

7.3

$$1. \text{cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$E(XY) = 1 \times 1 \times 0.1 = 0.1 \quad E(X) = 0.5 \quad E(Y) = 0.3 \quad E(X^2) = 0.5 \quad E(Y^2) = 0.3$$

$$\text{cov}(X, Y) = 0.1 - 0.5 \times 0.3 = -0.05$$

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma(X)\sigma(Y)} \quad \sigma(X) = \sqrt{D(X)} \quad \sigma(Y) = \sqrt{D(Y)}$$

$$D(X) = E(X^2) - E^2(X) = 0.5 - 0.25 = 0.25 \quad D(Y) = 0.21 \quad \sigma(X) = 0.5 \quad \sigma(Y) = \sqrt{0.21}$$

$$\rho_{X,Y} = \frac{-0.05}{0.5 \sqrt{0.21}} = \frac{-\sqrt{21}}{21}$$

5.

$$D(X+Y) = D(X) + 2E(X-E(X))(Y-E(Y)) + D(Y)$$

$$\text{cov}(X, Y) = E(X-E(X))(Y-E(Y))$$

$$\text{cov}(X, Y) = \rho_{X,Y} \sigma(X) \sigma(Y)$$

$$\sigma(X) = \sqrt{D(X)} = 5 \quad \sigma(Y) = \sqrt{D(Y)} = 6$$

$$\therefore \text{cov}(X, Y) = 6 \times 12 = 72$$

$$\rho_{X,Y} = \frac{72}{5 \times 6} = 2.4$$

$$\therefore D(X+Y) = 25 + 36 + 2 \times 72 = 155$$

$$D(X-Y) = D(X) - 2E(X-E(X))(Y-E(Y)) + D(Y) \\ = 25 + 36 - 2 \times 72 = 37$$

$$\cancel{D(X+Y) = E((X+Y)^2) - E^2(X+Y)}$$

$$\cancel{= E((X+Y)^2) - 2E(X+Y)E(X+Y) + E^2(X+Y)}$$

$$D(X+Y) = E[(X-E(X) + Y-E(Y))]^2$$

$$= E[(X-E(X))^2 + (Y-E(Y))^2 + 2(X-E(X))(Y-E(Y))]$$

$$= E(X-E(X))^2 + E(Y-E(Y))^2 + 2E(X-E(X))(Y-E(Y))$$

$$= D(X) + D(Y) + 2E(X-E(X))(Y-E(Y))$$

$$\hookrightarrow \text{cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$= \rho_{X,Y} \sigma(X) \sigma(Y)$$