Photoelectric absorption

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The three main processes involved in the interaction of high energy photons with atoms, nuclei and electrons are photoelectric absorption, Compton scattering and electron–positron pair production. These processes are important not only in the study of high energy astrophysical phenomena in a wide variety of different circumstances but also in the detection of high energy particles and photons. For example, photoelectric absorption is observed in the spectra of most X-ray sources at energies $\epsilon \lesssim 1$ keV. Thomson and Compton scattering appear in a myriad of guises from the processes occurring in stellar interiors, to the spectra of binary X-ray sources, and inverse Compton scattering figures prominently in sources in which there are intense radiation fields and high energy electrons. Pair production is bound to occur wherever there are significant fluxes of high energy γ-rays - evidence for the production of positrons by this process is provided by the detection of the 511 keV electron–positron annihilation line in our own Galaxy.

At low photon energies, $\hbar\omega \ll m_e c^2$, the dominant process by which photons interact with matter is photoelectric, or bound–free, absorption and is one of the principal sources of opacity in stellar interiors.

If the energies of the incident photons $\epsilon = \hbar\omega$ are greater than the energy of the X-ray atomic energy level E_{I}, an electron can be ejected from that level, the remaining energy $(\hbar\omega - E_{\mathrm{I}})$ being carried away as the kinetic energy of the ejected electron, the photoelectric effect. The photon energy at which $\hbar\omega = E_{\mathrm{I}}$ corresponds to an absorption edge in the spectrum of the radiation because ejection of electrons from this energy level is impossible if the photons are of lower energy. For photons with higher energies, the cross section for photoelectric absorption from this level decreases as roughly ω^{-3}.

The analytic solution for the absorption cross section for photons with energies $\hbar\omega \gg E_{\mathrm{I}}$ and $\hbar\omega \ll m_{\mathrm{e}}c^2$ due to the ejection of electrons from the K-shells of atoms, that is, from the 1s level, is

$$\sigma_{\mathrm{K}} = 4\sqrt{2}\sigma_{\mathrm{T}}\alpha^4 Z^5 \left(\frac{m_{\mathrm{e}}c^2}{\hbar\omega}\right)^{7/2} = \frac{e^{12} m_{\mathrm{e}}^{3/2} Z^5}{192\sqrt{2}\pi^5 \epsilon_0^6 \hbar^4 c} \left(\frac{1}{\hbar\omega}\right)^{7/2} \tag{1}$$

where $\alpha = e^2/4\pi\epsilon_0\hbar c$ is the fine structure constant and $\sigma_{\mathrm{T}} = 8\pi r_{\mathrm{e}}^2/3 = e^2/6\pi\epsilon_0^2 m_{\mathrm{e}}^2 c^4$ the Thomson cross-section. This cross-section takes account of the fact that there are 2 K-shell electrons in all elements except hydrogen, both 1s electrons contributing to the opacity of the material. The absorption cross section has a strong dependence upon the atomic number Z and so, although heavy elements are very much less abundant than hydrogen, the combination of the ω^{-3} dependence and the fifth-power dependence upon Z means that quite rare elements can make significant contributions to the total absorption cross-section at ultraviolet and X-ray energies.

The total absorption coefficient for X-rays, weighted by the cosmic abundance of the different elements,

$$\sigma_{\mathrm{e}}(\epsilon) = \frac{1}{n_{\mathrm{H}}} \sum_i n_i \sigma_i(\epsilon) \tag{2}$$

The K-edges, corresponding to the ejection of electrons from the 1s shell of the atom or ion, provide the dominant source of opacity. In low resolution X-ray spectral studies, these edges cannot be resolved individually as distinct features and a useful linear interpolation formula for the X-ray absorption coefficient, σ_{e}, and the corresponding optical depth, τ_{e} is

$$\tau_{\mathrm{e}}(\hbar\omega) = \int \sigma_{\mathrm{e}} N_{\mathrm{H}} \mathrm{d}l = 2 \times 10^{-26} \left(\frac{\hbar\omega}{1\mathrm{keV}}\right)^{-8/3} \int N_{\mathrm{H}} \mathrm{d}l \tag{3}$$

where the column depth $\int N_{\mathrm{H}} \mathrm{d}l$ is expressed in particles per square metre and N_{H} is the number density of hydrogen atoms in particles per cubic metre. For example, if the interstellar gas density were 10^6 hydrogen atoms m^{-3}, the optical depth of the medium is roughly unity for a path length of 1 kpc at 1 keV. Thus, the spectra of many X-ray sources turn over at about 1 keV because of interstellar photoelectric absorption. Because of the steep energy dependence of τ_{e}, photoelectric absorption is only important at energies $\hbar\omega \gg 1$ keV for sources with large column densities of matter between the source and the observer.