UNIVERSITY OF OXFORD
SOFTWARE ENGINEERING PROGRAMME

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Part-time postgraduate study in software engineering

Algorithmics, ALG

2nd – 6th October 2023
ASSIGNMENT

The purpose of this assignment is to test the extent to which you have achieved the learning objectives of the course. As such, your answer must be substantially your own original work. Where material has been quoted, reproduced, or co-authored, you should take care to identify the extent of that material, and the source or co-author.

Your answers to the questions on this assignment should be submitted using the Software Engineering Programme website — `www.softeng.ox.ac.uk` — following the submission guidelines. When submitting the assignment online, it is important that you formally complete all three assignment submission steps: step 1, read through the declaration; step 2, upload your files; step 3, check your files. **Please ensure your submission is anonymous: do not include your name or any other identifying information on the assignment, nor in accompanying material such as source code, nor within the file names of anything submitted.**

The deadline for submission is 12 noon on Tuesday, 21st November 2023. You are strongly encouraged to submit a version well before the deadline. You may update your submission as often as you like before the deadline, but no submissions or changes will be accepted after the deadline.

We hope to have preliminary results and comments available during the week commencing Monday, 8th January 2024. The final results and comments will be available after the subsequent examiners' meeting.

ANY QUERIES OR REQUESTS FOR CLARIFICATION REGARDING THIS ASSIGNMENT OR PROBLEMS INSTALLING SOFTWARE SHOULD, IN THE FIRST INSTANCE, BE DIRECTED TO THE PROGRAMME OFFICE WITHIN THE NEXT TWO WEEKS.

Introduction

The aim of this assignment is for you to demonstrate your understanding of algorithmics, as taught on the course.

The assignment consists of four parts: you should attempt to solve all four. One sub-task is marked with a star ($\star$), which indicates a higher level of difficulty — you might choose to work on the other questions first. In general, support your answers with reasoned arguments. In particular, even if the task takes the form of a polar question, a simple "yes" or "no" answer is not good enough. You have to justify your answer, for example, by providing an informal argument, a formal proof, or a counter-example.

1 Assignment

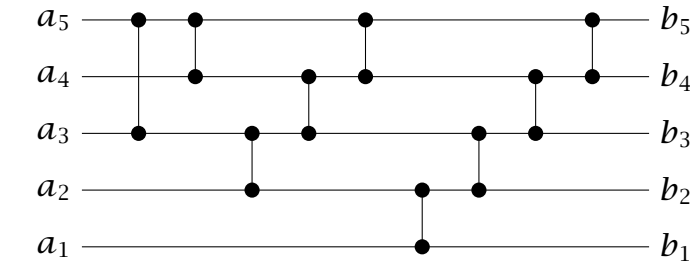
1.1 Comparison Networks

(a) Can you construct a *comparison* network — not necessarily a sorting network — that maps the input $a_1 = 0$, $a_2 = 1$, $a_3 = 0$, $a_4 = 1$, $a_5 = 1$ to the output $b_1 = 1$, $b_2 = 0$, $b_3 = 1$, $b_4 = 0$, $b_5 = 1$?

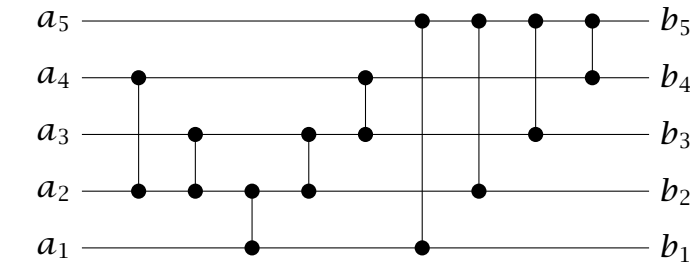
(b) Figure 1 displays several comparison networks. Determine size and depth of each network.

(c) Which networks are actually sorting networks? Justify your answer.

(d)$\star$ Given a comparison network with n inputs, how would you approach the general problem of deciding whether the network is a sorting network? How difficult or complex is this decision problem? Think about possible certificates that testify whether a given network is a sorting network or not. In case you find it useful, assume that the input elements are either 0 or 1. (Does this assumption simplify the problem? Does it restrict the generality of your findings?)



(i)



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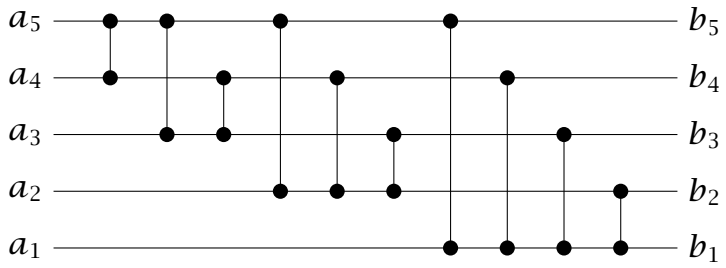

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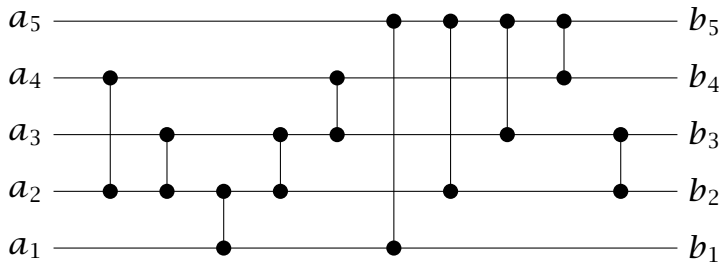


(iv)

Figure 1: Some example comparison networks.

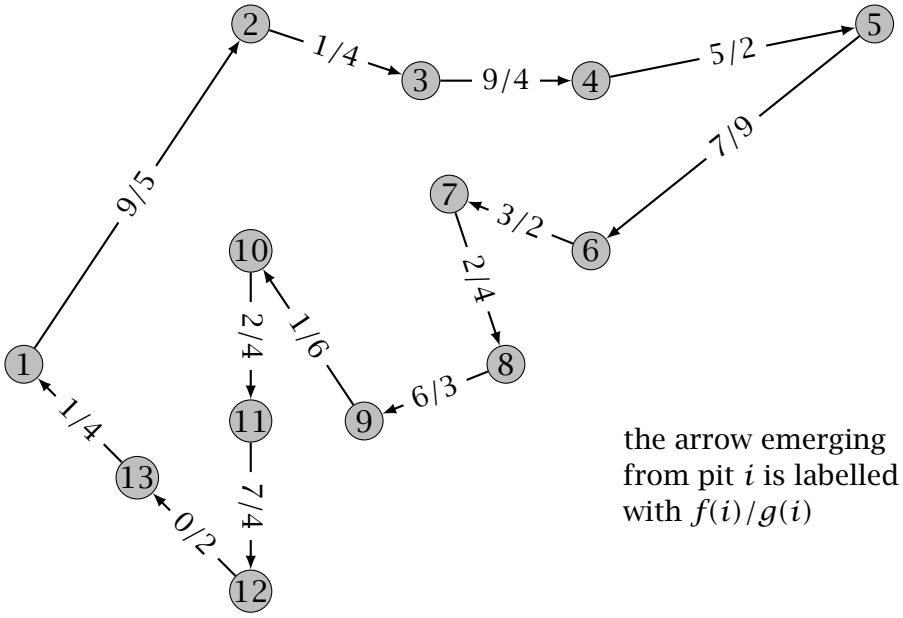


Figure 2: Pits along the Hockenheimring.

1.2 Searching Algorithms

There are n pits located along a circular race-track. The pits are numbered clockwise from 1 through n. There are $f(i)$ litres of petrol available at pit i. To race from pit i to its clockwise neighbour one needs $g(i)$ litres of petrol. Determine a pit from which it is possible to race a complete lap, starting with an empty but sufficiently large fuel tank. To guarantee the existence of such a starting pit you can assume that:

$$\sum_{1 \leqslant i \leqslant n} f(i) = \sum_{1 \leqslant i \leqslant n} g(i).$$

In case you need some data to play with, Figure 2 sketches an example race-track.

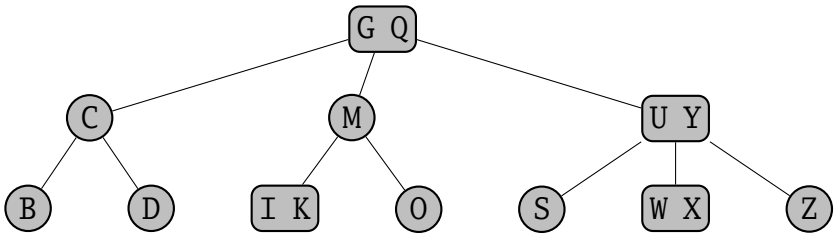
(a) Describe an algorithm that solves the problem. (You may wish to provide some pseudo-code or, perhaps, a concrete program in the programming language of your choice.)

(b) Show that your algorithm is correct, for example, by exhibiting a suitable invariant.

(c) Analyse the running-time of your algorithm. Is your algorithm asymptotically optimal?

1.3 2-3 Search Trees

Consider the 2-3 search tree shown below:



(a) Insert the element P into the tree.

(b) Insert the element V into the tree.

(c) Delete the element D from the tree.

(d) Delete the element Q from the tree.

Note that each part should be solved independent of the others (i.e. always insert into or delete from the original tree, *not* the tree from the previous part). For each insertion and deletion show the resulting tree and *all* the intermediate steps (see Slides 368 and 375 entitled "Rebalancing: Example").

1.4 Planning Wind Power Plants

Professor John Boredom is consulting for Hurricane Inc., a company specialised in the planning and construction of wind power plants. Currently, the company is planning a large transmission line running north to south through the state of Illusioning. The company wants to connect each wind power plant to the north-south main transmission line along a shortest route (either west or east), as shown in Figure 3. Given the x- and y-coordinates of the wind power plants, how should Professor John Boredom pick the location of the main transmission line so that the total length of the west-east lines is minimised? (Here are the locations of the wind power plants shown in Figure 3,

$$\begin{array}{l} \textit{val windfarm} = \\ \quad [\{x = 12, y = 22\}, \{x = 16, y = 38\}, \{x = 18, y = 30\}, \\ \quad\ \{x = 23, y = 23\}, \{x = 22, y = 35\}, \{x = 36, y = 26\}, \\ \quad\ \{x = 32, y = 36\}, \{x = 40, y = 35\}\,] \end{array}$$

just in case you would like to have some numbers to play with.)

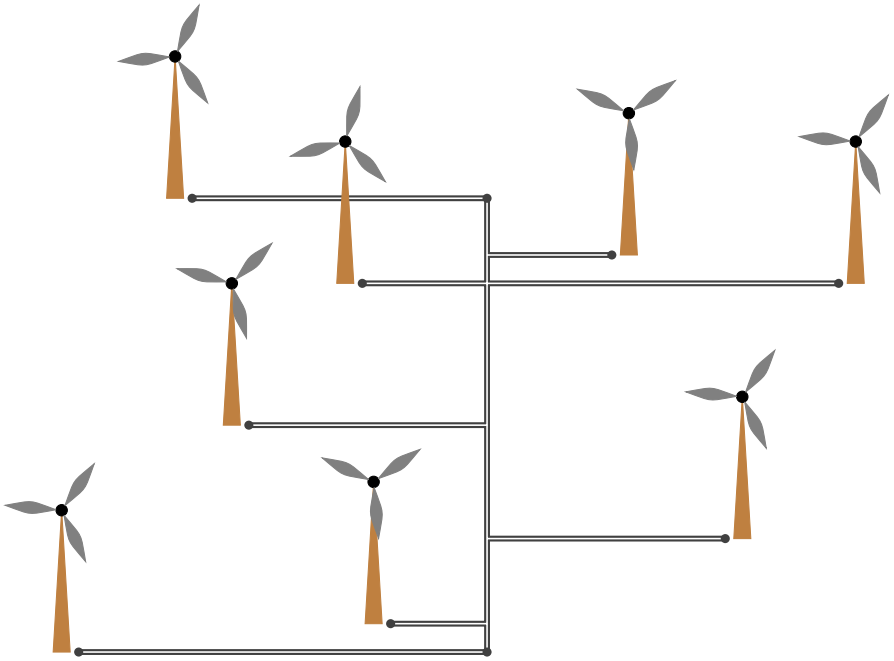


Figure 3: Wind Power Plants in Illusioning.

Write a short essay of no more than 1000 words on the subject of planning wind power plants. Try to systematically apply the different algorithmic methods discussed in the lectures. Perhaps you can also think of other approaches? For each approach, discuss the workings of the algorithm, both informally and formally, and estimate its running-time. Does the problem become harder if the rigid geometric layout is relaxed?

Support your arguments with diagrams, pictures, pseudo-code, or program snippets (in the programming language of your choice) as you see fit.

2 Guidance

You are explicitly encouraged to consult textbooks on algorithms and data structures. You may also be able to find (partial) solutions to the problems on the web. I recommend that you don't use them, or at least, don't rely on them: they are often of dubious quality. Whatever you do, do make sure you *make clear the source and the extent* of any derivative material.

Ralf Hinze
October 2023

3 Assessment criteria

The assessment will be based on the following criteria. The first and the second carry slightly more weight than the third and last.

- Have you understood fundamental algorithms and data structures e.g. for sorting and searching?
- Have you demonstrated an ability to apply algorithmic methods to a specific example problem?
- Do you have an appreciation of the limits of mechanisation both in practical and theoretical terms?
- Do you have the ability to present clear arguments supporting design decisions and discussing trade-offs, concisely and precisely?