THE UNIVERSITY OF NEW SOUTH WALES

MATH3871/MATH5970
Bayesian Inference and Computation

Introduction

This is the second assignment for course MATH3871/5960.

The assignment consists in two parts: a quiz and a programming part. The deadline for both is the 6th of November 2019 at 6PM (Sydney time). The deadline is strict.

For this assignment, it is possible to work in groups of maximum 4 people. However, it is not compulsory: groups may be formed by 1, 2, 3 or 4 people.

Each student should complete the quiz individually, while the programming part must be submitted once for each group. The members of the group should be clearly stated at the beginning of the code. Every results should be properly commented. Only .py or .R (or .rmd) files will be accepted. The programming part should run without errors to be evaluated.

The quiz worths 6 points, while the programming part worths 14 points.

The weight for this assignment on the final mark is 20%.

These instructions are strict.

Assignement 2

The annual number of serious accidents in the mines of Great Britain were recorded every year from 1851 to 1962. We want to build a statistical model to evaluate if during those 112 years there has been a change in the rate of occurrence of events, which could perhaps be due to legislative changes to protect security.

Let $(Y_1, \dots, Y_{m-1}, Y_m, Y_{m+1}, \dots, Y_n)$ be the number of annual incidents, with

$$\begin{aligned} Y_i &\sim Poi(\lambda) \qquad i = 1, 2, \dots, m \\ Y_j &\sim Poi(\phi) \qquad j = m+1, \dots, n \end{aligned}$$

The unknown parameters of the model are (ϕ, λ, m), while n is known and equal to 112.

Since the observations are supposed to follow a Poisson distribution, it is reasonable to consider Gamma priors for (ϕ, λ):

$$\begin{aligned} \lambda &\sim Gamma(\alpha, \beta) \\ \phi &\sim Gamma(a, b) \end{aligned}$$

On the other side, the parameter m can have a uniform distribution:

$$m \sim Unif(0, n-1)$$

The data are the following:

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y=c(4,5,4,1,0,4,3,4,0,6,
3,3,4,0,2,6,3,3,5,4,5,3,1,4,4,1,5,5,3,4,2,5,2,2,3,4,2,1,3,2,
1,1,1,1,1,3,0,0,1,0,1,1,0,0,3,1,0,3,2,2,0,1,1,1,0,1,0,1,0,0,
0,2,1,0,0,0,1,1,0,2,2,3,1,1,2,1,1,1,1,2,4,2,0,0,0,1,4,0,0,0,
1,0,0,0,0,0,1,0,0,1,0,0)
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Write an MCMC algorithm to perform a Bayesian analysis for this model. It could be useful to procede with the following steps

- write the likelihood function;
- write the full posterior distribution;
- code a Gibbs sampler with $N = 10000$ simulations and with a burnin of 1000 values;
- plot the simulates marginal chain for each parameter and comment;
- analyse the convergence of the chains with at least two tools;