

6.1

| | | | | | |
|-------|---------------|---------------|---------------|---------------|---------------|
| $x+2$ | 0 | $\frac{3}{2}$ | 2 | 4 | 6 |
| 概率 | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{6}$ | $\frac{1}{3}$ |

| | | | | | |
|--------|---------------|---------------|---------------|---------------|---------------|
| $-x+1$ | 3 | $\frac{3}{2}$ | 1 | -1 | -2 |
| 概率 | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{6}$ | $\frac{1}{3}$ |

| | | | | |
|-------|----------------|---------------|----------------|---------------|
| x^2 | 4 | $\frac{1}{4}$ | 0 | 16 |
| 概率 | $\frac{7}{24}$ | $\frac{1}{4}$ | $\frac{1}{64}$ | $\frac{1}{9}$ |

2. 易见 $Y = \frac{1}{2}X$ 不是一个连续随机变量, 它可能取值为 ~~1~~ 0, 1

$$P(Y=0) = P(X \leq 1) = \frac{1}{2} + P(X=0) = 2e^{-1}$$

$$P(Y=1) = P(X > 1) = 1 - P(X \leq 1) = 1 - 2e^{-1}$$

| | | |
|-----|-----------|---------------|
| Y | 0 | 1 |
| 概率 | $2e^{-1}$ | $1 - 2e^{-1}$ |

3.

(1) $z = 2x$ 为连续单调递增函数 $\therefore h(z) = \frac{z}{2}$ $f_z(z) = f_x(h(z)) |h'(z)|$

$$\therefore f_z(z) = \begin{cases} \frac{z}{2} & 0 < z < 2 \\ 0 & \text{其他} \end{cases}$$

(2) $z = -x+1$ 为连续单调递减函数 $h(z) = -z+1$ $f_z(z) = f_x(h(z)) |h'(z)|$

$$f_z(z) = \begin{cases} 2(1-z) & 0 < z < 1 \\ 0 & \text{其他} \end{cases}$$

(3) $z = x^2$ 在 $(0, 1)$ 上连续单调. $h(z) = \sqrt{z}$

$$f_z(z) = \begin{cases} 1 & 0 < z < 1 \\ 0 & \text{其他} \end{cases}$$

8. 设 $Y = \sigma X + a$ \therefore ~~$(\frac{y-a}{\sigma})^2$~~

$$\text{由于 } Y = \sigma X + a \text{ 为一个单调函数 } h(y) = \frac{y-a}{\sigma} \quad f_y(y) = f_x(h(y)) |h'(y)| = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-a)^2}{2\sigma^2}} \quad (-\infty < y < \infty)$$

$\therefore Y \sim N(a, \sigma^2)$ 仍记

9. $\because X$ 在 $[-1, 2]$ 上均匀分布 $\therefore f_X(x) = \begin{cases} \frac{1}{3} & -1 \leq x \leq 2 \\ 0 & \text{其他} \end{cases}$ $\therefore Y$ 分明 ~~不是~~ 是一个连续下随机变量

$$P(Y=-1) = P(X < 0) = \frac{1}{3} \quad P(Y=0) = P(X=0) = 0 \quad P(Y=1) = P(X \leq 1) = \frac{2}{3}$$

| | | | |
|-----|---------------|---|---------------|
| Y | -1 | 0 | 1 |
| 概率 | $\frac{1}{3}$ | 0 | $\frac{2}{3}$ |

0-6

1. (1)

| | | | | | |
|-------|---------------|---------------|---------------|---------------|---|
| $X+Y$ | 2 | 3 | 4 | 5 | 6 |
| 概率 | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | 0 |

| | | | | | |
|-------|---------------|---------------|---------------|---------------|---------------|
| $X-Y$ | -2 | -1 | 0 | 1 | 2 |
| 概率 | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |

| | | | |
|------|---------------|---------------|---------------|
| $2X$ | 2 | 4 | 6 |
| 概率 | $\frac{5}{8}$ | $\frac{1}{8}$ | $\frac{1}{4}$ |

| | | | | |
|------|---------------|---------------|---------------|---------------|
| XY | 1 | 2 | 3 | 6 |
| 概率 | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |

2. $Z = X+Y$

$X \sim B$

| | | |
|-------|---------------|---------------|
| X/Y | 0 | 1 |
| 概率 | $\frac{3}{4}$ | $\frac{1}{4}$ |

| | | | |
|-----|----------------|----------------|----------------|
| Z | 0 | 1 | 2 |
| 概率 | $\frac{9}{16}$ | $\frac{6}{16}$ | $\frac{1}{16}$ |

$Z = 2X$

| | | |
|-----|---------------|---------------|
| U | 0 | 2 |
| | $\frac{3}{4}$ | $\frac{1}{4}$ |

4. (1) $P(U=1) = P(X=1, Y=1) = \frac{1}{4}$

$P(U=2) = P(X=1, Y=2) + P(X=2, Y=1) = \frac{1}{3}$

$P(U=3) = P(X=3, Y=1) + P(X=1, Y=3) + P(X=2, Y=2) = \frac{5}{9}$

| | | | |
|-----|---------------|---------------|---------------|
| U | 0 | 2 | 3 |
| | $\frac{1}{9}$ | $\frac{1}{3}$ | $\frac{5}{9}$ |

(2) $P(V=1) = P(X=2, Y=1) + P(X=1, Y=2) + P(X=3, Y=1) + P(X=1, Y=3) = \frac{4}{9}$

| | | | |
|-----|---------------|---------------|---------------|
| V | 0 | 2 | 3 |
| | $\frac{1}{9}$ | $\frac{1}{3}$ | $\frac{1}{9}$ |

(3)

| | | | |
|-----|---------------|---------------|---------------|
| U | 1 | 2 | 3 |
| 1 | $\frac{1}{9}$ | 0 | 0 |
| 2 | $\frac{2}{9}$ | $\frac{1}{9}$ | 0 |
| 3 | $\frac{1}{9}$ | $\frac{2}{9}$ | $\frac{1}{9}$ |

$P(U=1, V=1) = P(\max\{X, Y\} = 1, \min\{X, Y\} = 1) = P(X=1, Y=1) = \frac{1}{9}$

同理:

$P(U=2, V=1) = \frac{2}{9}$

$P(U=2, V=2) = \frac{1}{9}$

$P(U=3, V=1) = \frac{2}{9}$

$P(U=3, V=2) = \frac{2}{9}$