Problem 3

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(i) Suppose there is a pair of w, b satisfying the constraints,
i.e., $y^{(i)}(w \cdot x^{(i)} + b) \geq 1$ for $i = 1, 2, 3, 4$
Since $y^{(1)} = y^{(2)} = 1$, $y^{(3)} = y^{(4)} = -1$,

$$\begin{cases} w_1 + b \geq 1 \\ 2w_0 + 3w_1 + b \geq 1 \\ 3w_1 + b \leq -1 \\ 2w_0 + w_1 + b \leq -1 \end{cases}$$

$$\Rightarrow \begin{cases} w_1 \geq 1 - b & (1) \\ 2w_0 + 3w_1 \geq 1 - b & (2) \\ 3w_1 \leq -1 - b & (3) \\ 2w_0 + w_1 \leq -1 - b & (4) \end{cases}$$

Combining (1)(2)(3)(4) we get an inequality:
$(-1 - b) + (b - 1) \geq 3w_1 + b - 1 \geq -2w_0 \geq 1 + b + w_1 \geq (1 + b) + (1 - b)$
$\Rightarrow -2 \geq 2$ which gives no solution.

Thus there is no w, b satisfying the constraints, i.e., no solution satisfies the constraints for the hard-margin SVM problem.

(ii) Let the hyperplane be $x + y - 4 = 0$,
i.e., $w_0 = w_1 = 1, b = -4$ which gives $w \cdot x^{(i)} + b = x_1 + x_2 - 4$.

$\xi^{(1)} = max(0, 1 - y^{(1)}(w \cdot x^{(1)} + b)) = max(0, 1 - (0 + 1 - 4)) = max(0, 4) = 4$,
$\xi^{(2)} = max(0, 1 - y^{(2)}(w \cdot x^{(2)} + b)) = max(0, 1 - (2 + 3 - 4)) = max(0, 0) = 0$,
$\xi^{(3)} = max(0, 1 - y^{(3)}(w \cdot x^{(1)} + b)) = max(0, 1 + (0 + 3 - 4)) = max(0, 0) = 0$,
$\xi^{(4)} = max(0, 1 - y^{(4)}(w \cdot x^{(1)} + b)) = max(0, 1 + (2 + 1 - 4)) = max(0, 0) = 0$,

Thus a solution is $(w_0, w_1, b, \xi^{(1)}, \xi^{(2)}, \xi^{(3)}, \xi^{(4)}) = (1, 1, -4, 4, 0, 0, 0)$.