FINM 32000: Homework 2

Due Thursday February 4, 2021 at 11:59pm

Problem 1

The Gold Dragon Coin (GDC) is a unit of currency in Westeros. Let S denote the GDC/USD exchange rate (the USD value of 1 GDC). Assume S has dynamics

$$\mathrm{d}S_t = (r - q)S_t\mathrm{d}t + \sigma(S_t, t)S_t\mathrm{d}W_t,$$

where W is Brownian motion under the [US] risk-neutral probability measure. The USD interest rate is $r = 0.06$, the GDC interest rate is $q = 0.01$, today's time-0 spot is $S_0 = 100$, and

$$\sigma(S, t) := \min[0.2 + 5(\log(S/100))^2 + 0.1e^{-t},\ 0.6].$$

(a) Find the time-0 price of an American-style put on the GDC. The put is struck at 95 and expires at time 0.75.

(b) Find the time-0 price of a European-style call on the put in part (a). The call is struck at 10 and expires at time 0.25. This call is an example of a *compound option.* At time 0.25 it gives you the right to buy the underlying put for 10. The underlying put will have the usual exercise privilege on the time interval $[0.25, 0.75]$, at strike 95.

All prices are, as usual, in USD unless stated otherwise.

Complete the coding of the function `pricer_compound_localvol_trinom` in the provided file `hw2.ipynb`. Use a trinomial tree.

Your code may reject N for which the call expiry fails to be represented in the tree. In choosing Δx, follow L2.9 and choose the "representative" volatility σ_{avg} to be $\sigma(S_0, 0)$.

The amount of work done by your algorithm in this problem should grow like N^2 as N grows (no proof required). If it grows like N^3 in this problem, then your algorithm has some major inefficiency.

Problem 2

(a) In the Black-Scholes model with interest rate r, no dividends, and volatility σ, approximate the time-0 *delta* of an at-the-money ($K = S_0$) vanilla call with expiry T, by applying a first-order Taylor expansion to the exact formula, and obtaining an explicit approximation formula in terms of the given parameters.

Then evaluate this approximation to two decimal places, assuming $\sigma = 0.2$ and $T = 0.25$ and $r = 0.01$.

(b) Suppose that some derivative contract (not necessarily a call or put) has a time-0 pricing function $C(S)$ with respect to an underlying S. Recall that the time-0 delta and gamma are defined as $\partial C/\partial S(S_0)$ and $\partial^2 C/\partial S^2(S_0)$ respectively.

Define the function

$$c(x) := C((1 + x)S_0).$$

Thus, $c(0.01)$ equals the contract's value, given a 1 percent increase in the underlying.

Define the contract's time-0 *dollar delta* to be

$$\frac{\partial c}{\partial x}(0)$$

and define the contract's time-0 *dollar gamma* to be

$$\frac{\partial^2 c}{\partial x^2}(0).$$

Suppose that at time 0, the underlying is at level $S_0 = 4$, and the derivative contract has price 2, *dollar delta* 3, and *dollar gamma* 6.

Use a second-order Taylor expansion (with zeroth, first, and second order terms) to approximate the time-0 value of the contract, given an underlying level of 3.6.

Either one of the following two solution methods is acceptable – your choice:

- ▷ Convert the dollar delta and dollar gamma into delta and gamma, and apply Taylor expansion to C.
- ▷ Use the unconverted dollar delta and dollar gamma, and apply Taylor expansion to c.