

4.3

1. $f(x) = \begin{cases} c(1-x) & 0 \leq x < 1 \\ 0 & \text{其他} \end{cases}$
 \therefore 为密度函数

(1) $\int_{-\infty}^{+\infty} f(x) dx = 1 = c \int_0^1 (1-x) dx = c(x - \frac{1}{2}x^2)|_0^1 = 1 \therefore c = 2$

(2) $P(X \leq \frac{1}{2}) = \int_{-\infty}^{\frac{1}{2}} 2(1-x) dx = \int_0^{\frac{1}{2}} 2(1-x) dx + \int_{-\infty}^0 0 dx = 2(x - \frac{1}{2}x^2)|_0^{\frac{1}{2}} = \frac{3}{4}$

(3) $P(X \leq \frac{1}{3})$ 同理可得 $= 2(x - \frac{1}{2}x^2)|_0^{\frac{1}{3}} = \frac{4}{9}$

2. $f(x) = \begin{cases} 2x & 0 < x < A \\ 0 & \text{其他} \end{cases}$

$\int_{-\infty}^{+\infty} f(x) dx = 1 = \int_0^A 2x dx = 1 = x^2|_0^A = 1 \therefore A = 1$
 $\therefore f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{其他} \end{cases}$

$P(X < 0.5) = \int_0^{0.5} 2x dx = x^2|_0^{0.5} = \frac{1}{4}$

X 的分布函数: $F(x) = \int_{-\infty}^{+\infty} f(x) dx = \begin{cases} x^2 & x \geq 1 \\ 0 & x < 0 \\ x^2 & 0 \leq x < 1 \end{cases}$



3. (1) $F(x) = \int_{-\infty}^{+\infty} f(x) dx = \begin{cases} 0 & x \leq 0 \\ 1 - (1+x)e^{-x} & x > 0 \end{cases}$
 $f(x) = \begin{cases} 0 & x \leq 0 \\ xe^{-x} & x > 0 \end{cases}$

(2) $P(X \leq 1) = \int_{-\infty}^1 f(x) dx = F(1) = 1 - 2e^{-1}$
 $P(X > 2) = (1 - F(2)) = 3e^{-2}$

4. $F(x) = \int_{-\infty}^{+\infty} f(x) dx = A + B \arctan x$
 $\lim_{x \rightarrow -\infty} F(x) = 0 \quad \lim_{x \rightarrow +\infty} F(x) = 1 \therefore A = -\frac{\pi}{2} \quad B = \frac{\pi}{2} \therefore F(x) = -\frac{\pi}{2} + \frac{\pi}{2} \arctan x$
 $P(-1 < X < 1) = F(1) - F(-1) = \frac{1}{2}$
 $\int_{-\infty}^{+\infty} f(x) dx = -\frac{\pi}{2} + \frac{\pi}{2} \arctan x \quad f(x) = \frac{1}{\pi(1+x^2)}$

11. $\mu = -1 \quad \sigma = 4$

$P(X < 2.44) = \Phi(\frac{-1+2.44}{4}) = \Phi(\frac{1.44}{4}) = \Phi(0.36) = 0.64$

$P(X > -1.5) = 1 - P(X \leq -1.5) = 1 - \Phi(\frac{-1-1.5}{4}) = 1 - \Phi(-\frac{2.5}{4}) = 1 - (1 - \Phi(\frac{2.5}{4})) = \Phi(\frac{2.5}{4}) = 0.5987$

$P(X < -2.8) = \Phi(\frac{-1-2.8}{4}) = \Phi(-\frac{3.8}{4}) = 1 - \Phi(\frac{3.8}{4}) = 1 - 0.6985 = 0.3015$

$P(|X| < 4) = P(-4 < X < 4) = \Phi(\frac{4+1}{4}) - \Phi(\frac{-4+1}{4}) = \Phi(1.25) - \Phi(-0.75) = 0.8944 - 0.2264 = 0.668$

$P(-5 < X < 2) = \Phi(\frac{2+1}{4}) - \Phi(\frac{-5+1}{4}) = \Phi(0.75) - \Phi(-1) = 0.7744 - 0.1587 = 0.6157$

$P(X-1 > 1) = P(X > 2) = 1 - P(X \leq 2) = 1 - \Phi(\frac{2+1}{4}) = 1 - \Phi(0.75) = 1 - 0.7744 = 0.2256$

$$P(X \leq a) = P(X > a) = \Phi\left(\frac{a+1}{4}\right) = 1 - \Phi\left(\frac{a+1}{4}\right) = \Phi\left(\frac{a+1}{4}\right) = \frac{1}{2} \quad \frac{a+1}{4} = 0 \quad a = -1.$$

13. $\mu = 2.05, \sigma = 0.1$

由题意可知: $P(1.8 \leq X \leq 2.2) = \Phi\left(\frac{2.2-2.05}{0.1}\right) - \Phi\left(\frac{1.8-2.05}{0.1}\right)$
 $= \Phi(1.5) - (1 - \Phi(2.5)) = \Phi(1.5) + \Phi(2.5) - 1 = 0.927$

5.2.

(1) ~~$P(X=0, Y=0) =$~~

(1)

X \ Y	0	$\frac{1}{2}$	1
0	$\frac{1}{6}$	0	$\frac{5}{12}$
-1	0	$\frac{1}{12}$	$\frac{1}{3}$

(2) $P(X=1, Y=2) = \frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$
 $P(X=1, Y=3) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$
 $P(X=2, Y=1) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
 $P(X=2, Y=3) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
 $P(X=3, Y=1) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$
 $P(X=3, Y=2) = \frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$
 $P(X=2, Y=2) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

X \ Y	1	2	3
1	0	$\frac{2}{12}$	$\frac{1}{12}$
2	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
3	$\frac{1}{12}$	$\frac{1}{6}$	0

$P(X=2) = P(X=2; Y=2) = \frac{1}{6}$