Blast-Wave Physics

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FIREBALLS AND RELATIVISTIC BLAST WAVES

[1] Consider an explosion that takes place in a uniform circumburst medium (CBM) with density n_0. Suppose that the event releases energy at a fixed rate over a timescale Δ_0/c, where Δ_0 is a characteristic size scale of the engine that releases the energy.

The apparent isotropic equivalent γ-ray energy E_0 released by a GRB explosion can exceed $\sim 10^{54}$ ergs, with apparent isotropic powers $\gtrsim 10^{51}$ ergs s^{-}1. In comparison, the rest mass energy of a Solar mass of material is $\simeq 2 \times 10^{54}$ ergs, and the bolometric luminosity of the universe is $\sim 10^{54}$ ergs s^{-}1.

Blast-Wave Deceleration

The blast-wave deceleration occurs when the relativistic blast-wave shell from an explosive event sweeps up CBM material at an external shock. The accumulation of swept-up material causes the shell to decelerate. For a uniform spherically symmetric CBM, the mass of swept-up material at radius x is $M_{\rm sw} = 4\pi\mu_0 m_p n_0 x^3/3$, where n_0 is the proton density and μ_0 is a factor accounting for the metallicity of the CBM.

The blast wave will start to undergo significant deceleration when an amount of energy comparable to the initial energy E_0 in the blast wave is swept up. Looked at from the comoving frame, each proton from the CBM carries with it an amount of energy $\Gamma_0 m_p c^2$ when captured by the blast wave. After capture and isotropization, the amount of energy carried by the blast wave from this swept-up proton is $\Gamma_0^2 m_p c^2$ as measured in the stationary frame. The condition $\Gamma_0^2 M_{sw} c^2 = E_0$ gives the deceleration radius

$$x_d \equiv \left(\frac{3E_0}{4\pi\Gamma_0^2 m_p c^2 n_0} \right)^{1/3} \simeq 2.6 \times 10^{16} \left(\frac{E_{52}}{\Gamma_{300}^2 n_0} \right)^{1/3} \text{ cm} , \tag{1}$$

where $E_0 = E_{52}/10^{52}$ ergs is the total explosion energy including rest mass energy, $\Gamma_{300} = \Gamma_0/300$, and n_0 is the CBM proton density in units of cm^{-3}.

Differential time elements in the stationary (starred), comoving (primed), and observer (unscripted) reference frames satisfy the relations

$$\mathrm{d}x = \beta c \mathrm{d}t_* = \beta\Gamma c \mathrm{d}t' = \beta c \frac{\mathrm{d}t}{(1+z)(1-\beta\mu)} , \tag{2}$$

where $\theta = \arccos\mu$ is the angle between the direction of outflow and the observer, and $\mathrm{d}t = (1+z)\mathrm{d}t'/\delta_D$.

$$\mathrm{d}t = \frac{(1+z)}{c} \mathrm{d}x(\beta^{-1} - \mu) \simeq \frac{(1+z)\mathrm{d}x}{\Gamma^2 c} . \tag{3}$$

The final term in this expression applies to relativistic flows ($\Gamma \gg 1$) observed at $\theta \simeq 1/\Gamma$, assuming that the average emitting region is located at cosine angle $\mu \simeq \beta$ to the line of sight.

The deceleration time as measured by an observer is

$$\begin{aligned} t_d &\equiv (1+z)\frac{x_d}{\beta_0 \Gamma_0^2 c} \\ &= (1+z)\left(\frac{3E_0}{4\pi n_0 m_p c^5 \Gamma_0^8}\right)^{1/3} \simeq \frac{9.6(1+z)}{\beta_0}\left(\frac{E_{52}}{\Gamma_{300}^8 n_0}\right)^{1/3} \text{ s }, \end{aligned} \quad (4)$$

where z is the redshift of the source, and the factor $\beta_0^{-1} = 1/\sqrt{1-\Gamma_0^{-2}}$ generalizes the original result for mildly relativistic and nonrelativistic supernova explosions. The Sedov radius, giving the distance traveled in a uniform CBM when the shell sweeps up an amount of mass energy comparable to the explosion energy, is given by

$$\ell_S = \left(\frac{3E_0}{4\pi m_p c^2 n_0}\right)^{1/3} = \Gamma_0^{2/3} x_d \quad (5)$$

$$\simeq 1.2 \times 10^{18} \left(\frac{E_{52}}{n_0}\right)^{1/3} \text{ cm} \simeq 6.6 \times 10^{18} \left(\frac{\mathcal{E}_\odot}{n_0}\right)^{1/3} \text{ cm }, \quad (6)$$

The final term is written in units of $\mathcal{E}_\odot = E_0/M_\odot c^2$, where $M_\odot$ is the mass of the Sun. For relativistic explosions, ℓ_S refers to the radius where the blast wave slows to mildly relativistic speeds, i.e., $\Gamma \sim 2$. The Sedov radius of a SN that ejects a $10 M_\odot$ envelope can reach several pc or more.

Blast-Wave Equation of Motion

The equation describing the momentum $P = \beta\gamma$ of the relativistic blast wave, which changes as a consequence of the blast wave sweeping up material from the surrounding medium and radiating internal energy, is derived.

Applying momentum conservation for the explosion and swept-up mass $m(x)$ gives

$$\begin{aligned} P\left[M_0 + \int_0^x \mathrm{d}\tilde{x}\left(\frac{\mathrm{d}m(\tilde{x})}{\mathrm{d}\tilde{x}}\right)\Gamma(\tilde{x})\right] &\simeq \beta\Gamma[M_0 + m(x)\Gamma(x)] \\ &\simeq \beta\Gamma[M_0 + kx^3\Gamma] \simeq \text{const }, \end{aligned} \quad (7)$$

giving the asymptotes $\Gamma \propto x^{-3/2}$ when $\Gamma_0 \gg \Gamma \gg 1$ and $\beta \propto x^{-3}$ when $\Gamma - 1 \ll 1$. For the radiative solution,

$$\begin{aligned} P\left[M_0 + \int_0^x \mathrm{d}\tilde{x} \left(\frac{\mathrm{d}m(\tilde{x})}{\mathrm{d}\tilde{x}}\right)\right] &\simeq \beta\Gamma[M_0 + m(x)] \\ &\simeq \beta\Gamma[M_0 + kx^3] \simeq \text{const} , \end{aligned} \tag{8}$$

giving the asymptotes $\Gamma \propto x^{-3}$ when $\Gamma_0 \gg \Gamma \gg 1$ and $\beta \propto x^{-3}$ when $\Gamma - 1 \ll 1$.

The limits for the adiabatic solution would seem to be obtained through total energy conservation from

$$\Gamma\left[M_0 + \int_0^x \mathrm{d}\tilde{x}\Gamma(\tilde{x}) \left(\frac{\mathrm{d}m(\tilde{x})}{\mathrm{d}\tilde{x}}\right)\right] \simeq \Gamma[M_0 + \Gamma m(x)] \simeq \Gamma(M_0 + kx^3\Gamma) \simeq \text{const} . \tag{9}$$

This gives the correct relativistic asymptote $\Gamma \propto x^{-3/2}$ when $\Gamma_0 \gg \Gamma \gg 1$, but implies that $\beta \propto x^{-3}$ when $\Gamma - 1 \ll 1$. The change in internal energy due to adiabatic losses becomes important in the nonrelativistic regime so that this estimate is not valid there.

RELATIVISTIC SHOCK HYDRODYNAMICS

[1] Assume idealized shock structures.

BEAMING BREAKS AND JETS

[1] An observer will receive most emission from those portions of a GRB blast wave that are within an angle $\sim 1/\Gamma$ to the direction to the observer. As the blast wave decelerates by sweeping up material from the CBM, a break in the light curve will occur when the jet opening half angle θ_j becomes smaller than $1/\Gamma$. This is due to a change from a spherical blast-wave geometry to a geometry defined by a localized emission region. Assuming that

the blast wave decelerates adiabatically in a uniform surrounding medium, the condition $\theta_j \simeq 1/\Gamma = \Gamma_0^{-1}(x_{\rm br}/x_d)^{3/2} = \Gamma_0^{-1}(t_{\rm br}/t_d)^{3/8}$ implies

$$t_{\rm br} \approx 45(1+z)\left(\frac{E_{52}}{n_0}\right)^{1/3}\theta_j^{8/3} \text{ days} , \tag{10}$$

from which the jet angle

$$\theta_j \approx 0.1\left[\frac{t_{\rm br}(d)}{(1+z)}\right]^{3/8}\left(\frac{n_0}{E_{52}}\right)^{1/8} \tag{11}$$

can be derived.

SYNCHROTRON SELF-COMPTON RADIATION

[1] Electrons cool in the comoving fluid frame by synchrotron and Compton losses. The Compton y-parameter

$$y_C \equiv \frac{L_C}{L_{\rm syn}} \simeq \frac{U_{\rm syn}}{U_B} , \tag{12}$$

gives the ratio of the (synchrotron-self) Compton and synchrotron powers, and the final expression holds for scattering in the Thomson regime.

The total internal energy $U = U_e + U_p + U_B + U_{\rm ph}$ in the shocked fluid shell is found in the form of nonthermal electron and protons/ions, magnetic field, and photons. The electron energy $U_e = \epsilon_e U$, and the magnetic field energy $U_B = \epsilon_B U$. The photon energy

$$U_{\rm ph} = U_{\rm syn} + U_C = \eta_e \epsilon_e U = \frac{\eta_e \epsilon_e U_B}{\epsilon_B} , \tag{13}$$

and η_e is the radiative efficiency to convert nonthermal electron energy into radiation. In the fast cooling regime, $\gamma_c < \gamma_{\rm min}$, and $\eta_e \simeq 1$. In the slowcooling regime, only electrons with $\gamma > \gamma_c$ cool efficiently. The fractional energy in electrons that stongly cool is $\sim (\gamma_{\rm min}/\gamma_c)^{p-2}$, so

$$\eta_e = \min\left[1, \left(\frac{\gamma_{\rm min}}{\gamma_c}\right)^{p-2}\right] \, . \tag{14}$$

$$\frac{U_{\rm syn}}{U_B} + \frac{U_C}{U_{\rm syn}}\frac{U_{\rm syn}}{U_B} = y_C + y_C^2 = \frac{\eta_e \epsilon_e}{\epsilon_B} \tag{15}$$

$$y_C \equiv \frac{L_C}{L_{\rm syn}} = \frac{-1+\sqrt{1+4\eta_e\epsilon_e/\epsilon_B}}{2} \to \begin{cases} \eta_e\epsilon_e/\epsilon_B, & \eta_e\epsilon_e/\epsilon_B \ll 1 \, , \\ \sqrt{\eta_e\epsilon_e/\epsilon_B}, & \eta_e\epsilon_e/\epsilon_B \gg 1 \, . \end{cases} \tag{16}$$

The inclusion of a Compton component in blast-wave afterglow modeling can be used to derive analytic and numerical spectra and light curves. A standard approximation to treat the Compton component analytically is to use the Thomson cross section truncated for scattering in the Klein-Nishina regime.

THEORY OF THE PROMPT PHASE

[1] The prompt phase of long-duration GRBs, when they are most luminous, lasts from seconds to minutes at $\sim$ 100 keV-MeV energies, as long as 90 minutes at $\geqslant$ 100 MeV energies, and up to $\sim 10^5$ after the start of a burst for keV X-ray flares found with Swift. In the internal shock model, an active central engine ejects waves of relativistic plasma that overtake and collide to form shocks. The shocks accelerate nonthermal particles that radiate high-energy photons. By contrast, in the external shock model, a single relativistic wave of particles interacts with inhomogeneities in the surrounding medium to accelerate particles that radiate the prompt γ-rays.

References

[1] C. D. Dermer and G. Menon. *High Energy Radiation from Black Holes: Gamma Rays, Cosmic Rays, and Neutrinos*. 2009.