Transformations and observables

November 28, 2018

[1] We associated an operator with every observable quantity through a sum over all states in which the system has a well-defined value of the observable $H = \sum_i E_i |E_i\rangle\langle E_i|$. This operator enabled us to calculate the expectation value of any function of the observable. Moreover, from the operator we could recover the observable's allowed values and the associated states because they are the operator's eigenvalues and eigenkets. These properties make an observable's operator a useful repository of information about the observable, a handy filing system. But they do not give the operator much physical meaning. Above all, they don't answer the question "what does an operator actually do when it operates?"

1 Transforming kets

[1] All physical information about any system is encapsulated in its ket $|\psi\rangle$, we must learn how $|\psi\rangle$ changes as we move and turn the system. The ket $|\psi\rangle$ that describes any of these objects contains information about the object's orientation in addition to its position and momentum.

1.1 Passive transformations

Imagine a whole family of coordinate systems set up throughout space, such that every physical point is at the origin of one coordinate system. We label by $\sum_{\boldsymbol{y}}$ the coordinate system whose origin coincides with the point labelled by $\boldsymbol{y}$ in our original coordinate system $\sum_0$, and we indicate the coordinate system used to obtain a wavefunction by making $\boldsymbol{y}$ a second argument of the wavefunction; $\psi_\mu(\boldsymbol{x};\boldsymbol{y})$ is the amplitude to find the system at the point labelled $\boldsymbol{x}$ in $\sum_{\boldsymbol{y}}$. Because the different coordinate systems vary smoothly with $\boldsymbol{y}$, we can use Taylor's theorem to express amplitudes in, say, $\sum_{\boldsymbol{a}+\boldsymbol{y}}$ in terms of amplitudes in $\sum_{\boldsymbol{y}}$. We have

$$\psi_\mu(\boldsymbol{x};\boldsymbol{a}+\boldsymbol{y}) = \exp\left(\boldsymbol{a}\cdot\frac{\partial}{\partial\boldsymbol{y}}\right)\psi_\mu(\boldsymbol{x};\boldsymbol{y}) \ . \tag{1}$$

References

[1] J. Binney and D. Skinner. *The Physics of Quantum Mechanics.* OUP Oxford, 2013.