Synchrotron Radiation

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[1] A particle (of charge e and mass m) with the total energy E moves in a homogeneous magnetic field of strength $\boldsymbol{H}$ along a spiral with the angular frequency

$$\omega_H = \frac{|e|H}{mc}\frac{mc^2}{E} = 1.76 \times 10^7 H \frac{mc^2}{E} \ , \tag{1}$$

the particles are electrons, and the field H is measured in oersteds (gausses).

If the particle is ultrarelativistic, it emits waves in the direction of its own velocity within the angle on the order of mc^2/E. If the angle χ between $\boldsymbol{v}$ and $\boldsymbol{H}$ is not too small, i.e. $\chi \gg mc^2/E$, the particle emits waves with many frequencies which are overtones of $\omega_H/\sin^2\chi$. At $E \gg mc^2$, the spectrum is practically continuous, and the radiation intensity maximum corresponds to a frequency

$$\begin{aligned}\nu_{\mathrm{m}} = \frac{\omega_m}{2\pi} &= 0.07\frac{|e|H_\perp}{mc}\left(\frac{E}{mc^2}\right)^2 \\ &= 1.2 \times 10^6 H_\perp \left(\frac{E}{mc^2}\right)^2 \\ &= 1.8 \times 10^{18} H_\perp [E(\mathrm{erg})]^2 \\ &= 4.6 \times 10^{-6} H_\perp [E(\mathrm{eV})]^2 \ \mathrm{Hz} \ ,\end{aligned} \tag{2}$$

$$\hbar\omega_{\rm m} = 1.9 \times 10^{-20} H_\perp [E({\rm eV})]^2 \ {\rm eV} \ , \tag{3}$$

where $H_\perp = H \sin\chi$ is a component of the field $\boldsymbol{H}$ perpendicular to particle velocity $\boldsymbol{v}$. In a typical interstellar field $H \sim H_\perp \sim 3 \times 10^{-6}$ Oe for particles with the energy $E \sim 10^9 - 10^{11}$ eV, the frequency $\nu_{\rm m}$ falls in the range of $10^7 - 10^{11}$ Hz which corresponds to the wavelength $\lambda_{\rm m} = c/\nu_{\rm m} \sim 30 \ {\rm m} \pm 0.3$ cm. Thus, the electron component of cosmic rays with $E > 10^9$ eV largely radiates in the radio-frequency range when in interstellar fields.

At frequency $\nu_{\rm m}$, the spectral density of the radioemission power is

$$p_{\rm m} \equiv p(\nu_{\rm m}) = 1.6 \times \frac{|e|^3 H_\perp}{mc^2} = 2.16 \times 10^{-22} H_\perp \ \frac{\rm erg}{\rm s \cdot Hz} \ . \tag{4}$$

If we consider a region in which emitting electrons at a concentration $N_{\rm e}$ are isotropically distributed by velocity, the corresponding emissivity (the spectral power of radiation per unit volume and unit solid angle) is given by

$$\varepsilon_{\nu_{\rm m}} = \frac{p_{\rm m} N_{\rm e}}{4\pi} = 1.7 \times 10^{-23} H_\perp N_{\rm e} \ \frac{\rm erg}{\rm cm^3 \cdot s \cdot sr \cdot Hz} \ . \tag{5}$$

where $H_\perp$ is the mean value of a field component perpendicular to the particle velocity for the radiating volume. The maximum radiation intensity for monochromatic electrons is

$$J_{\nu_{\rm m}} = \int \varepsilon_{\nu_{\rm m}} {\rm d}l = 1.7 \times 10^{-23} H_\perp N_{\rm e} L \ \frac{\rm erg}{\rm cm^2 \cdot s \cdot sr \cdot Hz} \ , \tag{6}$$

where L is the characteristic size of the region emitting radiowaves along the line of sight (i.e. $N_{\rm e} L = \int N_{\rm e} {\rm d}l$).

In the case of a discrete source of radioemission (e.g., a supernova remnant) which is at a distance R from us, the radiation flux is

$$\Phi_{\nu_{\rm m}} = \frac{p_{\rm m} N_{\rm e} V}{4\pi R^2} = 1.7 \times 10^{-23} \frac{H_\perp N_{\rm e} V}{R^2} \; \frac{\rm erg}{\rm cm^2 \cdot s \cdot sr \cdot Hz} \; , \tag{7}$$

V is the source volume.

The electron energy $E = 0.75 \times 10^{-9} \sqrt{\nu_{\rm m}/H_\perp}$ erg. The total electron energy in the source is

$$W_{\rm e} = E N_{\rm e} V = 4.4 \times 10^{13} \frac{\nu_{\rm m}^{1/2} \Phi_{\nu_{\rm m}} R^2}{H_\perp^{3/2}} \; . \tag{8}$$

Consider the electron spectrum of the power-law type, when the concentration of electrons in the interval dE has the form

$$N_{\rm e}(E){\rm d}E = K_{\rm e} E^{-\gamma_{\rm e}} {\rm d}E \; , \tag{9}$$

the intensity is

$$J_\nu = {\rm const} \cdot K_{\rm e} L H^{(\gamma_{\rm e}+1)/2} \nu^{(1-\gamma_{\rm e})/2} \; , \tag{10}$$

where H is a certain average value of the magnetic field along the line of sight. Measuring the dependence of J_ν on frequency ν immediately yields index $\gamma_{\rm e}$ while the value of J_ν itself allows coefficient $K_{\rm e}$ to be determined if L and H are known, that is to find the electron spectrum in the source and then the associated energy density

$$w_{\rm cr,e} = \int E N_{\rm e}(E){\rm d}E \; , \tag{11}$$

References

[1] V. L. Ginzburg. METHODOLOGICAL NOTES: Cosmic ray astrophysics (history and general review). *Physics Uspekhi*, 39:155–168, February 1996.