FINM33601:Fixed Income Derivatives Homework 1

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April 22nd, 2021

Question 1. **(1)** Accrual period is December 16, 2020 to March 16, 2021. The accrural period for US LIBOR has a T+2 convention.

(2) 90/360

(3) Prompt settlement since forward looking. Settlement payment will be made first day of the accrural period December 16, 2020.

(4) Since forward looking, the payment amount will be determined 3 months after contract is transacted, on December 14 2020.

(5) Sold FRA is being a lender. I will receive $\frac{N\cdot(r_{FRA}-r_{LIBOR})\tau}{1+R\cdot\tau}$, such that $N = 100000000$ is the notional amount in USD, $r_{FRA} = .0031$, $r_{LIBOR} = .0021925$ is the 3 month USD LIBOR rate on December 14,2020 and $\tau = \frac{90}{360}$.

$$\frac{N\cdot(r_{FRA}-r_{LIBOR})\tau}{1+R\cdot\tau} = \frac{100000000\cdot(.0031-.0021925)\cdot\frac{90}{360}}{1+.0021925\cdot\frac{90}{360}} = 22675.07\$$$

(6) You would beed to arrange for the deposit on December 18, 2020 to settle. According to Quandl , the annualized rate is if 0.0018504 so

$$0.0018504\cdot\frac{90}{360}\cdot 22675.07 = 10.49\$$$

(7) To be paid at end of the accrural period on December June 16,2021.

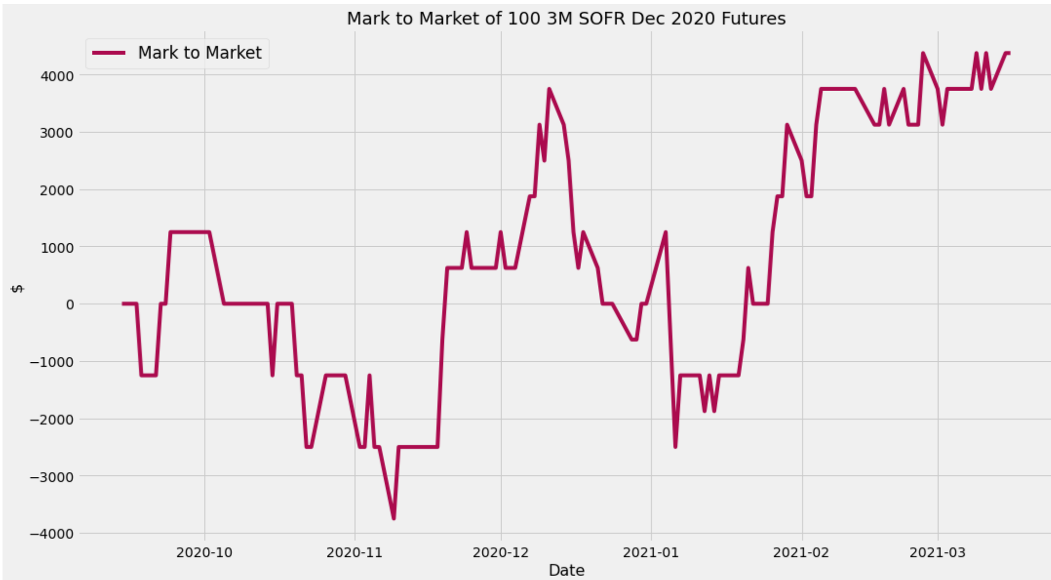
$$\frac{N\cdot(r_{FRA}-r_{LIBOR})\tau}{1} = \frac{100000000\cdot(.0031-.0021925)\cdot\frac{90}{360}}{1} = 22687.5\$$$

(8) At the end of the accrural period on March 16, 2021. We need to compound the daily SOFR rate each day for 3 months. Then annualize this return and compare to the strike rate of the contract. This is done in Python.

SOFR term rate: 0.05055

Payment: 4917.53$

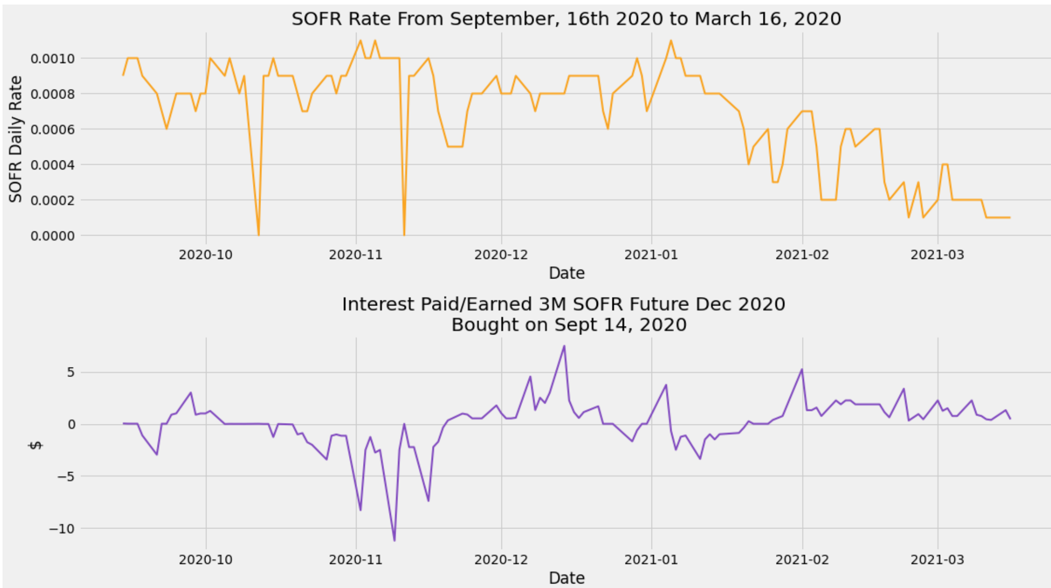
Question 2. (1) See figure bellow for Daily Mark to Market value experienced.



Time	Price	Market Rate
2020-09-14	99.93	0.00007
2021-03-16	99.9475	0.000525

(2) The final marging account balance without interest is: 4375.00$

(3) See graphs below.



Final Balance with interest: 4387.9$

FRA Settlement with market rate of future: 4034.72$

(4) Our balanced increased since we mostly had a positive position but only by 12.9 which is about 25 bps. The effect would have been larger if instead of SOFR decreasing over time, it would have been increasing. Looking at the first graph other than some data irregularities, there is a clear downward slopping trend in the SOFR rate which affected the interest earn on our position. If rate had increased, we would have hold more of a negative position and get paid more through SOFR, we have asymmetric risk. Also worth nothing, while the net interest gain is positive, in terms of absolute the largest interest exchanged was from us to the counter party.

Question 3. Lets define

$$V_{FIX}(0) = \sum_{i=1}^{n} c\tau_i P(0,t_i), \quad V_{FLOAT} = \sum_{i=1}^{n} R(0,t_{i-1},t_i)\tau_i P(0,t_i)$$

Let $0 < T < M < n, t_j = T, t_{j+1} = M$,

$\tau_i = \tau \ \forall i$ and $i \in [0,n] \cap \mathbb{Z}$,

Let $R(0,t_j,t_{j+1}) = R(0,T,M) = \mathcal{X}$,

Let's $V_{FIX}(0) = V_{FLOAT}(0)$,

but $\mathcal{X} \neq \frac{1}{\tau}\left(\frac{P(0,T)}{P(0,M)} - 1\right)$.

We can compute $V_{FIX}(0)$,

$$V_{FIX}(0) = \sum_{i=1}^{n}\left(\frac{P(0,t_0) - P(0,t_n)}{\sum_{i=1}^{n}\tau P(0,t_i)}\right)\tau P(0,t_i) = P(0,t_0) - P(0,t_n)$$

$$V_{FLOAT}(0) = \tau\left[\sum_{i=1}^{j}\frac{1}{\tau}\left(\frac{P(0,t_{i-1})}{P(0,t_i)} - 1\right)P(0,t_i) + \mathcal{X}P(0,M) + \sum_{i=j+1}^{n}\frac{1}{\tau}\left(\frac{P(0,t_{i-1})}{P(0,t_i)} - 1\right)P(0,t_i)\right]$$

$$V_{FLOAT}(0) = \sum_{i=1}^{n}\left(\frac{P(0,t_{i-1})}{P(0,t_i)} - 1\right)P(0,t_i) + \tau\mathcal{X}P(0,M) - \left(\frac{P(0,t_j)}{P(0,t_{j+1})} - 1\right)P(0,t_{j+1})$$

$$V_{FLOAT}(0) = \underbrace{\sum_{i=1}^{n}\left(\frac{P(0,t_{i-1})}{P(0,t_i)} - 1\right)P(0,t_i)}_{\alpha} + \underbrace{\tau\mathcal{X}P(0,M) - \left(\frac{P(0,T)}{P(0,M)} - 1\right)P(0,M)}_{\beta}$$

$$\alpha = \sum_{i=1}^{n}\frac{1}{\tau}\left(\frac{P(0,t_{i-1})}{P(0,t_i)} - 1\right)P(0,t_i) = \underbrace{\sum_{i=1}^{n}\left(P(0,t_{i-1}) - P(0,t_i)\right)}_{\text{telescoping sum!}} = P(0,t_0) - P(0,t_n) = V_{FIX}(0)$$

$$\Rightarrow V_{FLOAT}(0) = V_{FIX}(0) + \beta$$

If

$$\mathcal{X} < \frac{1}{\tau}\left(\frac{P(0,T)}{P(0,M)} - 1\right) \to V_{FLOAT}(0) < V_{FIX}(0) \qquad (1)$$

else, if

$$\mathcal{X} > \frac{1}{\tau}\left(\frac{P(0,T)}{P(0,M)} - 1\right) \to V_{FLOAT}(0) > V_{FIX}(0) \qquad (2)$$

But, we've set up the problem such that $V_{FLOAT}(0) = V_{FIX}(0)$. So in the first case the float is overvalued so we want to short the floating leg of the swap and take a long position in the fixed one. In case (2), we want to take the opposite of that trade. For the first case, we can also borrow at the forward rate, sell T bond and buy M bond. By selling the T bond and buying M bond we receive initial premium upfront. We pay off value of the T bond with we we borrowed and receive M bond at time B, and we end up lending at a higher rate from period between T and M. For the second case, we can lend the forward rate, buy T bond and sell M bond and we end up borrowing at a lower rate between T and M.