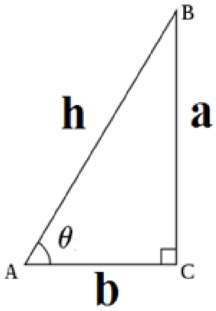
三角函数

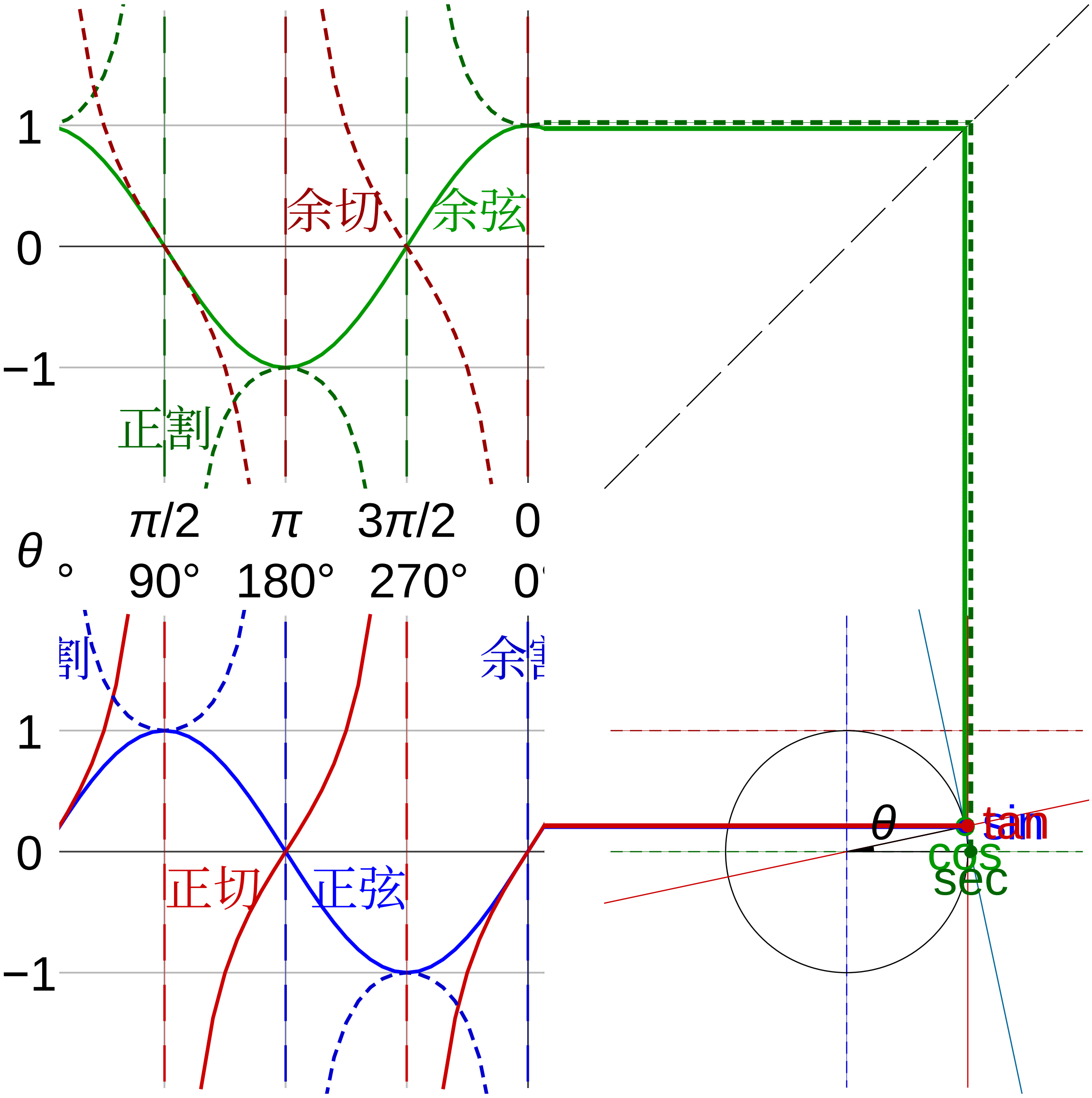
- sinx：正弦函数
- cosx：余弦函数
- tanx：正切函数
- cotx：余切函数
- secx：正割函数
- cscx：余割函数

几何定义



- $$cot\theta = \frac{b}{a} \tag{1}$$
- $$sec\theta = \frac{h}{b} \tag{2}$$
- $$csc\theta = \frac{h}{a} \tag{3}$$

图像演示



三角公式

- $$sin^2x + cos^2x = 1 \tag{4}$$
- $$tan^2x + 1 = sec^2 \tag{5}$$
- $$1 + cot^2 = csc^2 \tag{6}$$
- $$sin(x \pm y) = sinx * cosy \pm cosx * siny \tag{7}$$
- $$cos(x \pm y) = cosx * cosy \mp sinx * siny \tag{8}$$
- $$tan(x \pm y) = (tanx \pm tany)/(1 \mp tanx * tany) \tag{9}$$
- $$sin(2x) = 2sinx * cosx \tag{10}$$
- $$cos(2x) = cos^2x - sin^2x = 1 - 2sin^2x = 2cos^2x - 1 \tag{11}$$
- $$tan(2x) = \frac{2tanx}{(1 - tan^2x)} \tag{12}$$
- $$sin(\frac{x}{2}) = \pm\sqrt{\frac{1 - cosx}{2}} \tag{13}$$
- $$cos(\frac{x}{2}) = \pm\sqrt{\frac{1 + cosx}{2}} \tag{14}$$
- $$tan(\frac{x}{2}) = \pm\sqrt{\frac{1 - cosx}{1 + cosx}} = \frac{1 - cosx}{sinx} = \frac{sinx}{1 + cosx} \tag{15}$$
- $$sinx * cosy = \frac{sin(x + y) + sin(x - y)}{2} \tag{16}$$
- $$cosx * siny = \frac{sin(x + y) - sin(x - y)}{2} \tag{17}$$
- $$cosx * cosy = \frac{cos(x + y) + cos(x - y)}{2} \tag{18}$$
- $$sinx * siny = -\frac{cos(x + y) - cos(x - y)}{2} \tag{19}$$
- $$sinx + siny = 2 * sin(\frac{x + y}{2}) * cos(\frac{x - y}{2}) \tag{20}$$
- $$sinx - siny = 2 * cos(\frac{x + y}{2}) * sin(\frac{x - y}{2}) \tag{21}$$
- $$cosx + cosy = 2 * cos(\frac{x + y}{2}) * cos(\frac{x - y}{2}) \tag{22}$$
- $$cosx - cosy = -2 * sin(\frac{x + y}{2}) * sin(\frac{x - y}{2}) \tag{23}$$

正余弦定理

正弦定理

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} = 2R \tag{24}$$

余弦定理

$$c^2 = a^2 + b^2 - 2ab * cosC \tag{25}$$

连续、可导、可微关系及定义

$$\text{可微} <=> \text{可导} => \text{连续} <= \text{可微} \tag{26}$$

连续定义

$$lim_{x->x_0} f(x) = f(x_0) \tag{27}$$

可导定义

$$y'|_{x=x_0} = lim_{\Delta x->0}\frac{\Delta y}{\Delta x} = lim_{\Delta x->0}\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = lim_{x->x_0}\frac{f(x) - f(x_0)}{x - x_0} = lim_{h->0}\frac{f(x_0 + h) - f(x_0)}{h} \tag{28}$$

或者左右导数相等

$$lim_{x->x_0^-}\frac{f(x) - f(x_0)}{x - x_0} = lim_{x->x_0^+}\frac{f(x) - f(x_0)}{x - x_0} \tag{29}$$

可微定义

$$dy = A * \Delta x = f'(x_0) * \Delta x = f'(x_0)dx \tag{30}$$

基本求导公式

- $$(a^x)' = a^x lna \tag{31}$$
- $$(log_a x)' = \frac{1}{xlna} \tag{32}$$
- $$(tanx)' = sec^2 x \tag{33}$$
- $$(cotx)' = -csc^2 x \tag{34}$$
- $$(secx)' = secx * tanx \tag{35}$$
- $$(cscx)' = -cscx * cotx \tag{36}$$
- $$(arcsinx)' = \frac{1}{\sqrt{1-x^2}} \tag{37}$$
- $$(arccosx)' = -\frac{1}{\sqrt{1-x^2}} \tag{38}$$
- $$(arctanx)' = \frac{1}{1+x^2} \tag{39}$$
- $$(arccotx)' = -\frac{1}{1+x^2} \tag{40}$$

罗尔定理

$$\left.\begin{matrix} f(x)在[a,b]连续 \\ f(x)在(a,b)可导 \\ f(a) = f(b) \\ \exists \xi \in (a,b) \end{matrix}\right\} \implies f'(\xi) = 0 \tag{41}$$

拉格朗日中值定理

$$\left.\begin{matrix} f(x)在[a,b]连续 \\ f(x)在(a,b)可导 \\ \exists \xi \in (a,b) \end{matrix}\right\} \implies f(b) - f(a) = f'(\xi) * (b-a) \tag{42}$$

柯西中值定理

$$\left.\begin{matrix} f(x), F(x)在[a,b]连续 \\ f(x), F(x)在(a,b)可导 \\ x \in (a,b) \land F'(x) \neq 0 \\ \exists \xi \in (a,b) \end{matrix}\right\} \implies \frac{f(b)-f(a)}{F(b)-F(a)} = \frac{f'(\xi)}{F'(\xi)} \tag{43}$$

泰勒公式

$$f(x) = f(x_0) + \sum_{i=1}^{n} \frac{f^{(i)}(x_0)}{i!}(x-x_0)^i + o[(x-x_0)^n] \tag{44}$$

麦克劳林公式

$$f(x) = f(0) + \sum_{i=1}^{n} \frac{f^{(i)}(0)}{i!} x^i + o(x^n) \tag{45}$$

$$f(x) = f(0) + \sum_{i=1} \frac{\quad}{i!} x^{\cdot} + o(x^{\cdot}) \tag{45}$$

常用泰勒展开式

- $$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \frac{e^{\theta x}}{(n+1)!} x^{n+1} \tag{46}$$
- $$sinx = x - \frac{x^3}{3!} + \cdots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + (-1)^n \frac{cos(\theta x)}{(2n+1)!} x^{2n+1} \tag{47}$$
- $$cosx = 1 - \frac{x^2}{2!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + (-1)^{(} n+1) \frac{cos(\theta x)}{(2n+2)!} x^{2n+2} \tag{48}$$
- $$ln(1+x) = x - \frac{x^2}{2} + \cdots + (-1)^{n-1} \frac{x^n}{n} + (-1)^n \frac{x^{n+1}}{(n+1)(1+\theta x)^{n+1}} \tag{49}$$
- $$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \cdots + \frac{\alpha(\alpha-1)\ldots(\alpha-n+1)}{n!} x^n + \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)(a-n)}{(n+1)!} (1+\theta x)^{n-a-1} x^{n+1}$$

> 注意:
>
> $$\theta \in (0,1) \tag{51}$$

第一换元积分法

$$\int f[g(x)]g'(x)dx = \int f[g(x)]dg(x) = F[g(x)] + C \tag{52}$$

凑微分形式

- $$\int f(sinx)cosxdx = \int f(sinx)d(sinx) \tag{53}$$
- $$\int f(cosx)sinxdx = -\int f(cosx)d(cosx) \tag{54}$$
- $$\int f(tanx)\frac{1}{cos^2x}1dx = \int f(tanx)d(tanx) \tag{55}$$
- $$\int f(arcsinx)\frac{1}{\sqrt{(1-x^2)}}dx = \int f(arcsinx)d(arcsinx) \tag{56}$$
- $$\int f(arctanx)\frac{1}{\sqrt{(1+x^2)}}dx = \int f(arctanx)d(arctanx) \tag{57}$$

第二换元积分

$$\int f(x)dx = \int f[g(t)]g'(t)dt = F(t) + C \tag{58}$$

三种变量代换

- $$\sqrt{(a^2-x^2)} => x = asint(或x = acost) \tag{59}$$
- $$\sqrt{(a^2+x^2)} => x = atant \tag{60}$$
- $$\sqrt{(x^2-a^2)} => x = asect \tag{61}$$

分部积分

$$\int udv = u * v - \int vdu \tag{62}$$

> dv优先级
>
> $$e^x > sinx, cosx > x^n \tag{63}$$

不定积分公式

- $$\int sec^2xdx = tanx + C \tag{64}$$

- $$\int csc^2xdx = -cotx + C \tag{65}$$
- $$\int secxtanxdx = secx + C \tag{66}$$
- $$\int cscxcotxdx = -cscx + C \tag{67}$$
- $$\int \frac{1}{\sqrt{(1-x^2)}}dx = arcsinx + C \tag{68}$$
- $$\int \frac{1}{\sqrt{(a^2-x^2)}}dx = arcsin(\frac{x}{a}) + C \tag{69}$$
- $$\int \frac{1}{(1+x^2)}dx = arctanx + C \tag{70}$$
- $$\int \frac{1}{(a^2+x^2)}dx = \frac{1}{a}arctan(\frac{x}{a}) + C \tag{71}$$
- $$\int \frac{dx}{\sqrt{(x^2+a^2)}} = ln[x + \sqrt{(x^2+a^2)}] + C \tag{72}$$
- $$\int \frac{dx}{\sqrt{(x^2-a^2)}} = ln[x + \sqrt{(x^2-a^2)}] + C \tag{73}$$
- $$\int \frac{dx}{(x^2-a^2)} = \frac{1}{2a}ln|\frac{x-a}{x+a}| + C \tag{74}$$
- $$\int secxdx = ln|secx + tanx| + C \tag{75}$$
- $$\int cscdx = -ln|cscx + cotx| + C \tag{76}$$

定积分

$$\int_a^b f(x)dx = lim_{\lambda->0}\sum_{i=1}^{n} f(\xi_i) * \Delta x_i \quad \{\Delta x_i = \frac{b-a}{n}; \quad \xi_i = \frac{i}{n}\} \tag{77}$$

特殊形式

$$\int_0^1 f(x)dx = lim_{\lambda->0}\sum_{i=1}^{n} f(\xi_i) * \Delta x_i = lim_{n->\infty}\frac{1}{n}\sum_{i=1}^{n} f(\frac{i}{n}) \tag{78}$$

定积分存在的充分条件

区间 [a,b]，有：

- $$f(x)\text{连续} \implies \exists \int_a^b f(x)dx \tag{79}$$
- $$f(x)\text{有界} \wedge \text{只有有限个间断点} \implies \exists \int_a^b f(x)dx \tag{80}$$
- $$f(x)\text{只有有限个第一类间断点} \implies \exists \int_a^b f(x)dx \tag{81}$$

定积分性质

- $$\int_a^a f(x)dx = 0 \tag{82}$$
- $$\int_a^b f(x)dx = -\int_b^a f(x)dx \tag{83}$$
- $$\int_a^b \{\alpha f(x) + \beta g(x)\}dx = \alpha\int_a^b f(x)dx + \beta\int_a^b g(x)dx \tag{84}$$
- $$\int_a^c f(x)dx = \int_a^b f(x)dx - \int_b^c f(x)dx \tag{85}$$

- $$\int_a^b kdx = k(b-a) \tag{86}$$
- $$\begin{cases} f(x) \geq 0 \implies \int_a^b f(x)dx \geq 0 \\ f(x) \leq 0 \implies \int_a^b f(x)dx \leq 0 \end{cases} \tag{87}$$
- $$f(x) \leq g(x) \implies \int_a^b f(x)dx \leq \int_a^b g(x)dx \tag{88}$$
- $$m \leq f(x) \leq M \implies m(b-a) \leq \int_a^b f(x)dx \leq M(b-a) \tag{89}$$
- $$\mid \int_a^b f(x)dx \mid \leq \int_a^b \mid f(x) \mid dx \tag{90}$$
- $$\int_a^b f(x)dx = f(\xi)(b-a) \quad \{x \in [a,b], \quad a < \xi < b\} \quad (\text{中值定理}) \tag{91}$$
- $$\left.\begin{matrix} x \in [a,b] \\ f(x), g(x)\text{连续} \\ g(x)\text{不变号} \\ \xi \in [a,b] \end{matrix}\right\} \implies \int_a^b f(x)g(x)dx = f(\xi)\int_a^b g(x)dx \tag{92}$$
- $$\left.\begin{matrix} x \in [a,b] \\ f(x)\text{连续} \\ \psi_1(x), \psi_2(x)\text{可导} \end{matrix}\right\} \implies (\int_{\psi_1(x)}^{\psi_2(x)} f(t)dt)' = f[\psi_2(x)] * \psi_2'(x) - f[\psi_1(x)] * \psi_1'(x) \quad (\text{积分上限函数}) \tag{93}$$
- $$\int_a^b f(x)dx = F(x) \mid_a^b = F(b) - F(a) \quad (\text{牛顿} - \text{莱布尼兹}) \tag{94}$$
- $$\int_a^b f(x)dx = \int_\alpha^\beta f[\psi(t)] * \psi'(t)dt \quad (\text{换元法}) \tag{95}$$
- $$\int_a^b udv = uv \mid_a^b - \int_a^b vdu \quad (\text{分部积分}) \tag{96}$$
- $$\int_{-a}^a f(x)dx = \begin{cases} 0, & f(x) = -f(x) \\ 2\int_0^a f(x)dx, & f(x) = f(-x) \end{cases} \quad x \in [-a,a] \tag{97}$$
- $$\int_a^{a+T} f(x)dx = \int_0^T f(x)dx \qquad \{T\text{是}f(x\text{周期})\} \tag{98}$$

反常积分

无穷区间

$$\int_a^{+\infty} f(x)dx = lim_{t->+\infty}\int_a^t f(x)dx \tag{99}$$

$$\int_{-\infty}^b f(x)dx = lim_{t->-\infty}\int_t^b f(x)dx \tag{100}$$

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^0 f(x)dx + \int_0^{+\infty} f(x)dx \tag{101}$$

> 结论：极限（都）存在，f(x) 收敛，否则，发散。

$$\int_a^{+\infty} \frac{1}{x^p}dx \quad \begin{cases} p > 1, & \text{收敛} \\ p \leq 1, & \text{发散} \end{cases} \quad (a > 0) \tag{102}$$

无界函数

$$f(x)\text{在}(a,b]\text{连续} \implies \int_a^b f(x)dx = lim_{t->a^+}\int_t^b f(x)dx \tag{103}$$

$$f(x)\text{在}[a,b)\text{连续} \implies \int_a^b f(x)dx = lim_{t->b^-}\int_a^t f(x)dx \tag{104}$$

$$f(x)\text{在}[a,b]\text{上，除}c(a < c < b)\text{外连续} \implies \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \tag{105}$$

> 结论：极限（都）存在，f(x) 收敛，否则，发散。

$$\int_a^b \frac{1}{(x-a)^p}dx \quad \begin{cases} p \geq 1, & \text{发散} \\ p < 1, & \text{收敛} \end{cases} \tag{106}$$

$$\int_a^b \frac{1}{(b-x)^p}dx \quad \begin{cases} p \geq 1, & \text{发散} \\ p < 1, & \text{收敛} \end{cases} \tag{107}$$