Fundamental Definitions

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Luminosity

[1] By luminosity we mean the quantity of energy irradiated per second [erg s^{-1}]. The luminosity is not defined per unit of solid angle. The monochromatic luminosity $L(\nu)$ is the luminosity per unit of frequency ν (i.e. per Hz). The bolometric luminosity is integrated over frequency:

$$L = \int_0^\infty L(\nu)\mathrm{d}\nu \ . \tag{1}$$

Often we can define a luminosity integrated in a given energy (or frequency) range,

$$L_{[\nu_1-\nu_2]} = \int_{\nu_1}^{\nu_2} L(\nu)\mathrm{d}\nu \ . \tag{2}$$

Sun Luminosity: $L_\odot = 4 \times 10^{33}$ erg s^{-1}

Luminosity of a typical galaxy: $L_{\mathrm{gal}} \sim 10^{11} L_\odot$

Luminosity of the human body, assuming that we emit as a black-body at a temperature of $(273 + 36)$ K and that our skin has a surface of approximately $S = 2m^2$:

$$L_{\mathrm{body}} = S\sigma T^4 \sim 10^{10} \text{ erg/s} \sim 10^3 \ W \tag{3}$$

This is not what we loose, since we absorb from the ambient a power $L = S\sigma T^4_{\rm amb} \sim 8.3 \times 10^9$ erg/s if the ambient temperature is 20 C ($= 273 + 20$ K).

Flux

The flux [erg cm^{-2} s^{-1}] is the energy passing a surface of 1 cm^2 in one second. If a body emits a luminosity L and is located at a distance R, the flux is

$$F = \frac{L}{4\pi R^2} \tag{4}$$

$$F(\nu) = \frac{L(\nu)}{4\pi R^2} \tag{5}$$

$$F = \int_0^\infty F(\nu) {\rm d}\nu \tag{6}$$

Intensity

The intensity I is the energy per unit time passing through a unit surface located perpendicularly to the arrival direction of photons, per unit of solid angle. The solid angle appears: [erg cm^{-2} s^{-1} sterad^{-1}]. The monochromatic intensity $I(\nu)$ has units [erg cm^{-2} s^{-1} Hz^{-1} sterad^{-1}]. It always obeys the Lorentz transformation:

$$\frac{I(\nu)}{\nu^3} = \frac{I'(\nu')}{\nu'^3} = \text{invariant} . \tag{7}$$

Emissivity

The emissivity j is the quantity of energy emitted by a unit volume, in one unit of time, for a unit solid angle

$$j = \frac{\rm erg}{{\rm d}V {\rm d}t {\rm d}\Omega} \tag{8}$$

If the source is transparent, there is a simple relation between j and I :

$$I = jR \text{ (optically thin source)} \tag{9}$$

Radiative Energy Density

We can define it as the energy per unit volume produced by a luminous source, but we have to specify if it is per unit solid angle or not. Consider the bolometric intensity I. Along the light ray, construct the volume $\mathrm{d}V = c\mathrm{d}t\mathrm{d}A$ where $\mathrm{d}A$ (i.e. one cm^2) is the base of the little cylinder of height $c\mathrm{d}t$. The energy contained in this cylinder is

$$\mathrm{d}E = Ic\mathrm{d}t\mathrm{d}A\mathrm{d}\Omega \ . \tag{10}$$

In that cylinder I find the light coming from a given direction,

$$\mathrm{d}E = u(\Omega)c\mathrm{d}t\mathrm{d}A\mathrm{d}\Omega \ . \tag{11}$$

$$u(\Omega) = \frac{I}{c} \tag{12}$$

If I want the total u (i.e. summing the light coming from all directions) I must integrate over the entire solid angle.

RADIATIVE FLUX

[2] When the scale of a system greatly exceeds the wavelength of radiation (e.g., light shining through a keyhole), we can consider radiation to travel in straight lines (called rays) in free space or homogeneous media.

The energy flux: consider an element of area $\mathrm{d}A$ exposed to radiation for a time $\mathrm{d}t$. The amount of energy passing through the element should be proportional to $\mathrm{d}A\mathrm{d}z$, and we write it as $F\mathrm{d}A\mathrm{d}t$. The energy flux F is usually measured in erg s^{-1} cm^{-2}.

References

[1] G. Ghisellini, editor. *Radiative Processes in High Energy Astrophysics*, volume 873 of *Lecture Notes in Physics, Berlin Springer Verlag*, 2013.

[2] G. B. Rybicki and A. P. Lightman. *Radiative processes in astrophysics*. 1979.