Refinement

Concurrency and Distributed Systems

November 2023

Contents

Nondeterminism

nondeterminism, *n.*

1. *Philos.* The doctrine that human action is free and not determined by previous events.

2. *Math.* and *Computing.* A mode of computation in which, at certain points, there is a choice of ways to proceed which cannot be predicted in advance.

Oxford English Dictionary

Nondeterminism

We may wish to allow for two or more different patterns of behaviour, each of which would be regarded as satisfactory.

Alternatively, we may wish to describe a design at a higher level of abstraction, focussing upon only some of the events.

In either case, the resulting process description may be nondeterministic.

or it may not be…it depends upon how the information we are not considering affects the occurrence and availability of the events in our description

Semantics

If `<a,b>` is a trace of `P` then, after the event `a` has occurred, that process may allow `b`.

If `(<a>,{b})` is a failure of `P` then, after the event `a` has occurred, that process may refuse `b`.

A nondeterministic process is one that, after some trace, may allow some event but may also refuse that same event.

Example

```
EXT =
  a -> b -> STOP
  []
  c -> d -> STOP
```

is deterministic

```
INT =
  a -> b -> STOP
  |~|
  c -> d -> STOP
```

is nondeterministic

Example

```
failures(INT) = {
  (<>, {}), (<>,{a}), (<>,{b}), (<>,{c}), (<>,{d}), (<>,{a,b}),
            , (<>,{a,d}), (<>,{b,c}), (<>,{b,d}), (<>,{c,d}),
              , (<>,{a,b,d}),             , (<>,{b,c,d}),
  (<a>,{}), (<a>,{a}), (<a>,{c}), (<a>,{d}), (<a>,{a,c}),
  (<a>,{a,d}), (<a>,{c,d}), (<a>,{a,c,d}), (<a,b>,{}),
  (<a,b>,{a}), (<a,b>,{b}), (<a,b>,{c}), (<a,b>,{d}),
  (<a,b>,{a,b}), (<a,b>,{a,c}), (<a,b>,{a,d}), (<a,b>,{b,c}),
  (<a,b>,{b,d}), (<a,b>,{c,d}), (<a,b>,{a,b,c}), (<a,b>,{a,b,d}),
  (<a,b>,{a,c,d}), (<a,b>,{b,c,d}), (<a,b>,{a,b,c,d}), (<c>,{}),
  (<c>,{a}), (<a>,{b}), (<a>,{c}), (<c>,{a,b}), (<c>,{a,c}),
  (<c>,{b,c}), (<c>,{a,b,c}), (<c,d>,{}), (<c,d>,{a}), (<c,d>,{b}),
  (<c,d>,{c}), (<c,d>,{d}), (<c,d>,{a,b}), (<c,d>,{a,c}),
  (<c,d>,{a,d}), (<c,d>,{b,c}), (<c,d>,{b,d}), (<c,d>,{c,d}),
  (<c,d>,{a,b,c}), (<c,d>,{a,b,d}), (<c,d>,{a,c,d}), (<c,d>,{b,c,d}),
  (<c,d>,{a,b,c,d}) }
```

Example

```
failures(EXT) = {
  (<>, {}),         , (<>,{b}),         , (<>,{d}),           ,
            ,           ,           , (<>,{b,d}),           ,
              ,             ,             ,             ,
  (<a>,{}), (<a>,{a}), (<a>,{c}), (<a>,{d}), (<a>,{a,c}),
  (<a>,{a,d}), (<a>,{c,d}), (<a>,{a,c,d}), (<a,b>,{}),
  (<a,b>,{a}), (<a,b>,{b}), (<a,b>,{c}), (<a,b>,{d}),
  (<a,b>,{a,b}), (<a,b>,{a,c}), (<a,b>,{a,d}), (<a,b>,{b,c}),
  (<a,b>,{b,d}), (<a,b>,{c,d}), (<a,b>,{a,b,c}), (<a,b>,{a,b,d}),
  (<a,b>,{a,c,d}), (<a,b>,{b,c,d}), (<a,b>,{a,b,c,d}), (<c>,{}),
  (<c>,{a}), (<a>,{b}), (<a>,{c}), (<c>,{a,b}), (<c>,{a,c}),
  (<c>,{b,c}), (<c>,{a,b,c}), (<c,d>,{}), (<c,d>,{a}), (<c,d>,{b}),
  (<c,d>,{c}), (<c,d>,{d}), (<c,d>,{a,b}), (<c,d>,{a,c}),
  (<c,d>,{a,d}), (<c,d>,{b,c}), (<c,d>,{b,d}), (<c,d>,{c,d}),
  (<c,d>,{a,b,c}), (<c,d>,{a,b,d}), (<c,d>,{a,c,d}), (<c,d>,{b,c,d}),
  (<c,d>,{a,b,c,d}) }
```

Question

Internal choice is sometimes referred to as ‘nondeterministic choice’.

In general, a process of the form `P|~|Q` will indeed be nondeterministic, although this isn’t always the case.

Can you give a simple example?

Question

Similarly, an external choice between two processes need not be deterministic.

Can you give a simple example?

nondeterminism is a semantic property

Determinism checking

The tool allows us to check whether a process `P` is deterministic, using the following assertion:

```
assert P :[deterministic[F]]
```

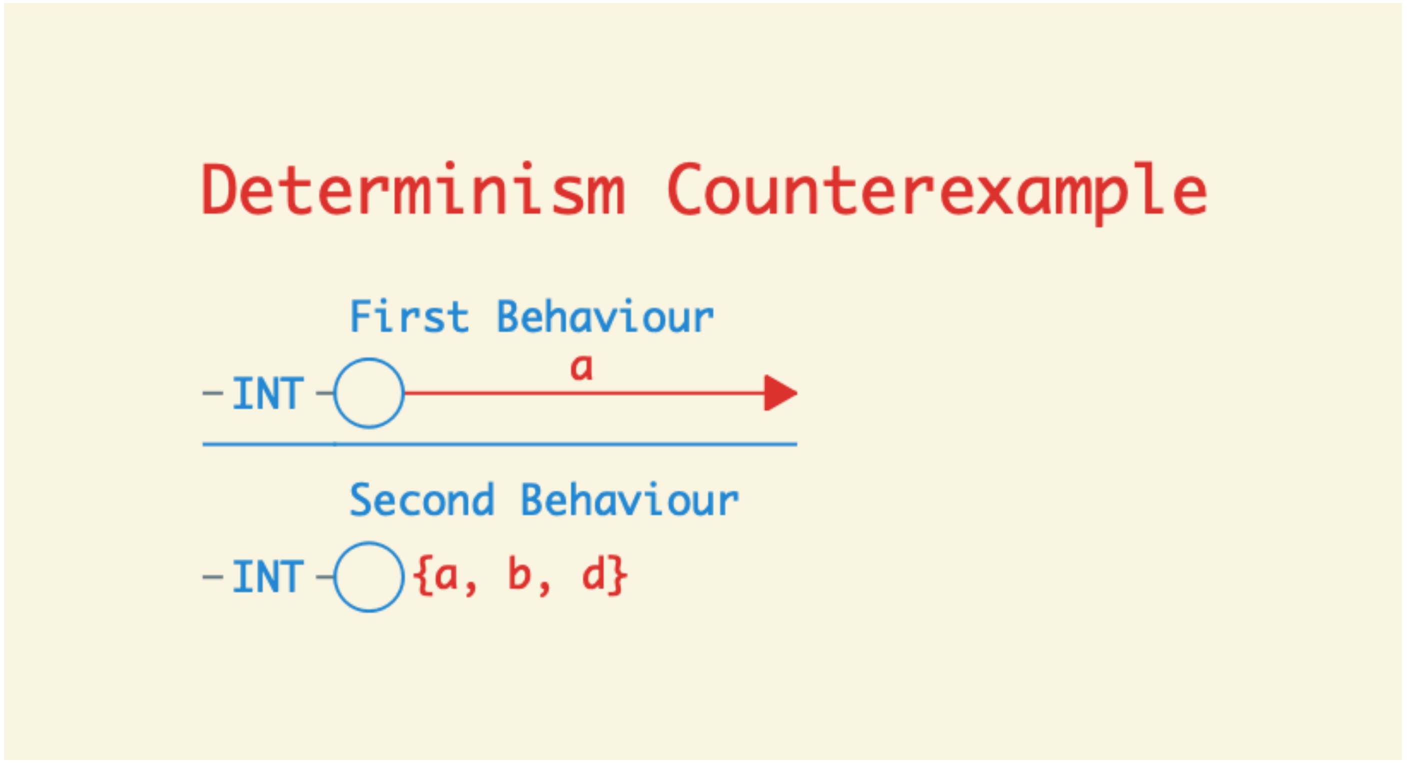
This assertion will use the failures model `F`.

Example

```
assert EXT :[deterministic[F]]

assert INT :[deterministic[F]]
```

The second assertion will fail.

Example


Determinism Counterexample
First Behaviour
INT
a
Second Behaviour
INT
{a, b, d}

Using different models

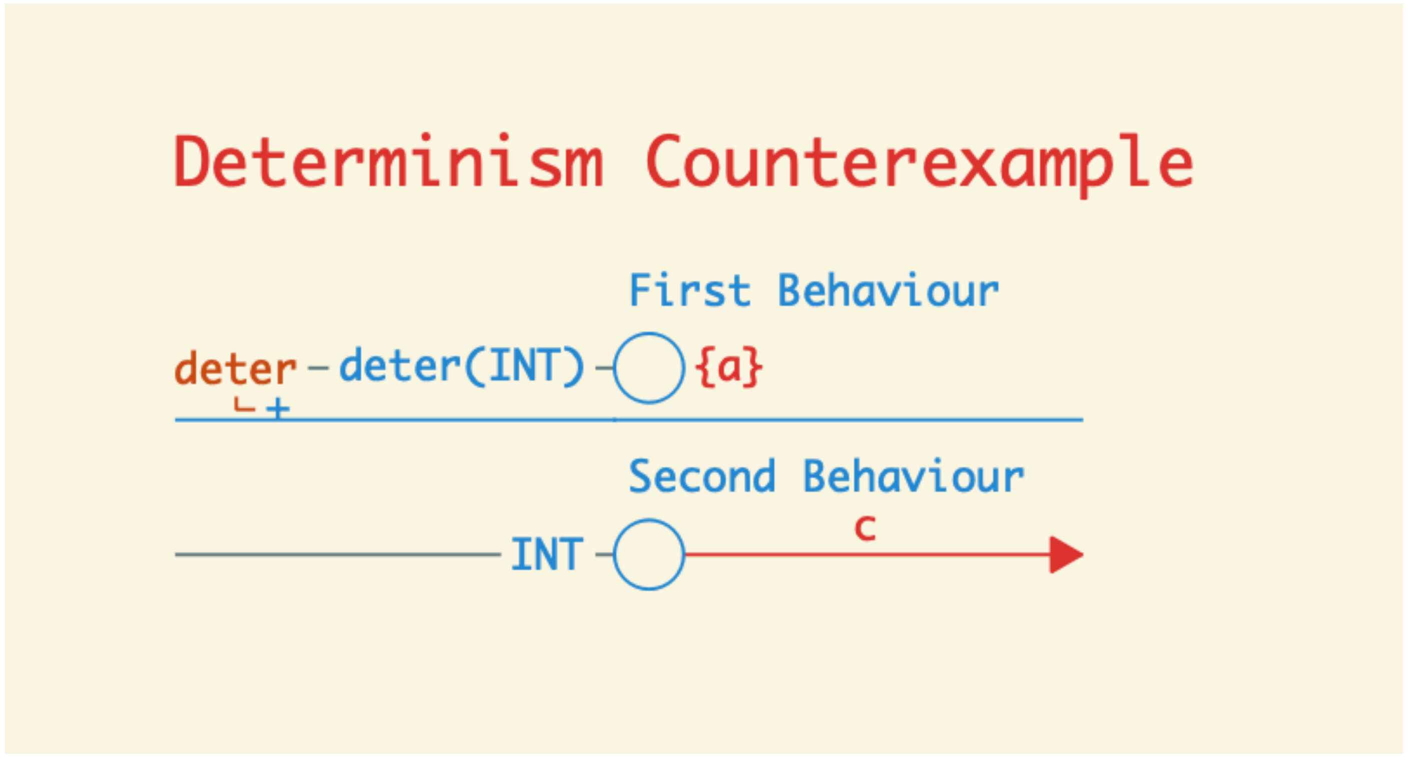
If we omit the model parameter `[F]`, or replace it with the default `[FD]`, then the check will use the failures-divergences model instead. It will then check for divergences at the same time.

We may prefer to check for this separately, using the assertion

```
assert P :[divergence free]
```

The algorithm employed for the failures-divergences model is more efficient, but the explanation provided is less accessible.

Example


Determinism Counterexample
First Behaviour
deter
deter(INT)
{a}
+
Second Behaviour
INT
c

Testing

We can establish properties of processes by checking to see whether a particular trace or failure is included in the semantics.

This can help to confirm that a process has the intended meaning, whether we are using that process as a specification, an assumption, or as part of a design.

May

If `<a,b>` is a trace of `P` then, after the event `a` has occurred, that process may allow `b`.

If `(<a>,{b})` is a failure of `P` then, after the event `a` has occurred, that process may refuse `b`.

At least one of the two premisses must be true. A process must either allow or refuse each event.

Must

If `<a,b>` is not a trace of `P` then, after the event `a` has occurred, that process must refuse `b`.

If `(<a>,{b})` is not a failure of `P` then, after the event `a` has occurred, that process must allow `b`.

Both premisses may be false, in which case the process is nondeterministic.

I can haz trace

The tool allows us to check whether a trace `tr` is a trace of process `P` using the following assertion:

```
assert P :[has trace[T]]: tr
```

I can haz failure?

There is no assertion that can be used to check for the presence of a specific failure in the semantics of a process.

However, we can check whether a trace is deterministically available: that is, none of the events could also have been refused along the way.

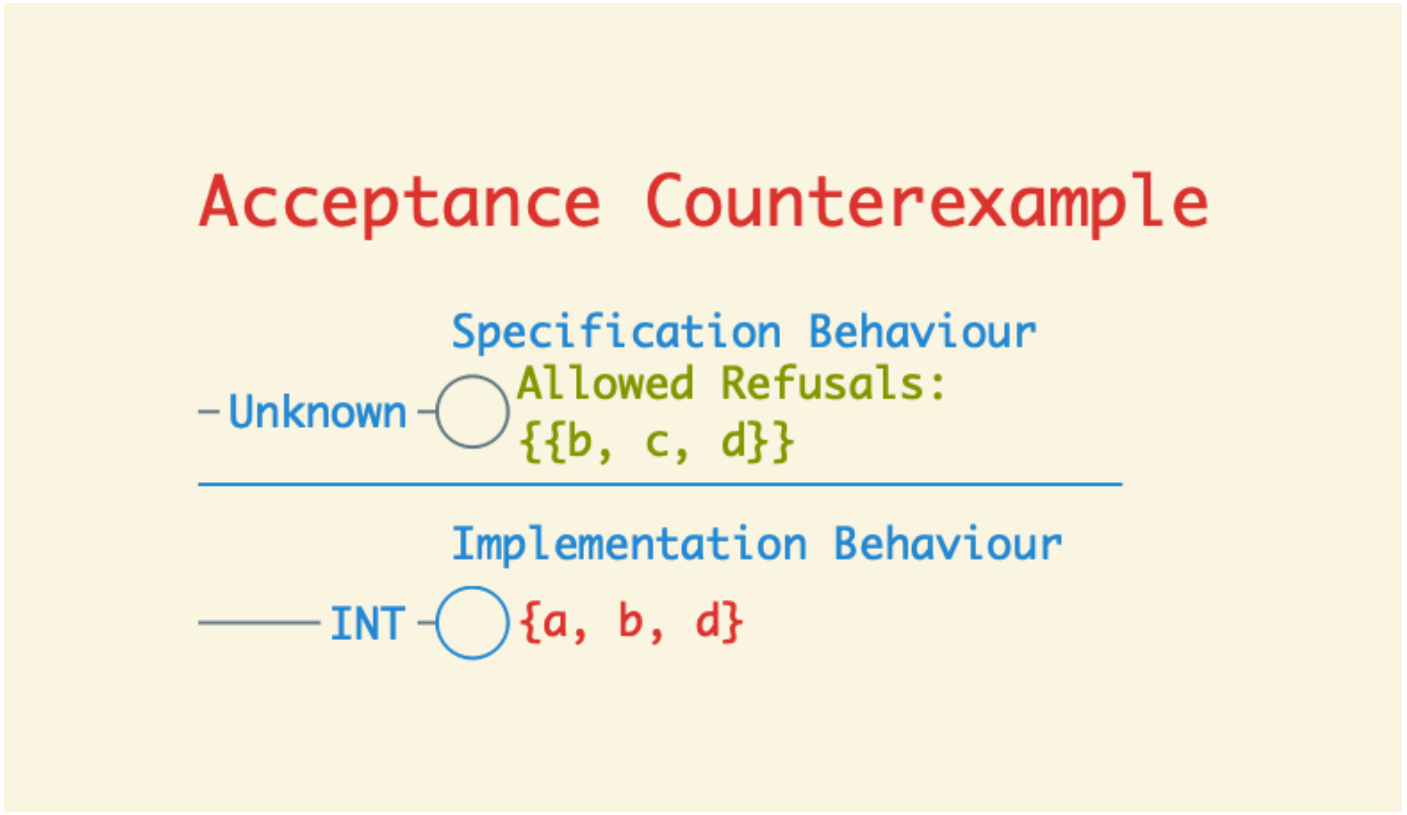
```
assert P :[has trace[F]]: tr
```

Example

```
assert EXT :[has trace[T]]: <a,b>

assert INT :[has trace[T]]: <a,b>

assert EXT :[has trace[F]]: <a,b>

assert INT :[has trace[F]]: <a,b>
```

The fourth assertion will fail.

Example


Acceptance Counterexample
Specification Behaviour
Unknown
Allowed Refusals:
{{b, c, d}}
Implementation Behaviour
INT
{a, b, d}

Comparing processes

Most of the time, however, we will wish to compare two processes—to test one process against another.

If each process has a finite number of states, then it will be possible to check all of the possible behaviours with a single test.

even if the sets of traces and failures are infinite

Refinement

If every behaviour of process `P` is also a behaviour of process `Q`, then we say that `P` is a refinement of `Q`.

We write this as:

```
Q [M= P
```

where `M` is one of our semantic models:

- `T` traces
- `F` failures
- `FD` failures-divergences

every behaviour of `P` is allowed by `Q`

Example

`failures(EXT)` is a subset of `failures(INT)`, and so

```
  INT [F= EXT
```

as we move from right to left, we are refining, eliminating behaviours, and—for the failures model—increasing determinism

Definition

refinement, *n.*

1. The refining of a substance or product; esp. the removing of impurities or unwanted elements.
2. The improvement, modification, or clarification of a faculty, product, mechanism.

Oxford English Dictionary

Trace refinement

If `P` and `Q` are processes, then we write

```
P [T= Q
```

to indicate that `P` is trace-refined by `Q` or—equally—that `Q` is a trace refinement of `P`.

Trace refinement is simply the removal of possible traces.

in the traces model, we cannot see nondeterminism

Trace refinement (restated)

If P and Q are processes, then

```
P [T= Q
```

if and only if every trace of Q is also a trace of P.

In moving from P to Q, we have simply removed some* of the traces from P.

*or possibly none

Example

```
(a -> b -> STOP [] c -> d -> STOP)

[T=

(a -> b -> STOP)
```

Example

```
a -> b -> STOP

[T=

a -> STOP
```

Using trace refinement

If `Spec` describes a set of good traces, then the refinement

```
  Spec [T= Impl
```

holds if and only if every trace of `Impl` is good.

Safety

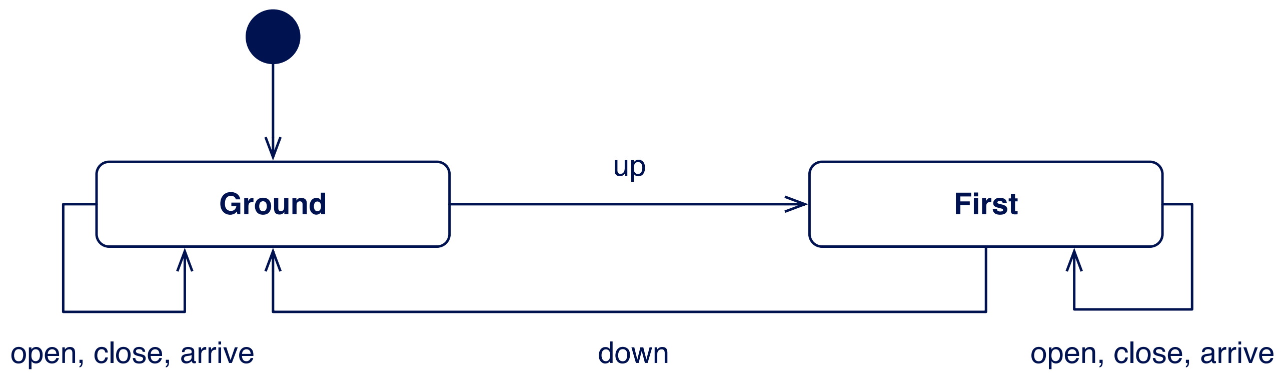
Trace refinement allows us to check what are sometimes called safety properties.

These are assertions that ‘nothing bad happens’. For our process models, this can be interpreted as

- no transaction takes place when it shouldn’t – or, equally
- every trace is a good trace

just because a system satisfies this kind of ‘safety property’, it doesn’t mean that it is ‘safe’ in other senses of the word: refusing to perform a transaction might also be ‘a bad thing’ that happens.

Example



focussing on the ups and downs

Example

```
Spec =
  let
    Ground =
      up -> First
      []
    ( [] e : {open,close,arrive} @ e -> Ground )

    First =
      down -> Ground
      []
    ( [] e : {open,close,arrive} @ e -> First )
  within
    Ground
```

Example

We can use the tool to check that this specification is satisfied by our lift controller design:

```
assert Spec [T= LiftController
```

what goes up must come down before it goes up again

Example

```
Spec [T= STOP
```

STOP will satisfy any safety property

a lift that never moves is safe, in this respect

Trace equivalence

For any processes `P` and `Q`, if

```
P [T= Q
```

and

```
Q [T= P
```

then

```
traces(P) = traces(Q)
```

this does not mean that `P = Q` unless…

Determinism

If `P` and `Q` are both deterministic, then

```
  traces(P) = traces(Q)
```

is enough to show that `P = Q`.

Example

If P and Q are defined by

```
P = a -> b -> P [] c -> d -> P
Q = c -> d -> Q [] a -> b -> Q
```

then the four assertions

```
assert P :[deterministic]
assert Q :[deterministic]
assert P [T= Q
assert Q [T= P
```

would be enough to confirm that `P = Q`.

Failures refinement

If `P` and `Q` are processes, then we write

```
P [F= Q
```

to indicate that `P` is failures-refined by `Q` or – equally – that `Q` is a failures refinement of `P`.

In the failures semantics, we know which failures correspond to nondeterministic choices, and failures refinement is precisely the removal of nondeterminism.

Failures refinement

If P and Q are processes, then

```
P [F= Q
```

if and only if every failure of Q is also a failure of P.

In moving from P to Q, we may have removed some of the failures, but only those that correspond to one half of a nondeterministic choice.

Example

```
(a -> b -> STOP |~| c -> d -> STOP)

[F=

 a -> b -> STOP
```

Example

```
(a -> b -> STOP [] c -> d -> STOP)
```

is not failures-refined by

```
 a -> b -> STOP
```

Using failures refinement

If `Spec` describes a set of good failures, then the refinement

```
Spec [F= Impl
```

holds if and only if every failure of `Impl` is good.

Using failures refinement

If `Spec` describes a set of good failures, then the refinement

```
Spec [F= Impl
```

holds if and only if every trace allowed by `Impl` is good, and every set of events that may be refused by `Impl` is good.

good = included as a (possibly nondeterministic) option in `Spec`

Liveness

Failures refinement allows us to check what are sometimes called liveness properties.

These are assertions that ‘something good happens eventually’. For our process models, this can be interpreted as

- no set of transactions is refused when it shouldn’t be – or, equally
- no additional requirement is imposed, in terms of other transactions that would have to happen first

Example

In the case of the `LiftController` design, we may wish to insist that the lift doors can be opened at any time.

```
assert

Spec1 =
  let
    CanOpen = open -> CanOpen
  within
    CanOpen
```

However,

```
not Spec1 [F= LiftController
```

for two reasons!

(1) other things can happen; (2) it isn't true, anyway

Example

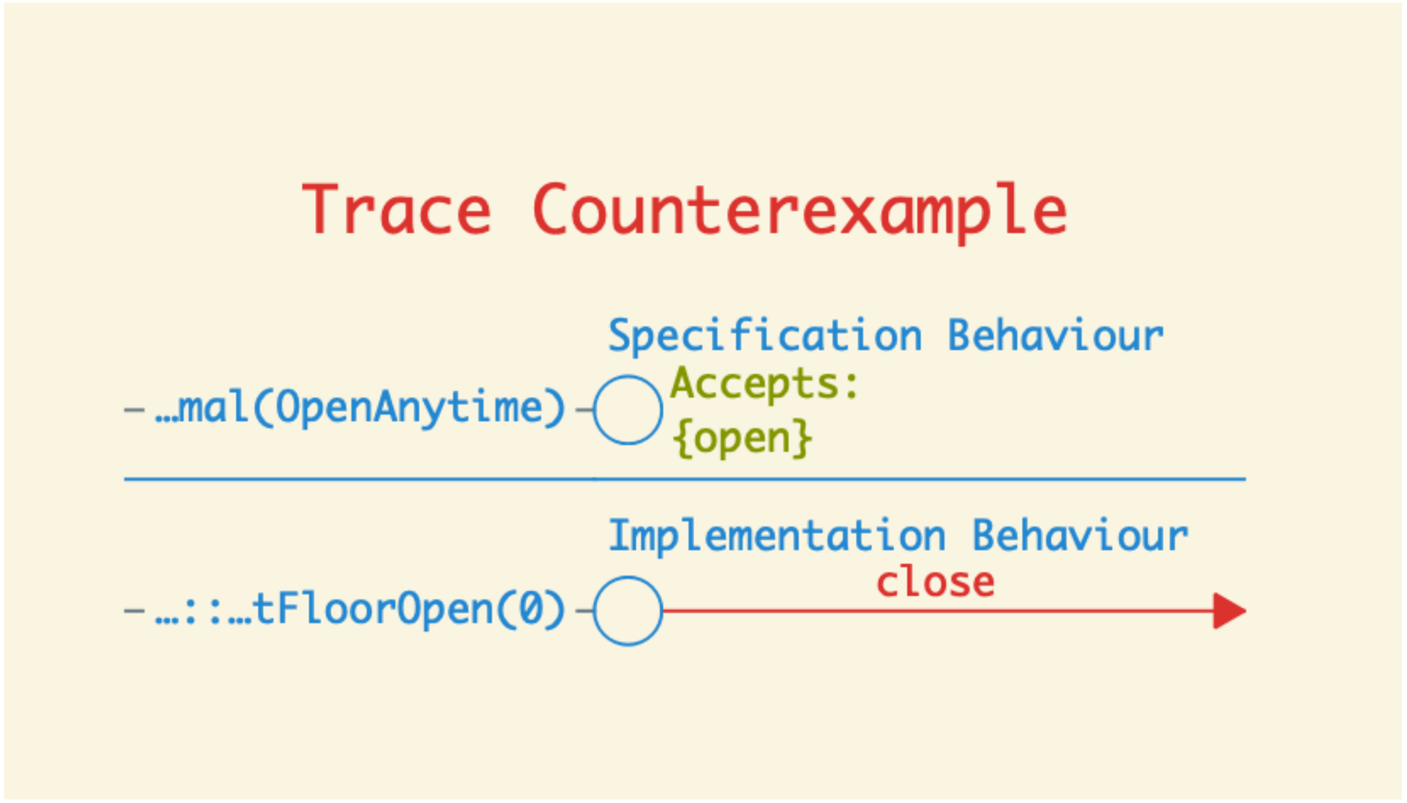
If we use the tool to check the assertion

```
  assert Spec1 [F= LiftController
```

then it will fail.

The specification is too strong to be satisfied

Example

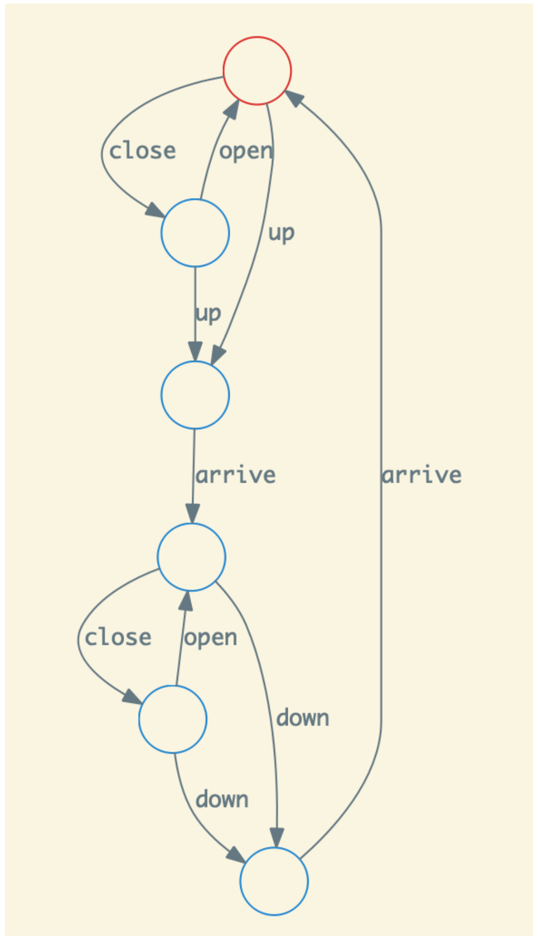

Trace Counterexample
…mal(OpenAnytime)
Specification Behaviour
Accepts:
{open}
…::…tFloorOpen(0)
Implementation Behaviour
close

Example

To see what is happening, as `LiftController` has relatively few states, we may examine the state graph:

```
:graph LiftController
```

Example

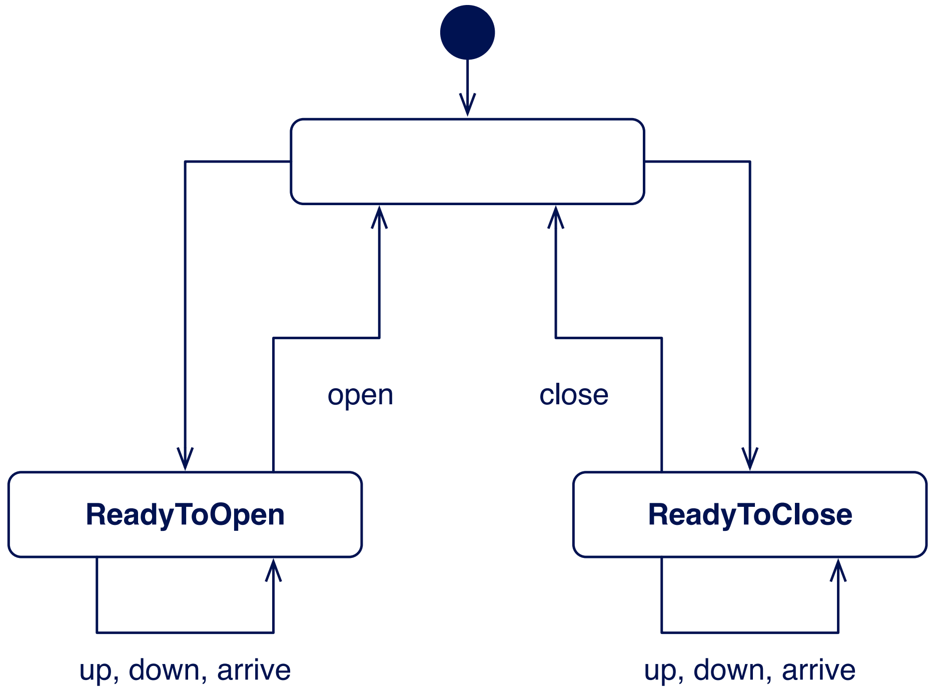


Example

In the specification, we need to allow that other events may occur.

```
OpenOrClose =
  let
    Start =
      ReadyToOpen |~| ReadyToClose

    ReadyToOpen =
      open -> OpenOrClose
      []
      (|~| e : {up,down,arrive} @ e -> ReadyToOpen)
```

Example


ReadyToOpen
ReadyToClose
open
close
up, down, arrive
up, down, arrive

Example

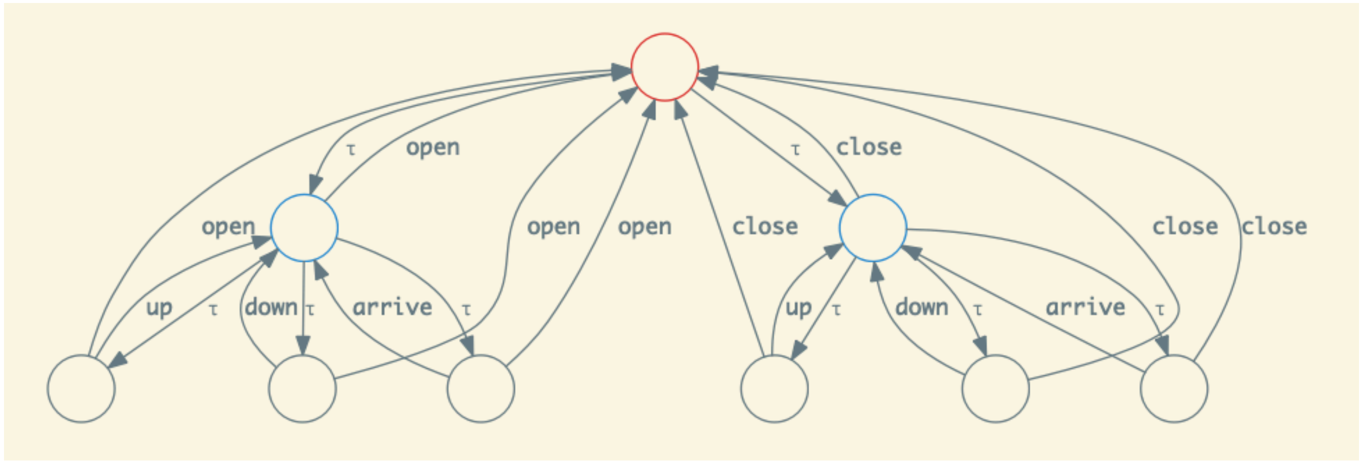
The specification is still too strong, and the assertion

```
  assert OpenOrClose [FD= LiftController
```

fails.

We need more nondeterminism.

Example

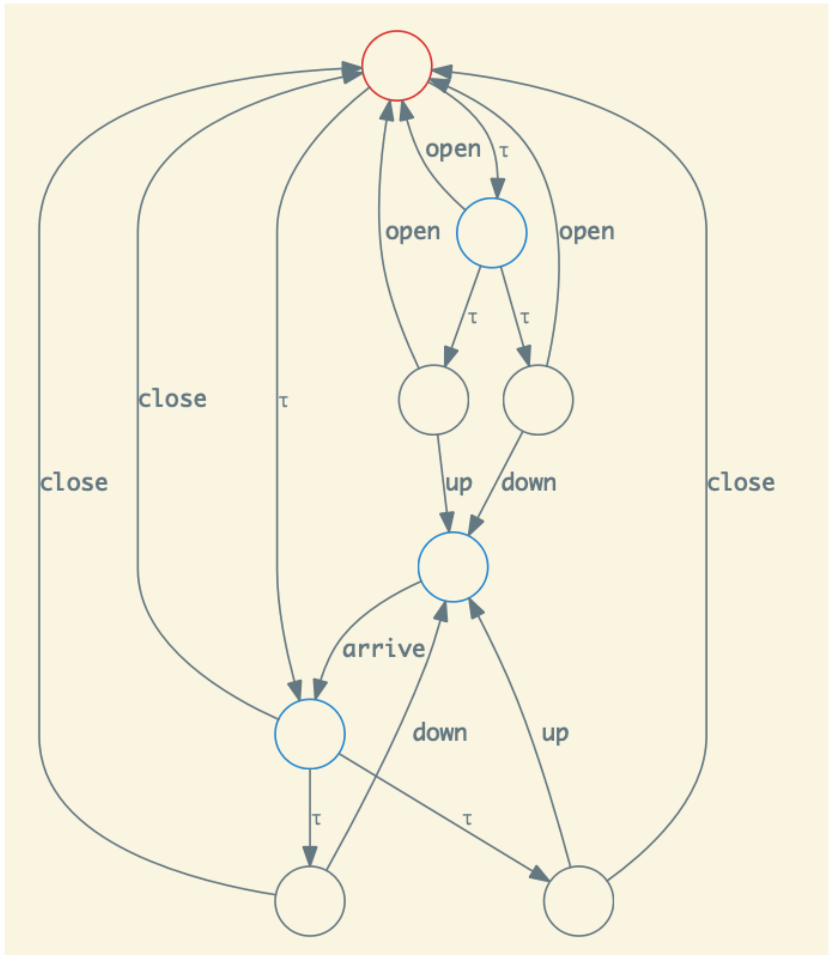


(note that silent transitions are decorated with a tau symbol)

Example

```
OpenOrCloseUnlessMoving =
let
  Start =
    ReadyToOpen |~| ReadyToClose

  ReadyToOpen =
    open -> OpenOrCloseUnlessMoving
    []
    (|~| e : {up,down} @ e -> Moving)
  ReadyToClose =
    close -> OpenOrCloseUnlessMoving
    []
    (|~| e : {up,down} @ e -> Moving)

  Moving = arrive -> ReadyToClose
within
  Start
```

this will do the trick—hard to draw, though!


open
τ
open
open
τ
τ
close
τ
close
up
down
close
arrive
down
up
τ
τ

Footnote: refusal sets

In the failures model, we consider whether or not a process, after allowing a particular trace, can refuse any one of a set of events. This allows us to properly model the behaviour of processes in a concurrent context.

If we were instead to consider whether or not the process might refuse each different event—one event at a time, rather than in combination—then we could build a semantics that would work for event prefix, external choice, internal choice, and parallel composition.

Such a semantics would let us down badly, however, when it came to abstraction or hiding.

With refusal sets, we get the semantics we need.

Compositionality

If we establish that `P` is refined by `Q`, then that result remains valid in any context.

A refinement of any component produces a refinement of the system. If `P [= Q`, then

- `a -> P [= a -> Q`
- `P [] R [= Q [] R`
- `P |~| R [= Q |~| R`
- `P || R [= Q || R`
- `P \ A [= Q \ A`

Summary

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