UNIVERSITÀ DEGLI STUDI
DI MILANO

Corso di

Visione Artificiale

Laurea Magistrale in Informatica (F94)

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Where are we?

First part:
Image formation and Early vision

- Image formation
  - **Geometric Camera Models**
  - Color spaces
- Image Processing
  - Punctual and spatial processing
  - Feature Extraction
- Reconstruction
  - Camera calibration
  - Stereo Vision
  - Structure from Motion and RGB-d Cameras
  - Optical flow and Tracking

Second part:
Machine learning for CV

- Linear Neural Network
- Multi Layer Perceptron
- Convolutional Neural Networks
- Recurrent Neural Networks
- Transformers
- Variational Auto-Encoders
- Generative Adversarial Networks
- Graph Neural Networks
- Self-supervised learning
- Vision Language Models

PINHOLE CAMERA

Ideal camera model

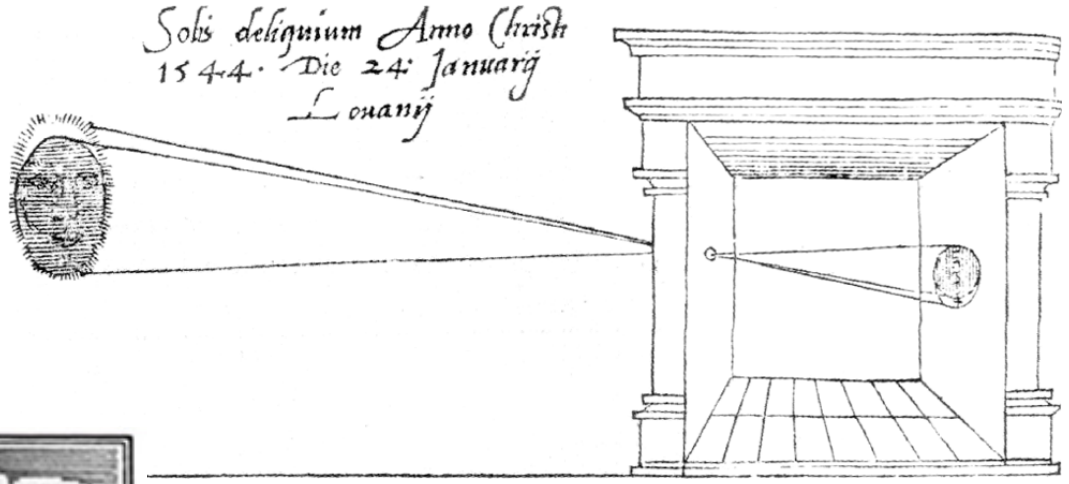
Chapter 1 – Forsyth Ponce

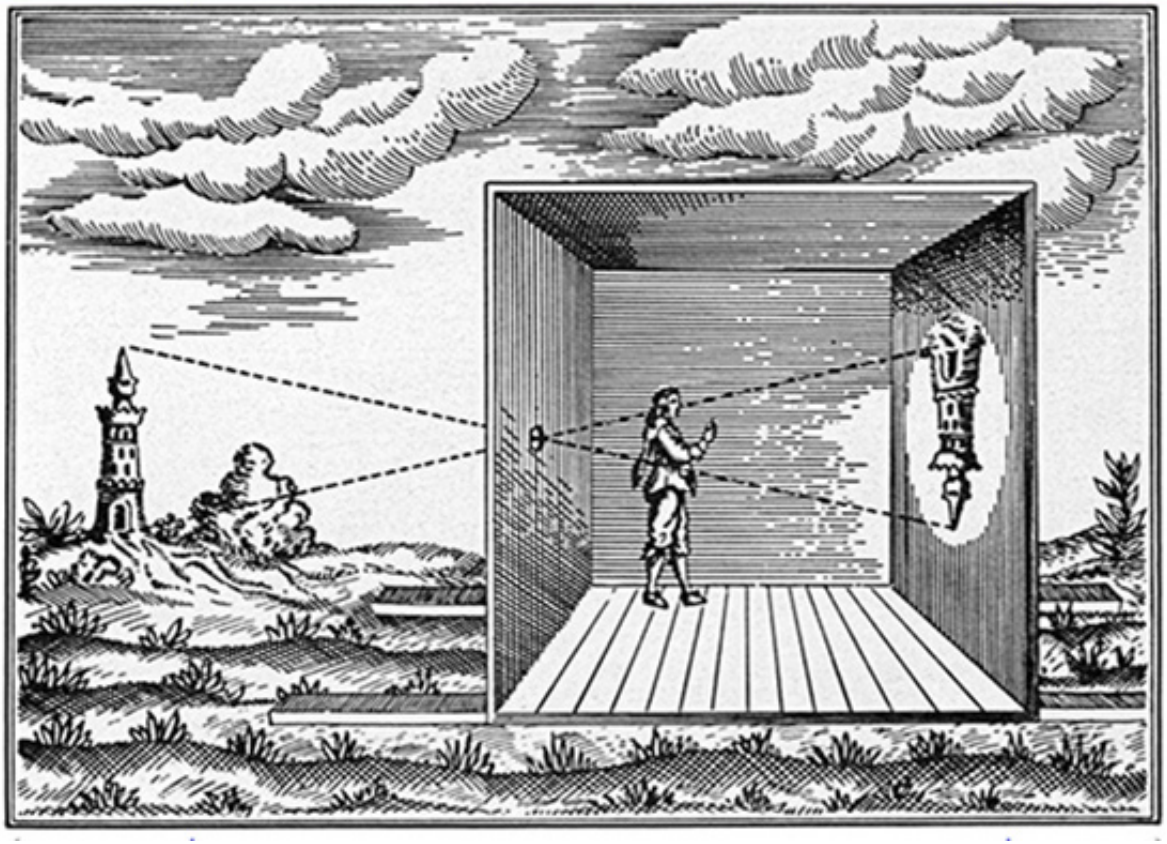
credits, F. Pedersini, S. Nayar

Pinhole camera model

"Camera obscura"

- *Brunelleschi / Leonardo da Vinci, 1544*
- generation system of a perspective projection of the front scene (like the human eye)





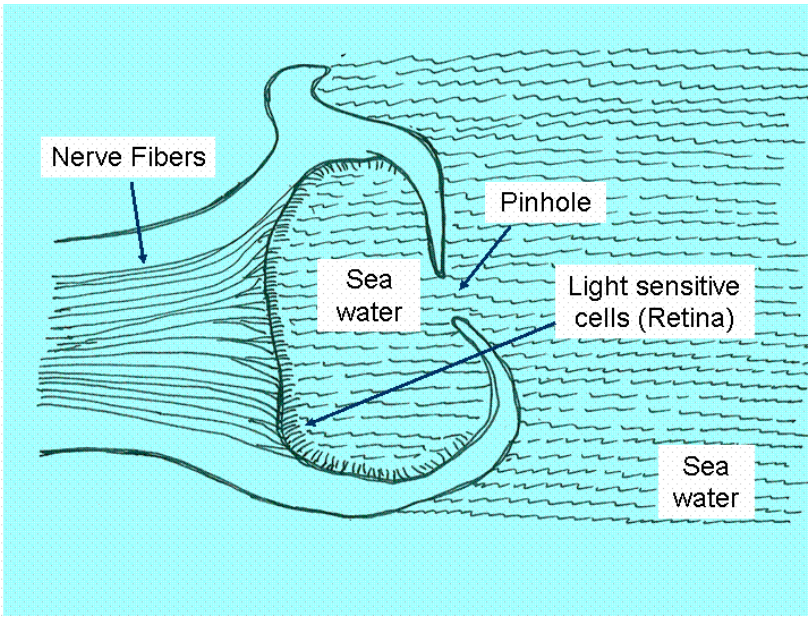
Perspective Projection:

Function that maps
a 3D space
onto a 2D plane
(image plane)

$$f: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$$

Pinhole eye of Nautilus pompilius

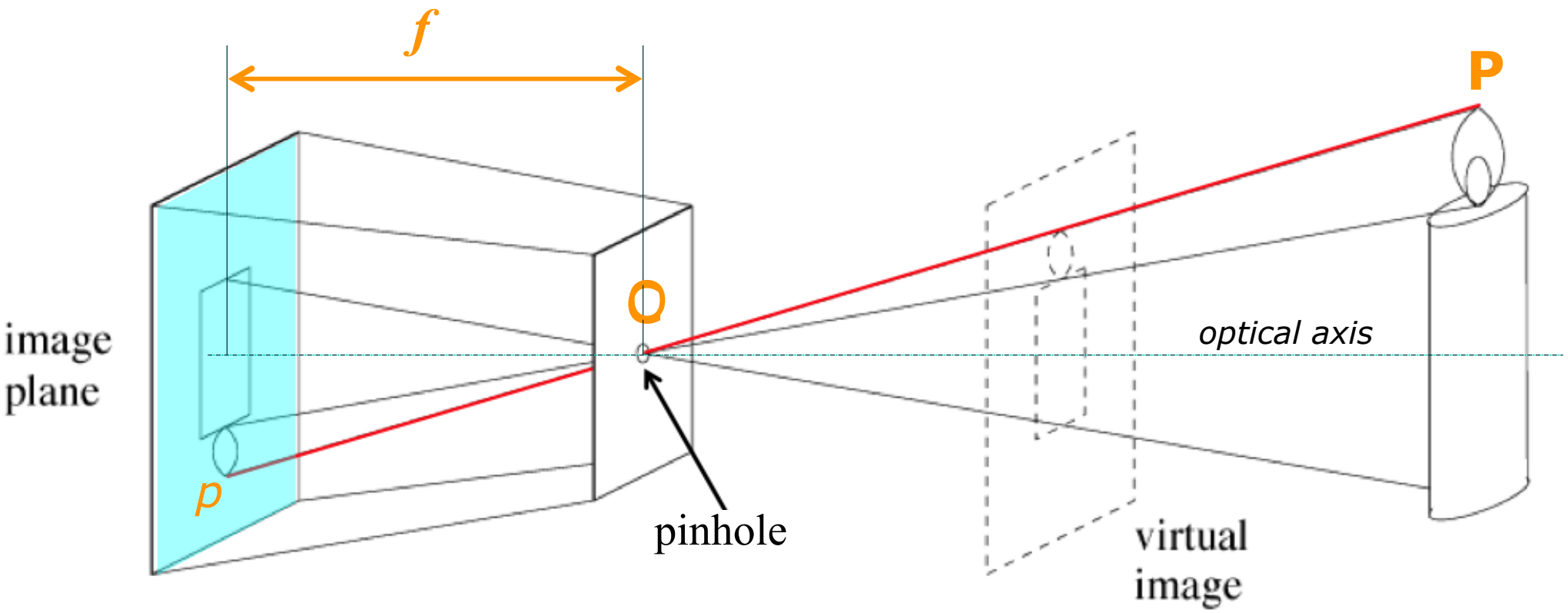
- Nautilus pompilius' eye does not have a lens:
- It uses a large pinhole to create an image



Perspective – the pinhole model

Pinhole camera model:

- from each point of the scene P, only the beam passing through P and hole O (ideally a point) reaches the image plane in p

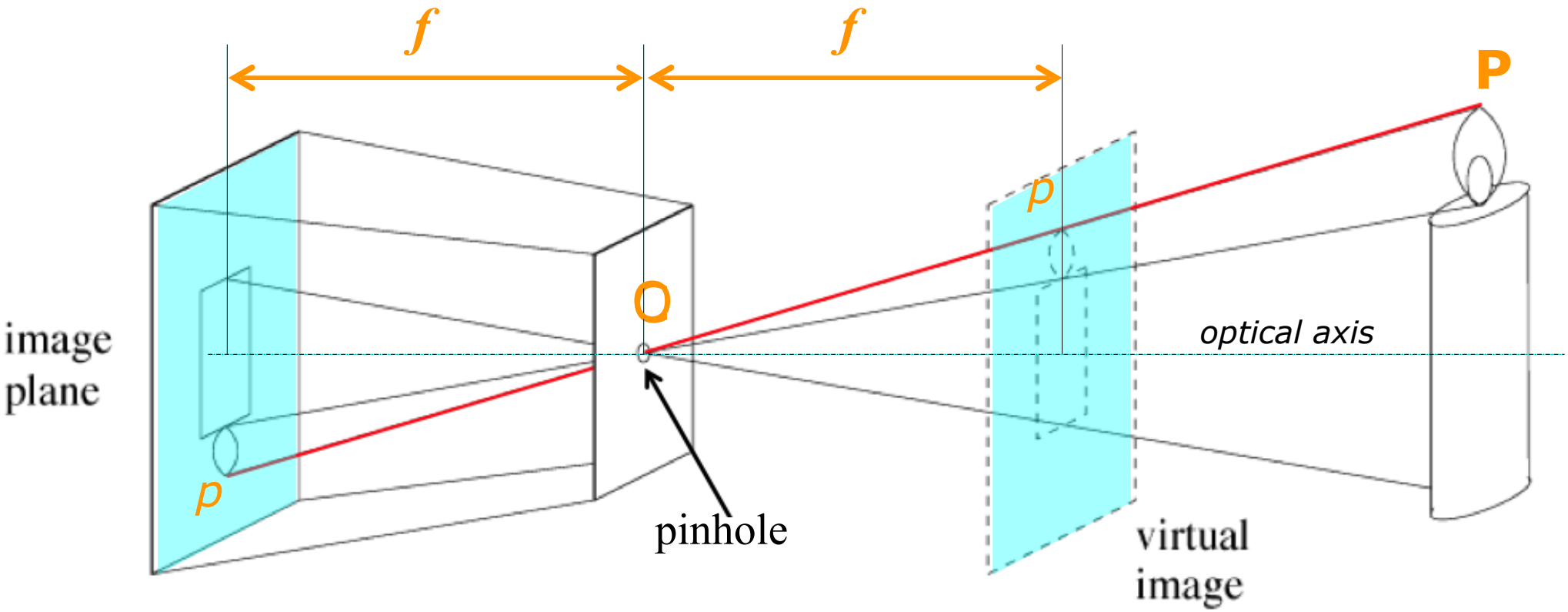


Perspective – the pinhole model

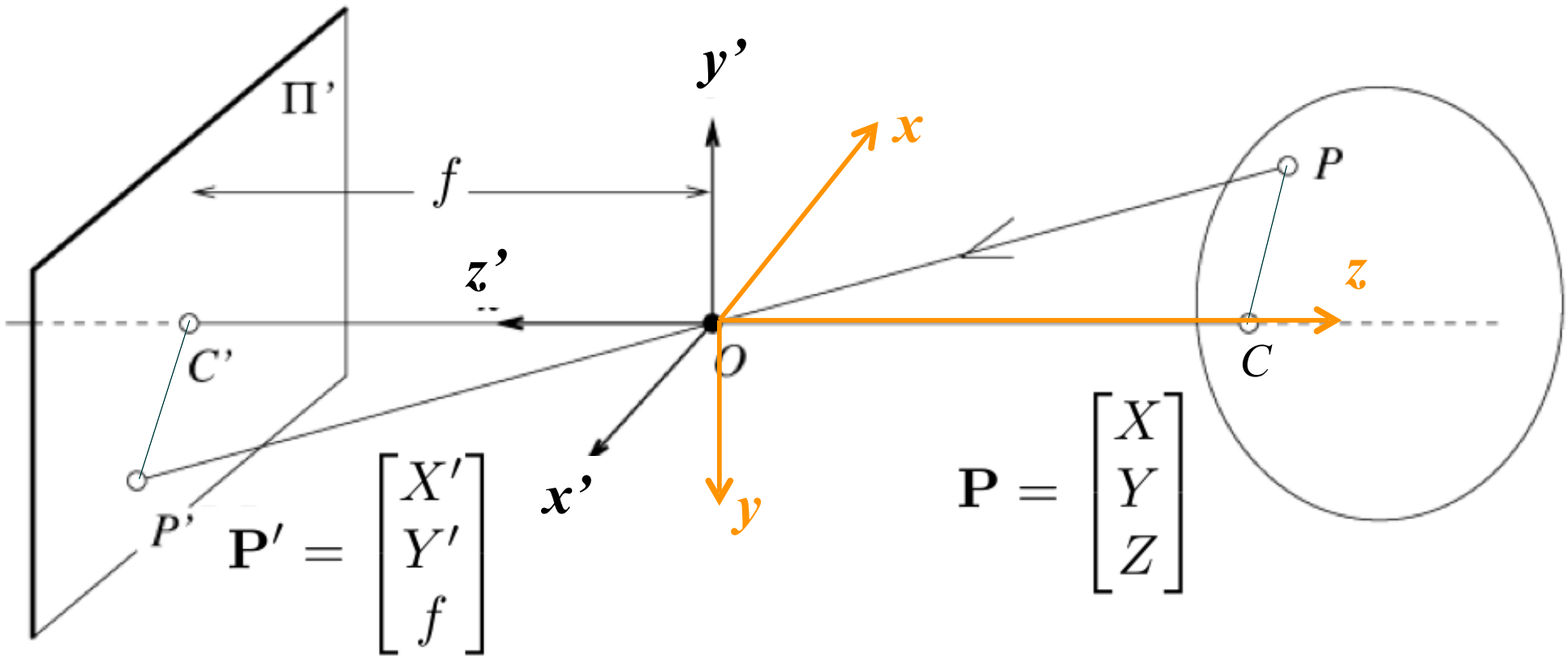
Pinhole: real and virtual image

- **real image** inverted, **virtual image** not inverted and same distance *f* wrt the pinhole O (geometrically equivalent)

- ***f* – focal length:** distance pinhole – image plane

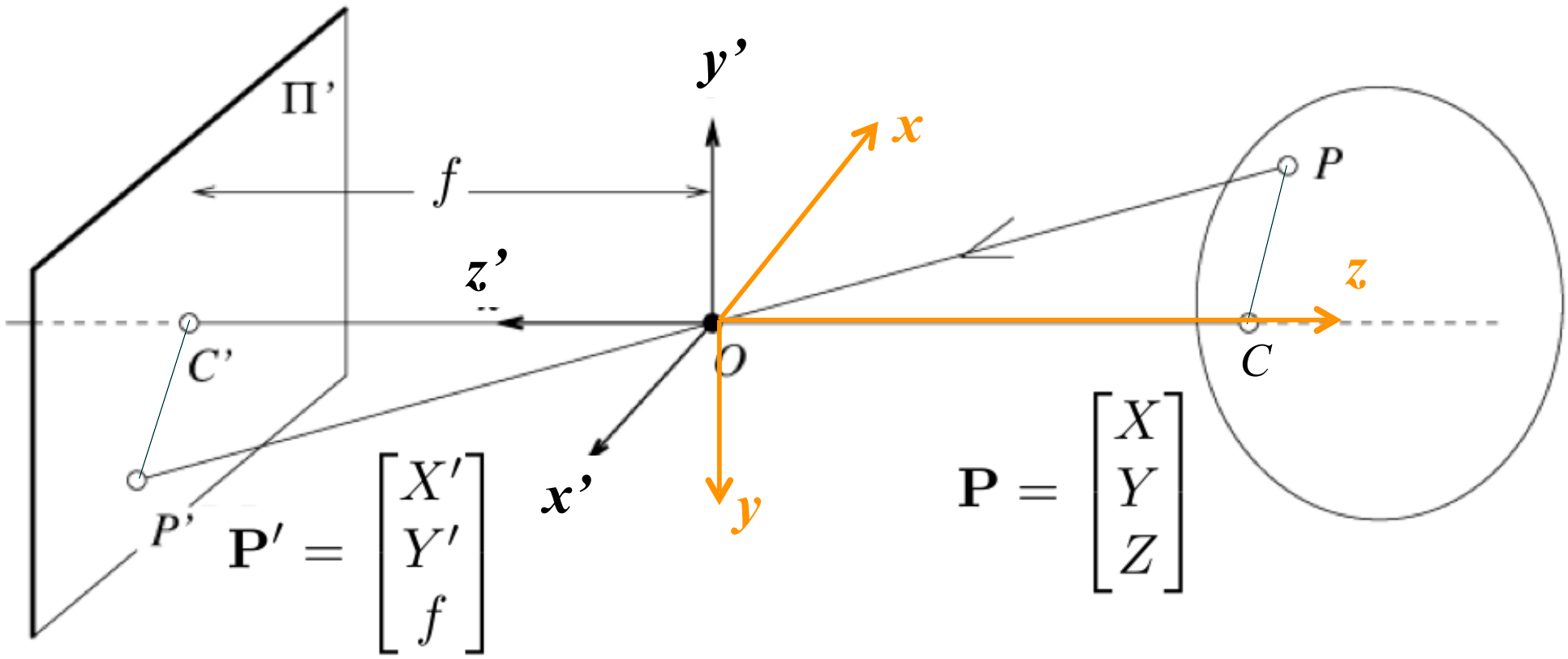


The pinhole mathematical model



- O: Origin
- **[x y z]**: World Coordinate System (**WCS**)
- **[x'y'z']**: Camera Coordinate System (**CCS**)
- Π′: Image plane

The pinhole mathematical model, cont.

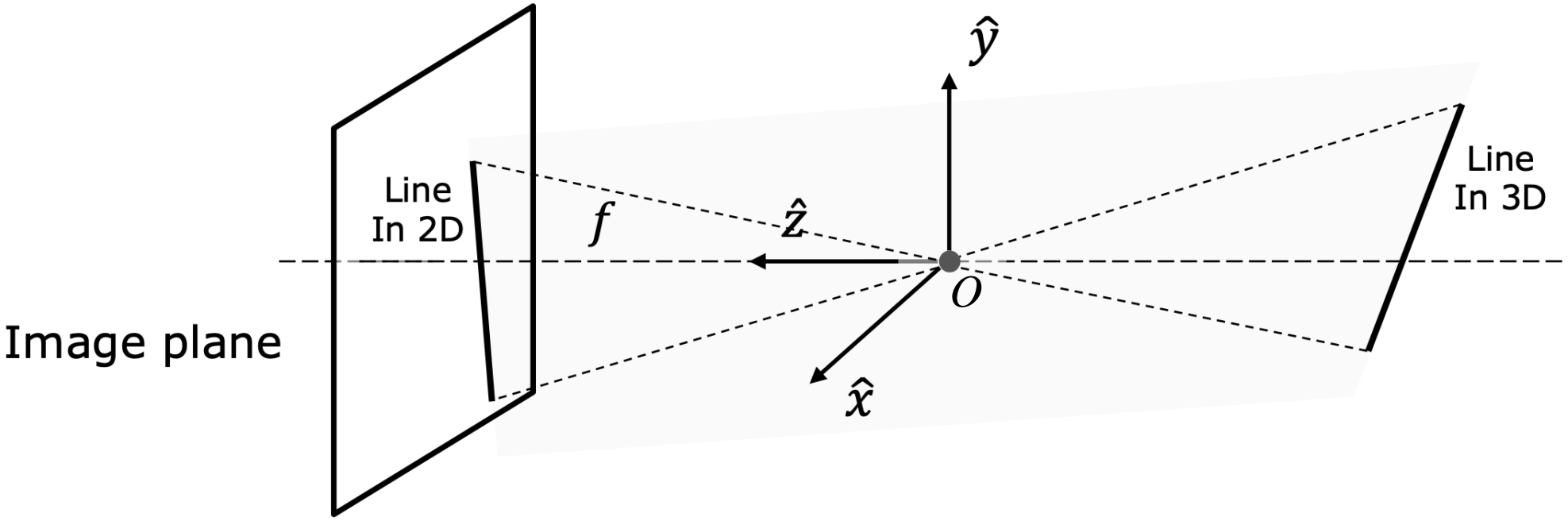


Mathematical model (wrt Image Plane):

- **The triangles OPC and OP'C' are similar**

Perspective equations: $X' = f\frac{X}{Z}\,; \quad Y' = f\frac{Y}{Z}$

Perspective Properties: Projection of a line

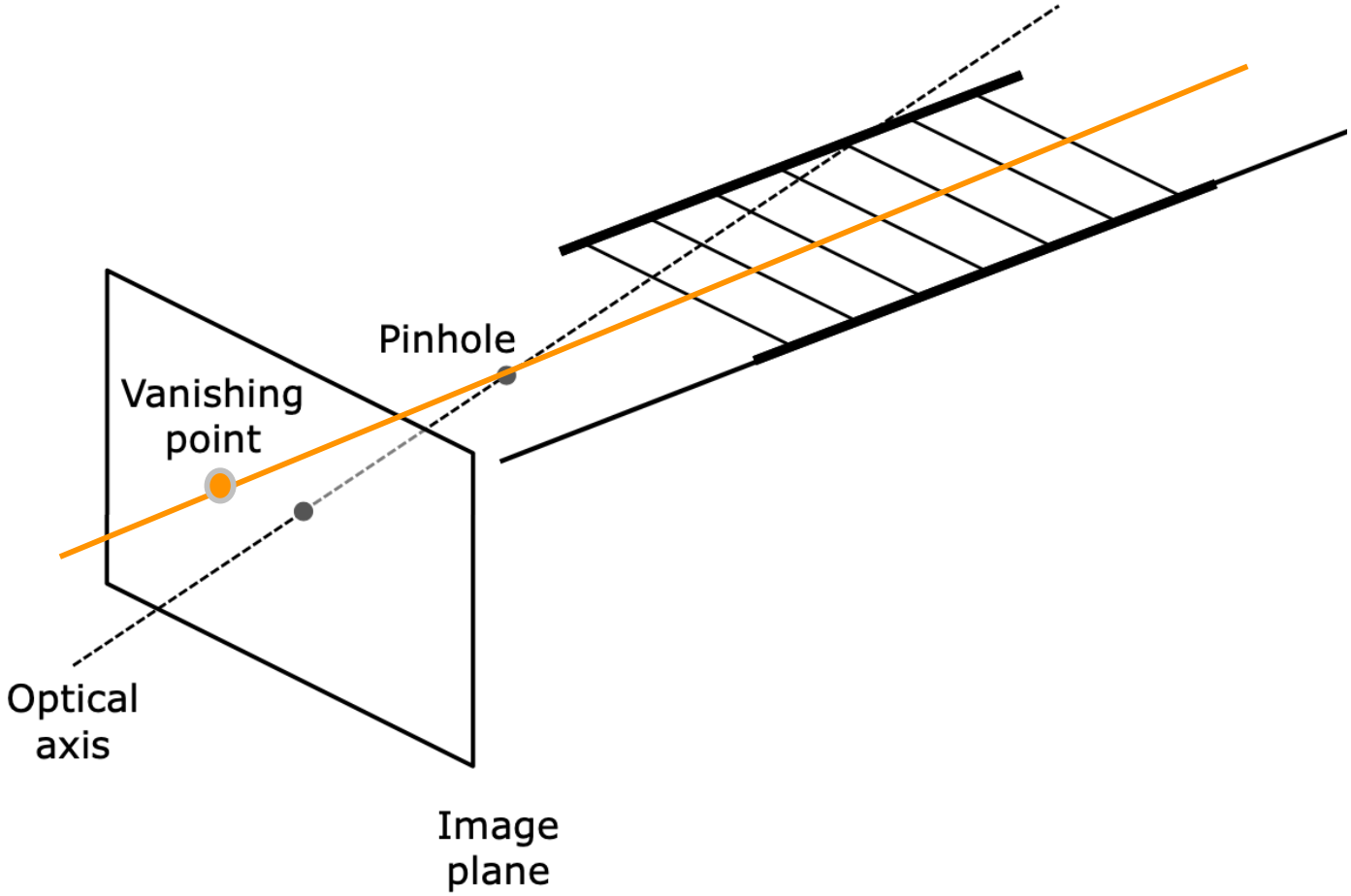


- Straight line in scene remains straight in image

Perspective Properties: Vanishing point

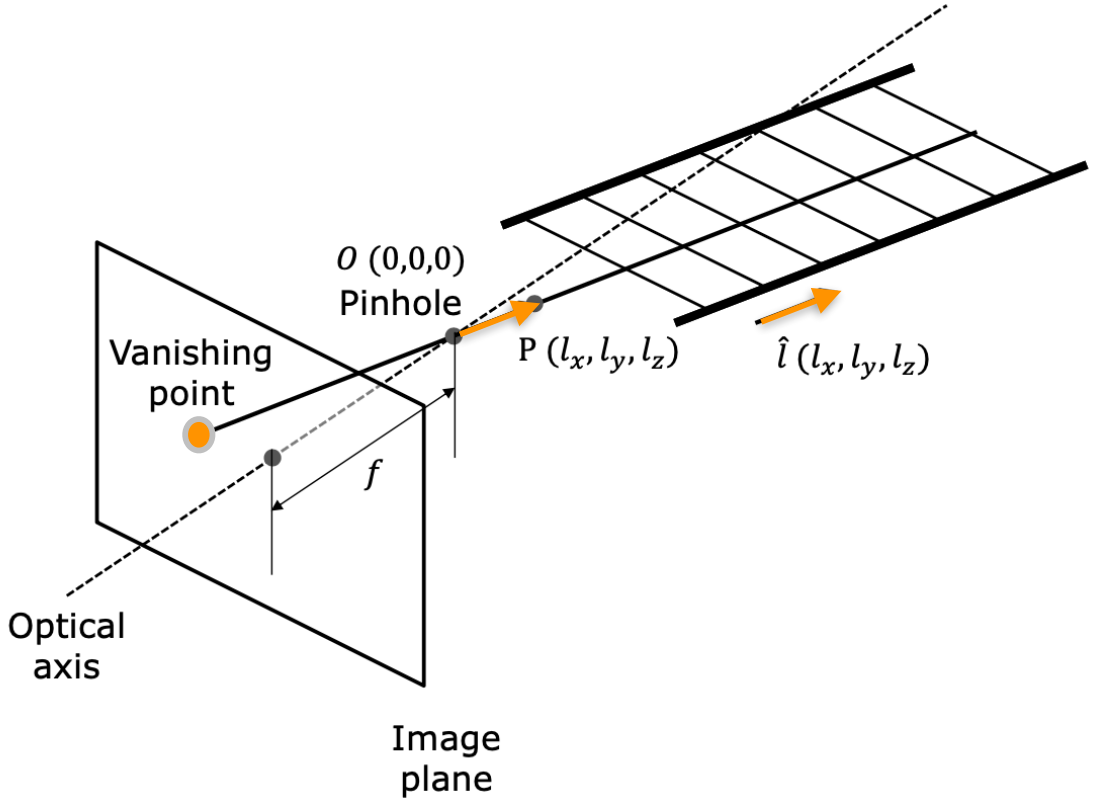
- Parallel scene lines converge at a single image point: the **vanishing point**
- Its location depends on orientation of parallel scene lines

Perspective Properties: Finding the vanishing point



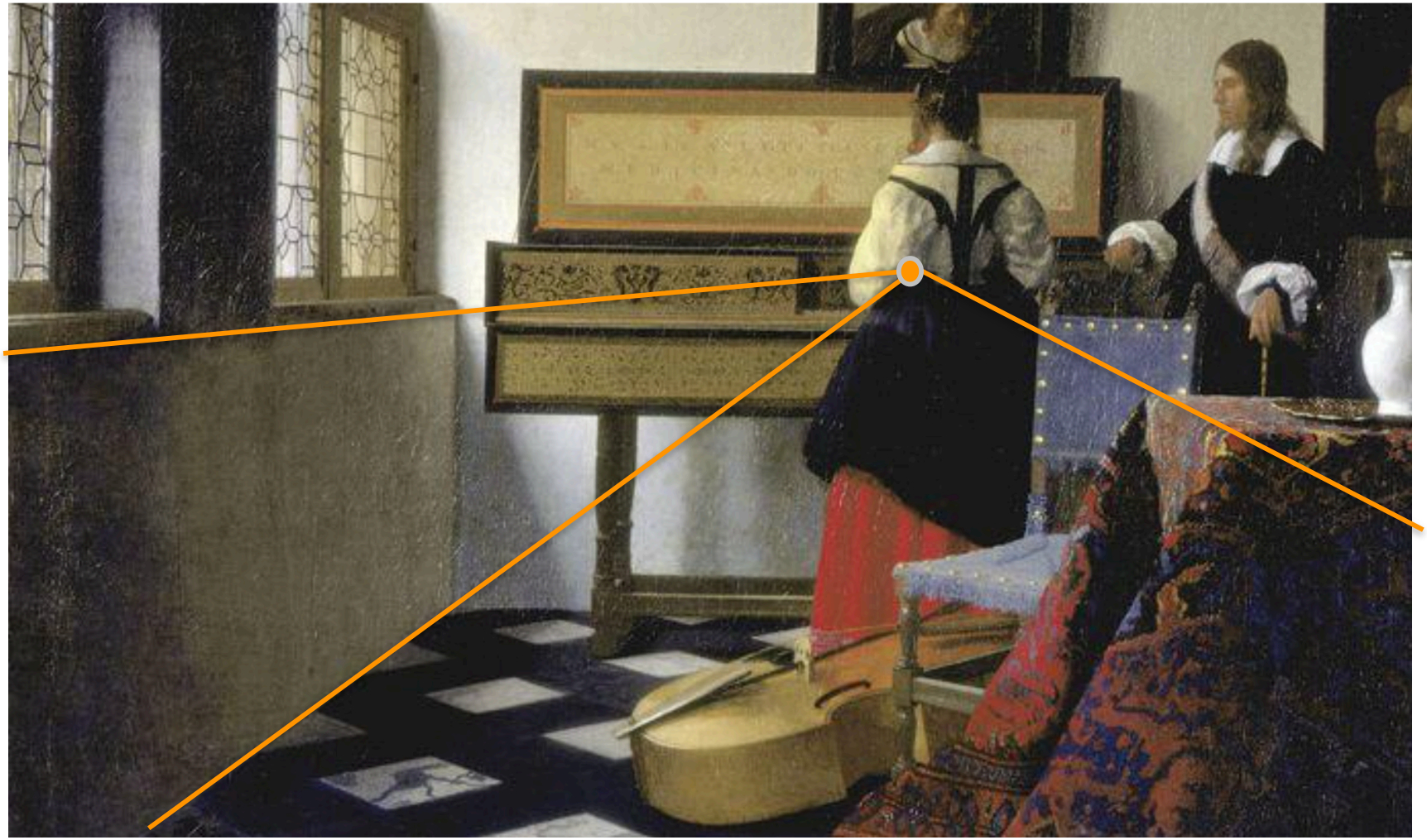
- **Intuitively:**
  - trace a line parallel to the track AND passing through the Pinhole.
  - the point where this line pierces the image is the location of the vanishing point

Perspective Properties: Finding the vanishing point



- **Mathematically**:
  - define the direction of the line $\hat{l} = (l_x, l_y, l_z)$
  - determine P as the point at distance $\hat{l}$ from the origin
  - **vanishing point**: projection of point P: $\left(f\frac{l_x}{l_z}, f\frac{l_y}{l_z} \right)$

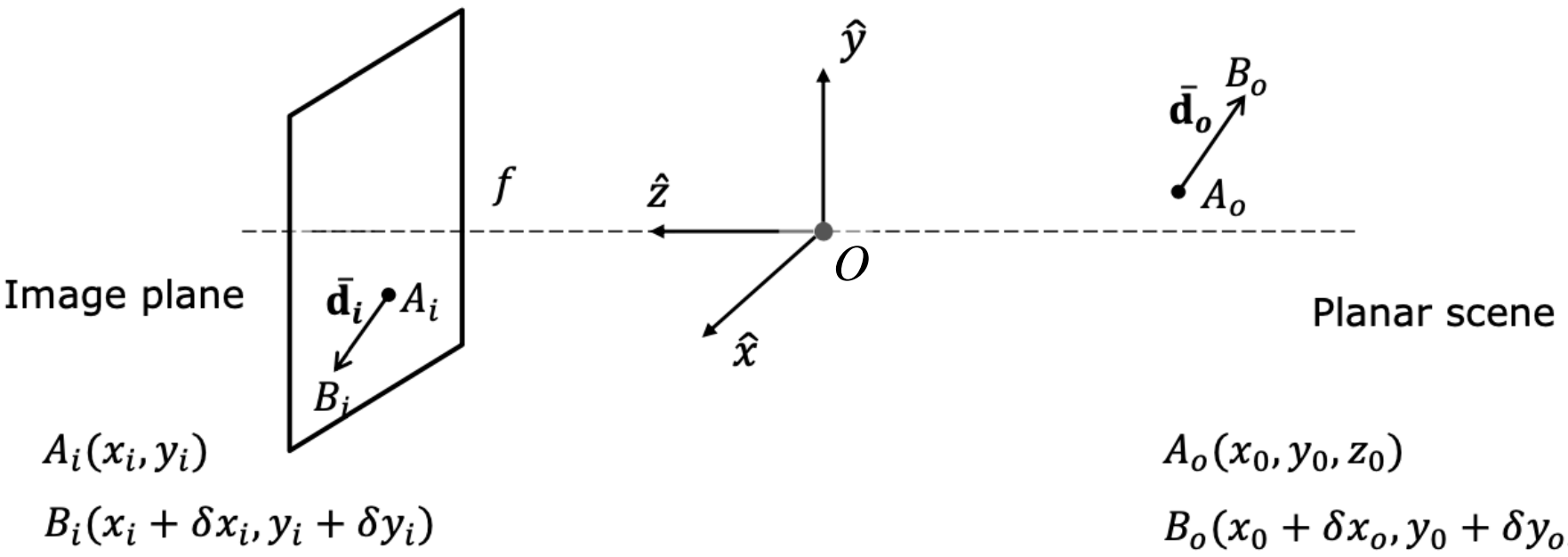
Vanishing point used in art



The Music Lesson, Johannes Vermeer, c. 1662-1664

- To make the attention converge to the most important subject in the scene

Perspective Properties: Image magnification



Magnification: ratio between the apparent size of an object in an image ($\|\mathbf{d}_i\|$) relative to its actual size in real life ($\|\mathbf{d}_o\|$)

$$|m| = \frac{\|\mathbf{d}_i\|}{\|\mathbf{d}_0\|} = \frac{\sqrt{\delta x_i^2 + \delta y_i^2}}{\sqrt{\delta x_0^2 + \delta y_0^2}}$$

Perspective Properties: Image magnification

- recap of the quantity at stake:
  - world coords:

$$A_o = (x_o, y_o, z_o)$$
$$B_o = (x_o + \delta x_o, y_o + \delta y_o, z_o)$$

$$|m| = \frac{\|\mathbf{d}_i\|}{\|\mathbf{d}_0\|} = \frac{\sqrt{\delta x_i^2 + \delta y_i^2}}{\sqrt{\delta x_0^2 + \delta y_0^2}}$$

  - image coords:

$$A_i = (x_i, y_i)$$
$$B_i = (x_i + \delta x_i, y_i + \delta y_i)$$

- to compute m, let's start by rewriting the image coords using perspective projection:

$$x_i = f\frac{x_0}{z_0}, \quad y_i = f\frac{y_0}{z_0},$$

$$x_i + \delta x_i = f\frac{x_0 + \delta x_0}{z_0}, \quad y_i + \delta y_i = f\frac{y_0 + \delta y_0}{z_0}$$

Perspective Properties: Image magnification

Substituting x_i

1. $x_i = f\frac{x_0}{z_0}$

$$x_i + \delta x_i = f\frac{x_0 + \delta x_0}{z_0}$$

2. $f\frac{x_0}{z_0} + \delta x_i = f\frac{x_0}{z_0} + f\frac{\delta x_0}{z_0}$

3. $\delta x_i = f\frac{\delta x_0}{z_0}$

The same wrt y_i

1. $y_i = f\frac{y_0}{z_0}$

$$y_i + \delta y_i = f\frac{y_0 + \delta y_0}{z_0}$$

2. $f\frac{y_0}{z_0} + \delta y_i = f\frac{y_0}{z_0} + f\frac{\delta y_0}{z_0}$

3. $\delta y_i = f\frac{\delta y_0}{z_0}$

Perspective Properties: Image magnification

Given: $\delta x_i = f\frac{\delta x_0}{z_0}, \quad \delta y_i = f\frac{\delta y_0}{z_0}$

Magnification becomes: $$|m| = \frac{\|\mathbf{d}_i\|}{\|\mathbf{d}_0\|} = \frac{\sqrt{\delta x_i^2 + \delta y_i^2}}{\sqrt{\delta x_0^2 + \delta y_0^2}} = \left|\frac{f}{z_0}\right|$$

Perspective Properties: Image magnification

- $m = \frac{f}{z_0} \rightarrow$ Image size inversely proportional to depth

Perspective Properties: Image magnification

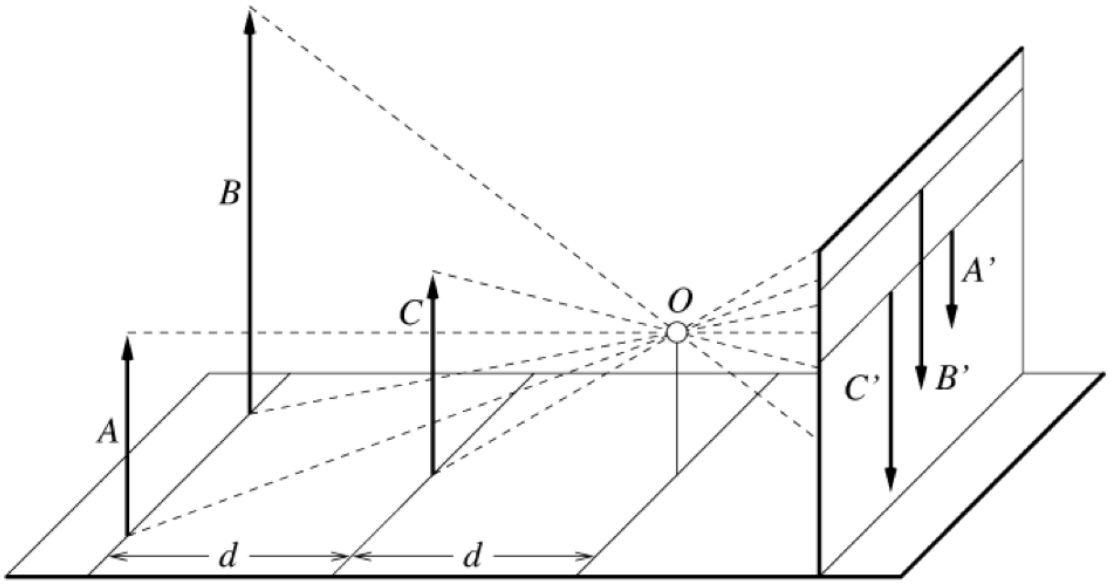
- Remark: m can be assumed to be constant if the range of scene depth Δz (size of the object) is much smaller than the average scene depth z

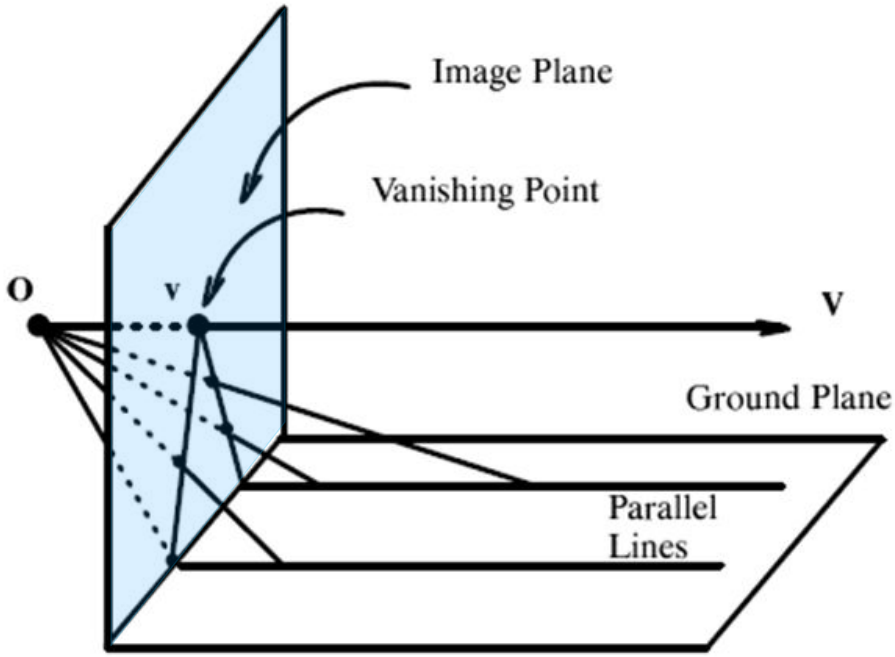
- That's not the case when the size of an object is significant compared to its distance from the camera.

- Eg: selfies:the nose is more magnified than other parts of the head as it is closed to the camera

Perspective properties (central perspective) [RECAP]

- **Image Size** inversely proportional to the distance
- **Straight lines** remain straight
- **Corners** are not preserved
- **Parallel lines →** converging lines





vanishing point

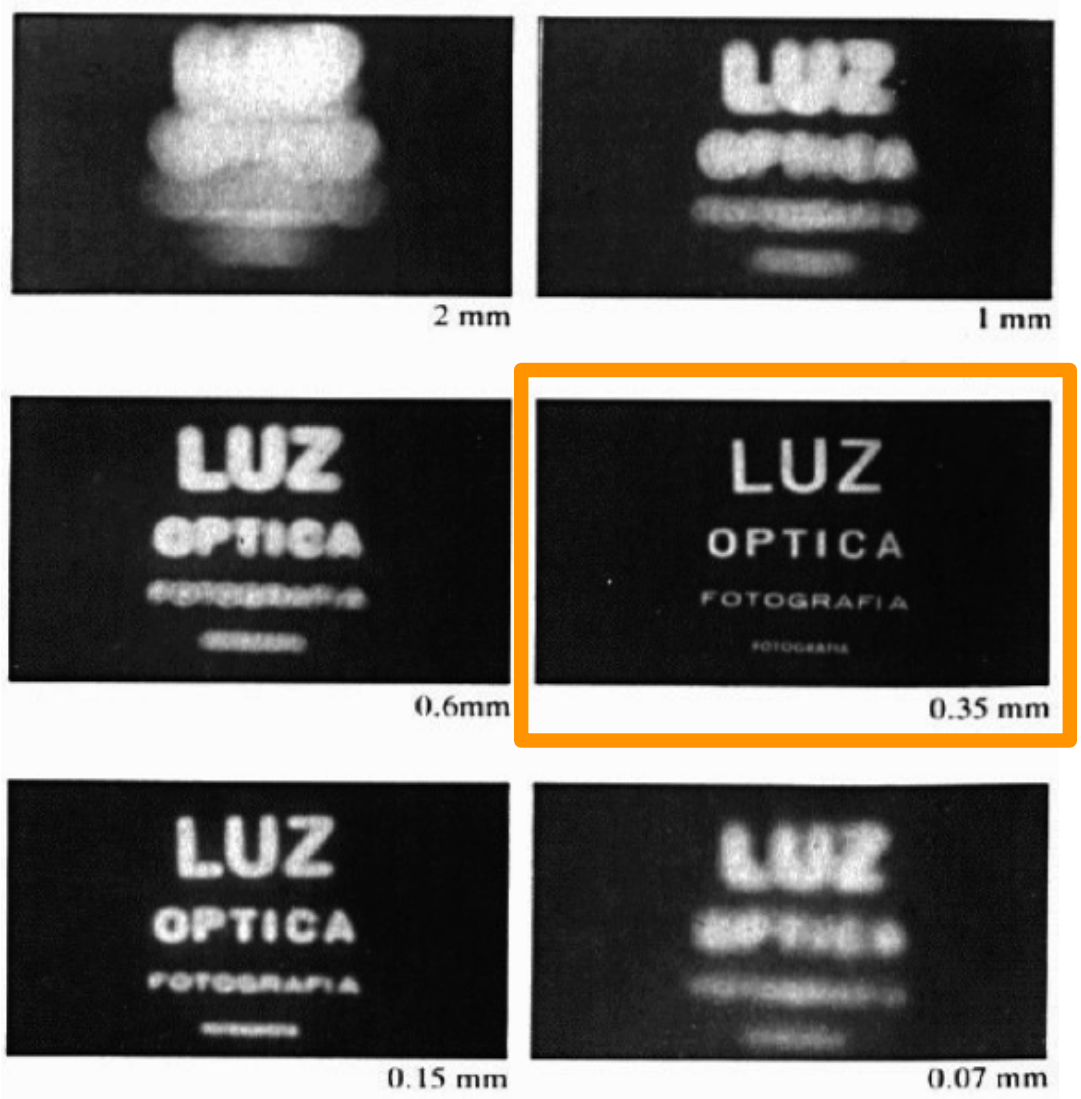
What is the ideal pinhole size?

Dependency on the Pinhole diameter:

- **reducing:**
  - More and more focused images (but there is the diffraction limit)
  - Scarse light energy
- **augmenting:**
  - More light
  - More and more blurred images
- **Ideal pinhole diameter**

$$d \approx 2\sqrt{f\lambda}$$

  - f : focal length
  - λ :wavelength



What about the exposure time?

- focus everywhere with:
- $f = 73mm, d = 0.2mm$
- **Exposure:** $T = 12s!!!$

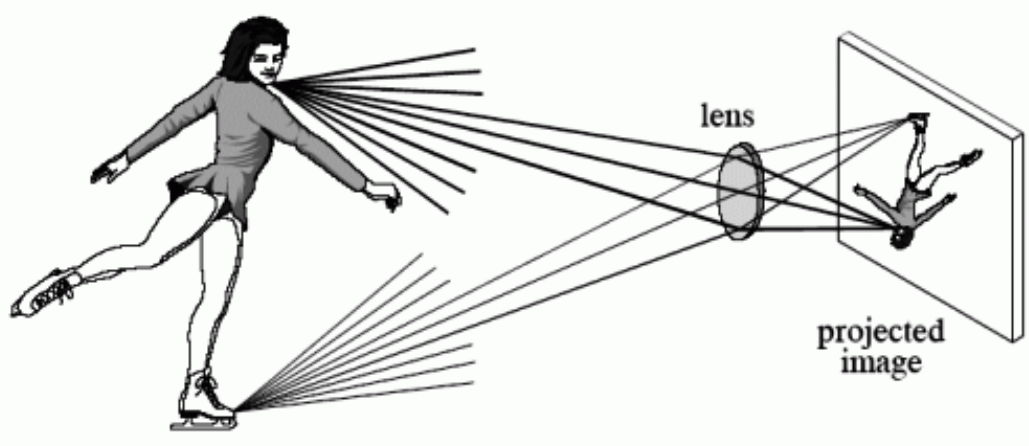
- Too long: we need lens

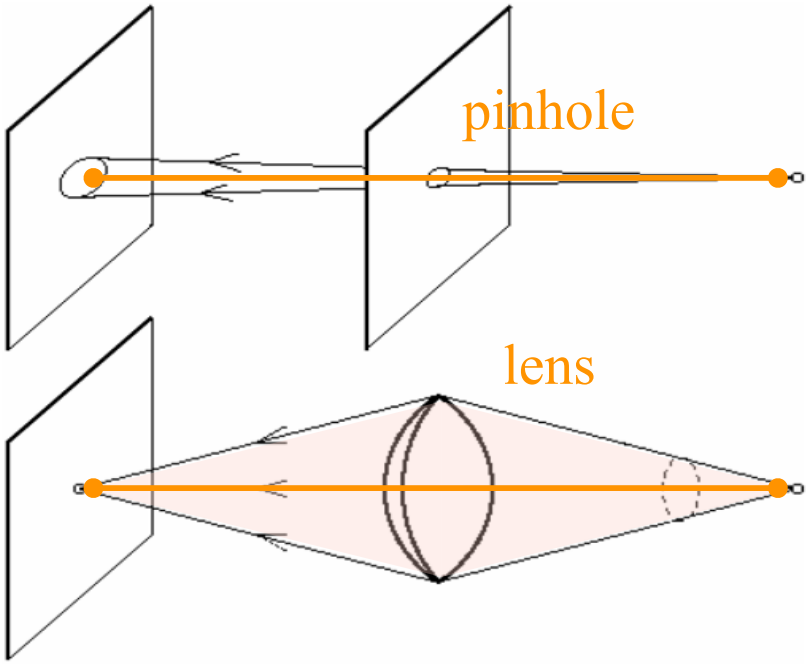
THIN LENS CAMERA

Chapter 1 – Forsyth Ponce

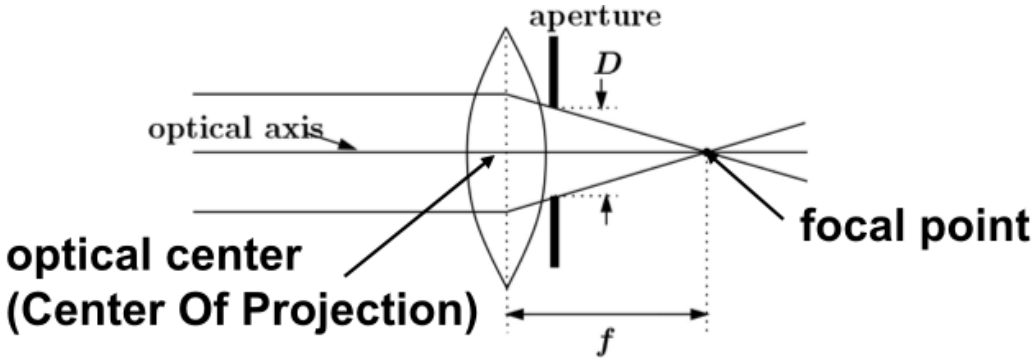
Pinhole vs. Lens

A **pinhole** camera captures a beam of light (object-image straight line)
A **lens** captures all the rays of light that reach its opening





Lens Aperture:
diameter/focal length ratio



$$\text{Aperture: } A = \frac{D}{f}$$

Camera with lens

Geometrical model of a **''thin lens''** (converging lens)

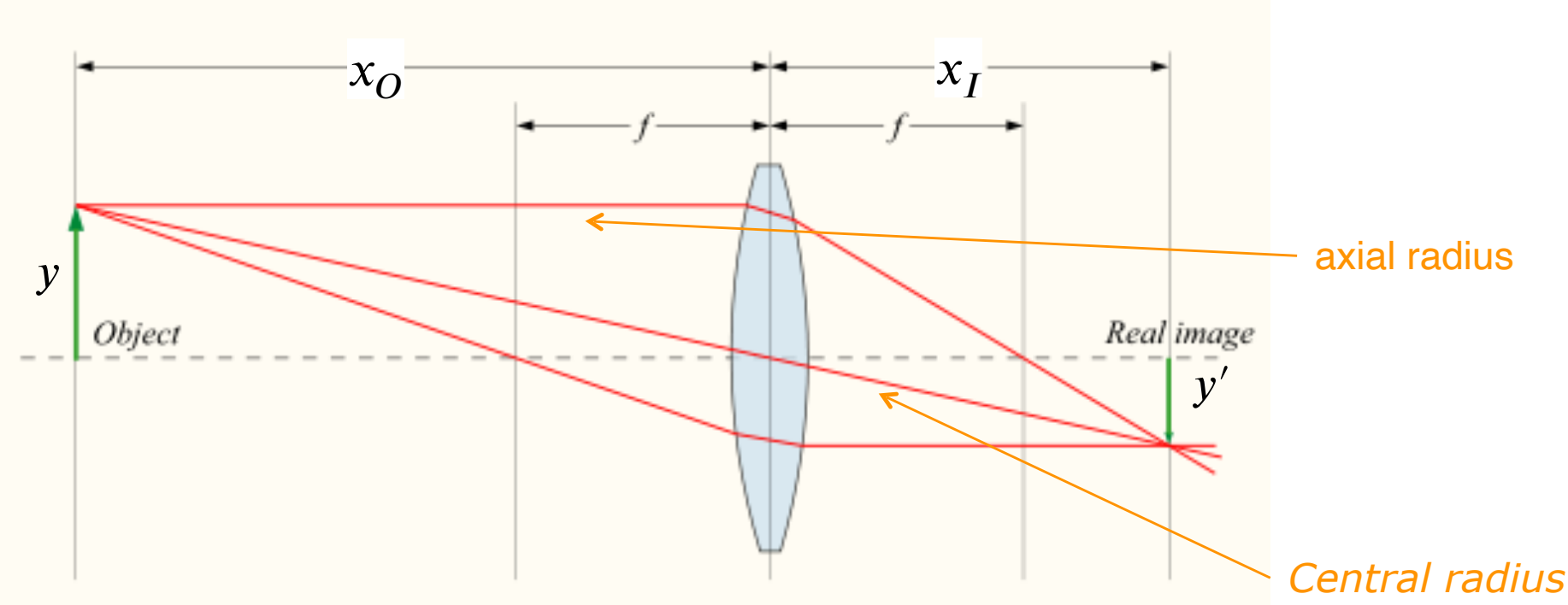
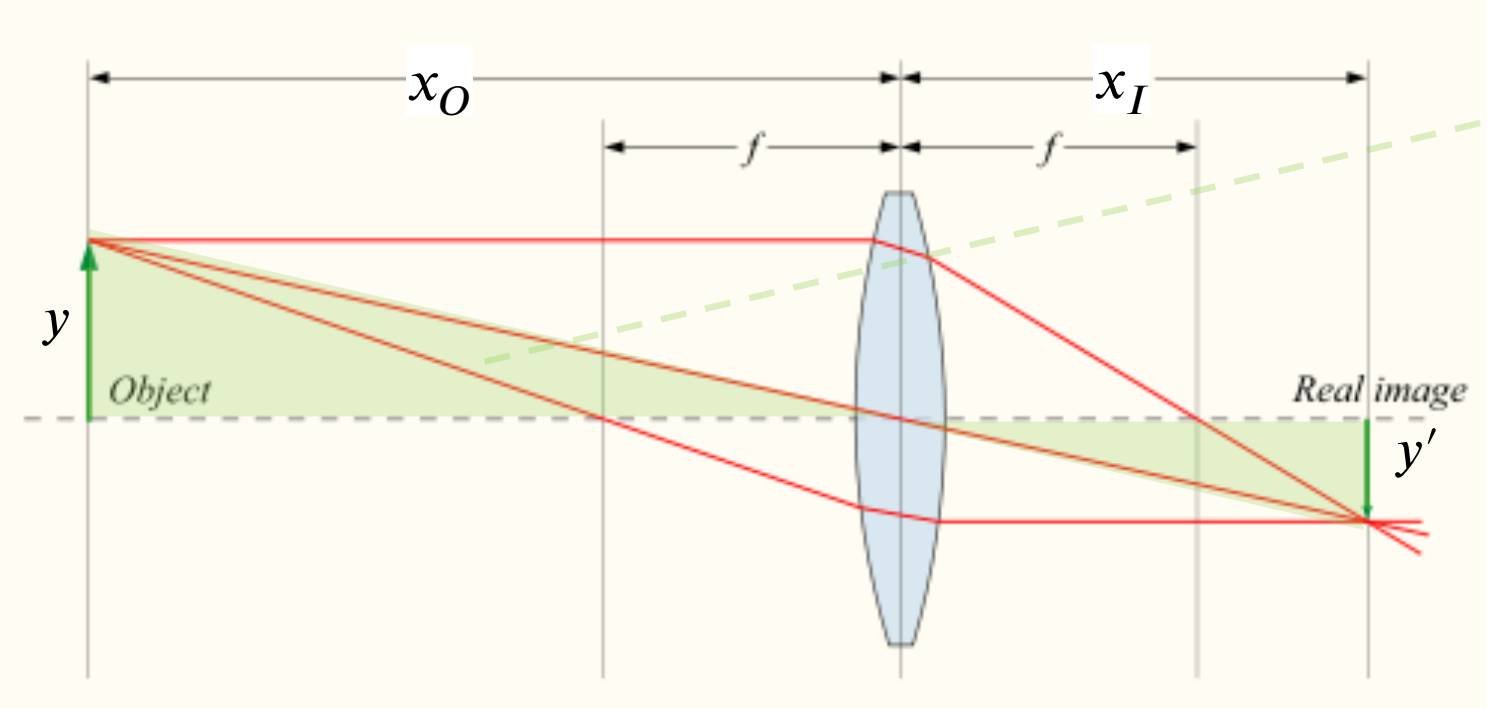


Image construction

- The central radius passes the lens centre without refraction
- The axial radius is parallel to the optical axis, and is refracted passing through the focal point

Camera with lens

1. From the similarity between the **green** triangles we have:

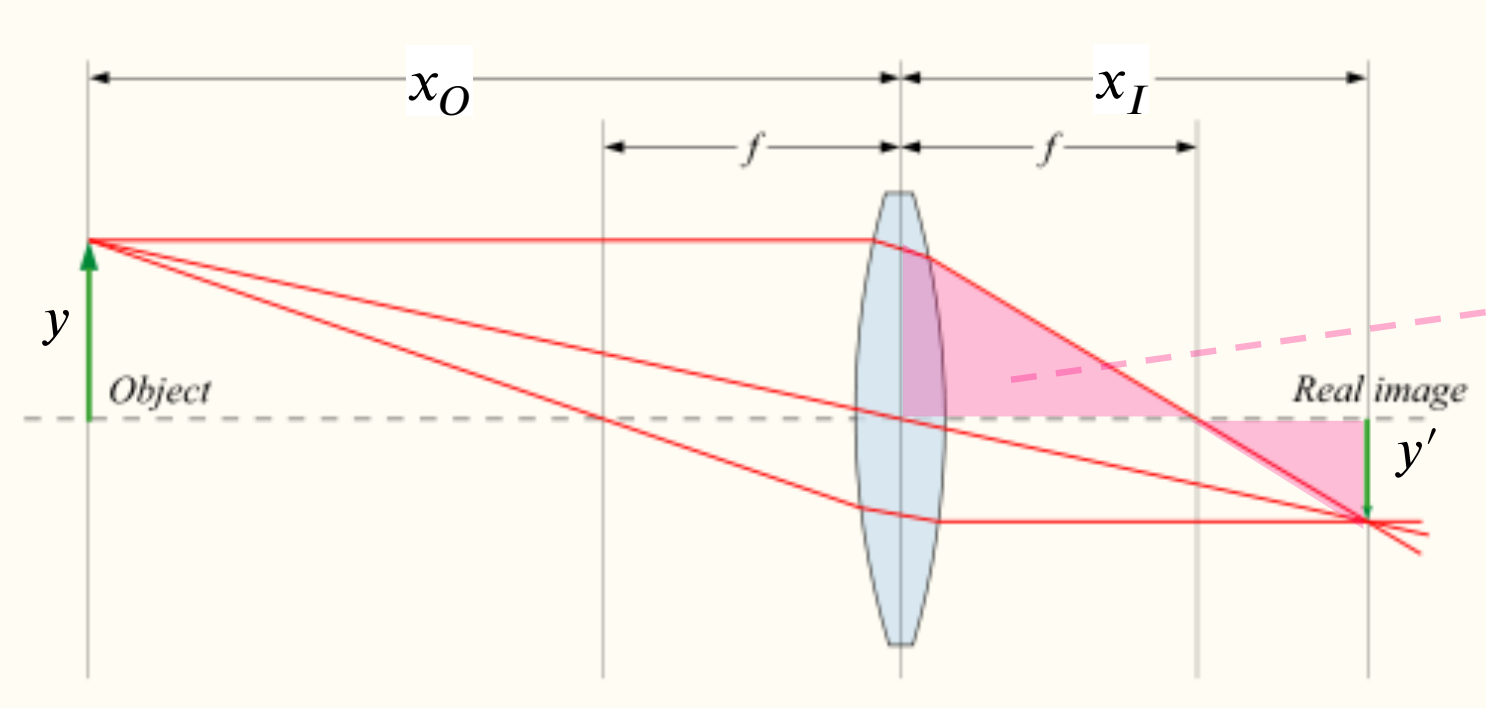


$$\frac{y'}{y} = \frac{x_I}{x_O}$$

(It's m: the magnification)

Camera with lens

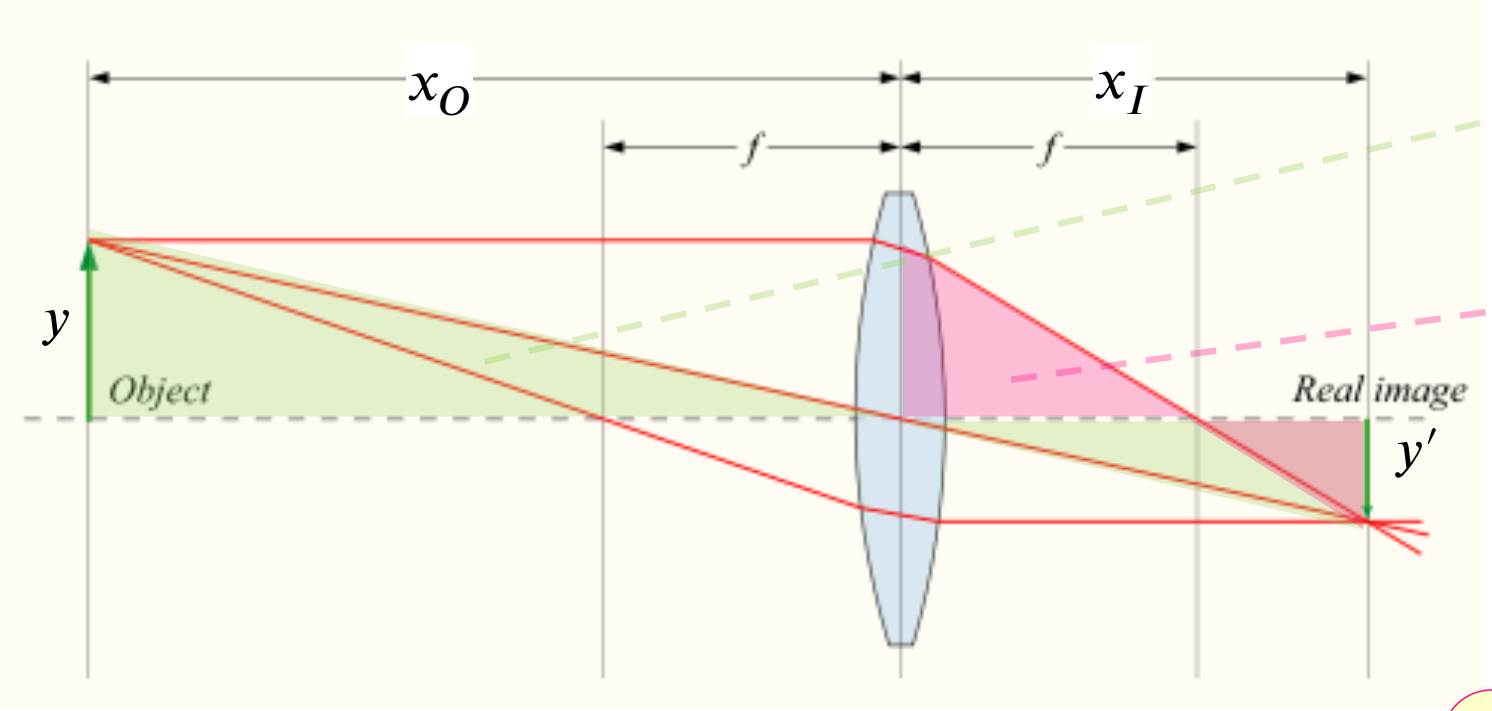
2. From the similarity between the **pink** triangles we have:



$$\frac{y'}{y} = \frac{x_I - f}{f}$$

Camera with lens: Gaussian lens law

3. By equating the two equations and dividing by x_I:



$$\frac{y'}{y} = \frac{x_I}{x_O}$$

$$\frac{y'}{y} = \frac{x_I - f}{f}$$

$$\frac{x_I}{x_O} = \frac{x_I - f}{f} = \frac{x_I}{f} - 1 \quad \xrightarrow{: \frac{1}{x_I}} \quad \frac{1}{x_O} = \frac{1}{f} - \frac{1}{x_I}$$

Thin lens equation

$$\frac{1}{x_I} + \frac{1}{x_O} = \frac{1}{f}$$

Camera with lens: Gaussian lens law

Thin lens equation

$$\frac{1}{x_I} + \frac{1}{x_O} = \frac{1}{f}$$

Sumup: with the thin lens the geometrical model does not change, while we have <u>infocus distance depending on</u> x_o

Examples:

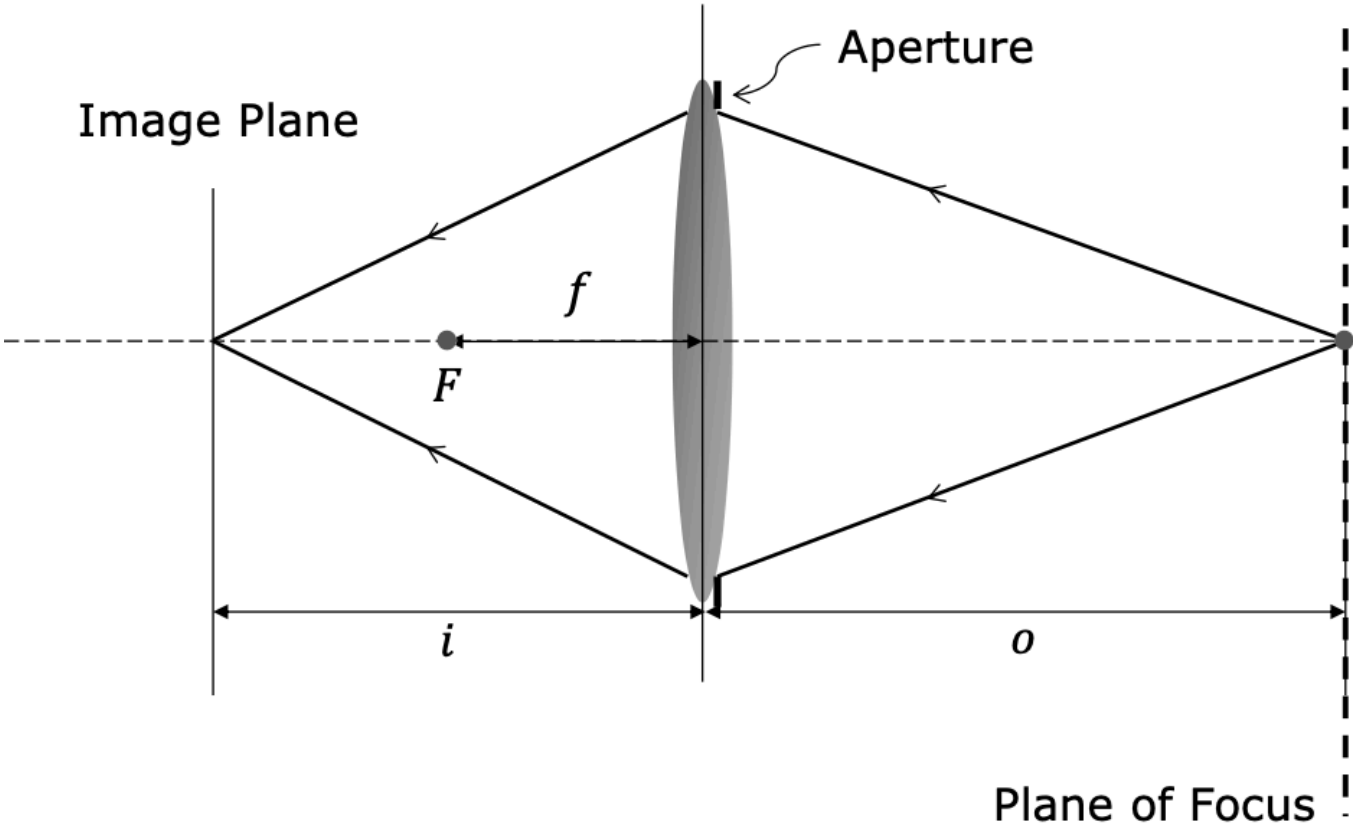
$$x_O \to \infty \quad \Longrightarrow \quad \frac{1}{x_I} = \frac{1}{f} \quad \Longrightarrow \quad x_I = f$$

$$x_O = 2f \quad \Longrightarrow \quad \frac{1}{x_I} + \frac{1}{2f} = \frac{1}{f} \quad \Longrightarrow \quad x_I = 2f$$

If $f = 50mm$, $x_o = 300mm$, then image distance is $x_i = 60mm$

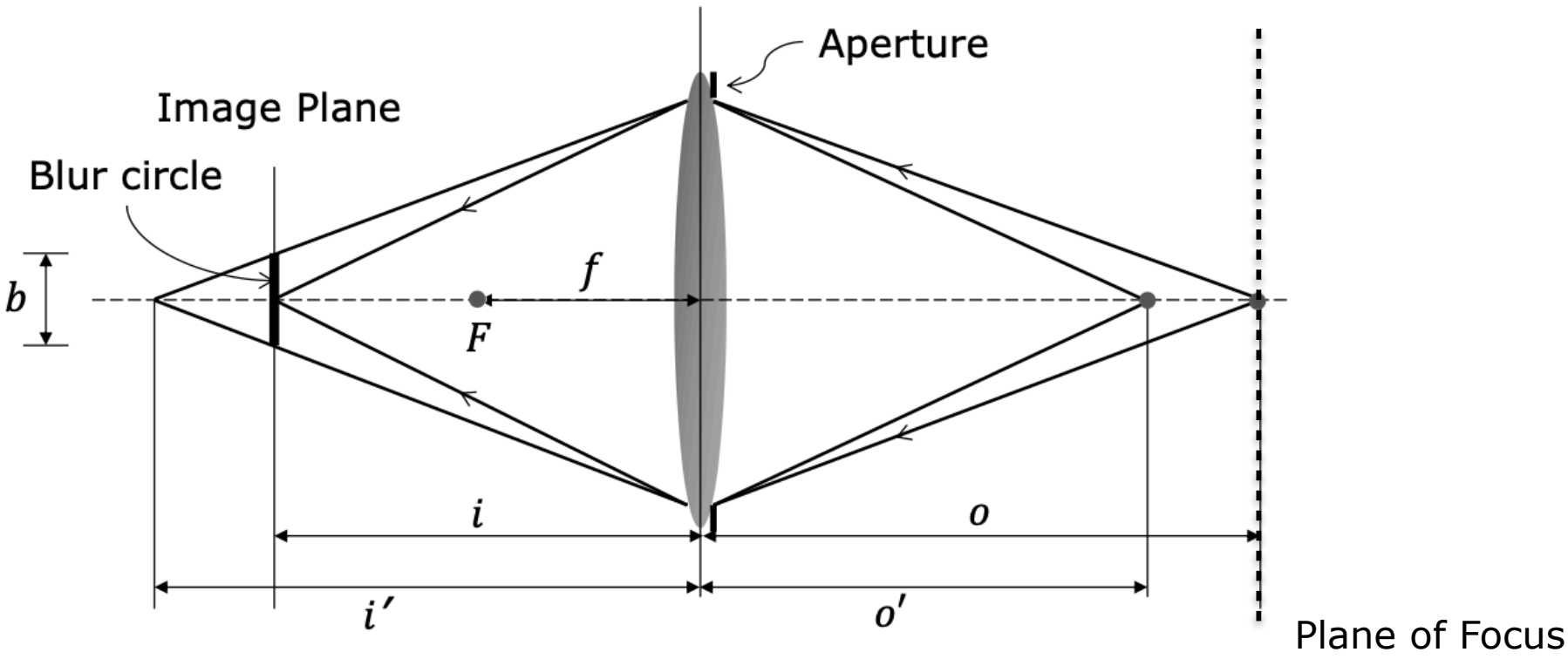
Lens defocus

➔ With a **lens** instead of a **pinhole** not everything is in focus (with a pinhole, yes)



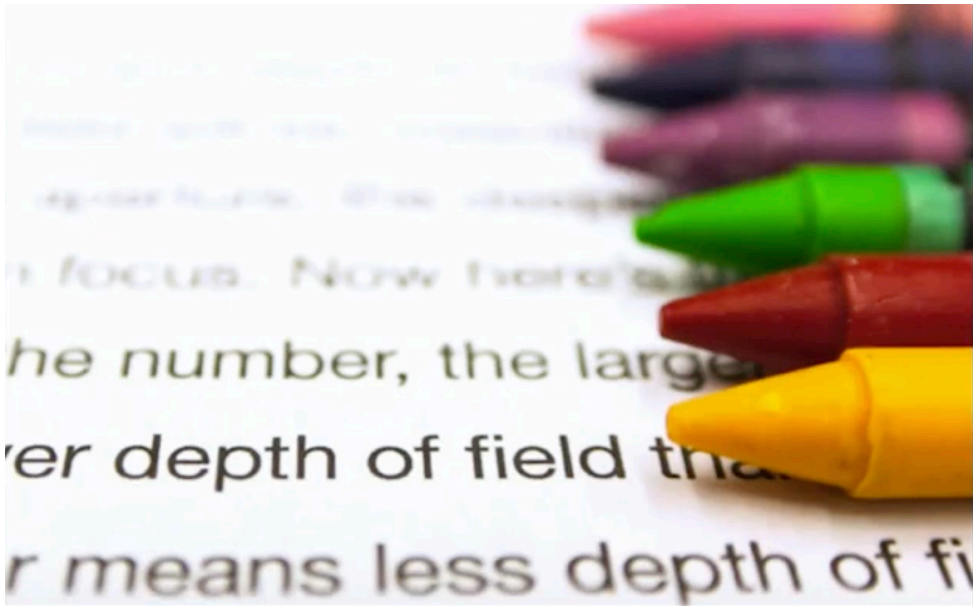
- fixing all the camera parameters, only the points on the plane of focus will be in focus on the image plane

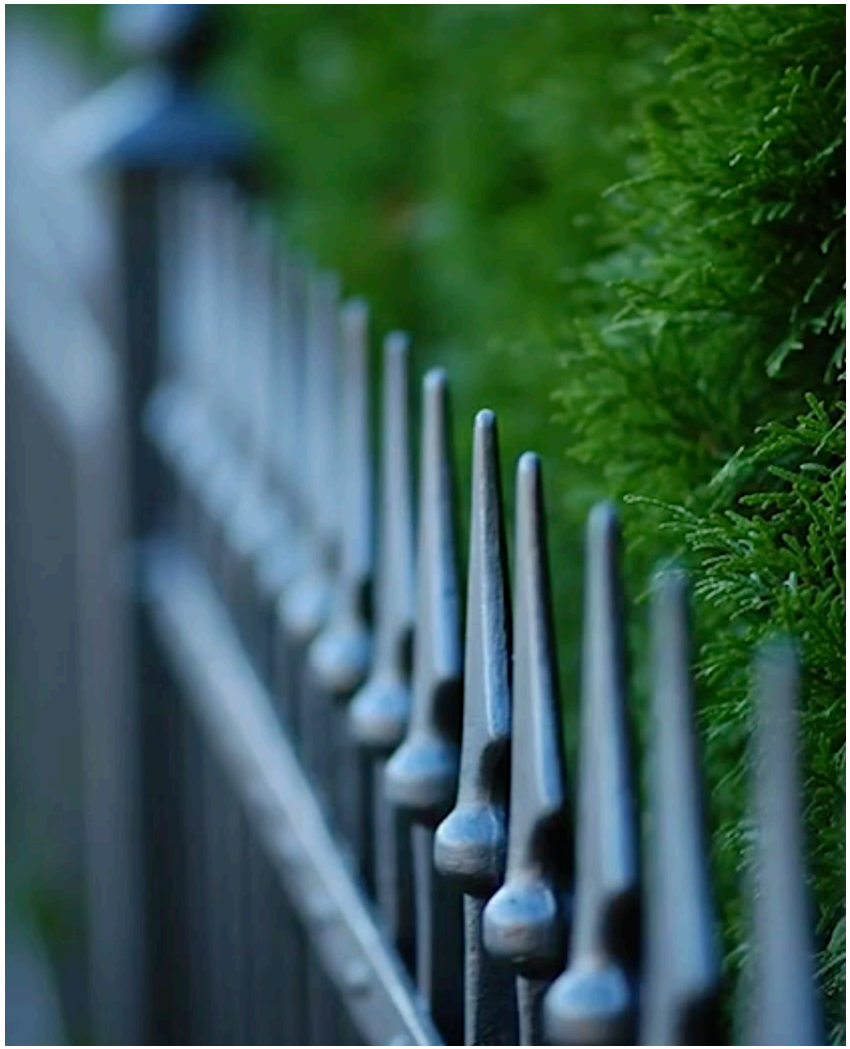
Lens defocus



- taking a point o' outside the Plane of focus, closer to the lens
- its image is formed behind the image plane
- on the image plane: a **blur circle**
- $b \propto$ Aperture

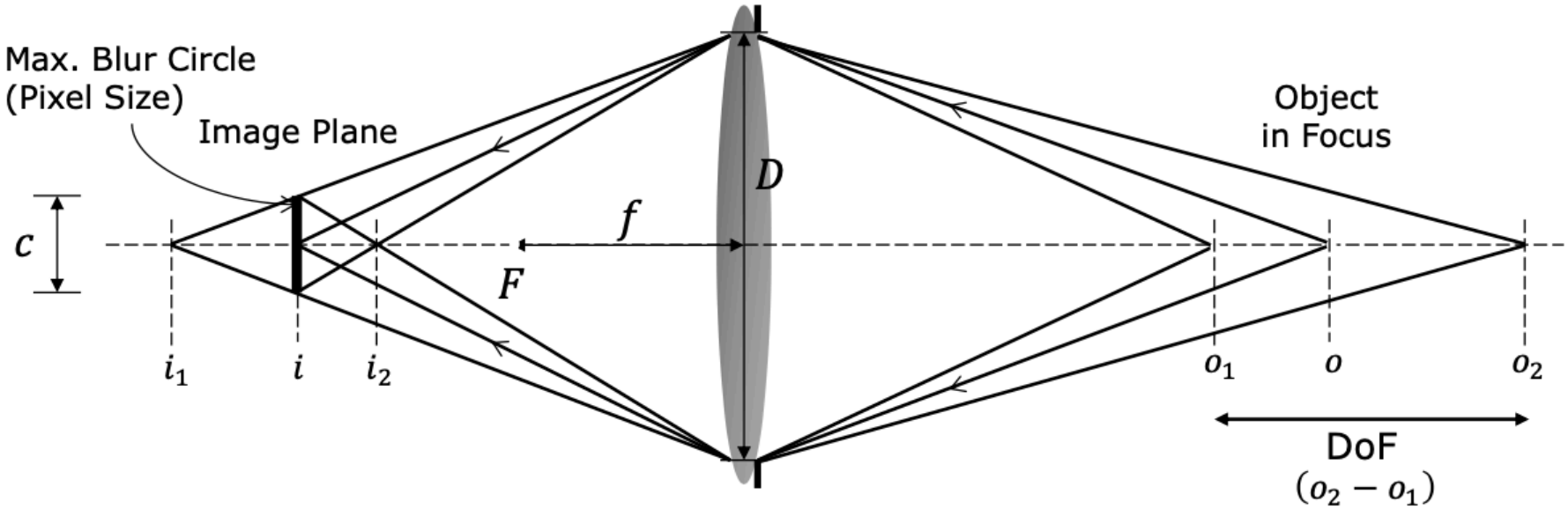
Depth of field (DoF)





DoF: range of object distances over which the image is "sufficiently well" focused, i.e., range over which blur b, is less than pixel size

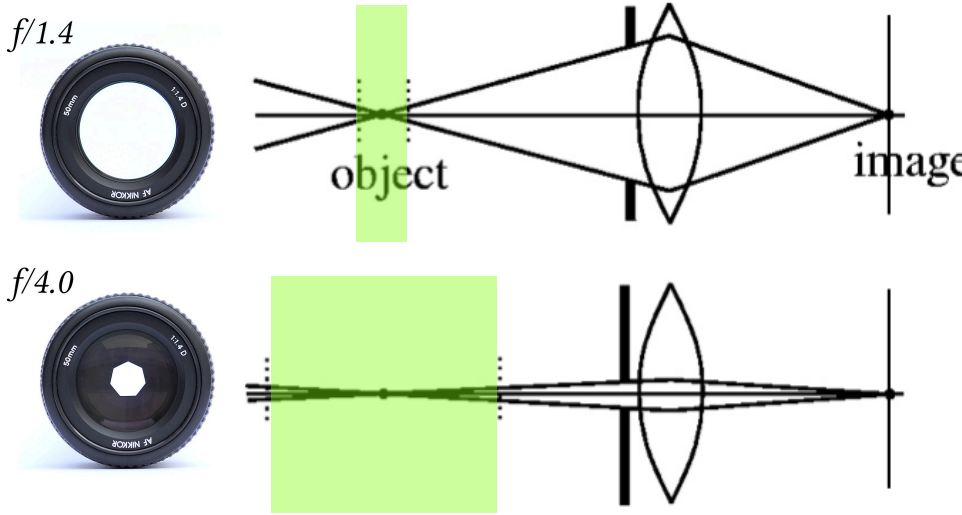
Depth of field (DoF)



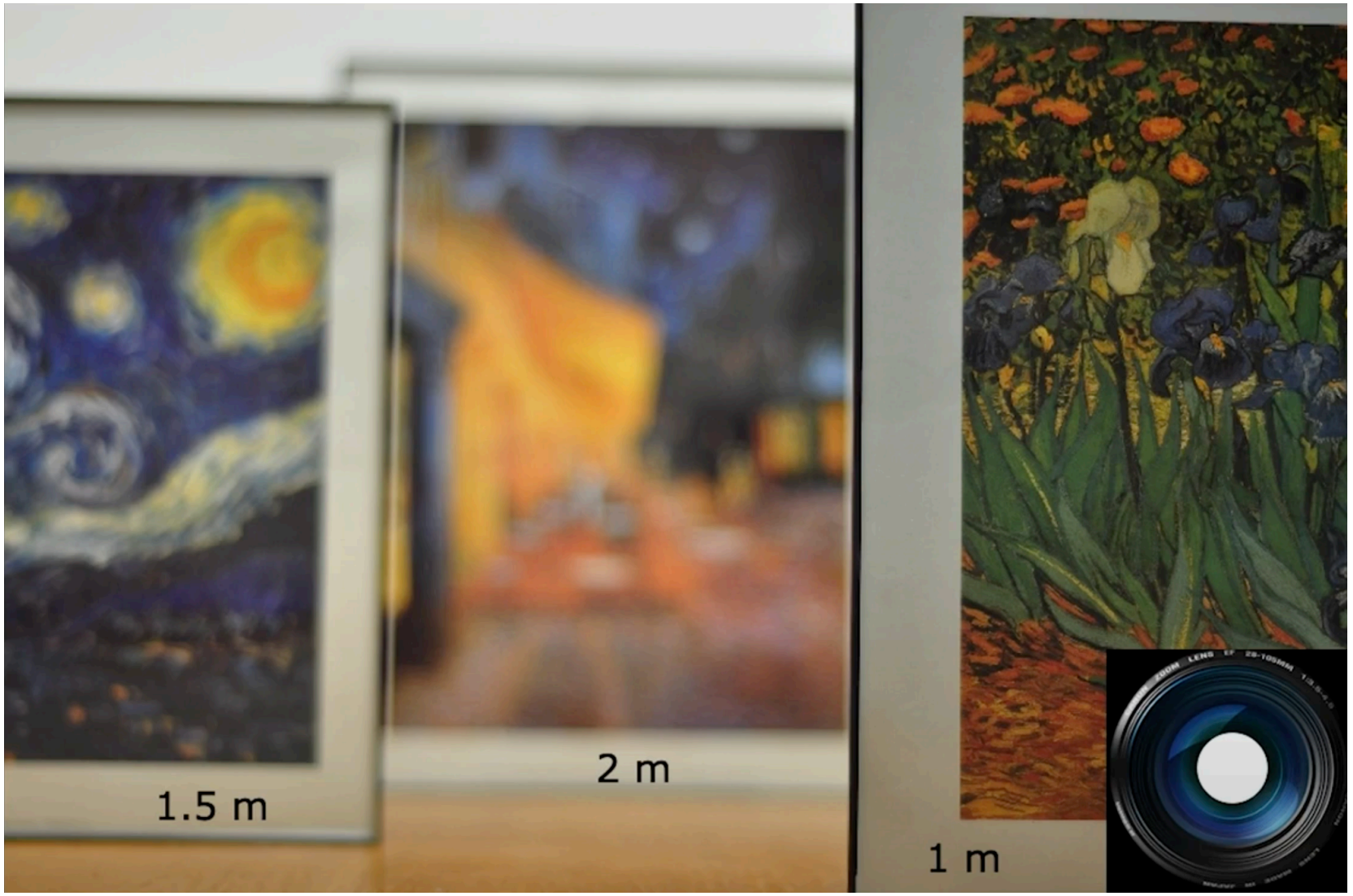
If o_1 and o_2 are the nearest and farthest distances respectively for which blur circle is maximum c, then:

$$DoF = o_2 - o_1 \propto \frac{1}{A}$$

c is taken as the pixel dimension

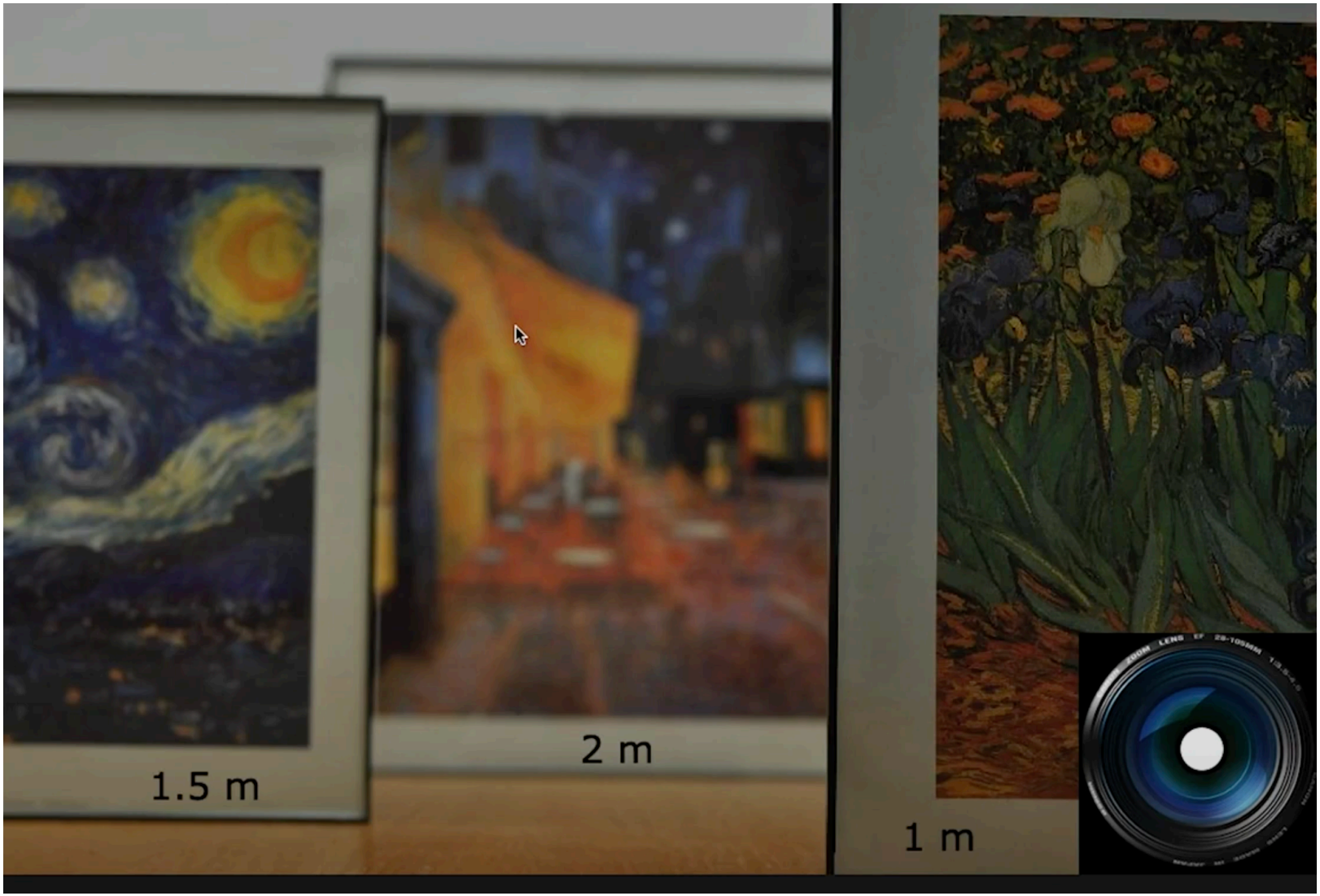


Aperture size: DoF vs Brightness



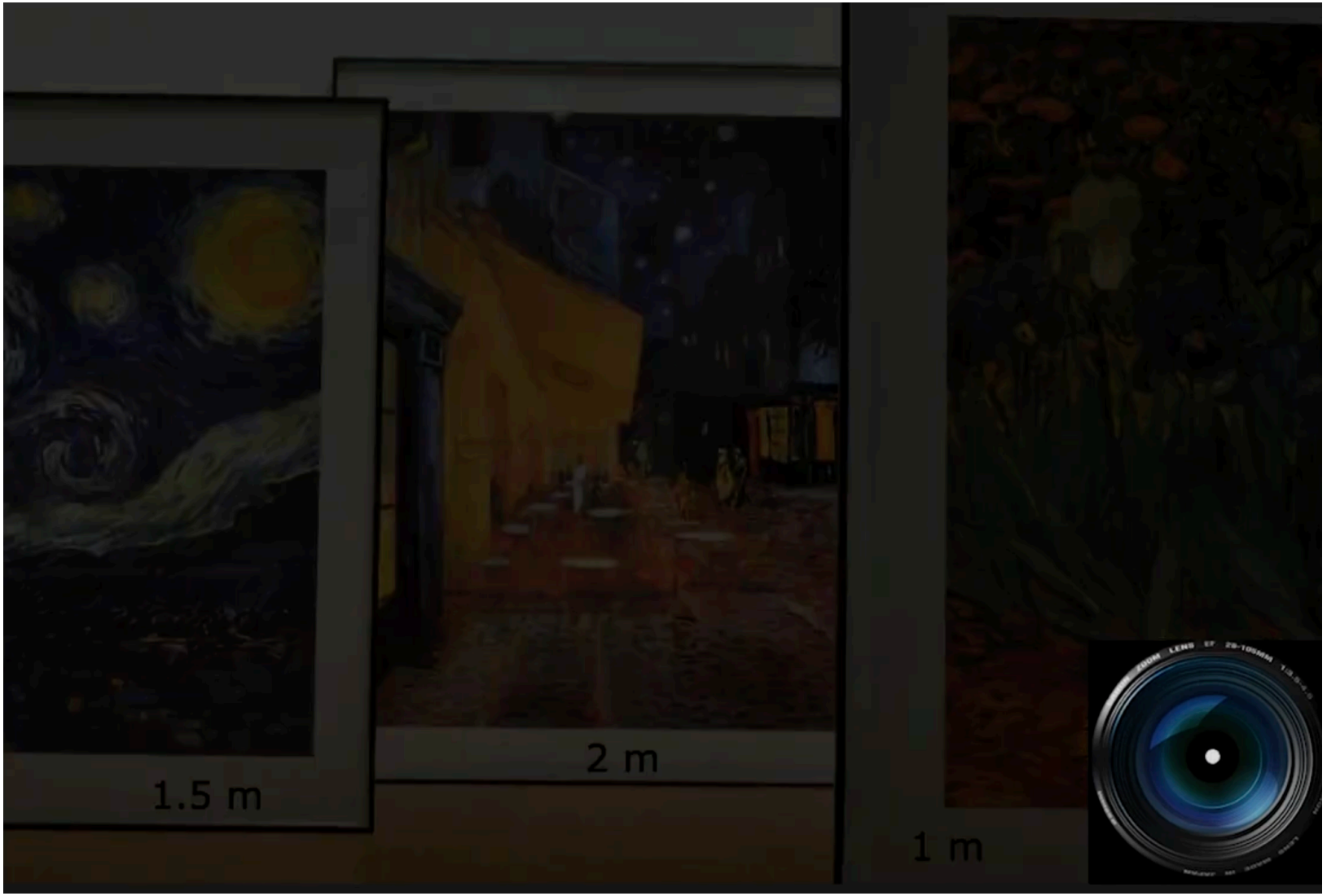
Focal Length 50 mm, Focus = 1 m, Aperture D = 25 mm

Aperture size: DoF vs Brightness



Focal Length 50 mm, Focus = 1 m, Aperture D = 12.5 mm

Aperture size: DoF vs Brightness



Focal Length 50 mm, Focus = 1 m, Aperture D = 3.125 mm

Aperture size: DoF vs Brightness

- Large Aperture
  - Bright Image or Short Exposure Time
  - Shallow Depth of Field

- Small Aperture
  - Dark Image or Long Exposure Time
  - Large Depth of Field

THICK LENS CAMERA

Real camera

Chapter 1 – Forsyth Ponce

Lens distortions

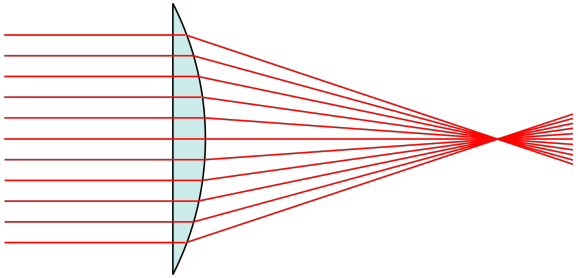
Optical aberrations of a lens

A real (thick) lens does not behave exactly like a thin lens

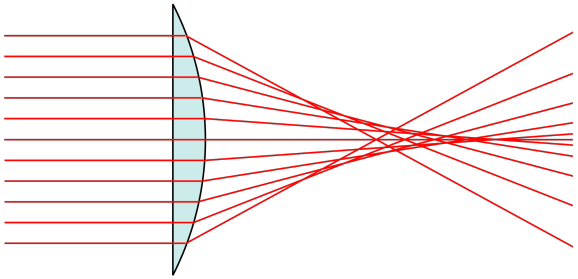
- **Spherical Aberration, Coma, Astigmatism**
  - Rays refracted by the lens form a circle of confusion on the image plane
  - **effect: unfocused image**

- **Chromatic aberration**
  - different lens behavior for different colors (wavelengths)
  - **effect: color separation**

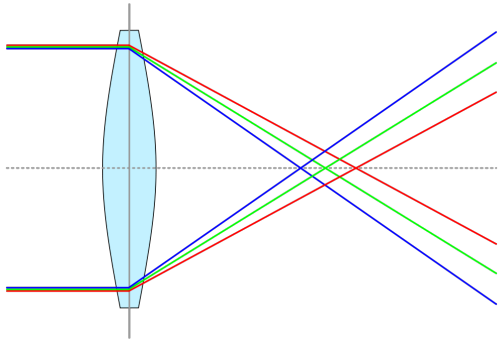
- **Radial Distortion**
  - Image magnification changes as the distance from the central point
  - **effect: image deformation!**



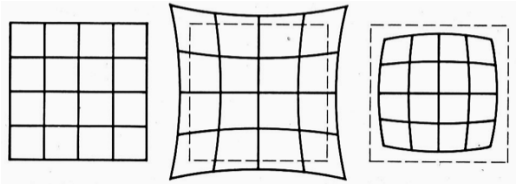
Ideal (thin) lens



Spherical Aberration



Chromatic aberration



Radial distortion

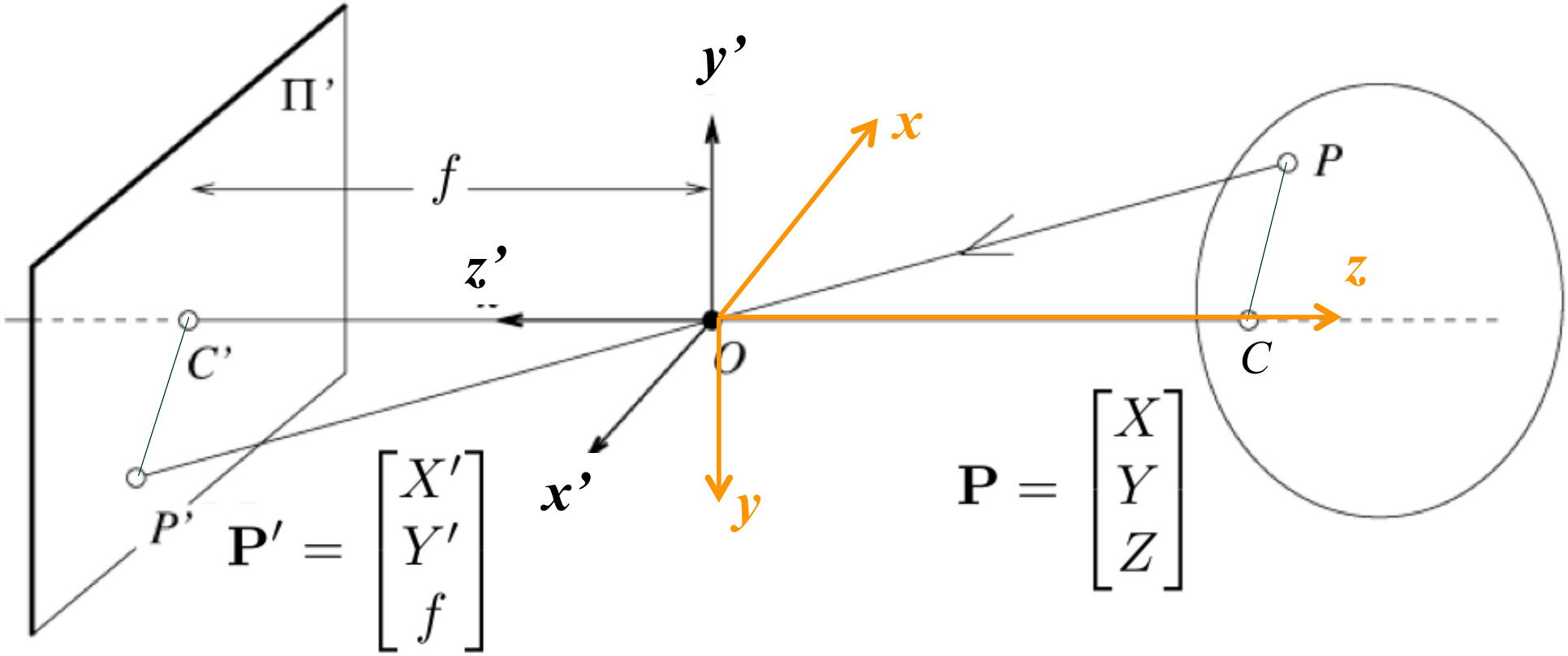
IMAGE FORMATION

Linear camera model (”pinhole”)

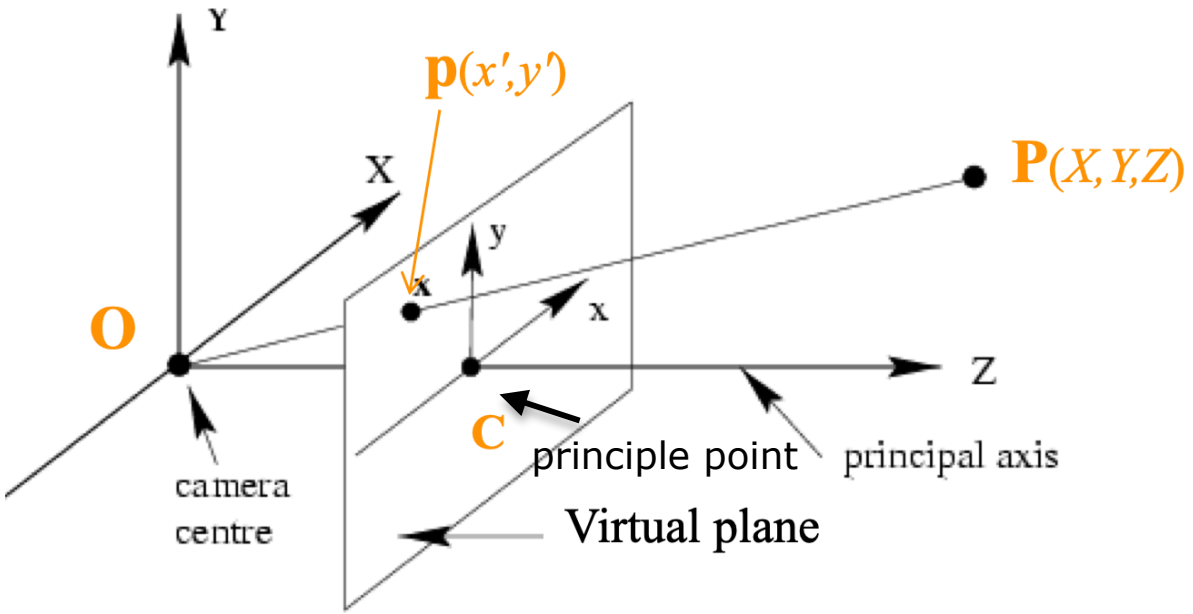
Chapter 1 – Forsyth Ponce

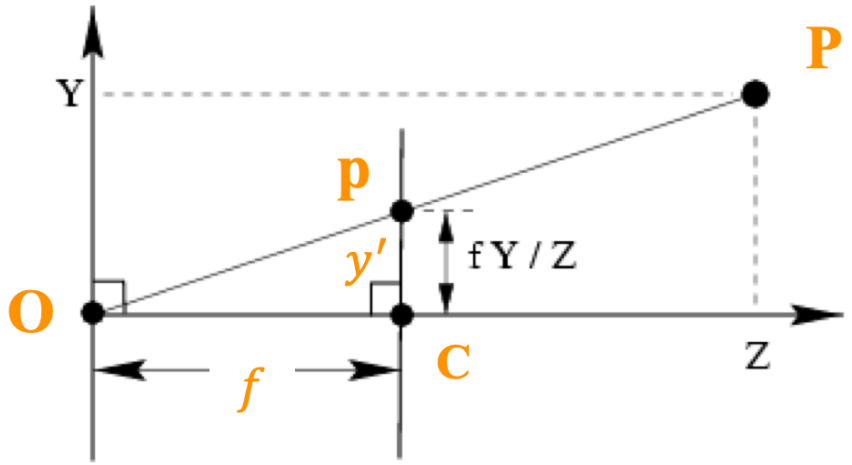
Forward Imaging Model

- Model that relates points in the real world P with their image points P' on the image plane



The pinhole mathematical model





- From the perspective equation it holds $\longrightarrow$ $x' = f\frac{X}{Z}\,;\quad y' = f\frac{Y}{Z}$

- In Matricial form:

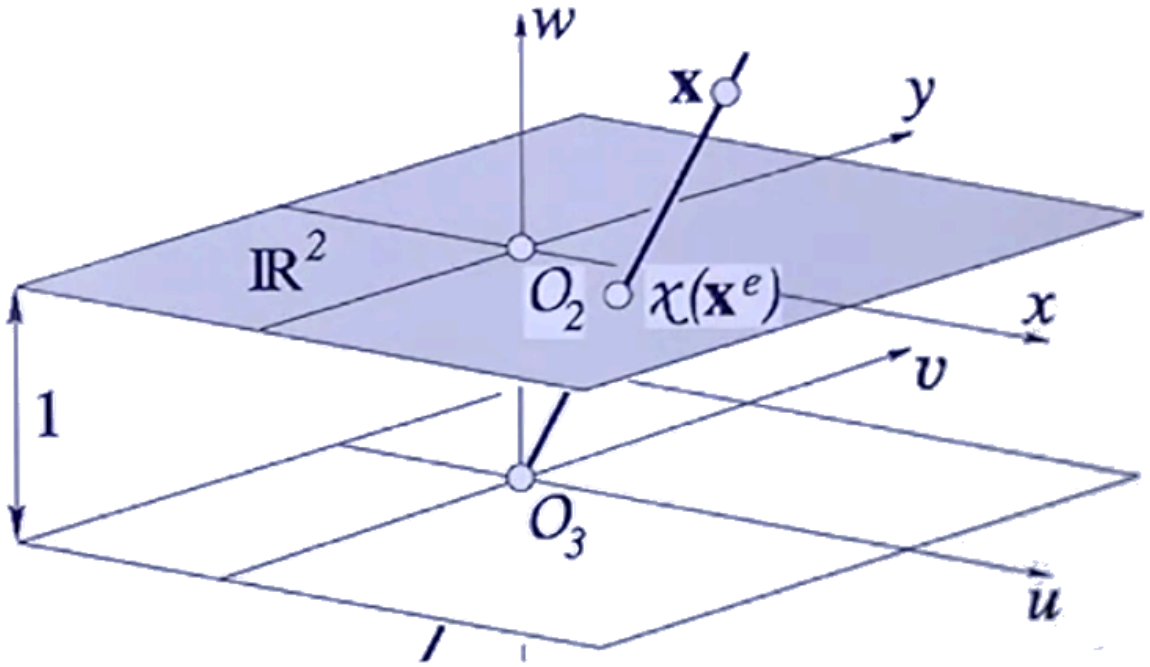
$$\mathbf{p} = \begin{bmatrix} x' \\ y' \end{bmatrix} \in \mathbb{R}^2\,;\quad \mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \in \mathbb{R}^3 \quad \longrightarrow \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{f}{Z} & 0 & 0 \\ 0 & \frac{f}{Z} & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad : \quad \mathbf{p} = \mathbf{M} \bullet \mathbf{P}$$

- Remark: **non linear!**

Homogeneous coordinates

- The homogeneous representation of a 2D point $\mathbf{x} = (x, y)$ is a 3D point $\tilde{\mathbf{x}} = (u, v, w)$, where the third coordinate $\tilde{w} \neq 0$ is called fictitious such that:

$$x = \frac{u}{w}, \quad y = \frac{v}{w}$$



- It holds:

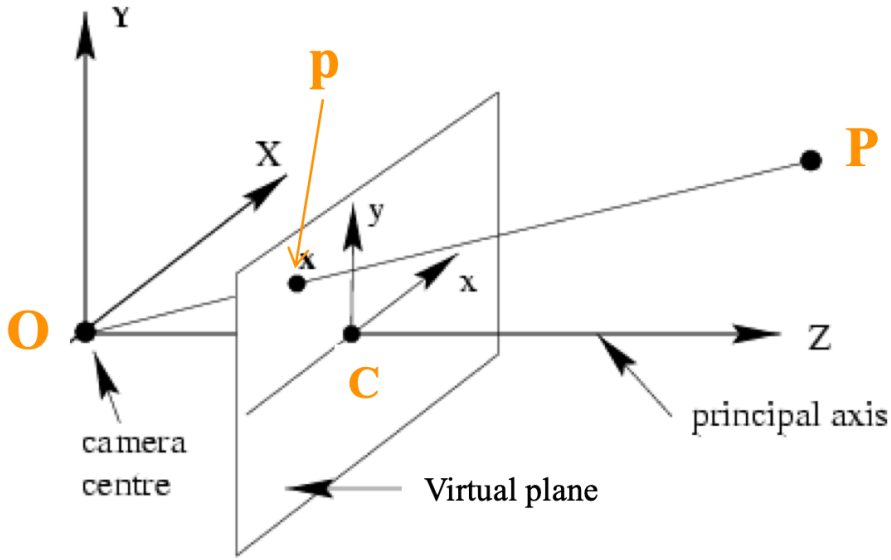
Euclidean $\boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ Homogeneous $\mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$

- Similarly, for any n-dimensional point

Perspective projection in homogeneous coordinates

Euclidean space *Projective space*

$$\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \tilde{\mathbf{p}} = \lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \tilde{P} = \lambda \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



for $\lambda = 1$:

Scale factor: Z

$$\tilde{\mathbf{p}} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{f}{Z}X \\ \frac{f}{Z}Y \\ 1 \end{bmatrix} = \begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = K \cdot \tilde{\mathbf{P}}$$

Projection: Matrix product **K** [3×4]

In homogeneous coordinates the perspective projection becomes linear!

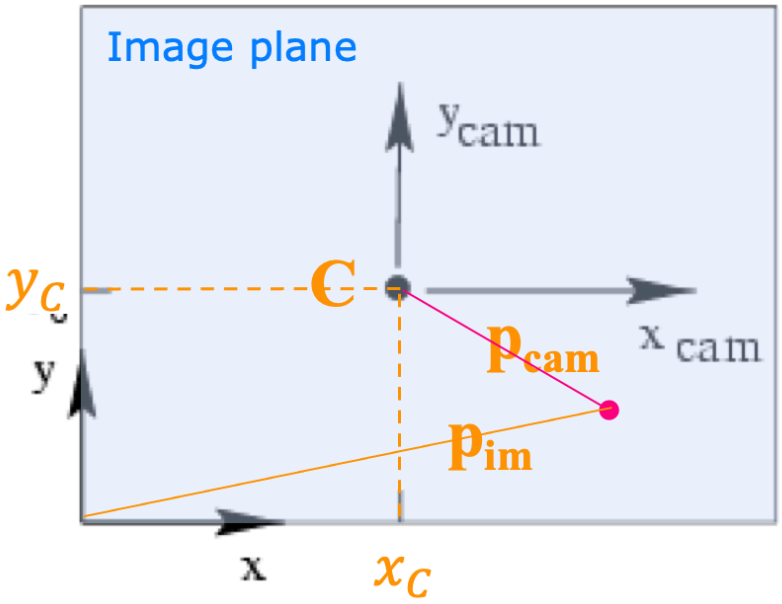
Perspective projection – a more complete camera model

1. Image coordinates (2D):

- Camera coords: p_{cam} – wrt *pinhole*
- Image coords: p_{im} – wrt the image plane (camera sensor)

Properties Image coords:

- axes parallel to the sensor axes
- unit of measurement: **pixel**



2. Optical centre/Principle point

- $C=\langle x_C, y_C \rangle$

$$\mathbf{p}_{\text{im}} = \mathbf{p}_{\text{cam}} + \mathbf{C} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x_C \\ y_C \end{bmatrix} \Rightarrow \tilde{\mathbf{p}}_{\text{im}} = \begin{bmatrix} x + x_C \\ y + y_C \\ 1 \end{bmatrix} = \begin{bmatrix} fX/Z + x_C \\ fY/Z + y_C \\ 1 \end{bmatrix}$$

Scale factor: Z

K_I: Intrinsic calibration matrix

$$\tilde{P} \overset{\text{projection}}{\longmapsto} \tilde{\mathbf{p}}_{\text{im}} = \begin{bmatrix} \frac{fX}{Z} + x_C \\ \frac{fY}{Z} + y_C \\ 1 \end{bmatrix} \equiv \begin{bmatrix} fX + x_C Z \\ fY + y_C Z \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & x_C & 0 \\ 0 & f & y_C & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{K}_I \cdot \tilde{\mathbf{P}}$$

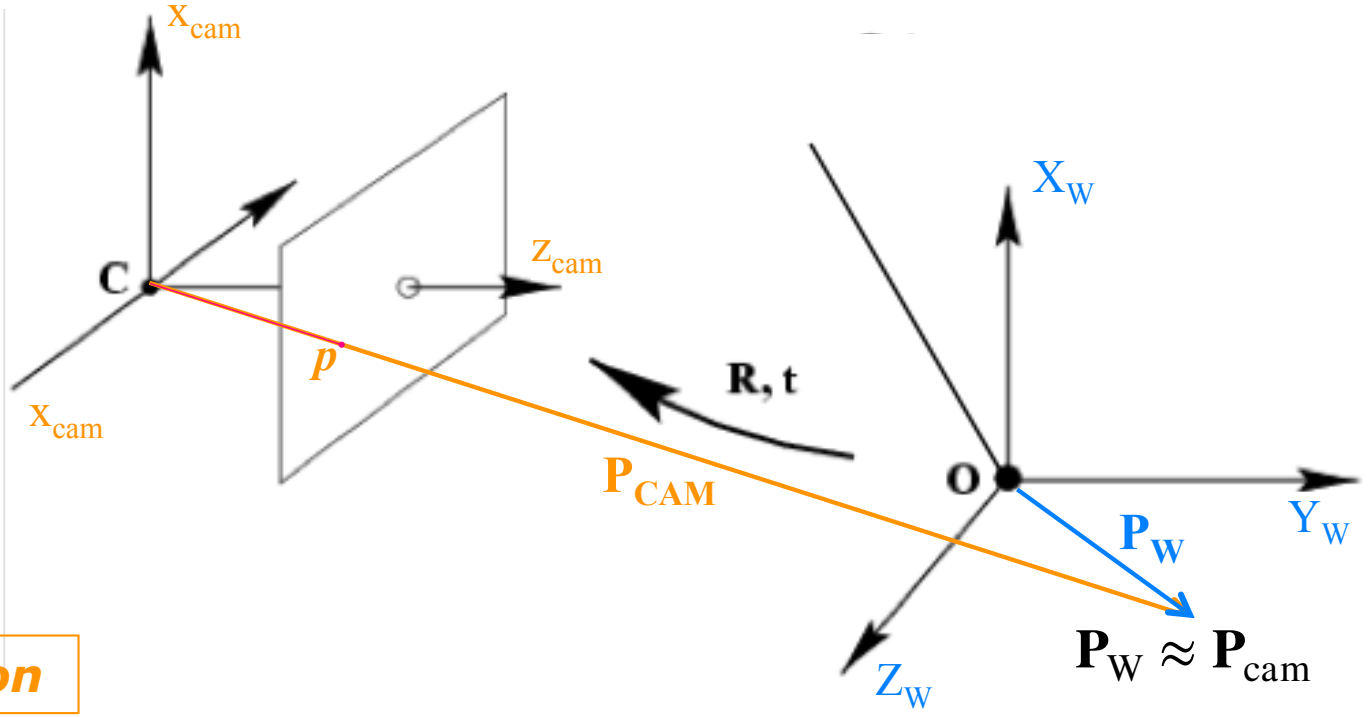
Perspective projection – a more complete camera model

3. Reference System change (World coordinates → Camera coordinates)

- **world coords** – 3D ref. System expressed wrt the **scene**
- **camera coords** – 3D ref. System expressed wrt the **camera**

- **System change:**
  3D roto-translation

$$\mathbf{P}_{\text{cam}} = \begin{bmatrix} x_{\text{cam}} \\ y_{\text{cam}} \\ z_{\text{cam}} \end{bmatrix}; \quad \mathbf{P}_W = \begin{bmatrix} x_W \\ y_W \\ z_W \end{bmatrix}$$



Rotation *Translation*

$$\mathbf{P}_{\text{cam}} = \mathbf{R} \bullet \mathbf{P}_W + \mathbf{T} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_W \\ y_W \\ z_W \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{cases} x_{\text{cam}} = \mathbf{r}_1 \bullet \mathbf{P}_W + t_x \\ y_{\text{cam}} = \mathbf{r}_2 \bullet \mathbf{P}_W + t_y \\ z_{\text{cam}} = \mathbf{r}_3 \bullet \mathbf{P}_W + t_z \end{cases}$$

Rotation matrix

R: 3D rotation matrix composed of 3 rotations, on the 3 axes x,y,z

- described by 3 rotation angles (Eulero angles)

$$\mathbf{R} = \mathbf{R}(\varphi,\vartheta,\rho) = \mathbf{R}(\varphi)\cdot\mathbf{R}(\vartheta)\cdot\mathbf{R}(\rho) =$$

$$= \underbrace{\begin{bmatrix} \cos(\varphi) & -\sin(\varphi) & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Around } \mathbf{Z}} \cdot \underbrace{\begin{bmatrix} \cos(\vartheta) & 0 & -\sin(\vartheta) \\ 0 & 1 & 0 \\ \sin(\vartheta) & 0 & \cos(\vartheta) \end{bmatrix}}_{\text{Around } \mathbf{Y}} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\rho) & -\sin(\rho) \\ 0 & \sin(\rho) & \cos(\rho) \end{bmatrix}}_{\text{Around } \mathbf{X}}$$

Rotation order ←

R properties:

- The rotation order is **not commutative**!
- **Ortonormal Matrix**
  - *Determinant =1*
  - *Rows and cols ortonormals to each other*
- **Inverse = Transpose**

$$\det(\mathbf{R}) = 1; \quad \mathbf{R}^T = \mathbf{R}^{-1}$$

$$\mathbf{r_i}\cdot\mathbf{r_i} = 1 \quad , \quad \mathbf{r_i} = \left[r_{i1}, r_{i2}, r_{i3}\right]$$

$$\mathbf{r_i}\times\mathbf{r_i} = 0$$

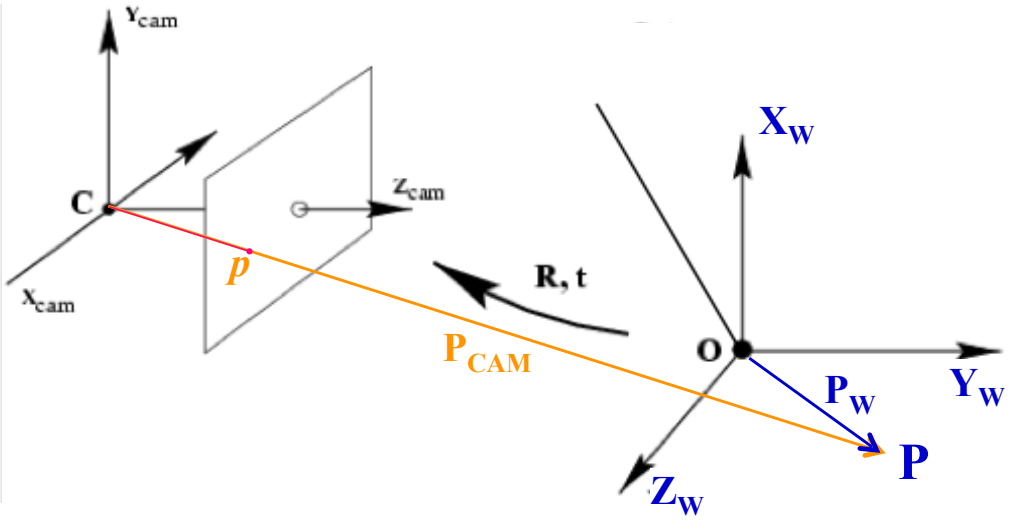
Perspective projection – a more complete camera model

Geometric relationship between Camera coordinates and World coordinates in homogeneous coords:

$$\tilde{\mathbf{P}}_{\mathbf{CAM}} = \begin{bmatrix} X_{CAM} \\ Y_{CAM} \\ Z_{CAM} \\ 1 \end{bmatrix} = \left[\begin{array}{ccc:c} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix} = \left[\begin{array}{c:c} \mathbf{R} & \mathbf{T} \\ \hdashline \mathbf{0} & 1 \end{array}\right] \cdot \tilde{\mathbf{P}}_{\mathbf{W}} = \mathbf{E} \cdot \tilde{\mathbf{P}}_{\mathbf{W}}$$

$$\mathbf{E} = \left[\begin{array}{c:c} \mathbf{R} & \mathbf{T} \\ \hdashline \mathbf{0} & 1 \end{array}\right]$$

E: Extrinsic Roto-translation matrix [4 × 4]



Complete perspective projection camera model

By combining all relationships, we obtain the complete perspective projection model (in homogeneous coordinates):

- Roto-translation:

$$\tilde{\mathbf{P}}_{\mathbf{W}} \mapsto \tilde{\mathbf{P}}_{\mathbf{CAM}} = \left[\begin{array}{c|c} \mathbf{R} & \mathbf{T} \\ \hline \mathbf{0} & 1 \end{array}\right] \cdot \tilde{\mathbf{P}}_{\mathbf{W}} = \mathbf{E} \cdot \tilde{\mathbf{P}}_{\mathbf{W}}$$

- Perspective projection:

$$\tilde{\mathbf{P}}_{\mathbf{CAM}} \mapsto \tilde{\mathbf{p}}_{IM} = \begin{bmatrix} x_{IMM} \\ y_{IMM} \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & x_C & 0 \\ 0 & f & y_C & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \tilde{\mathbf{P}}_{\mathbf{CAM}} = \mathbf{K}_{\mathbf{I}} \cdot \tilde{\mathbf{P}}_{\mathbf{CAM}}$$

- **Complete projective model:**

$$\tilde{\mathbf{P}}_{\mathbf{W}} \mapsto \tilde{\mathbf{p}}_{IM} = \begin{bmatrix} f & 0 & x_C & 0 \\ 0 & f & y_C & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \left[\begin{array}{c|c} \mathbf{R} & \mathbf{T} \\ \hline \mathbf{0} & 1 \end{array}\right] \cdot \tilde{\mathbf{P}}_{\mathbf{W}} = \mathbf{K}_{\mathbf{I}} \cdot \mathbf{E} \cdot \tilde{\mathbf{P}}_{\mathbf{W}} = \mathbf{M} \cdot \tilde{\mathbf{P}}_{\mathbf{W}}$$

Intrinsic *calibration matrix [3x4]*

Extrinsic *rototranslation matrix [4x4]*

Projection Matrix [3×4]

Complete perspective projection camera model

$$\tilde{\mathbf{p}}_{IM} = \begin{bmatrix} f & 0 & x_C & 0 \\ 0 & f & y_C & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \left[\begin{array}{c|c} \mathbf{R} & \mathbf{T} \\ \hline \mathbf{0} & 1 \end{array}\right] \cdot \tilde{\mathbf{P}}_{\mathbf{W}} = \mathbf{M}(\xi) \cdot \tilde{\mathbf{P}}_{\mathbf{W}}$$

- **Linear model in 11 parameters (M_{3x4}, up to scale)**
  - **only 9 params are independent:**

$$\xi = [\mathbf{R}, \quad \mathbf{T}, \quad f, \quad \mathbf{C}] = [\varphi, \vartheta, \rho, \quad t_X, t_Y, t_Z, \quad f, \quad x_C, y_C]$$

- **Extrinsic Parameters:** depend on the relative position camera-scene
  - **Rotation**: Euler angles: **R** = [φ, θ, ρ]
  - **Translation**: translation vector: **T** = [t_X, t_Y, t_Z]

- **Intrinsic Parameters:** depend on the camera characteristics
  - **Focal length**: f
  - **Optical Centre position**: C= <x_C, y_C>

IMAGE FORMATION

Geometric Camera Models

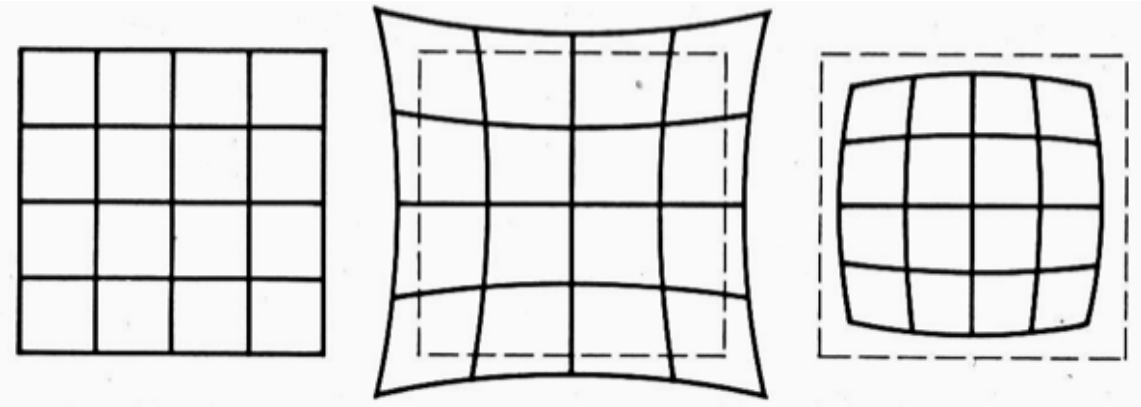
Real camera model

Chapter 1 – Forsyth Ponce

Radial distortion

Image magnification changes as the distance from the central point

- **effect: image deformation:** shift of the image point with respect to the linear geometric model



No distortion
k = 0

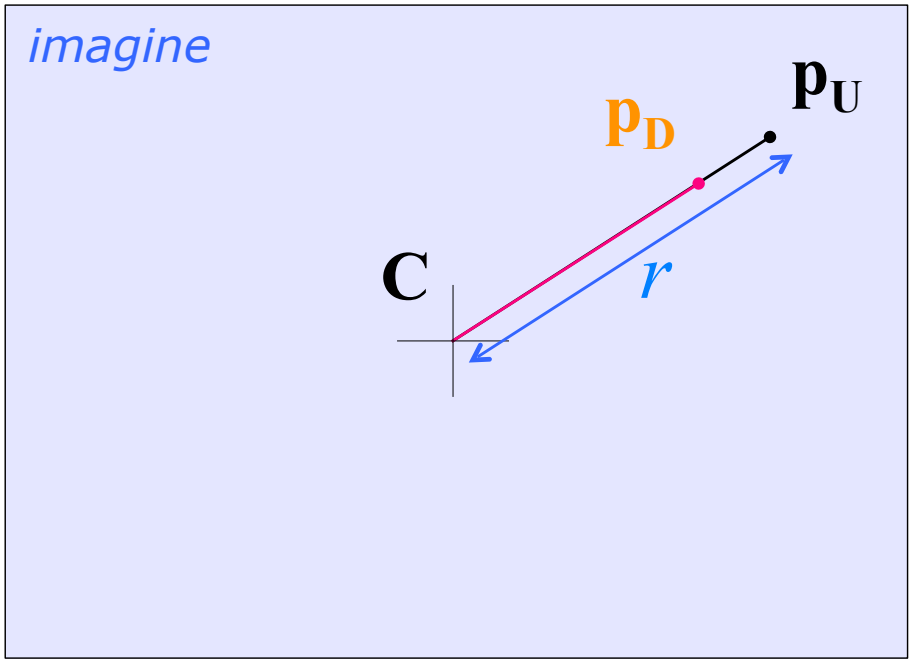
"pincushion" distortion
k < 0

"barrel" distortion
k > 0

➔ **It has to be inserted into the camera geometric model**

Geometrical model with radial distortion

- Radial distortion:
  ➔ varies along the ray length from the optical centre to the image point



Data:

$\mathbf{p_U}=(x_U, y_U)$: Undistorted position
(obtained by the linear model)

$\mathbf{p_D}=(x_D, y_D)$: Real position, Distorted

$$r^2 = \left|\mathbf{p}_U\right|^2 = x_U^2 + y_U^2$$

$$\mathbf{p}_D = \begin{bmatrix} x_D \\ y_D \end{bmatrix} = \mathbf{p}_U \left(1 + k_1 r^2 + k_2 r^4 + \cdots\right) = \begin{bmatrix} x_U \left(1 + k_1 r^2 + k_2 r^4 + \cdots\right) \\ y_U \left(1 + k_1 r^2 + k_2 r^4 + \cdots\right) \end{bmatrix}$$

- $\mathbf{p_U}$ is obtained with the linear model, then compute $\mathbf{p_D}$.

Radial distortion introduces **non-linearity** into the camera model!

Complete geometrical model with radial distortion

- **Roto-translation:**

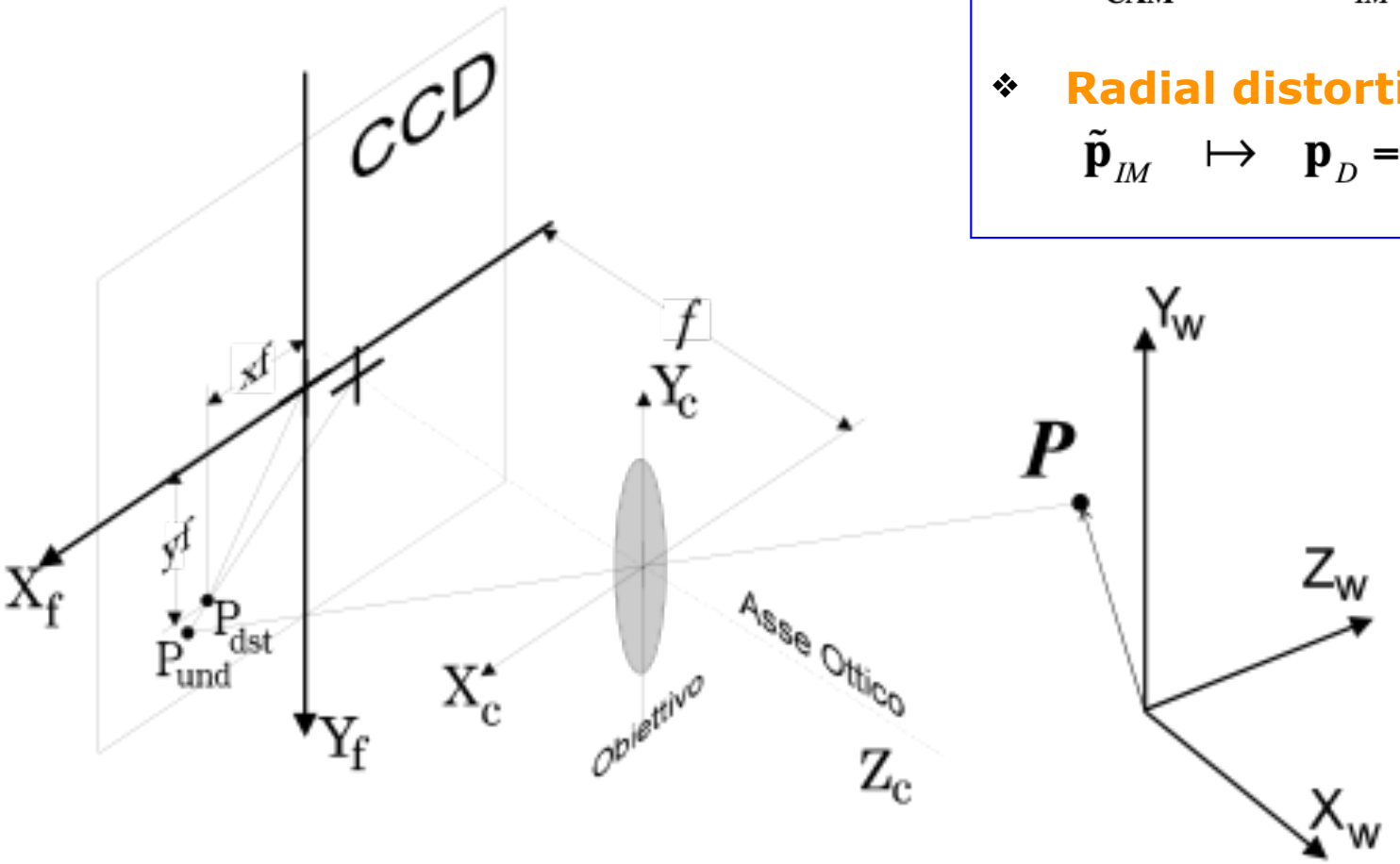
$$\tilde{\mathbf{P}}_{\mathbf{W}} \mapsto \tilde{\mathbf{P}}_{\mathbf{CAM}} = \mathbf{E} \cdot \tilde{\mathbf{P}}_{\mathbf{W}}$$

- **Perspective Projection:**

$$\tilde{\mathbf{P}}_{\mathbf{CAM}} \mapsto \tilde{\mathbf{p}}_{IM} = \mathbf{K}_{\mathbf{I}} \cdot \tilde{\mathbf{P}}_{\mathbf{CAM}}$$

- **Radial distortion correction**

$$\tilde{\mathbf{p}}_{IM} \mapsto \mathbf{p}_D = \mathbf{p}_U \left(1 + k_1 r^2 + k_2 r^4 + \cdots\right)$$



Where are we?

First part:
Image formation and
Early vision

- Image formation
  - Geometric Camera Models
  - **Color spaces**
- Image Processing
  - Punctual and spatial processing
  - Feature Extraction
- Reconstruction
  - Camera calibration
  - Stereo Vision
  - Structure from Motion and RGB-d Cameras
  - Optical flow and Tracking

Second part:
Machine learning for CV

- Linear Neural Network
- Multi Layer Perceptron
- Convolutional Neural Networks
- Recurrent Neural Networks
- Transformers
- Variational Auto-Encoders
- Generative Adversarial Networks
- Graph Neural Networks
- Self-supervised learning
- Vision Language Models

IMAGE ACQUISITION AND REPRESENTATION

Image acquisition

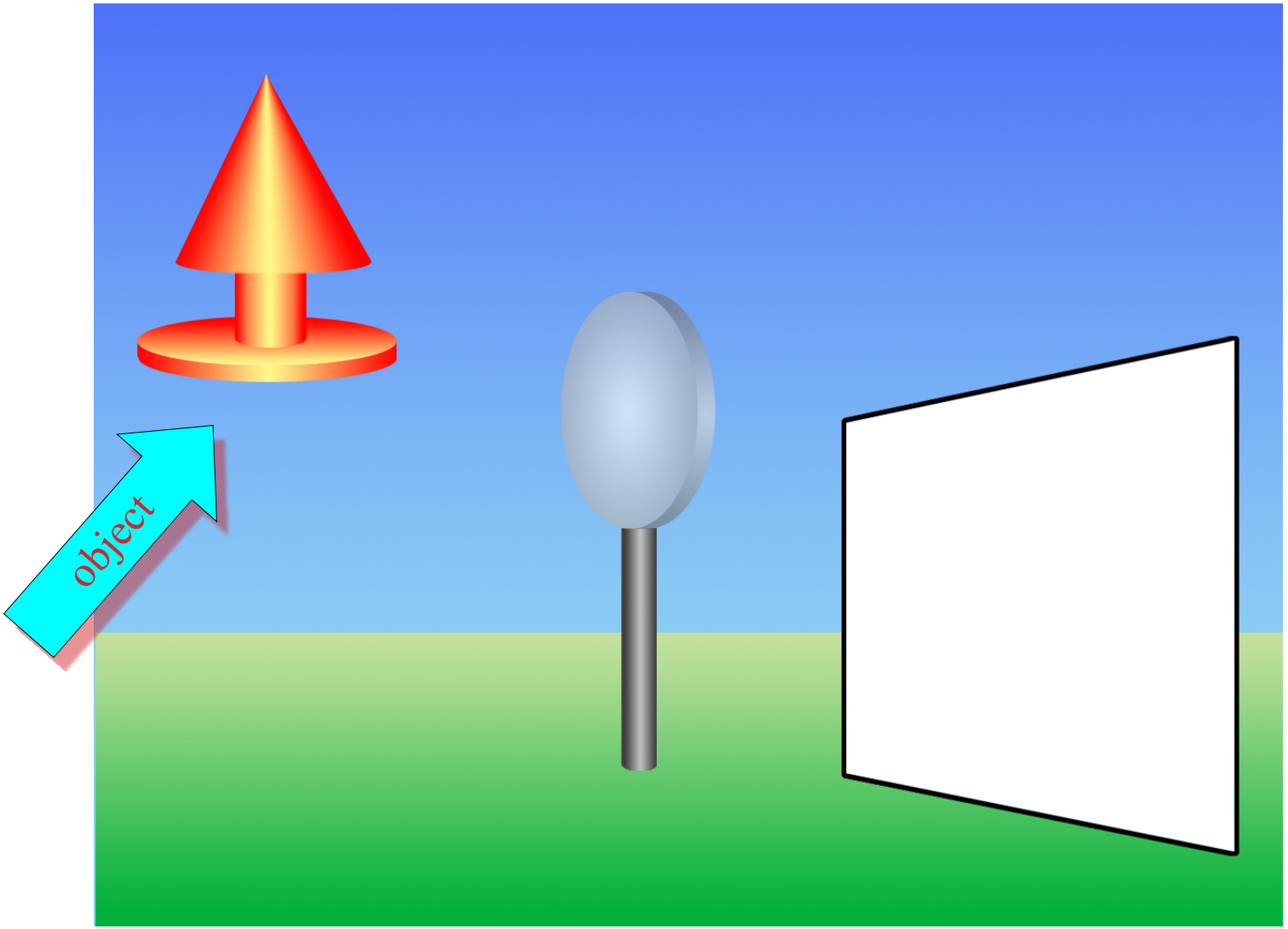


Image acquisition

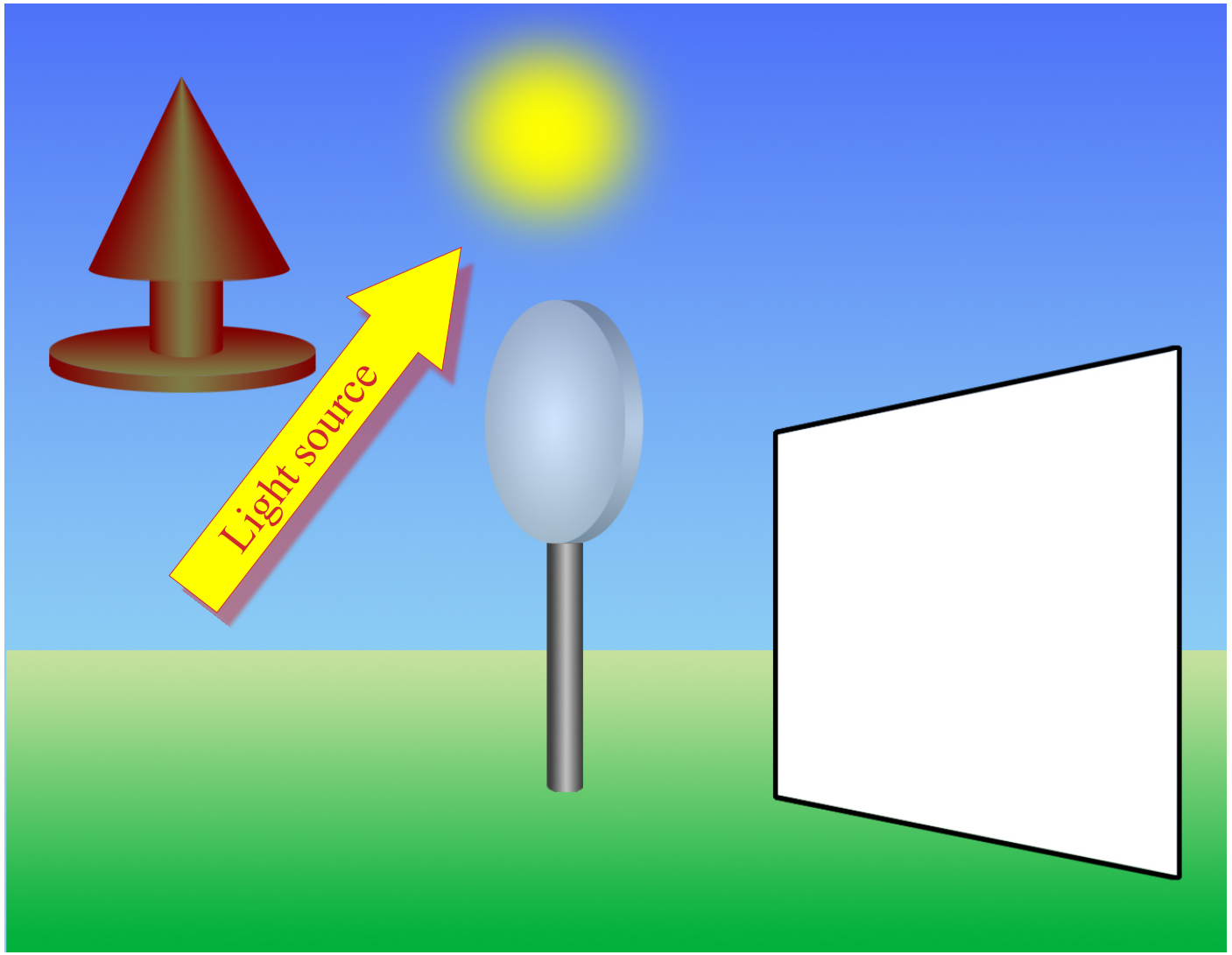


Image acquisition

projection

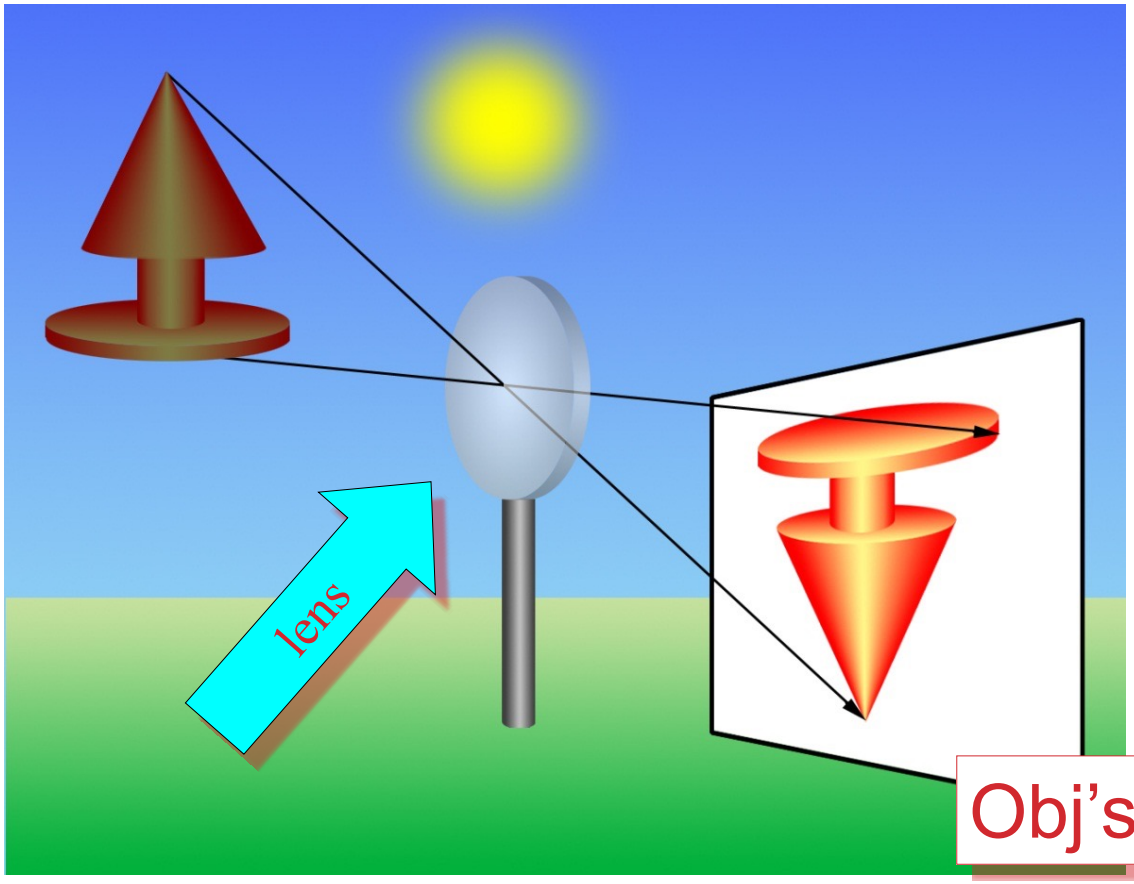


Image acquisition: sampling and quantization

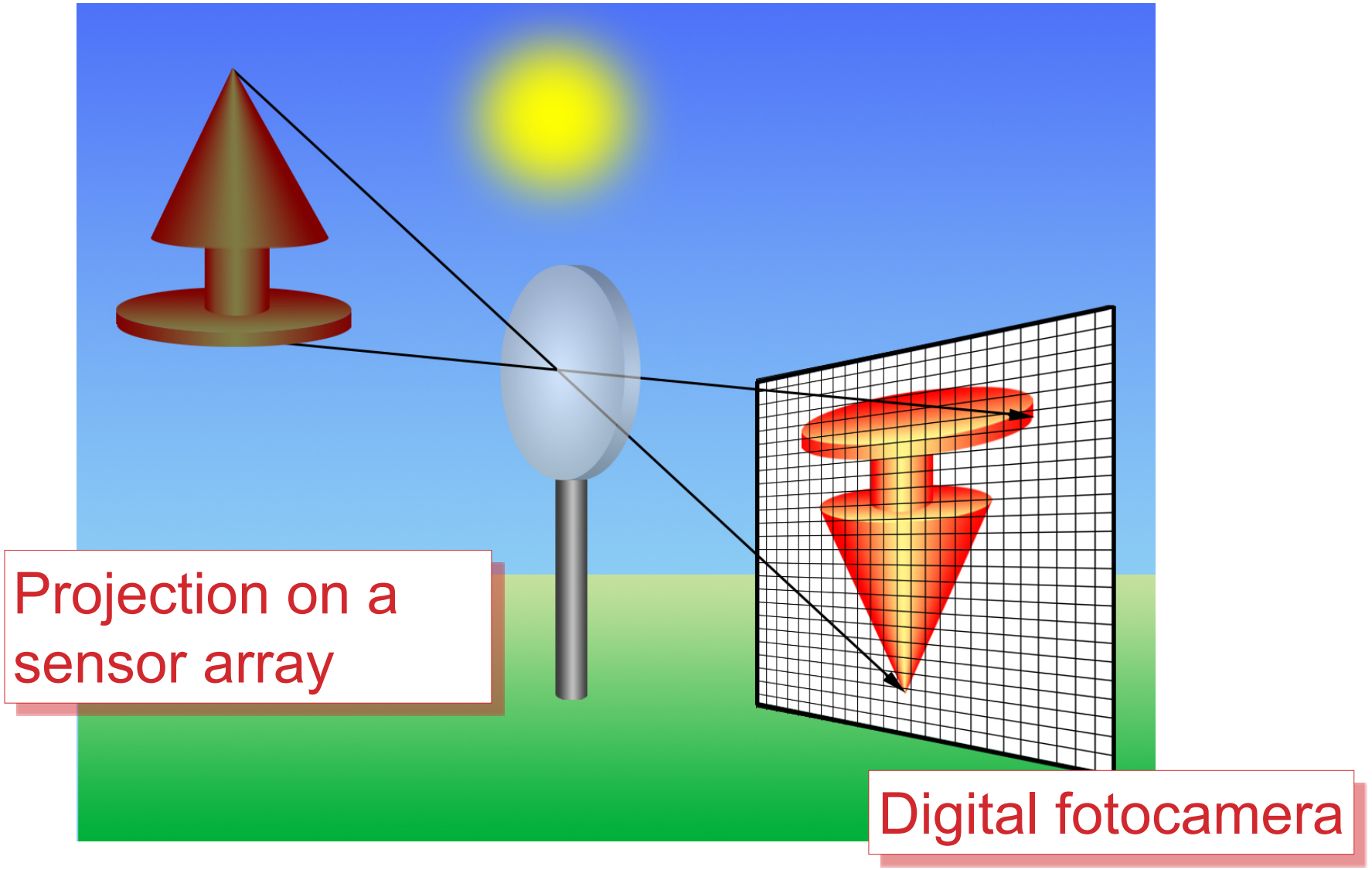
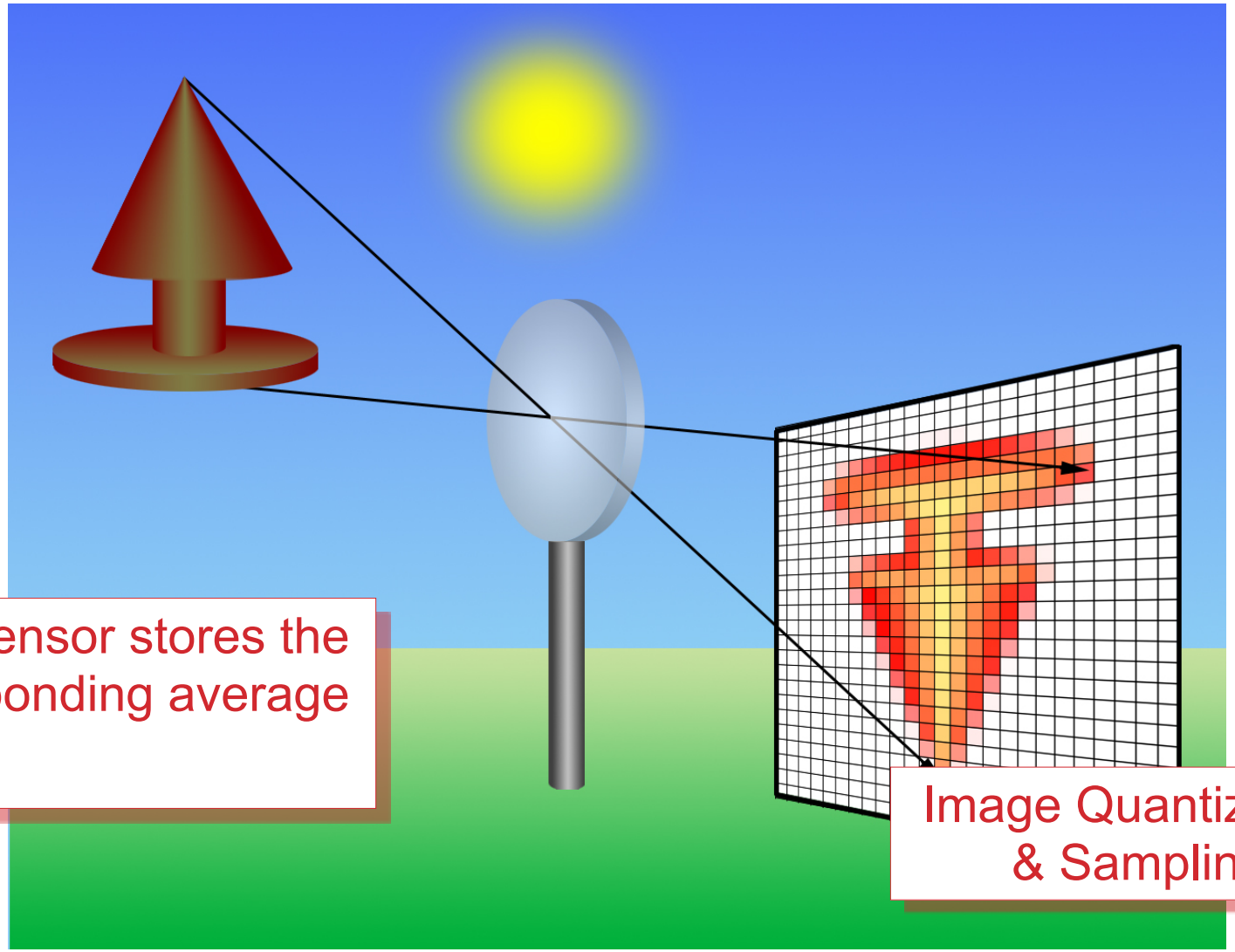


Image acquisition



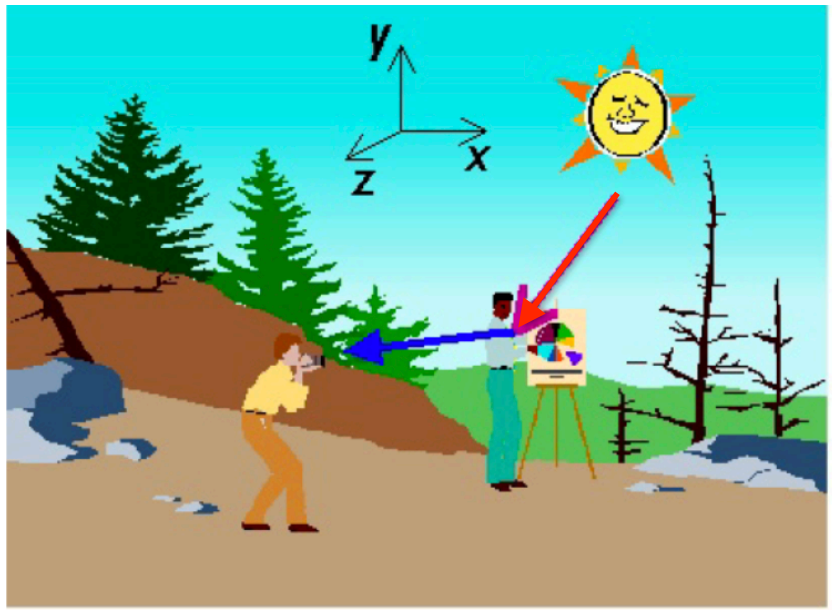
Each sensor stores the corresponding average color

Image Quantization & Sampling

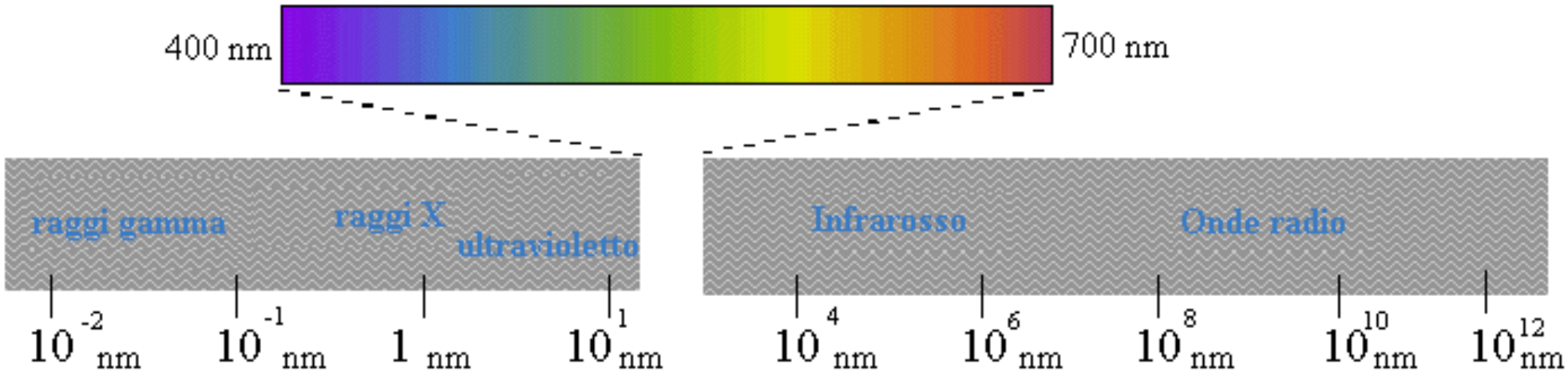
Image formation

- Light source (irradiance) is characterized by its 3d position (x,y,z) and its wavelengths λ

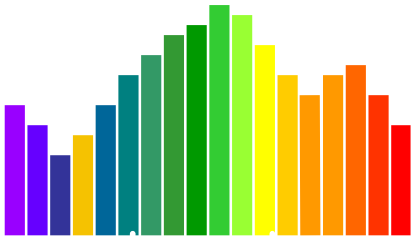
$$E(x, y, z, \lambda)$$



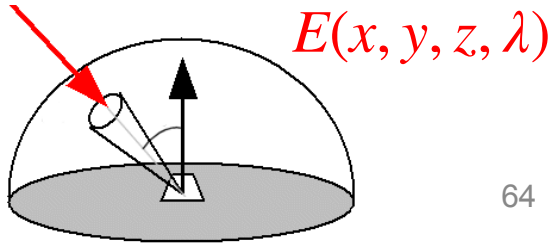
Light source: Electromagnetic Spectrum



- Each light source has its **Spectrum**: range and power of wavelengths of electromagnetic radiation that it emits $[W]$



- **Irradiance**: amount of electromagnetic radiation that is received by a surface per unit area per unit time $[W/m^2]$

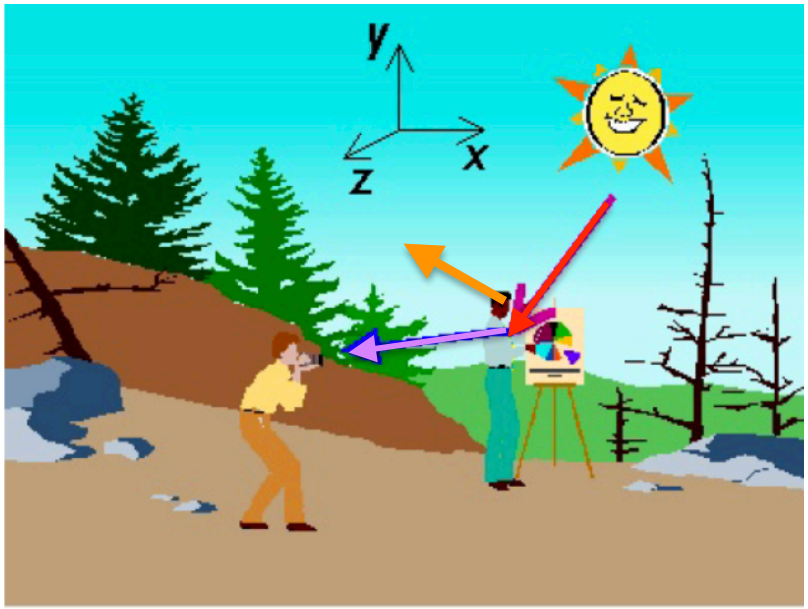


https://www.daviddarling.info/encyclopedia/I/irradiance.html

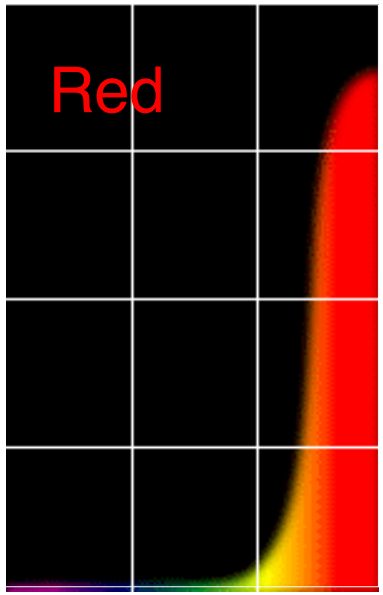
Image formation

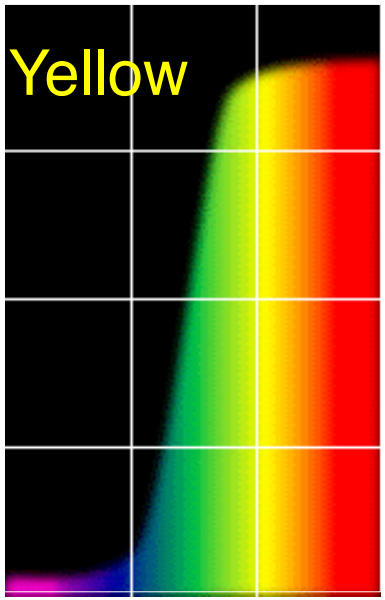
- Light source (irradiance) is characterized by its 3d position (x,y,z) and its wavelengths λ
- Each point in the scene has a reflectivity function

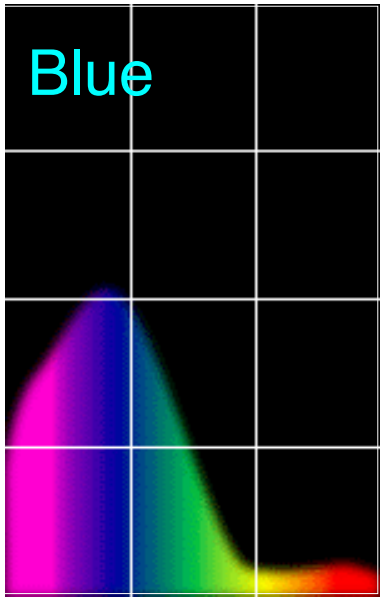
$$r(x, y, z, \lambda)$$



Reflectance spectra







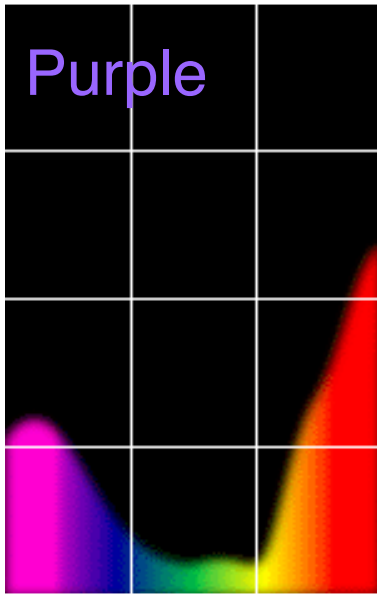
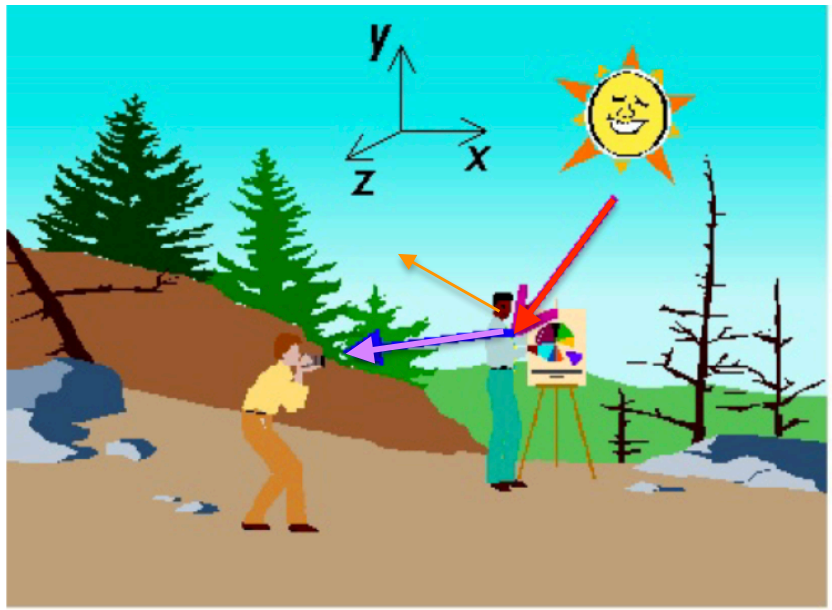


Image formation

- Light source (irradiance) is characterized by its 3d position (x,y,z) and its wavelengths λ: $E(x, y, z, \lambda)$
- Each point in the scene has a reflectivity function $r(x, y, z, \lambda)$
- These quantities are generally combined in a multiplicative way, producing a reflected light $c(x, y, z, \lambda)$

  $c(x, y, z, \lambda)$ is the input to the camera!

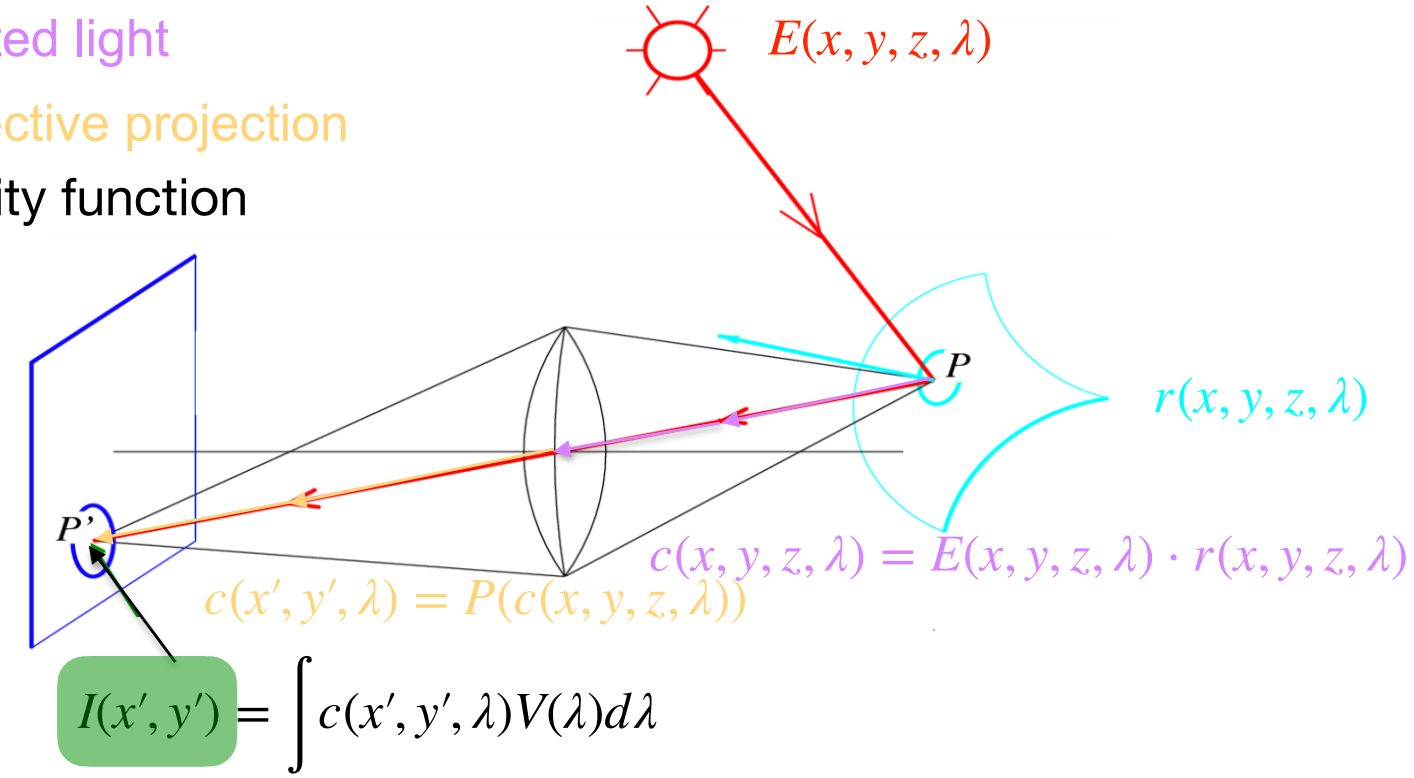


$$c(x, y, z, \lambda) = E(x, y, z, \lambda) \cdot r(x, y, z, \lambda)$$

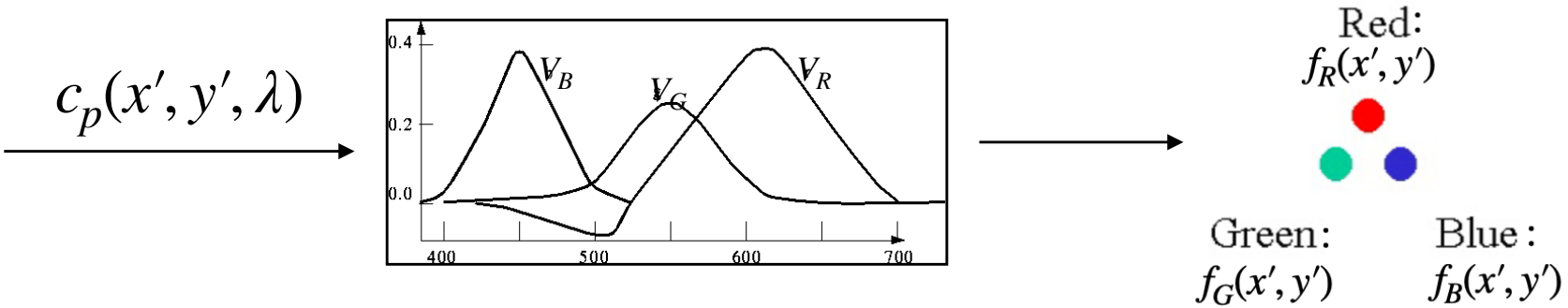
$$\text{Camera}(c(x, y, z, \lambda))$$

Image formation

- **What determines the pixel value $I(x', y')$?**
  - Light source properties
  - Surface properties
  - Reflected light
  - Perspective projection
- Sensitivity function



Sensitivity functions



Sensitivity functions

$$f_R(x', y') = \int c_p(x', y', \lambda) \cdot V_R(\lambda) d\lambda$$

$$f_G(x', y') = \int c_p(x', y', \lambda) \cdot V_G(\lambda) d\lambda$$

$$f_B(x', y') = \int c_p(x', y', \lambda) \cdot V_B(\lambda) d\lambda$$

- **Spectral sensitivity** refers to a sensor's response to different wavelengths of light.
- It is described by a **sensitivity function**, which quantifies how much a sensor detects light at each wavelength.

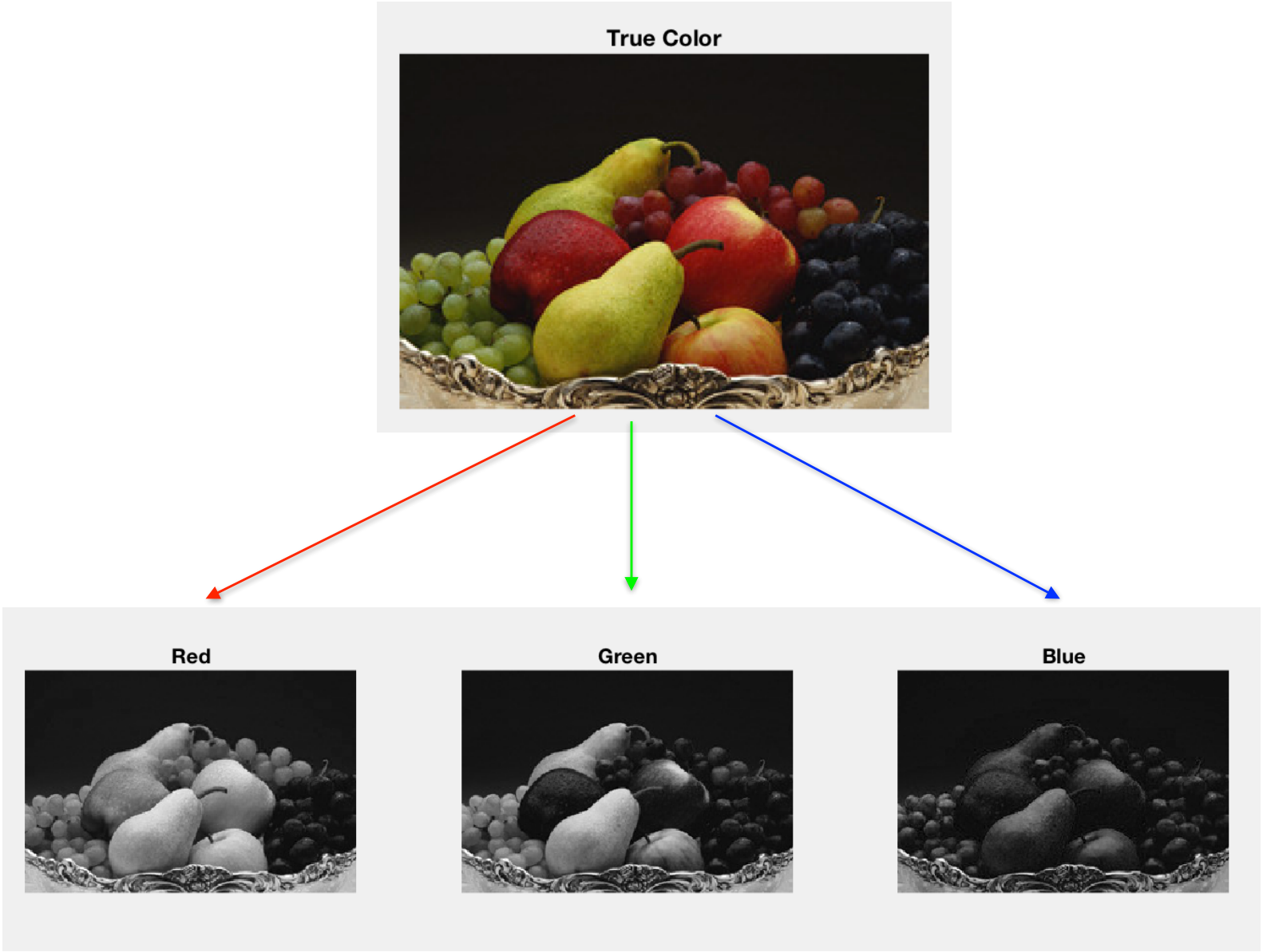
Image formation: summing up

- The image function $f_{\mathrm{C}}(x', y')$ $(C = R,\ G,\ B)$ is formed as:

$$f_{\mathrm{C}}(x', y') = \int c_p(x', y', \lambda) V_{\mathrm{C}}(\lambda) d\lambda \tag{2}$$

- It is the result of:
  1. Incident light $E(x, y, z, \lambda)$ at the point (x, y, z) in the scene,
  2. The reflectivity function $r(x, y, z, \lambda)$ of this point,
  3. The formation of the reflected light $c(x, y, z, \lambda) = E(x, y, z, \lambda) \times r(x, y, z, \lambda)$,
  4. The projection of the reflected light $c(x, y, z, \lambda)$ from the *three* dimensional world coordinates to *two* dimensional camera coordinates which forms $c_p(x', y', \lambda)$,
  5. The sensitivity function(s) of the camera $V_{\mathrm{C}}(\lambda)$.

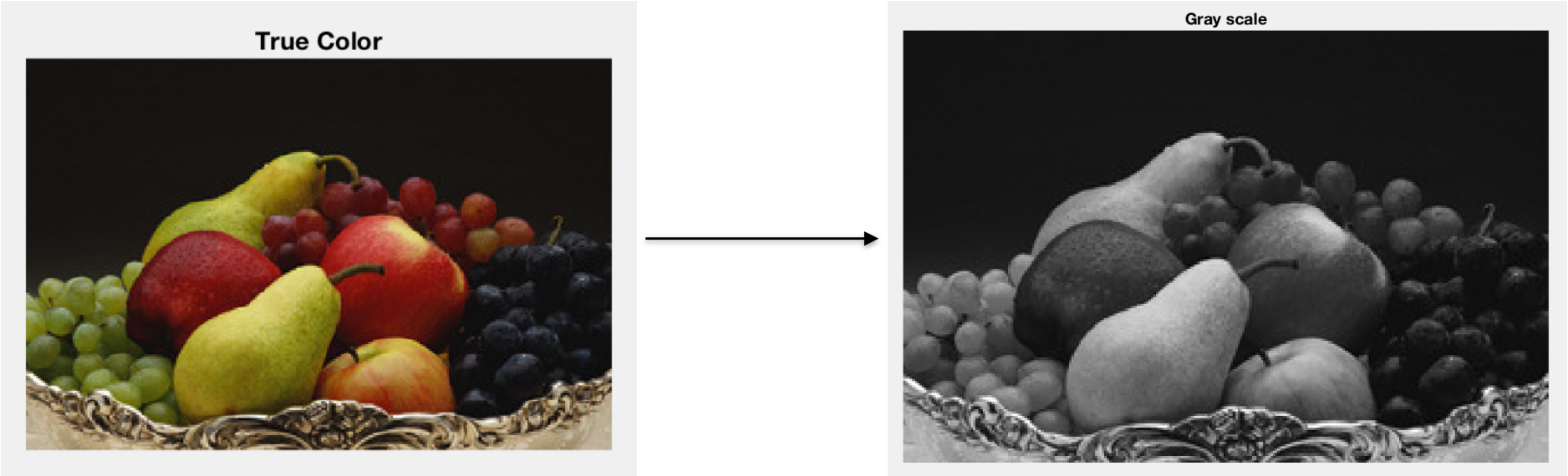
A true color image



Luminance image

- For each pixel we have the corresponding amount of light (luminance):

$$Y = 0.299R + 0.587G + 0.114B$$



Other Color spaces: HSV

HSV:

- **H**: Hue
- **S**: Saturation
- **V**: Value or Intensity

RGB → HSV (nonlinear conversion):

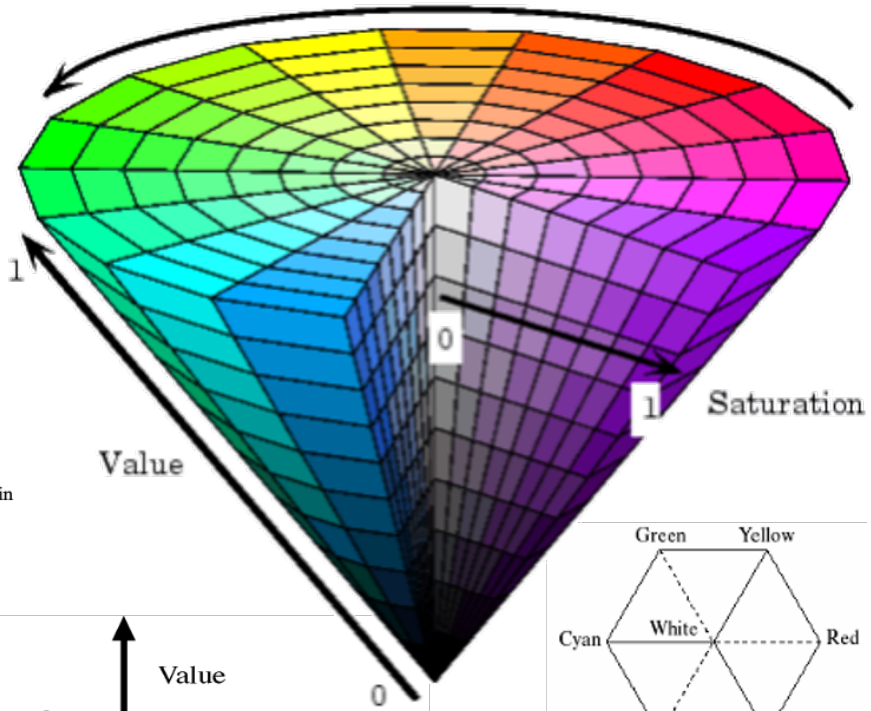
.

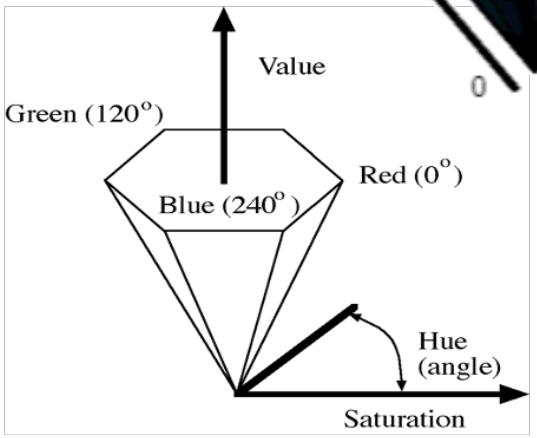
$$R' = \frac{R}{255}, \quad G' = \frac{G}{255}, \quad B' = \frac{B}{255}$$

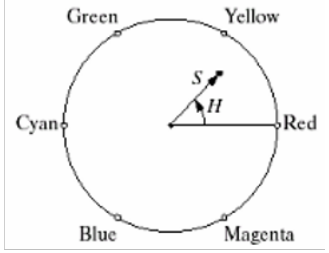
$$C_{\max} = \max\left(R', G', B'\right), \quad C_{\min} = \min\left(R', G', B'\right), \quad \Delta = C_{\max} - C_{\min}$$

$$H = \begin{cases} 0°, & \Delta = 0 \\ 60° \cdot \left(\frac{G' - B'}{\Delta} \bmod 6 \right), & C_{\max} = R' \\ 60° \cdot \left(\frac{B' - R'}{\Delta} + 2 \right), & C_{\max} = G' \\ 60° \cdot \left(\frac{R' - G'}{\Delta} + 4 \right), & C_{\max} = B' \end{cases}$$

$$S = \begin{cases} 0, & C_{\max} = 0 \\ \frac{\Delta}{C_{\max}}, & C_{\max} \neq 0 \end{cases} \quad ; \quad V = C_{\max}$$







Other Color spaces: CIE-Lab

CIE L*a*b* (1976)

- **perceptually uniform** space: a given numerical change corresponds to a similar perceived change in color

- L* : perceptual lightness
- a*, b* : spread over the 4 unique colors of human vision (red, green, blue, yellow)
- Non linear conversion s.t. Koenderink: **"an awful mix of magical numbers and arbitrary functions that somehow 'fit' the eye measure"**