Revisão

1) Resolver o sistema por eliminação de Gauss-Jordan.

$$\begin{cases} 2x_1 + 2x_2 + 2x_3 = 0 \\ -2x_1 + 5x_2 + 2x_3 = 1 \\ 8x_1 + x_2 + 4x_3 = -1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right] = \frac{1}{2}L_1 \qquad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right] \begin{array}{l} \\ = L_2 + 2L_1 \\ = L_3 - 8L_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{array}\right] = \frac{1}{7}L_2 \qquad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & -7 & -4 & -1 \end{array}\right] \begin{array}{l} \\ \\ = L_3 + 7L_2 \end{array}$$

$$\underbrace{\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right]}_{\text{matriz escalonada}} = L_1 - L_2 \qquad \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \underbrace{\left[\begin{array}{ccc|c} 1 & 0 & 3/7 & -1/7 \\ 0 & 1 & 4/7 & 1/7 \end{array}\right]}_{\text{matriz escalonada reduzida}} \end{array}$$

- Reescrevemos o sistema associado a matriz escalonada reduzida:

$$\begin{cases} x_1 \quad + 3/7\, x_3 = -1/7 \\ \quad x_2 + 4/7\, x_3 = 1/7 \end{cases}$$

Isolando as variáveis associadas aos pivôs, temos:

$$x_1 = -\frac{1}{7} - \frac{3}{7}x_3 \rightarrow \text{variável livre}$$

$$x_2 = \frac{1}{7} - \frac{4}{7}x_3$$

Logo, o sistema tem múltiplas soluções. Para $x_3 = 0$, temos $x_1 = -\frac{1}{7}$ e $x_2 = \frac{1}{7}$.

/ /

(2) Resolva o sistema por eliminação gaussiana e retro-substituição.

$$\begin{cases} 2x_1 + 3x_2 + 2x_3 = 1 \\ -2x_1 + x_2 + 5x_3 = 2 \\ x_1 + 3x_2 - 4x_3 = -1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 2 & 1 \\ -2 & 1 & 5 & 2 \\ 1 & 3 & -4 & -1 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 3 & -4 & -1 \\ 2 & 3 & 2 & 1 \\ -2 & 1 & 5 & 2 \end{array}\right] \begin{array}{l} \\ = L_2 + (-2)L_1 \\ = L_3 + 2L_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & -1 \\ 0 & -3 & 10 & 3 \\ 0 & 7 & -3 & 0 \end{array}\right] = -\frac{1}{3}L_2 \quad \left[\begin{array}{ccc|c} 1 & 3 & -4 & -1 \\ 0 & 1 & -\frac{10}{3} & -1 \\ 0 & 7 & -3 & 0 \end{array}\right] = L_3 - 7L_2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & -1 \\ 0 & 1 & -\frac{10}{3} & -1 \\ 0 & 0 & 61/3 & 7 \end{array}\right] = \frac{3}{61}L_3 \quad \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \underbrace{\left[\begin{array}{ccc|c} 1 & 3 & -4 & -1 \\ 0 & 1 & -\frac{10}{3} & -1 \\ 0 & 0 & 1 & 21/61 \end{array}\right]}_{\text{matriz escalonada}} \end{array}$$

- Reescrevendo o sistema:

$$\begin{cases} x_1 + 3x_2 - 4x_3 = -1 \\ x_2 - \frac{10}{3}x_3 = -1 \\ \boxed{x_3 = 21/61} \end{cases}$$

//

- Substituindo x_3 na L_2, temos:

$$x_2 - \frac{10}{3}\left(\frac{21}{61}\right) = -1 \Rightarrow x_2 - \frac{70}{61} = -1 \Rightarrow \boxed{x_2 = 9/61}$$

//

- Substituindo x_3 e x_2 na L_1, temos:

$$x_1 + 3\left(\frac{9}{61}\right) - 4\left(\frac{21}{61}\right) = -1 \Rightarrow x_1 - \frac{57}{61} = -1 \Rightarrow \boxed{x_1 = \frac{-4}{61}}$$

//

3) Determine se o sistema é linear:

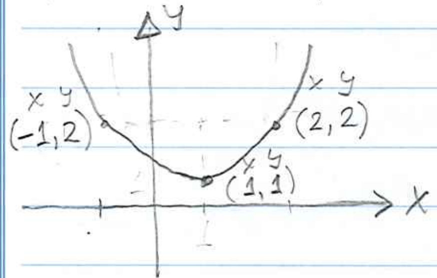
a) $\begin{cases} a^{-\operatorname{sen} \frac{3\pi}{2}} + 2b = 2 \\ 2a + 3b = 1 \end{cases} \Leftrightarrow a^{-(-1)} + 2b = 2 \Leftrightarrow a + 2b = 2$

Logo, é linear.

b) $\begin{cases} x^{\log_2 2} + y = 2 \\ x^2 x^{\cos\pi} + 2y = 1 \end{cases} \begin{matrix} \Leftrightarrow x + y = 2 \\ \Leftrightarrow x^2 x^{-1}\ 2y = 1 \Leftrightarrow x + 2y = 1 \end{matrix}$

Logo, é linear.

4) Determine a equação da parábola $y = ax^2 + bx + c$ que passa pelos pontos dados na figura abaixo.



• substitui cada ponto na equação $y = ax^2 + bx + c$ para obter um sistema linear.

$$\begin{cases} 2 = a(-1)^2 + b(-1) + c \\ 1 = a(1)^2 + b(1) + c \\ 2 = a(2)^2 + b(2) + c \end{cases} \Rightarrow \begin{cases} a - b + c = 2 \\ a + b + c = 1 \\ 4a + 2b + c = 2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 4 & 2 & 1 & 2 \end{array}\right] \begin{matrix} \\ = L_2 - L_1 \\ = L_3 - 4L_1 \end{matrix} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 2 & 0 & -1 \\ 0 & 6 & -3 & -6 \end{array}\right] = \frac{1}{2} L_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 6 & -3 & -6 \end{array}\right] = L_3 - 6L_2 \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & -3 & -3 \end{array}\right] = -\frac{1}{3} L_3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 1 \end{array}\right] = L_1 - L_3 \left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 1 \end{array}\right] = L_1 + L_2$$

$$\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 1 \end{array}\right] \end{array} \Rightarrow \begin{cases} a = 1/2 \\ b = -1/2 \\ c = 1 \end{cases}$$

Logo: $\boxed{y = \frac{1}{2}x^2 - \frac{1}{2}x + 1}$ //

5 Supondo que as matrizes dadas são quadradas. Simplifique:

a) $(AB)\, B^{-1}A^{-1}C(D + C^{-1})D^{-1} \Rightarrow \underbrace{AB\,(AB)^{-1}}_{I} C(\underbrace{DD^{-1}}_{I} + C^{-1}D^{-1})$

$\Rightarrow IC(I + C^{-1}D^{-1}) \Rightarrow C + \underbrace{CC^{-1}}_{I}D^{-1} \Rightarrow C + D^{-1}$ //

b) $C(D + C^{-1})D^{-1} + C(C^{-1} - D)D^{-1} \Rightarrow$

$\Rightarrow C(\underbrace{DD^{-1}}_{I} + C^{-1}D^{-1}) + C(C^{-1}D^{-1} - \underbrace{DD^{-1}}_{I})$

$\Rightarrow C(I + C^{-1}D^{-1}) + C(C^{-1}D^{-1} - I)$

$\Rightarrow C + \underbrace{CC^{-1}}_{I}D^{-1} + \underbrace{CC^{-1}}_{I}D^{-1} - C$

$\Rightarrow \cancel{C} + D^{-1} + D^{-1} - \cancel{C} \Rightarrow (1+1)D^{-1} = 2D^{-1}$ //

6 Sejam as matrizes $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 \\ 5 & 3 \\ 1 & 2 \end{bmatrix}$ e $C = \begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}$. Calcule:

a) $AB = \begin{bmatrix} 15 & 17 \\ 6 & 5 \end{bmatrix}$

b) $A + B^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 5 & 1 \\ 5 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 7 & 4 \\ 5 & 4 & 3 \end{bmatrix}$

c) $3C^{-1}$ $\quad \det(C) = 1.5 - 2.1 = 3$

$$C^{-1} = \frac{1}{3}\begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 5/3 & -1/3 \\ -2/3 & 1/3 \end{bmatrix}$$

$$3C^{-1} = 3\begin{bmatrix} 5/3 & -1/3 \\ -2/3 & 1/3 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix}$$

d) Calcule p e q, tal que $\begin{bmatrix} p+q & 1 \\ 2 & p-q \end{bmatrix} = C$

$$\begin{bmatrix} p+q & 1 \\ 2 & p-q \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix} \Rightarrow \begin{cases} p+q = 1 \\ p-q = 5 \end{cases} \begin{matrix} \Rightarrow q = 1-p \\ \Rightarrow p-(1-p) = 5 \end{matrix}$$

$\Rightarrow p - 1 + p = 5 \Rightarrow 2p = 6 \Rightarrow \boxed{p = 3}$ //

• $q = 1 - p \Rightarrow q = 1 - 3 \Rightarrow \boxed{q = -2}$ //