FINM33601:Fixed Income Derivatives — Homework 3

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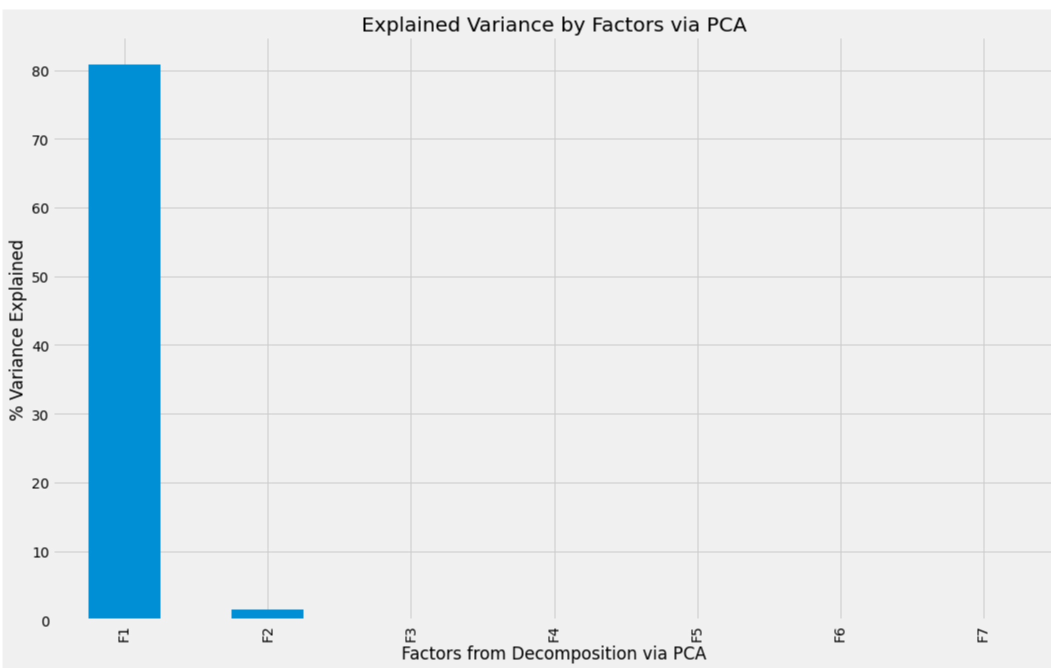
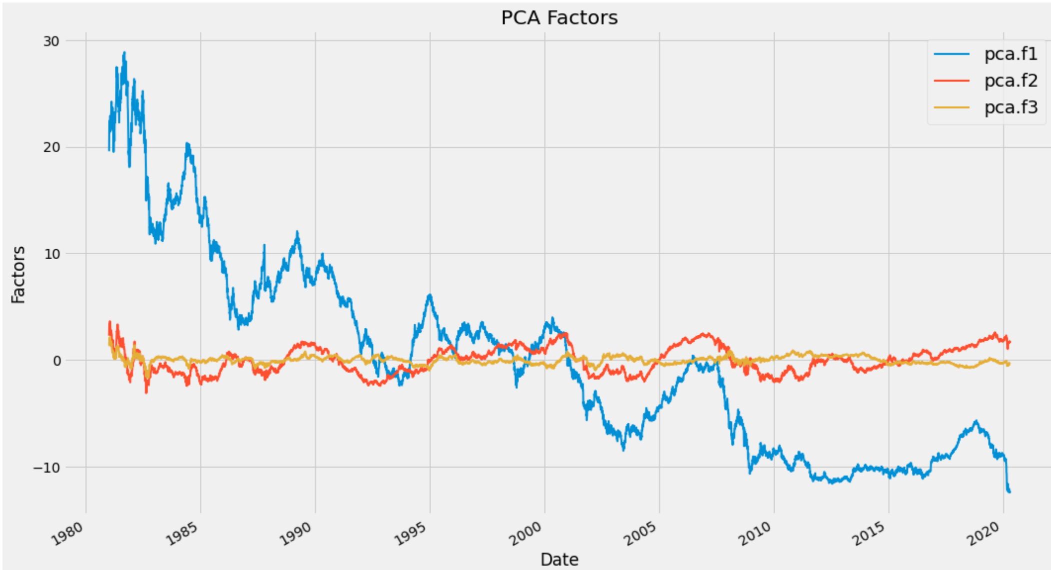
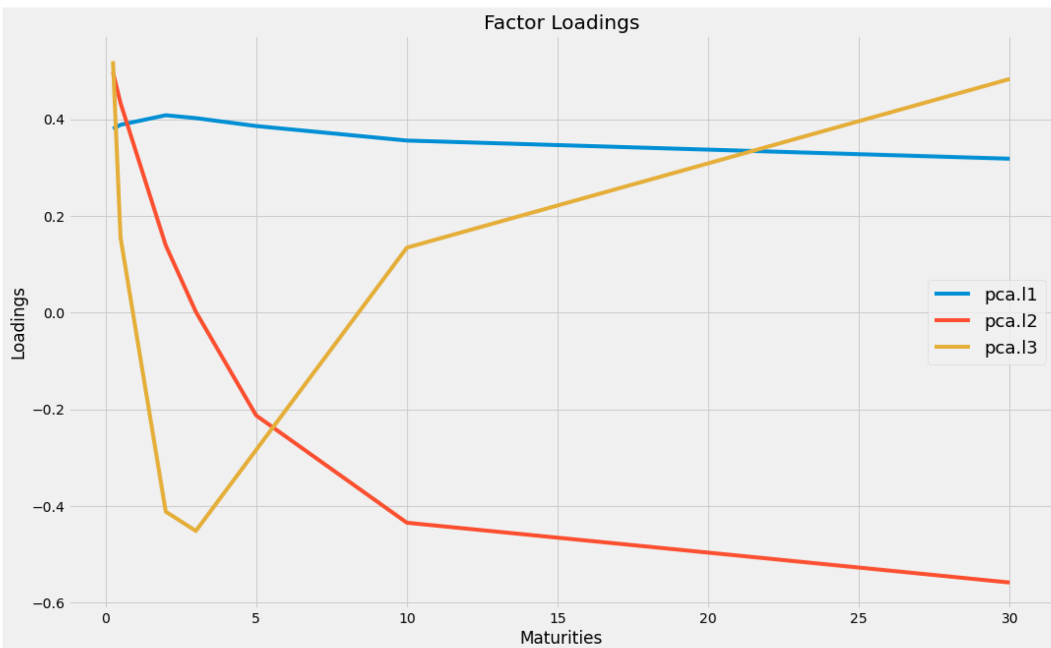
Question 1. Table below for Data.

	USGG3M	USGG6M	USGG2YR	USGG3YR	USGG5YR	USGG10YR	USGG30YR
DATE							
1981-01-05	13.52	13.09	12.289	12.28	12.294	12.152	11.672
1981-01-06	13.58	13.16	12.429	12.31	12.214	12.112	11.672
1981-01-07	14.50	13.90	12.929	12.78	12.614	12.382	11.892
1981-01-08	14.76	14.00	13.099	12.95	12.684	12.352	11.912
1981-01-09	15.20	14.30	13.539	13.28	12.884	12.572	12.132

Question 2. See charts below for the Variance Explained by the factor decomposition via PCA and the factors value overtime of the PCA and factor loadings.







2.3 We can observe that the first loading remains stable and quite consistent throughout the maturities while the other two change a lot. The second factor continues to decreases up to the last maturities. The third factor starts by decreasing rapidly then slightly before 5 years of maturity starts to increase up to 30. They all start positive but the factor 2 and 3 go negative and the third factor comes back positive around 8 years. The first factor affects most of the variance so it has a lot more weight on the movement of the rates. Depending on the signs of the factors and the value of the zero level factor, from the factor loadings we can interpret how the yield curve moves. Given that factor

1 is decreasing and positive before 2000 and negative afterwards and that in absolute term it is larger than f2 and f3 for most of it. We have that the first loading factor is positive stable across the board so as f1 decrease the interest rate will decrease and will be pushing toward negative value but will be offset from the long term zero loading. The key is to understand that the yield curve is the dot product of the factors and the loadings, so to have an increasing effect for a factor it needs to have the loading on the same sign. As for the size, it is measured by the scalar value of the multiplication of the factor and it's loading. The factors sort of act as a weight to the loadings in affecting the yield curve.

Question 3. See multiple tables below with captions to see what part of the volatility we are measuring.

Table 1: Correlation Matrix Factors Using Whole Data for the First 3 Factors

	pca.f1	**pca.f2**	**pca.f3**
pca.f1	1	8.3347e-16	1.0000e-15
pca.f2	8.3347e-16	1	2.0248e-16
pca.f3	1.0000e-15	2.0248e-16	1

Table 2: Standard Deviations Factors Using Whole Data for the First 3 Factors

pca.f1	8.9892
pca.f2	1.2306
pca.f3	0.3592

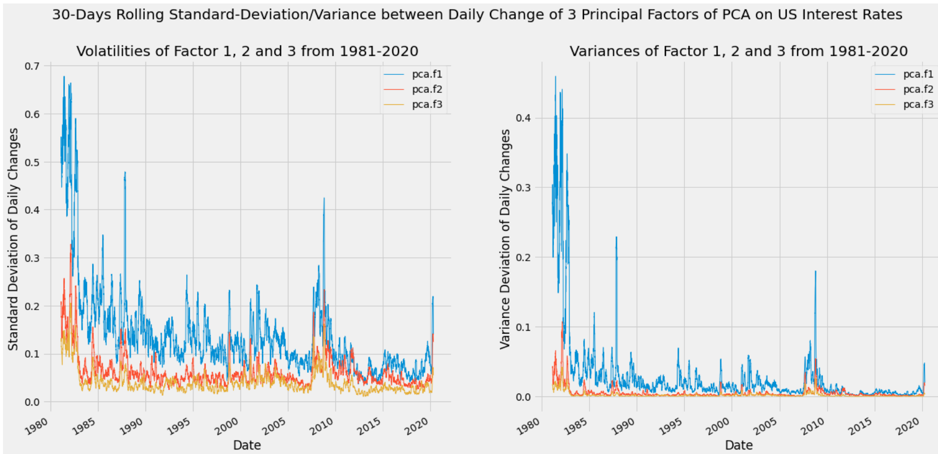
Table 3: Correlation Matrix Factor Differentiation Using Whole Data for the First 3 Factors

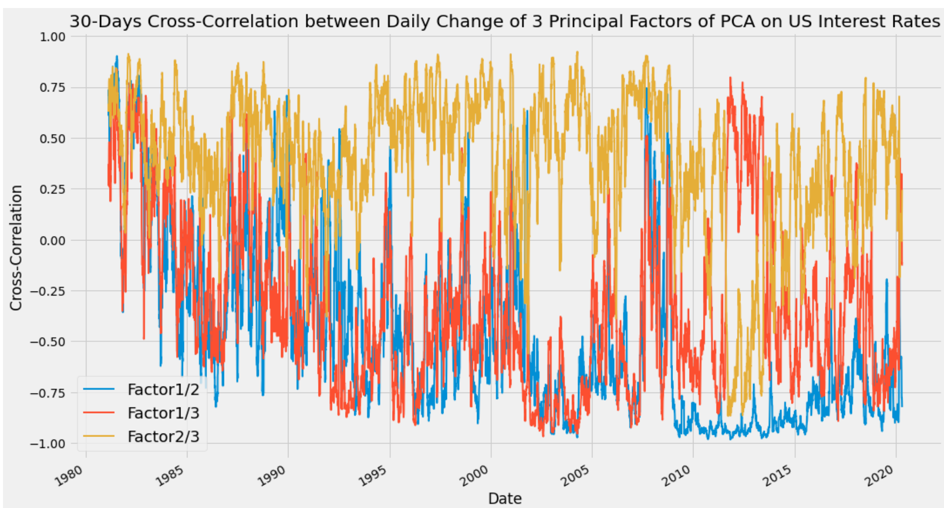
	pca.f1	**pca.f2**	**pca.f3**
pca.f1	1.0000	-0.0096	-0.0469
pca.f2	-0.0096	1.0000	0.5234
pca.f3	-0.0469	0.5234	1.0000

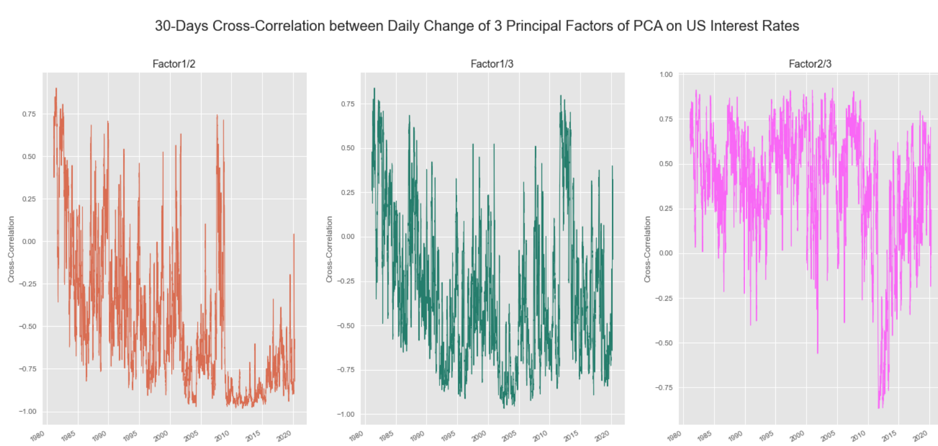
Table 4: Standard Deviations Factors Differentiation Using Whole Data for the First 3 Factors

pca.f1	0.1732
pca.f2	0.0734
pca.f3	0.0478

3.2


30-Days Rolling Standard-Deviation/Variance between Daily Change of 3 Principal Factors of PCA on US Interest Rates
Volatilities of Factor 1, 2 and 3 from 1981-2020
Variances of Factor 1, 2 and 3 from 1981-2020
Standard Deviation of Daily Changes
Variance Deviation of Daily Changes
pca.f1
pca.f2
pca.f3
Date


30-Days Cross-Correlation between Daily Change of 3 Principal Factors of PCA on US Interest Rates
Cross-Correlation
Factor1/2
Factor1/3
Factor2/3
Date


30-Days Cross-Correlation between Daily Change of 3 Principal Factors of PCA on US Interest Rates
Factor1/2
Factor1/3
Factor2/3
Cross-Correlation
Date

Question 4. See tables below for historical estimates of volatilities of the first 3 factor corresponding to the last month of the observed period (April 2020).

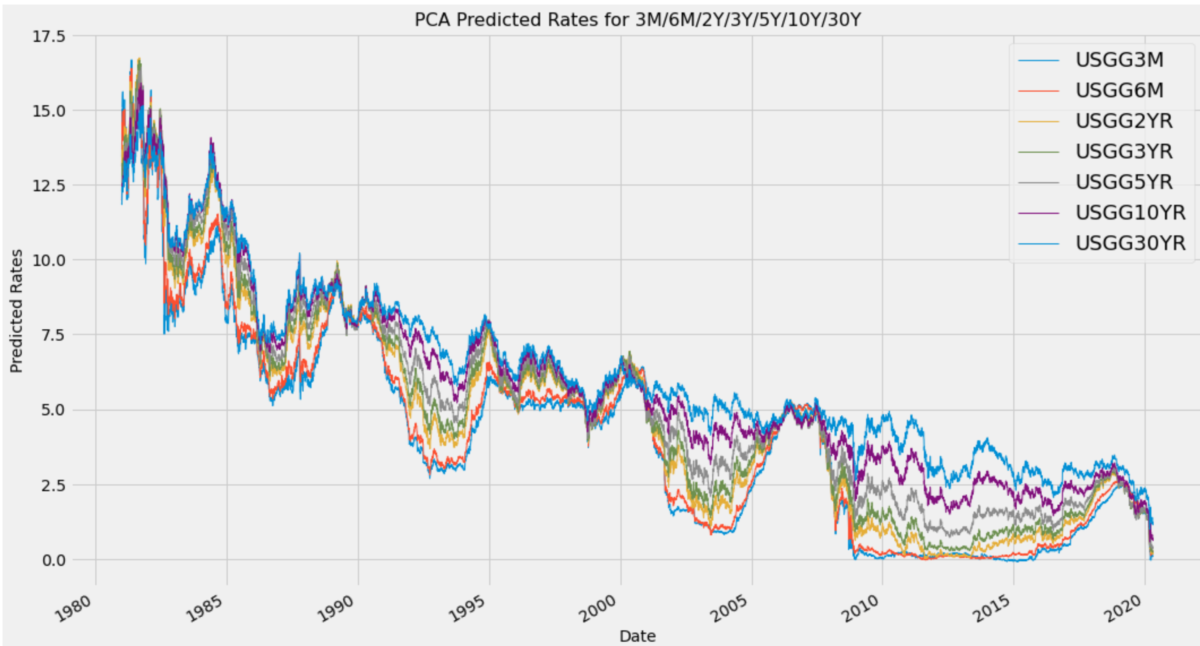
Table 5: Standard Deviations Factors Differentiation Using Last Month Data (April 2020) for the First 3 Factors

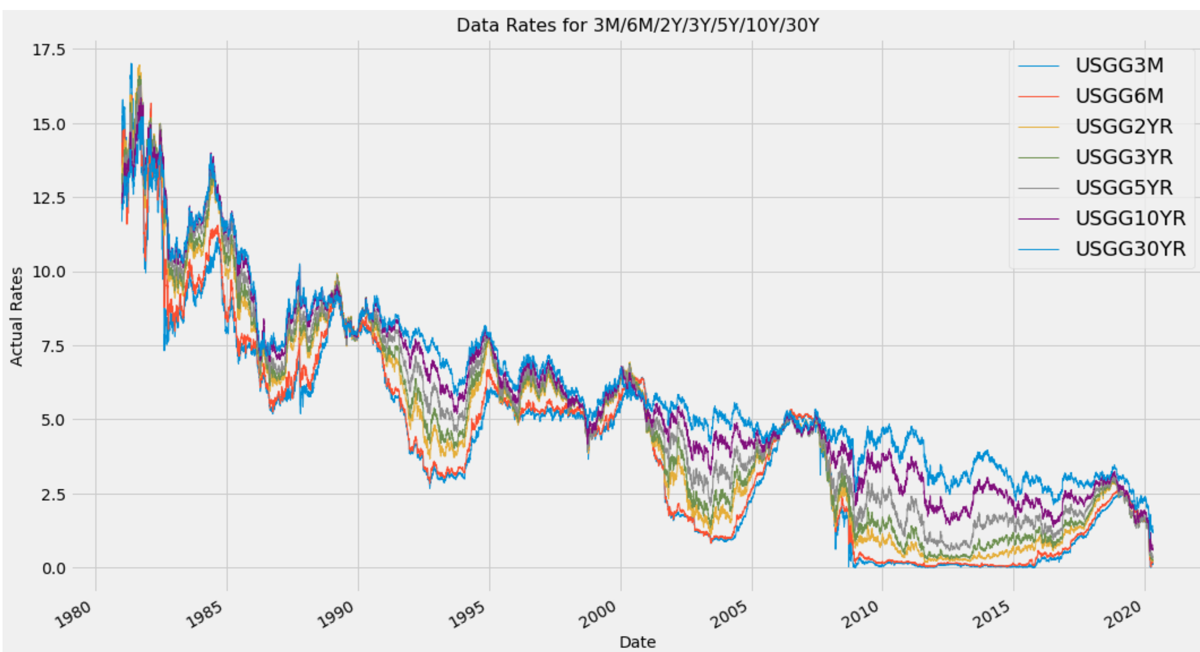
pca.f1	0.0730
pca.f2	0.0472
pca.f3	0.0382

Table 6: Correlation Matrix Factor Differientiation Using Last Month Data (April 2020) for the First 3 Factors

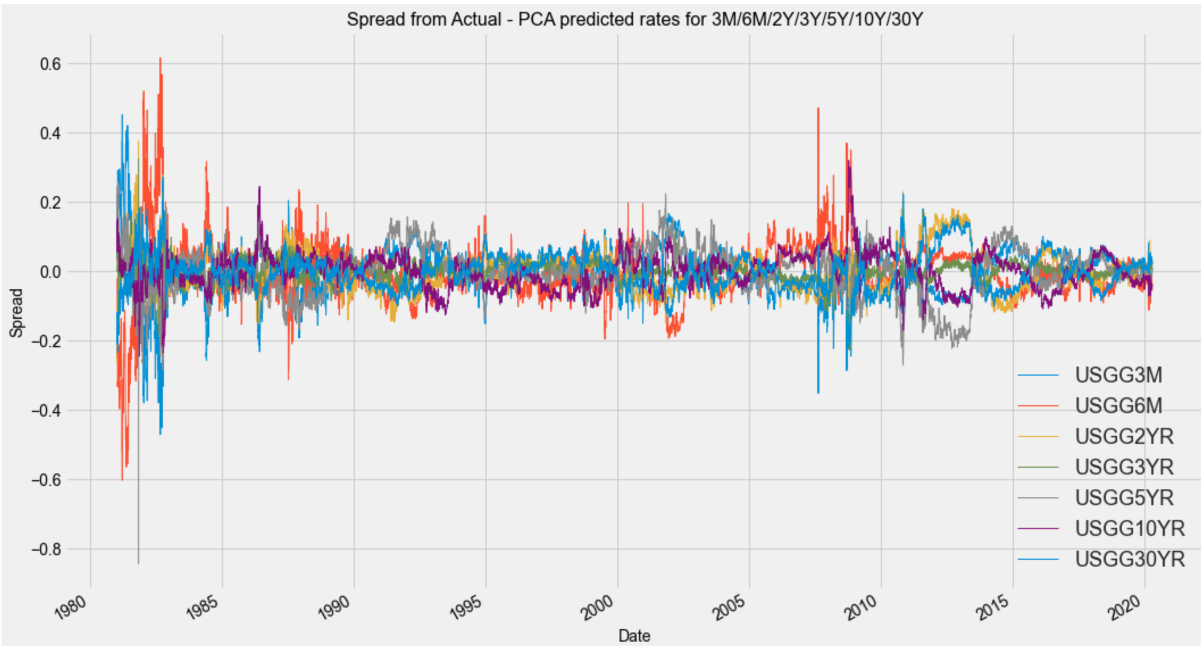
	pca.f1	**pca.f2**	**pca.f3**
pca.f1	1.0000	-0.7122	0.5265
pca.f2	-0.7122	1.0000	-0.3946
pca.f3	0.5265	-0.3946	1.0000

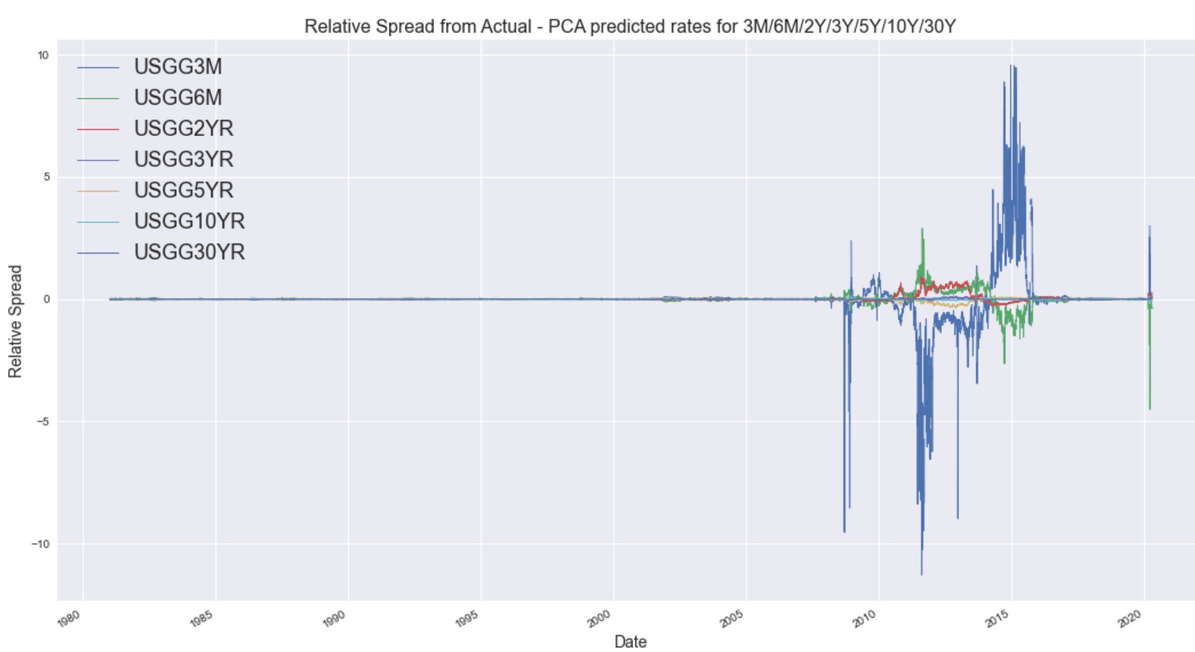
Question 5. See the two charts below to observe how the predicted rates versus the actual.





The next two charts shows the absolute difference between the predicted rates and the actual rates and the relative difference between them. Notice how the PCA has a harder time predicting accurately the rates during the financial crisis and afterwards for shorter terms yields.





Question 6. Here are the results for the loadings displayed in array form:

Results for loading 0:
a : [0.394 0.]
b : [-2.395 6.452]
Results for loading 1:
a : [0.141 0. 22.381]
b : [0.145 0.284 -0.284]
Results for loading 2:
a : [0.016 0.397]
b : [-0.853 1.417]
Results for loading 3:
a : [0.149 0.068 0.15 2.161]
b : [-3.434 3.749 -1.784 2.479]

Question 7.

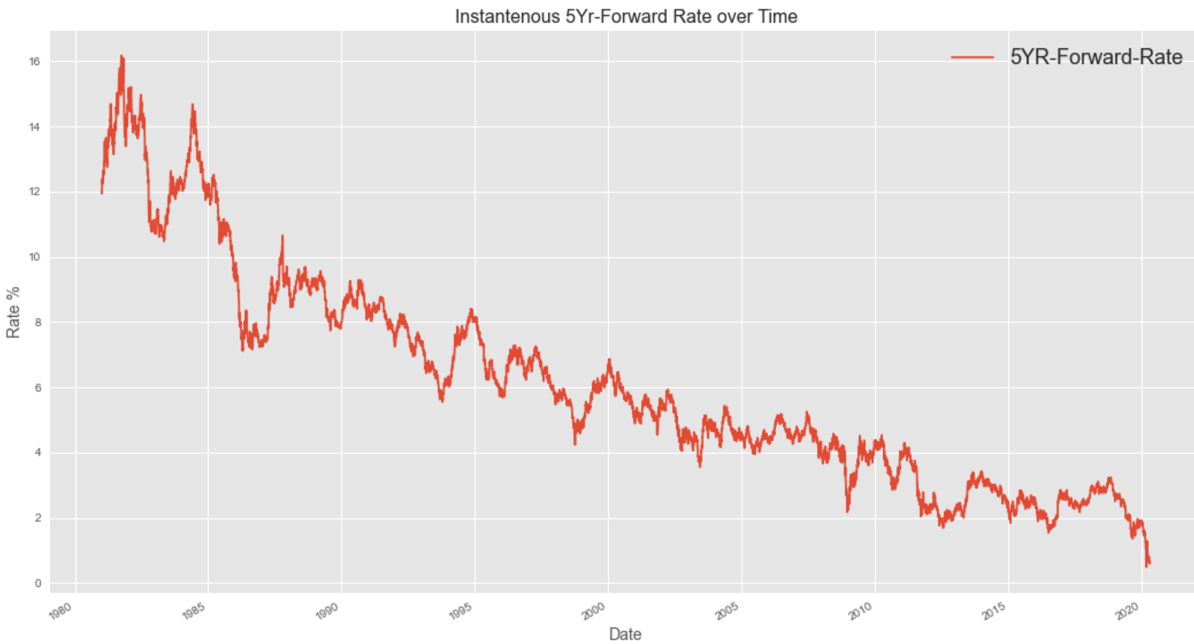
$$F(t,\tau) = B_0'(\tau) + \sum_{i=1}^{3} B_i'(\tau) \cdot f_i(t)$$

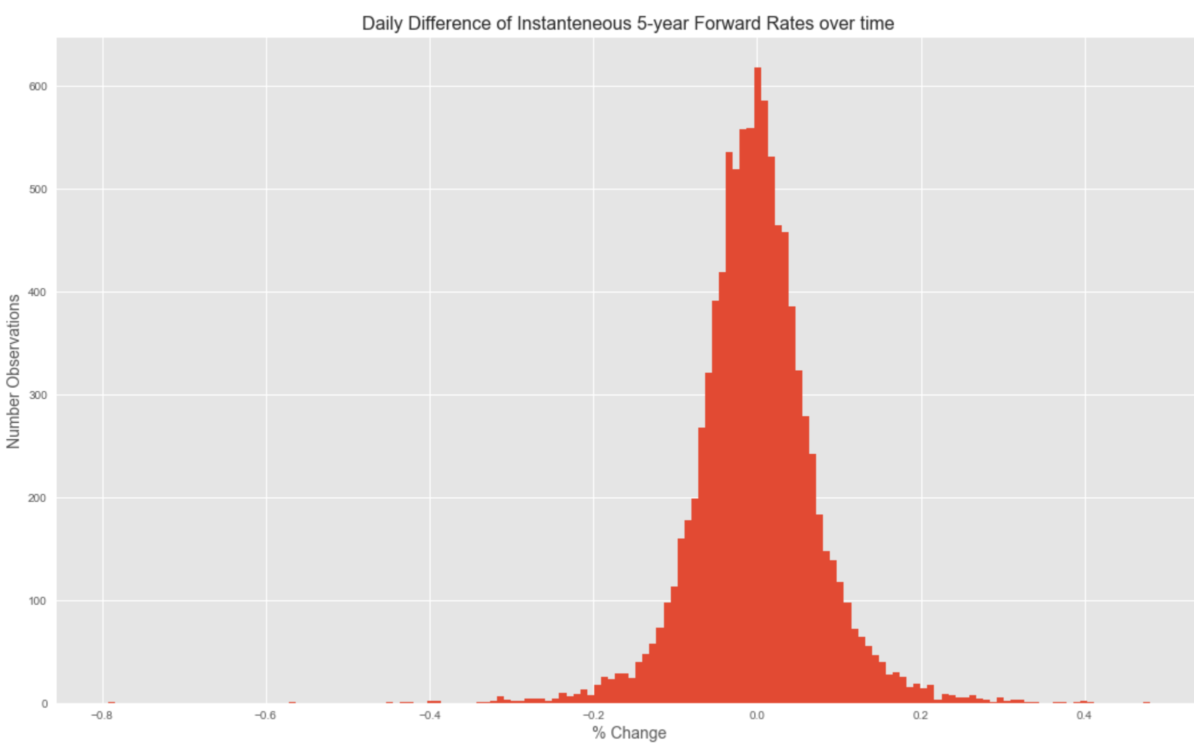
$$B_i(\tau) = l_i(\tau)\tau = \tau \left[\sum_{j=1}^{m_i} b_{ij} \frac{1 - \exp\left(-a_{ij}\tau\right)}{a_{ij}\tau} \right]$$

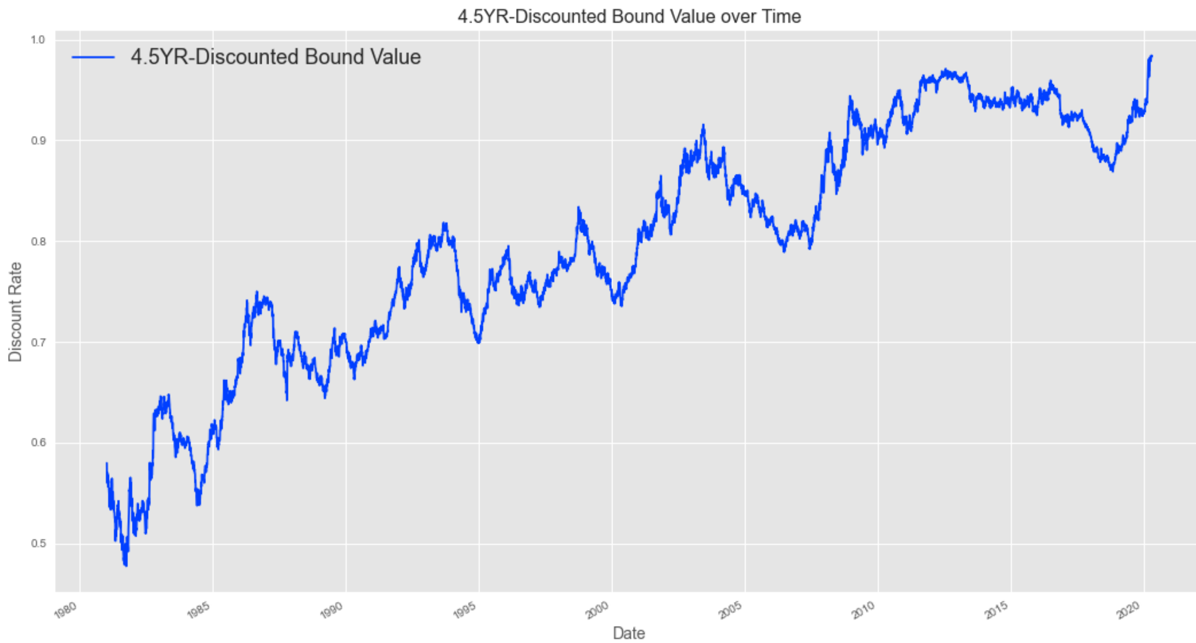
$$B_i'(\tau) = \sum_{j=1}^{m_i} b_{ij} e^{-a_{ij}\tau)}$$

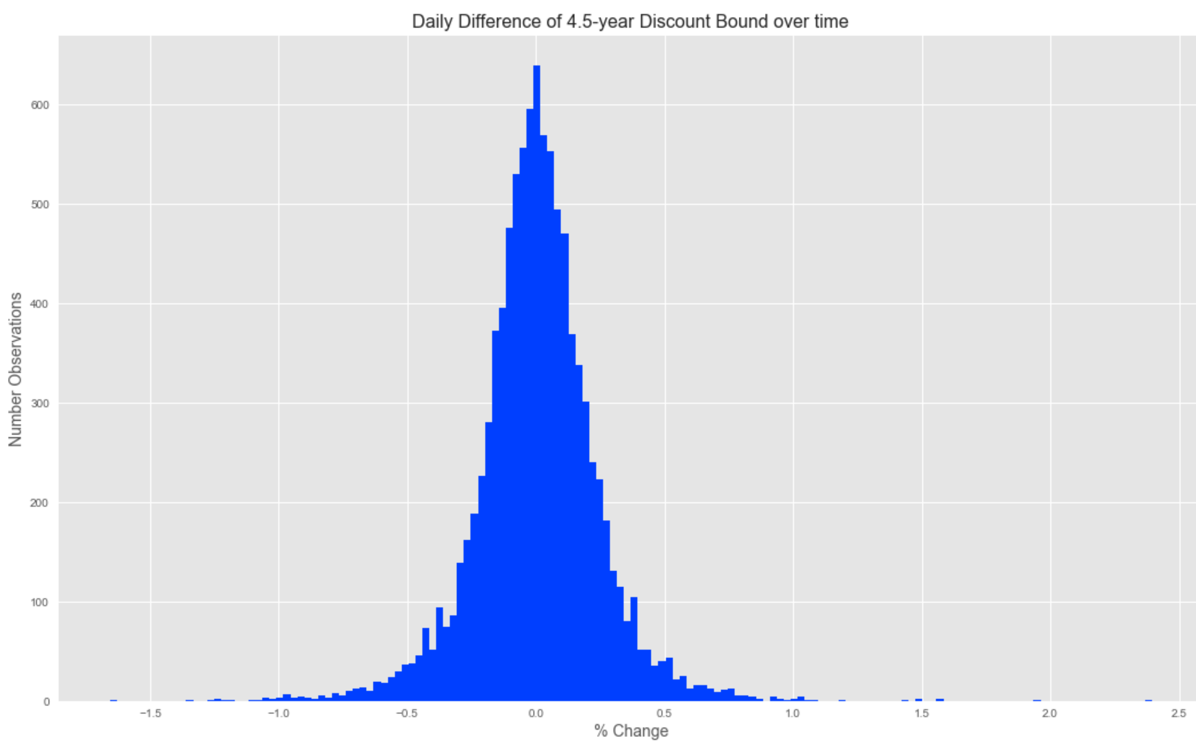
$$F(t,\tau) = B_0(\tau) + \sum_{i=1}^{3} \left(\sum_{j=1}^{m_i} b_{ij} e^{-a_{ij}\tau} \right) \cdot f_i(t)$$

Using the form above, I was able to generate the follow charts for the 5-Yr Forward Rate and the 4.5-Yr Discounted Bonds.

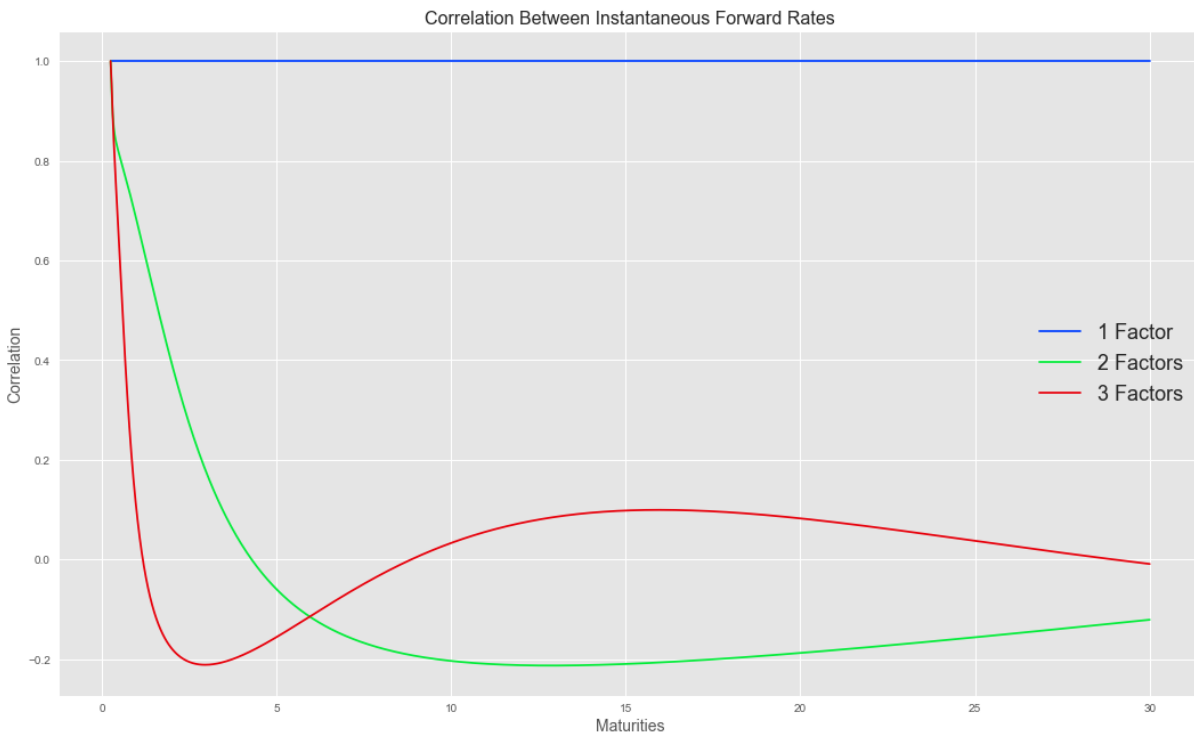

Instantenous 5Yr-Forward Rate over Time
5YR-Forward-Rate
Rate %
Date
0
2
4
6
8
10
12
14
16
1980
1985
1990
1995
2000
2005
2010
2015
2020


Daily Difference of Instanteneous 5-year Forward Rates over time
Number Observations
% Change
0
100
200
300
400
500
600
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4


4.5YR-Discounted Bound Value over Time
4.5YR-Discounted Bound Value
Discount Rate
Date
1.0
0.9
0.8
0.7
0.6
0.5
1980
1985
1990
1995
2000
2005
2010
2015
2020


Daily Difference of 4.5-year Discount Bound over time
Number Observations
% Change
600
500
400
300
200
100
0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5

Question 8. See Python code for calculation. Decided to run it over multiple different tenors instead of those given to smooth out the curve.



Question 9. For this question, I assumed that the values given in the last questions are the parameters of the yield curve generated with an FRA rate of 2.968021%. Given that the date we are meant to compute is not given in the yield curve. I chose to use an interpolation technique between the closest two dates for the factors. We can see in the table below how the factors were close by, so it shouldn't change the final answers too much from having the actual information.

Table 7: PCA Factors for the given dates below

Date	**pca.f1**	**pca.f2**	**pca.f3**
2014-04-17	-10.0496	-0.8433	0.1722
2014-04-21	-10.0499	-0.8333	0.1859

Result of the interpolation is this:

Table 8: PCA Factors April 18th, 2014

Date	**pca.f1**	**pca.f2**	**pca.f3**
2014-04-18	-10.0497	-0.8408	0.1756

To find the forward-rate of a 3x9-FRA, using

$$F(t, \tau_1, \tau_2) = \frac{(B_0(\tau_2) - B_0(\tau_1))}{\tau_2 - \tau_1} + \sum_{i=1}^{3} \frac{(B_i(\tau_2) - B_i(\tau_1))}{\tau_2 - \tau_1} \times f_i(t)$$

With the factors above and $\tau_2 = 9/12,\ \tau_1 = 3/12$. 6 months Forward Rate starting in 3 months from 2014/04/18 using interpolation between 2014/04/17 and 2014/04/21

$$F(t, 3/12, 9/12) = -0.0391\%$$