Math 401, Quiz 1, 02/14/2023

Instructions: Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem. **Write everything on the answer booklet, including your name**.

1. Let the consumption matrix be

$$M = \begin{bmatrix} 0.6 & 0.8 \\ 0.2 & 0.6 \end{bmatrix}$$

(a) Assume the economy is closed, find a nonzero production vector $\overrightarrow{p}$.

(b) Give an example of external demand vector $\overrightarrow{d}$ such that $(I_2 - M)\overrightarrow{p} = \overrightarrow{d}$ does not have a solution.

(a) $I - M = \begin{pmatrix} 0.4 & -0.8 \\ -0.2 & 0.4 \end{pmatrix}$

$(I-M)\cdot\vec{p} = 0$, have infinitely many solution,

one of them is $\vec{p} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(b) For any $\vec{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$

$(I-M)\cdot\vec{p} = \begin{pmatrix} 0.4p_1 - 0.8p_2 \\ -0.2p_1 + 0.4p_2 \end{pmatrix}$

The first component is always -2 times the second component, so whenever $\vec{d}$ does not satisfy this, there is no solution

e.g. $\vec{d} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

2. Assume an economy as two sectors producing product 1 and product 2, suppose we are given

$$(I_2 - M)^{-1} = \begin{bmatrix} 5 & 5 \\ 3 & 5 \end{bmatrix}$$

(a) If the demand for sector 1 changes by +1, how the production in sector 2 must change? And how the production in sector 1 must change?

(b) Find the consumption matrix M.

(a) sector 2 must produce 3 more units

sector 1 must produce 5 more units.

(b) $I - M = \begin{pmatrix} 5 & 5 \\ 3 & 5 \end{pmatrix}^{-1} = \frac{1}{10}\begin{pmatrix} 5 & -5 \\ -3 & 5 \end{pmatrix}$

$= \begin{pmatrix} 0.5 & -0.5 \\ -0.3 & 0.5 \end{pmatrix}$

Thus $M = \begin{pmatrix} 0.5 & 0.5 \\ 0.3 & 0.5 \end{pmatrix}$

3. In two dimensions find the matrix transformation composed of one translation followed by one rotation which moves the line segment joining (5,7) and (7,9) so that the point (5,7) moves to the origin and the segment lies along the positive x-axis. **You can leave your answer as a product of several matrices, no need to simplify.**

① Move (5.7) to origin.

(7.9) becomes (2.2)

② rotate (2.2) to positive x axis.

$R(-\frac{\pi}{4})$

$$\begin{pmatrix} \cos(-\frac{\pi}{4}) & -\sin(-\frac{\pi}{4}) & 0 \\ \sin(-\frac{\pi}{4}) & \cos(-\frac{\pi}{4}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{pmatrix}$$

4. Consider the axis $\mathcal{A}$ pointing in the direction of the vector

$$\begin{bmatrix} 0.5 \\ 0.5 \\ 0 \end{bmatrix}$$

By rotating around the z-axis by an appropriate amount this axis can be placed on top of the x axis. Use this fact to find the rotation matrix which will rotate by an angle of $\pi/6$ around $\mathcal{A}$. **You can leave your answer as a product of several matrices, no need to simplify**.

① rotate $\begin{bmatrix} 0.5 \\ 0.5 \\ 0 \end{bmatrix}$ to x axis.

$R_z(-\frac{\pi}{4})$

② rotate around x axis by $\frac{\pi}{6}$.

$R_x(\frac{\pi}{6})$

③ Rotate x axis back to $\begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix}$

$R_z(\frac{\pi}{4})$

$$\begin{pmatrix} \cos(\frac{\pi}{4}) & -\sin(\frac{\pi}{4}) & 0 & 0 \\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} & 0 \\ 0 & \sin\frac{\pi}{6} & \cos\frac{\pi}{6} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(-\frac{\pi}{4}) & -\sin(-\frac{\pi}{4}) & 0 & 0 \\ \sin(-\frac{\pi}{4}) & \cos(-\frac{\pi}{4}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5. In three dimensions find the image of the three points

$$(2, 3, 4), (-3, 3, 10), (5, 4, -5)$$

under the perspective projection with center of the perspective at $z = 50$. In other words, find the (x_0, y_0) coordinates in the xy-plane for each point.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{50} & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \\ 1 \end{pmatrix} \to \begin{pmatrix} 2 \\ 3 \\ 0 \\ 1-\frac{2}{25} \end{pmatrix}$$

similarly

$$\begin{pmatrix} -3 \\ 3 \\ 10 \\ 1 \end{pmatrix} \to \begin{pmatrix} -3 \\ 3 \\ 0 \\ 1-\frac{1}{5} \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ 4 \\ -5 \\ 1 \end{pmatrix} \to \begin{pmatrix} 5 \\ 4 \\ 0 \\ 1+\frac{1}{10} \end{pmatrix}$$

Thus divide by the fourth component for each vector.

$$\left(\frac{2}{\frac{23}{25}}, \frac{3}{\frac{23}{25}}\right). \quad \left(\frac{-3}{\frac{4}{5}}, \frac{3}{\frac{4}{5}}\right). \quad \left(\frac{5}{\frac{11}{10}}, \frac{4}{\frac{11}{10}}\right)$$