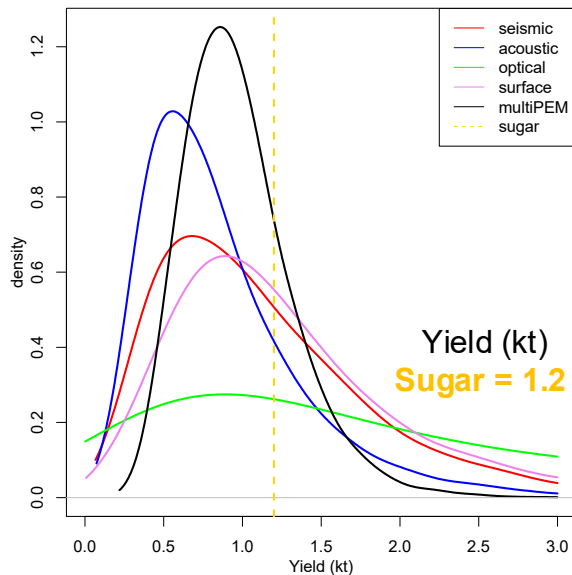
1,000 parameter values
imputed from posterior
distribution using only
benchmark data

New event Yield/HOB

20 posterior samples
per imputation using
only new event data
20,000 final samples

Bayesian Results: Marginal Posterior

IYDT



The MultiPEM result may not achieve uncertainty reduction relative to all single phenomenology results. Variance reduction depends on the consistency of single-phenomenology inferences.

Discussion

- MultiPEM characterization provides a probabilistic statement about new event quantities of interest
 - Confidence/credible intervals/regions
- In addition to new event device parameter characterization, calibration parameters can also be characterized at the benchmark stage
 - Example: Chemical-to-nuclear equivalency (C2N)

Sensor Type	Log C2N	95% CI LB	95% CI UB
Seismic (S)	1.27 (1.06)	0.24	2.3
Acoustic (A)	0.84 (0.96)	-0.1	1.78
S & A	1.03 (0.72)	0.33	1.73

- Assumed value (Ford et al.): 0.69