2022年4月总结

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目录

1 二阶椭圆方程正则性

本学期开始讲二阶椭圆方程正则性，课程安排如下

- Review of Sobolev space
  - L^p 空间
  - 研究 Sobolev 空间的两条路
  - 比较数分中的推广
  - Sobolev 空间的一些基本性质
- De Giorgi's theory
  - De Giorgi's theory
  - Nash-Morse-De Giorgi iteration
  - Application in Yamabe flow
- Harmous map and H-system
- Techniques from harmonic analysis and Gaugle theory
  - Techniques from harmonic analysis and Gaugle theory
  - Caderm-zygmmd theory
- Regularity of harmonic maps
- CZ theory or LP estimate

1.1 4 月 12 号

本节课首先回顾了以下定理的证明思路

Theorem 1. *Let $0 \leq u \in H^1\left(B_{4r}(0)\right)$ be a weak solution of (1.8) with (2.1) and let $q > 0$. Then with $C = C(q, \lambda, \Lambda, n) > 0$ there holds*

$$\sup_{|x|<r} u(x) \leq C\left(r^{-n} \int_{B_{4r}(0)} u^q dx\right)^{\frac{1}{q}}.$$

然后接着给出了了以下定理的证明

Theorem 2. *Suppose $0 \leq u \in H^1\left(B_{4\sqrt{n}r}(0)\right)$ is a weak super-solution of (1.8) with (2.1). Then for suitable $0 < q < \frac{n}{n-2}$ with $C = C(\lambda, \Lambda, n) > 0$, there holds*

$$\inf_{|x|<r} u(x) \geq C^{-1}\left(r^{-n} \int_{B_{2r}(0)} u^q dx\right)^{\frac{1}{q}},$$

再综合上述两个结果得到了如下的 Harnack 不等式

Theorem 3. *Let* $0 \leq u \in H^1(\Omega)$ *be a weak solution of (1.8) with (2.1). Then for any* $B_r(x_0) \subset B_{4\sqrt{n}r}(x_0) \subset \Omega$ *with* $C = C(n, \lambda, \Lambda) > 0$*, there holds*

$$\sup_{B_r(x_0)} u \leq C \inf_{B_r(x_0)} u.$$

1.2 4 月 19 号

本次课主要综合之前介绍的所有结果证明了 De Giorgi 理论

Theorem 4. *Let* $u \in H^1(\Omega)$ *be a weak solution of (1.8) with uniformly elliptic coefficients* $a_{ij} = a_{ji} \in L^\infty(\Omega)$*. Then* $u \in C^\alpha_{loc}(\Omega)$ *for some* $\alpha \in (0, 1]$.

然后开始介绍 Yamabe 问题中与二阶椭圆方程有关的部分，首先介绍了 Yamabe 问题中的相关概念，然后尝试证明了以下定理

Theorem 5. *On any* (M, g_0) *as above, there exists a conformal metric of constant scalar curvature.*

1.3 4 月 26 号

本节课接着介绍 Yamabe 问题中的相关结论，首先继续上节课以下定理的证明

Theorem 6. *On any* (M, g_0) *as above, there exists a conformal metric of constant scalar curvature.*

然后证明如下定理

Theorem 7. *Suppose* $0 < u \in C^\infty(M, g_0)$ *satisfies*

$$L_{g_0} u = -c(n)\Delta_{g_0} u + R_0 u \geq 0.$$

Then for some $q > 0$ *with a constant* $C > 0$ *there holds*

$$C \inf_M u \geq \|u\|_{L^q(M)}.$$

2 微分几何讨论班

本月我接着讲解讨论班的以下内容

2.1 外微分式的积分

本节首先介绍了向量空间的定向，从 2，3 维出发，逐步推广到 n 维，给出了定向的概念；然后介绍了可定向微分流形的概念，并给出了单位球面是可定向，莫比乌斯圈是不可定向的证明；再介绍了定向微分流形的判定定理，即具有第二可数公理的 n 维光滑流形 M 是可定向的，当且仅当在 M 上存在一个处处不为零的 n 次外微分式；最后给出了 n 次外微分式在 n 维有向光滑流形上的积分。

2.2 Stokes 定理

本节首先回顾了我们之前学习的 Newton-Leibniz 公式、Green 公式、Gauss 公式，并将这三者进行对比，提炼出一般规律，得出了 Stokes 定理的形式和概念；然后站在微分几何的角度，给出了带边区域和它的边界的概念；最后证明了 Stokes 定理。

3 分析问题项目结项

之前参与的分析问题项目接近尾声，最终摘要如下

3.1 Boundary extensions of mappings between metric spaces

Abstract

In this paper, we consider boundary extensions of two classes of mappings between metric measure spaces. These two mapping classes include in particular the well-studied geometric mappings such as quasiregular mappings and mappings with exponentially integrable distortion. Our main results extend the corresponding results of Äkkinen and Guo [Ann. Mat. Pure. Appl. 2017] to the setting of metric measure spaces.

Keywords: Uniform domain, φ-length domain, Dyadic-Whitney decomposition, limits along John curves, quasiregular mappings.

2010 Mathematics Subject Classification: 49N60; 58E20

4 毕业设计课题

毕业设计我找了数学与交叉科学研究中心的李邢武老师，现在每周四在参与比勒菲尔德大学的 Frank-Riedel 老师的讨论班，毕业设计涉及的领域是递归效用与投资理论 (Recursive Utility and Investment) 涉及的参考文献如下：

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