Weekly Assignment 2:
Breadth-First Search

September 2025

1. Apply the BFS algorithm to the directed graph displayed in Figure 1 using the node with label 2 as source. Specify:

    - The content of the queue, after initialization and after each iteration of the loop,
    - For any discovered node, its predecessor and the iteration when it was discovered

    You may assume that the adjacent vertices of a node are visited in the order of their sequence number.

    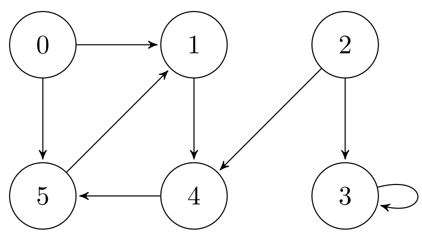

    Figure 1: Directed graph

2. Suppose we have a string s containing lowercase English letters. Design an algorithm, using a stack, that removes all adjacent and equal letters from s. The final string after removing all adjacent and equal letters should be returned. For example, input $s =$ "*aacacbbcab*" leads to output "*cb*". You can use the following functions:

    - `push`(e) takes an element e and adds it to the end of the stack.
    - `pop` return and removes the last element from the stack.
    - `peek` returns the last element on the stack without removing it from the stack.
    - `toStr` returns the stack in String format (the first element in the stack is also the first element in the string).

3. Consider a directed graph G and a vertex $s \in V$. Let C be the set of cycles in G that visit vertex s. Give an efficient algorithm that takes a graph G and a vertex v in G and prints a cycle from C in case C is nonempty, and `false` otherwise. You can use the following functions:

    - `New`($queue$) returning an empty queue.
    - `Enqueue`(Q, $element$) adding $element$ to queue Q.
    - `Dequeue`(Q) removing and returning and $element$ from Q.
    - `Print`(w) to print the vertex id

Explain why your algorithm is correct.

4. Given a graph $G = (V, E)$, a 2-coloring is a function $c\colon V \to \{blue, red\}$ assigning colors to vertices such that $(u, v) \in E$ implies $c(u) \neq c(v)$, i.e., adjacent vertices have different colors. Let G be an undirected graph. An odd-length cycle is a sequence $v_1, v_2, \ldots, v_k$ of vertices of G such that $(v_i, v_{i+1}) \in E$, for $i = 1, \ldots, k-1$, $(v_k, v_1) \in E$ and k is odd. See Figure 2 for an example of a graph with a valid 2-coloring and a graph with an odd-length cycle.

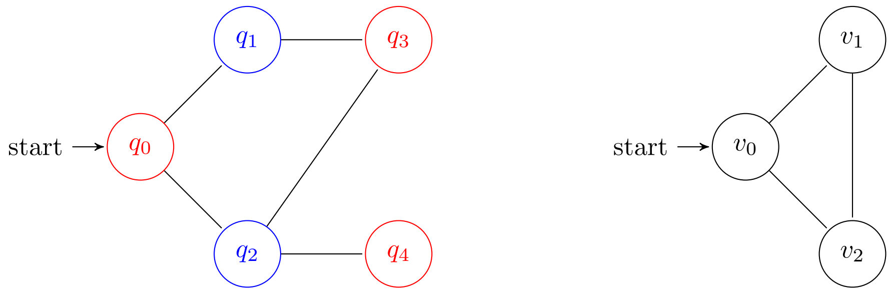

Figure 2: A graph with a 2-coloring and a graph with odd-length cycle v_0, v_1, v_2.

a. Show that, if G has an odd-length cycle, then there is no 2-coloring.

b. Assume G has no odd-length cycles. Use BFS algorithm to find an appropriate 2-coloring for G and proof that it is correct. Hints:

   i. Assume that your algorithm does not produce a valid coloring, i.e., there is a pair of adjacent vertices with the same color, and prove the claim by contradiction;

   ii. Prove and use the following fact: two vertices with the same color must both have even or odd shortest distance from the BFS source.

5. Give an example of a directed graph $G = (V, E)$, a source vertex $s \in V$, and a set of tree edges $E_\pi \subseteq E$ such that for each vertex $v \in V$, the unique simple path in the graph (V, E_π) from s to v is a shortest path in G, yet the set of edges E_π cannot be produced by running BFS on G, no matter how the vertices are ordered in each adjacency list.