Two Pointer Method

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Question:

You are given an integer array height of length n. There are n vertical lines drawn such that the two endpoints of the ith line are (i, 0) and (i, height[i]).

Find two lines that together with the x-axis form a container, such that the container contains the most water.

Return the maximum amount of water a container can store.

Notice that you may not slant the container.

Answer:

Let $i = 0$, $j = \text{height.size}() - 1$, $M = \mathbf{Area}(i, j)$.
Then execute the following loop:

Loop:

If $i >= j$, exit loop.
If $\text{height}_i <= \text{height}_j$, let $i = i + 1$.
Else if $\text{height}_i > \text{height}_j$, let $j = j - 1$.
If $\mathbf{Area}(i, j) > M$, let $M = \mathbf{Area}(i, j)$.

Proof:

Let $L := \text{height.size}()$, $h_i := \text{height}_i \forall 0 \leq i \leq L$, and $A(i, j) := \mathbf{Area}(i, j)$.
Then $\exists 0 \leq i_{max} < j_{max} \leq L$ such that $A_{max} = A(i_{max}, j_{max}) = \sup_{0 \leq i < j \leq L} \{A(i, j)\}$.
Since A_{max} is the maximum, no $i' < i_{max}$ can have $h_{i'} \geq h_{i_{max}}$, otherwise $A(i', j_{max}) > A_{max}$.
Similarly, no $j' > j_{max}$ can have $h_{j'} \geq h_{j_{max}}$, otherwise $A(i_{max}, j') > A_{max}$.
Therefore $\forall i' < i_{max}$, $h_{i'} < h_{i_{max}}$, and $\forall j' > j_{max}$, $h_{j'} < h_{j_{max}}$.