电磁波的发射

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1 Scalar and vector potential

1.1 Gauge transformation

1.1.1 gauge freedom

1.1.2 Coulomb Gauge

1.1.3 Lorenz Gauge

2 Retarded potential

$$\phi(\boldsymbol{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\boldsymbol{r}', t_r)}{R} \mathrm{d}\tau' \tag{1}$$

$$\boldsymbol{A}(\boldsymbol{r}, t) = \frac{\mu_0}{4\pi\epsilon_0} \int \frac{\boldsymbol{J}(\boldsymbol{r}', t_r)}{R} \mathrm{d}\tau' \tag{2}$$

2.1 Jefimenko's equations

$$\boldsymbol{E}(\boldsymbol{r},t) = \frac{1}{4\pi\epsilon_0}\int\left[\frac{\rho(\boldsymbol{r}',t_r)}{R^2}\hat{\boldsymbol{R}} + \frac{\dot{\rho}(\boldsymbol{r}',t_r)}{cR}\hat{\boldsymbol{R}} - \frac{\dot{\boldsymbol{J}}(\boldsymbol{r}',t_r)}{c^2R}\right]\mathrm{d}\tau' \tag{3}$$

$$\boldsymbol{B}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\int\left[\frac{\boldsymbol{J}(\boldsymbol{r}',t_r)}{R^2} + \frac{\dot{\boldsymbol{J}}(\boldsymbol{r}',t_r)}{cR}\right]\times\hat{\boldsymbol{R}}\mathrm{d}\tau' \tag{4}$$

3 Liénard-Wiechert Potential

The Liénard-Wiechert potential for a moving point charge

$$\phi(\boldsymbol{r},t) = \frac{1}{4\pi\epsilon_0}\frac{qc}{(1-\boldsymbol{n}\cdot\boldsymbol{\beta})R} \tag{5}$$

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\frac{qc\boldsymbol{v}}{(1-\boldsymbol{n}\cdot\boldsymbol{\beta})R} = \frac{\boldsymbol{v}}{c^2}\phi(\boldsymbol{r},t) \tag{6}$$

4 Fields of a moving point charge

The field of a point charge q in arbitrary motion in SI unit

$$\boldsymbol{E}(\boldsymbol{r},t) = \frac{q}{4\pi\epsilon_0}\left[\frac{\boldsymbol{n}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3R^2}\right]_{\mathrm{ret}} + \frac{q}{4\pi\epsilon_0 c}\left[\frac{\boldsymbol{n}\times\{(\boldsymbol{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}}{(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3R}\right]_{\mathrm{ret}} \tag{7}$$

$$\boldsymbol{B} = \frac{1}{c}[\boldsymbol{n}\times\boldsymbol{E}]_{\mathrm{ret}} \tag{8}$$

and in Gaussian (CGS) units

$$\boldsymbol{E}(\boldsymbol{r},t) = q\left[\frac{\boldsymbol{n}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3R^2}\right]_{\mathrm{ret}} + \frac{q}{c}\left[\frac{\boldsymbol{n}\times\{(\boldsymbol{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}}{(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3R}\right]_{\mathrm{ret}} \tag{9}$$

$$\boldsymbol{B} = [\boldsymbol{n}\times\boldsymbol{E}]_{\mathrm{ret}} \tag{10}$$

The first term is velocity field and the second is acceleration field.

The Poynting vector is

$$\boldsymbol{S} = \frac{1}{\mu_0}(\boldsymbol{E} \times \boldsymbol{B}) = \frac{1}{\mu_0 c}[\boldsymbol{E} \times (\boldsymbol{n} \times \boldsymbol{E})] = \frac{1}{\mu_0 c}[E^2 \boldsymbol{n} - (\boldsymbol{n} \cdot \boldsymbol{E})\boldsymbol{E}] \tag{11}$$