黑洞

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1 Black holes in the nuclei of galaxies

Dead stars with masses greater than $3M_\odot$ must be black holes,

The only properties which isolated black holes can possess are mass, angular momentum and electric charge.

For a non-rotating black hole, a *Schwarzschild black hole*, there is a spherical surface about the black hole from which electromagnetic radiation suffers an infinite gravitational redshift, as observed from outside this surface. This surface of infinite redshift has radius

$$r_g = \frac{2GM}{c^2} = 3\left(\frac{M}{M_\odot}\right)\ \text{km} \tag{1}$$

and is known as the Schwarzschild radius. Radiation with frequency ν_0 emitted at radius r from the black hole suffers a gravitational redshift, so that the frequency of the radiation as observed at an infinite distance from the black hole ν_∞ is

$$\nu_\infty = \nu_0\left(1 - \frac{2GM}{rc^2}\right)^{1/2} = \nu_0\left(1 - \frac{r_g}{r}\right)^{1/2} \tag{2}$$

There is a last stable circular orbit about a Schwarzschild black hole at radius $r = 3r_g$. Within this radius, test particles spiral inevitably into the black hole, contributing to its mass and angular momentum. As will be shown below, the speed of a test particle on the last stable circular orbit of a Schwarzschild black hole is $v_\phi = c/2$.

In the case of black holes with finite angular momentum J, the *Kerr black holes*, the surface of infinite redshift occurs at radius

$$r_\infty = \frac{GM}{c^2} + \left[\left(\frac{GM}{c^2}\right)^2 - \left(\frac{J}{Mc}\right)^2\right]^{1/2} \tag{3}$$

There is a maximum angular momentum which a rotating black hole can possess, $J_{\max} = GM^2/c$. The radius of the surface of infinite redshift for a maximally rotating black hole then occurs at $r_\infty = GM/c^2 = r_g/2$, that is, half the Schwarzschild radius of a non-rotating black hole.

There is a last stable orbit about a Kerr black hole, but now test particles can orbit in either the corotating or counter-rotating directions with respect to the angular momentum axis of the black hole. For a maximally rotating Kerr black hole, the last stable circular orbit for corotating test particles coincides with r_∞, that is $r = GM/c^2$, one sixth of the corresponding radius for a non-rotating, Schwarzschild black hole.

The binding energies of particles on the last stable orbit for Schwarzschild and maximally rotating Kerr black holes, relative to their rest mass energies, are

$$\text{Schwarzschild} \quad \left[1 - \sqrt{\frac{8}{9}}\right] \qquad \text{Kerr} \quad \left[1 - \sqrt{\frac{1}{3}}\right] \tag{4}$$

corresponding to 5.72% and 42.3% of their rest mass energies, respectively. A fraction of the rotational energy of a rotating black hole can also be made available to the external

Universe. In terms of the rest-mass energy of the black hole, this fraction is

$$1 - 2^{-1/2}\left\{1 + [1 - (J/J_{\max})^2]^{1/2}\right\}^{1/2} \tag{5}$$

amounting to 29% for a maximally rotating Kerr black hole, $J = J_{\max}$.

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