

# Standard Code Library

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## 数学

### 快速求逆元 (内含 exgcd)

使用条件:  $x \in [0, mod)$  并且  $x$  与  $mod$  互质

```
LL exgcd(LL a, LL b, LL &x, LL &y) {
    if(!b) return x = 1, y = 0, a;
    else {
        LL d = exgcd(b, a % b, x, y);
        LL t = x; x = y;
        y = t - a / b * y;
        return d;
    }
}

LL inv(LL a, LL p) {
    LL d, x, y;
    exgcd(a, p, d, x, y);
    return d == 1 ? (x + p) % p : -1;
}
```

### 中国剩余定理

返回结果:  $x \equiv r_i(mod\ p_i)$  ( $0 \leq i < n$ )

```
LL china(int n, int *a, int *m) {
    LL M = 1, d, x = 0, y;
    for(int i = 0; i < n; i++)
        M *= m[i];
    for(int i = 0; i < n; i++) {
        LL w = M / m[i];
        d = exgcd(m[i], w, d, y);
        y = (y % M + M) % M;
        x = (x + y * w % M * a[i]) % M;
    }
    while(x < 0) x += M;
    return x;
}
```

```
//merge Ax=B and ax=b to A'x=B'
void merge(LL &A, LL &B, LL a, LL b) {
    LL x, y;
    sol(A, -a, b - B, x, y);
    A = lcm(A, a);
    B = (a * y + b) % A;
}
```

```
B = (B + A) % A;
}
```

### 小步大步

返回结果:  $a^x = b \pmod{p}$  使用条件:  $p$  为质数

```
LL BSGS(LL a, LL b, LL p) {
    LL m = 0; for(; m * m <= p; m++);
    map<LL, int> hash; hash[1] = 0;
    LL e = 1, amv = inv(pw(a, m, p), p);
    for(int i = 1; i < m; i++) {
        e = e * a % p;
        if(!hash.count(e))
            hash[e] = i;
        else break;
    }
    for(int i = 0; i < m; i++) {
        if(hash.count(b))
            return hash[b] + i * m;
        b = b * amv % p;
    }
    return -1;
}

LL solve2(LL a, LL b, LL p) {
    //a^x=b (mod p)
    b %= p;
    LL e = 1 % p;
    for(int i = 0; i < 100; i++) {
        if(e == b) return i;
        e = e * a % p;
    }
    int r = 0;
    while(gcd(a, p) != 1) {
        LL d = gcd(a, p);
        if(b % d) return -1;
        p /= d; b /= d; b = b * inv(a / d, p);
        r++;
    }
    LL res = BSGS(a, b, p);
    if(res == -1) return -1;
    return res + r;
}
```

### Miller Rabin 素数测试

```
const int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
bool check(long long n, int base) {
    long long n2 = n - 1, res; int s = 0;
    while(n2 % 2 == 0) n2 >>= 1, s++;
    res = pw(base, n2, n);
    if((res == 1) || (res == n - 1)) return 1;
    while(s--) {
        res = mul(res, res, n);
        if(res == n - 1) return 1;
    }
    return 0; // n is not a strong pseudo prime
}

bool isprime(const long long &n) {
```

```

if(n == 2) return true;
if(n < 2 || n % 2 == 0) return false;
for(int i = 0; i < 12 && BASE[i] < n; i++)
    if(!check(n, BASE[i])) return false;
return true;
}

```

Pollard Rho 大数分解

```

LL prho(LL n, LL c) {
    LL i = 1, k = 2, x = rand() % (n - 1) + 1, y = x;
    while(1) {
        i++; x = (x * x % n + c) % n;
        LL d = __gcd((y - x + n) % n, n);
        if(d > 1 && d < n) return d;
        if(y == x) return n;
        if(i == k) y = x, k <= 1;
    }
}
void factor(LL n, vector<LL>&fat) {
    if(n == 1) return;
    if(isprime(n)) {fat.push_back(n); return;}
    LL p = n;
    while(p >= n) p = prho(p, rand() % (n - 1) + 1);
    factor(p, fat); factor(n / p, fat);
}

```

快速数论变换 (zky)

返回结果:  $c_i = \sum_{0 \leq j \leq i} a_j \cdot b_{i-j} \pmod{n}$  ( $0 \leq i < n$ )

```

/*{(mod,G)}={(81788929,7),(101711873,3),(167772161,3),
(377487361,7),(998244353,3),(1224736769,3),
(1300234241,3),(1484783617,5)}*/
int mo = 998244353, G = 3;
void NTT(int a[], int n, int f) {
    for(register int i = 0; i < n; i++)
        if(i < rev[i]) swap(a[i], a[rev[i]]);
    for(register int i = 2; i <= n; i <= 1) {
        static int exp[maxn];
        exp[0] = 1;
        exp[1] = pw(G, (mo - 1) / i);
        if(f == -1) exp[1] = pw(exp[1], mo - 2);
        for(register int k = 2; k < (i >> 1); k++)
            exp[k] = 1LL * exp[k - 1] * exp[1] % mo;
        for(register int j = 0; j < n; j += i) {
            for(register int k = 0; k < (i >> 1); k++) {
                register int &pA = a[j + k], &pB = a[j + k + (i >> 1)];
                register int A = pA, B = 1LL * pB * exp[k] % mo;
                pA = (A + B) % mo; pB = (A - B + mo) % mo;
            }
        }
    }
    if(f == -1) {
        int rv = pw(n, mo - 2) % mo;
        for(int i = 0; i < n; i++) a[i] = 1LL * a[i] * rv % mo;
    }
}

```

```

}
void mul(int m, int a[], int b[], int c[]) {
    int n = 1, len = 0;
    while(n < m) n <= 1, len++;
    for(int i = 1; i < n; i++)
        rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len - 1));
    NTT(a, n, 1); NTT(b, n, 1);
    for(int i = 0; i < n; i++) c[i] = 1LL * a[i] * b[i] % mo;
    NTT(c, n, -1);
}

```

原根

```

vector<LL>fct;
bool check(LL x, LL g) {
    for(int i = 0; i < fct.size(); i++)
        if(pw(g, (x - 1) / fct[i], x) == 1)
            return 0;
    return 1;
}
LL findrt(LL x) {
    LL tmp = x - 1;
    for(int i = 2; i * i <= tmp; i++) {
        if(tmp % i == 0) {
            fct.push_back(i);
            while(tmp % i == 0) tmp /= i;
        }
    }
    if(tmp > 1) fct.push_back(tmp);
    // x is 1,2,4,p^n,2p^n
    // x has phi(phi(x)) primitive roots
    for(int i = 2; i < int(1e9); i++)
        if(check(x, i)) return i;
    return -1;
}

```

线性递推

```

//已知  $a_0, a_1, \dots, a_{m-1}$ 
 $a_n = c_0 * a_{n-m} + \dots + c_{m-1} * a_{n-1}$ 
求  $a_n = v_0 * a_0 + v_1 * a_1 + \dots + v_{m-1} * a_{m-1}$ 
void linear_recurrence(long long n, int m, int a[], int c[], int p) {
    long long v[M] = {1 % p}, u[M << 1], msk = !!n;
    for(long long i(n); i > 1; i >= 1) msk <= 1;
    for(long long x(0); msk; msk >= 1, x <= 1) {
        fill_n(u, m << 1, 0);
        int b(!!(n & msk));
        x |= b;
        if(x < m) u[x] = 1 % p;
        else {
            for(int i(0); i < m; i++)
                for(int j(0), t(i + b); j < m; j++, t++)
                    u[t] = (u[t] + v[i] * v[j]) % p;
            for(int i((m << 1) - 1); i >= m; i--)
                for(int j(0), t(i - m); j < m; j++, t++)
                    u[t] = (u[t] + c[j] * u[i]) % p;
        }
    }
}

```

```

    copy(u, u + m, v);
}
//a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] * a[m - 1].
for(int i(m); i < 2 * m; i++) {
    a[i] = 0;
    for(int j(0); j < m; j++)
        a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
}
for(int j(0); j < m; j++) {
    b[j] = 0;
    for(int i(0); i < m; i++) b[j] = (b[j] + v[i] * a[i + j]) % p;
}
for(int j(0); j < m; j++) a[j] = b[j];
}

```

直线下整点个数

返回结果:  $\sum_{0 \leq i < n} \lfloor \frac{a+b \cdot i}{m} \rfloor$     使用条件:  $n, m > 0, a, b \geq 0$     时间复杂度:  $\mathcal{O}(n \log n)$

```

LL solve(LL n, LL a, LL b, LL m) {
    if(b == 0)
        return n * (a / m);
    if(a >= m || b >= m)
        return n * (a / m) + (n - 1) * n / 2 * (b / m) + solve(n, a % m, b % m, m);
    return solve((a + b * n) / m, (a + b * n) % m, m, b);
}

```

高斯消元

```

int Gauss(){//求秩
    int r, now = -1;
    int ans = 0;
    for(int i = 0; i < n; i++){
        r = now + 1;
        for(int j = now + 1; j < m; j++){
            if(fabs(A[j][i]) > fabs(A[r][i])) r = j;
        }
        if(!sgn(A[r][i])) continue;
        ans++, now++;
        if(r != now) for(int j = 0; j < n; j++) swap(A[r][j], A[now][j]);
        for(int k = now + 1; k < m; k++){
            double t = A[k][i] / A[now][i];
            for(int j = 0; j < n; j++)
                A[k][j] -= t * A[now][j];
        }
    }
    return ans;
}

```

1e9+7 FFT

```

// double 精度对  $10^9 + 7$  取模最多可以做到  $2^{20}$ 
const int MOD = 1000003;
const double PI = acos(-1);
typedef complex<double> Complex;
const int N = 65536, L = 15, MASK = (1 << L) - 1;
Complex w[N];
void FFTInit() {
    for (int i = 0; i < N; ++i)

```

```

        w[i] = Complex(cos(2 * i * PI / N), sin(2 * i * PI / N));
}
void FFT(Complex p[], int n) {
    for (int i = 1, j = 0; i < n - 1; ++i) {
        for (int s = n; j ^= s >>= 1, ~j & s;);
        if (i < j) swap(p[i], p[j]);
    }
    for (int d = 0; (1 << d) < n; ++d) {
        int m = 1 << d, m2 = m * 2, rm = n >> (d + 1);
        for (int i = 0; i < n; i += m2) {
            for (int j = 0; j < m; ++j) {
                Complex &p1 = p[i + j + m], &p2 = p[i + j];
                Complex t = w[rm * j] * p1;
                p1 = p2 - t, p2 = p2 + t;
            }
        }
    }
}

```

```

Complex A[N], B[N], C[N], D[N];
void mul(int a[N], int b[N]) {
    for (int i = 0; i < N; ++i) {
        A[i] = Complex(a[i] >> L, a[i] & MASK);
        B[i] = Complex(b[i] >> L, b[i] & MASK);
    }
    FFT(A, N), FFT(B, N);
    for (int i = 0; i < N; ++i) {
        int j = (N - i) % N;
        Complex da = (A[i] - conj(A[j])) * Complex(0, -0.5),
                db = (A[i] + conj(A[j])) * Complex(0.5, 0),
                dc = (B[i] - conj(B[j])) * Complex(0, -0.5),
                dd = (B[i] + conj(B[j])) * Complex(0.5, 0);
        C[j] = da * dd + da * dc * Complex(0, 1);
        D[j] = db * dd + db * dc * Complex(0, 1);
    }
    FFT(C, N), FFT(D, N);
    for (int i = 0; i < N; ++i) {
        long long da = (long long)(C[i].imag() / N + 0.5) % MOD,
                db = (long long)(C[i].real() / N + 0.5) % MOD,
                dc = (long long)(D[i].imag() / N + 0.5) % MOD,
                dd = (long long)(D[i].real() / N + 0.5) % MOD;
        a[i] = ((dd << (L * 2)) + ((db + dc) << L) + da) % MOD;
    }
}

```

FWT

```

void FWT(LL *a, int n) {
    for(int h = 2; h <= n; h <= 1)
        for(int j = 0; j < n; j += h)
            for(int k = j; k < j + h / 2; k++) {
                LL u = a[k], v = a[k + h / 2];
                // xor: a[k] = (u + v) % MOD; a[k + h / 2] = (u - v + mo) % MOD;
                // and: a[k] = (u + v) % MOD; a[k + h / 2] = v;
                // or: a[k] = u; a[k + h / 2] = (u + v) % MOD;
            }
}
void IFWT(LL *a, int n) {
    for(int h = 2; h <= n; h <= 1)

```

```

for(int j = 0; j < n; j += h)
    for(int k = j; k < j + h / 2; k++) {
        LL u = a[k], v = a[k + h / 2];
        // xor: a[k] = mul((u + v) % MOD, inv2);
        // a[k + h / 2] = mul((u - v + MOD) % MOD, inv2);
        // and: a[k] = (u - v + MOD) % MOD; a[k + h / 2] = v;
        // or: a[k] = u; a[k + h / 2] = (u - v + MOD) % MOD;
    }
}

void multiply(LL *a, LL *b, LL *c, int len) {
    int l = 1; while(l < len) l <<= 1;
    len = l; FWT(a, len); FWT(b, len);
    for(int i = 0; i < len; i++) c[i] = mul(a[i], b[i]);
    IFWT(c, len);
}

```

自适应辛普森

```

double area(const double &left, const double &right) {
    double mid = (left + right) / 2;
    return (right - left) * (calc(left) + 4 * calc(mid) + calc(right)) / 6;
}

double simpson(const double &left, const double &right,
    const double &eps, const double &area_sum) {
    double mid = (left + right) / 2;
    double area_left = area(left, mid);
    double area_right = area(mid, right);
    double area_total = area_left + area_right;
    if (std::abs(area_total - area_sum) < 15 * eps)
        return area_total + (area_total - area_sum) / 15;
    return simpson(left, mid, eps / 2, area_left)
        + simpson(mid, right, eps / 2, area_right);
}

double simpson(const double &left, const double &right, const double &eps) {
    return simpson(left, right, eps, area(left, right));
}

```

多项式求根

```

const double eps=1e-12;
double a[10][10];
typedef vector<double> vd;
int sgn(double x) { return x < -eps ? -1 : x > eps; }
double mypow(double x, int num){
    double ans=1.0;
    for(int i=1;i<=num;++i) ans*=x;
    return ans;
}

double f(int n, double x){
    double ans=0;
    for(int i=n;i>=0;--i) ans+=a[n][i]*mypow(x,i);
    return ans;
}

double getRoot(int n, double l, double r){
    if(sgn(f(n,l))==0)return l;
    if(sgn(f(n,r))==0)return r;
    double temp;

```

```

    if(sgn(f(n,l))>0)temp=-1; else temp=1;
    for(int i=1;i<=10000;++i){
        double m=(l+r)/2;
        double mid=f(n,m);
        if(sgn(mid)==0) return m;
        if(mid*temp<0)l=m; else r=m;
    }
    return (l+r)/2;
}

vd did(int n){
    vd ret;
    if(n==1){
        ret.push_back(-1e10);
        ret.push_back(-a[n][0]/a[n][1]);
        ret.push_back(1e10);
        return ret;
    }
    vd mid=did(n-1);
    ret.push_back(-1e10);
    for(int i=0;i+1<mid.size();++i){
        int t1=sgn(f(n,mid[i])),t2=sgn(f(n,mid[i+1]));
        if(t1*t2>0)continue;
        ret.push_back(getRoot(n,mid[i],mid[i+1]));
    }
    ret.push_back(1e10);
    return ret;
}

int main(){
    int n; scanf("%d",&n);
    for(int i=n;i>=0;--i) scanf("%lf",&a[n][i]);
    for(int i=n-1;i>=0;--i)
        for(int j=0;j<=i;++j)a[i][j]=a[i+1][j+1]*(j+1);
    vd ans=did(n);
    sort(ans.begin(),ans.end());
    for(int i=1;i+1<ans.size();++i)printf("%.10f\n",ans[i]);
    return 0;
}

```

数据结构

```

lct
struct LCT {
    int fa[N], c[N][2], rev[N], sz[N];
    void update(int o) {
        sz[o] = sz[c[o][0]] + sz[c[o][1]] + 1;
    }
    void pushdown(int o) {
        if(!rev[o]) return;
        rev[o] = 0;
        rev[c[o][0]] ^= 1;
        rev[c[o][1]] ^= 1;
        swap(c[o][0], c[o][1]);
    }
    bool ch(int o) {
        return o == c[fa[o]][1];
    }
}

```

```

bool isroot(int o) {
    return c[fa[o]][0] != o && c[fa[o]][1] != o;
}
void setc(int x, int y, bool d) {
    if(x) fa[x] = y;
    if(y) c[y][d] = x;
}
void rotate(int x) {
    if(isroot(x)) return;
    int p = fa[x], d = ch(x);
    if(isroot(p)) fa[x] = fa[p];
    else setc(x, fa[p], ch(p));
    setc(c[x][d^1], p, d);
    setc(p, x, d^1);
    update(p); update(x);
}
void splay(int x) {
    static int q[N], top;
    int y = q[top = 1] = x;
    while(!isroot(y)) q[++top] = y = fa[y];
    while(top) pushdown(q[top--]);
    while(!isroot(x)) {
        if(!isroot(fa[x]))
            rotate(ch(fa[x]) == ch(x) ? fa[x] : x);
        rotate(x);
    }
}
void access(int x) {
    for(int y = 0; x; y = x, x = fa[x])
        splay(x), c[x][1] = y, update(x);
}
void makeroot(int x) {
    access(x), splay(x), rev(x) ^= 1;
}
void link(int x, int y) {
    makeroot(x), fa[x] = y, splay(x);
}
void cut(int x, int y) {
    makeroot(x); access(y);
    splay(y); c[y][0] = fa[x] = 0;
}
};

```

树上莫队

```

struct Query{
    int l, r, extra, i;
    friend bool operator < (const Query &a, const Query &b) {
        if(bid[a.l] != bid[b.l]) return bid[a.l] < bid[b.l];
        return a.r < b.r;
    }
} q[M];
int dfs_clock, st[N], ed[N], col[N << 1], id[N << 1];
void dfs(int x, int p){
    col[st[x] = ++dfs_clock] = w[x];
    id[st[x]] = x;

```

```

    for(auto y: g[x])
        if(y != p) dfs(y, x);
    col[ed[x] = ++dfs_clock] = w[x];
    id[ed[x]] = x;
}
void prepare(){
    dfs_clock = 0;
    dfs(1, 0);
    int BS = (int)sqrt(dfs_clock + 0.5);
    for(int i = 1; i <= dfs_clock; i++)
        bid[i] = (i + BS - 1) / BS;
    for(int i = 1; i <= m; i++){
        int a = q[i].l, b = q[i].r, c = lca(a, b);
        if(st[a] > st[b]) swap(a, b);
        if(c == a){
            q[i].l = st[a];
            q[i].r = st[b];
            q[i].extra = 0;
        }
        else{
            q[i].l = ed[a];
            q[i].r = st[b];
            q[i].extra = c;
        }
    }
    sort(q + 1, q + m + 1);
}
int curans, ans[M], cnt[N];
bool state[N];
void rev(int x){
    int &c = cnt[col[x]];
    curans -= !!c;
    c += (state[id[x]] ^= 1) ? 1 : -1;
    curans += !!c;
}
void solve(){
    prepare();
    curans = 0;
    memset(cnt, 0, sizeof(cnt));
    memset(state, 0, sizeof(state));
    int l = 1, r = 0;
    for(int i = 1; i <= m; i++){
        while(l < q[i].l) rev(l++);
        while(l > q[i].l) rev(--l);
        while(r < q[i].r) rev(++r);
        while(r > q[i].r) rev(r--);
        if(q[i].extra) rev(st[q[i].extra]);
        ans[q[i].i] = curans;
        if(q[i].extra) rev(st[q[i].extra]);
    }
}

```

树状数组 kth

```

int find(int k){
    int cnt=0,ans=0;

```

```

for(int i=22;i>=0;i--){
    ans+=(1<<i);
    if(ans>n || cnt+d[ans]>=k)ans-=(1<<i);
    else cnt+=d[ans];
}
return ans+1;
}

```

虚树

```

int a[maxn*2],sta[maxn*2],top=0,k;
void build(){
    top=0;
    sort(a,a+k,bydfn);
    k=unique(a,a+k)-a;
    sta[top++]=1;_n=k;
    for(int i=0;i<k;i++){
        int LCA=lca(a[i],sta[top-1]);
        while(dep[LCA]<dep[sta[top-1]]){
            if(dep[LCA]>=dep[sta[top-2]]){
                add_edge(LCA,sta[top-1]);
                if(sta[top-1]!=LCA) sta[top++]=LCA;
                break;
            }add_edge(sta[top-2],sta[top-1]); top--;
        }if(sta[top-1]!=a[i]) sta[top++]=a[i];
    }
    while(top>1) add_edge(sta[top-2],sta[top-1]),top--;
    for(int i=0;i<k;i++)inr[a[i]]=1;
}

```

图论

点双连通分量 (lyx)

```

#define SZ(x) ((int)x.size())
const int N = 400005, M = 200005; //N 开 2 倍点数
vector<int> g[N], bcc[N], G[N];
int bccno[N], bcc_cnt;
bool iscut[N];
struct Edge {
    int u, v;
} stk[M << 2];
int top, dfn[N], low[N], dfs_clock; // 注意栈大小为边数 4 倍
void dfs(int x, int fa)
{
    low[x] = dfn[x] = ++dfs_clock;
    int child = 0;
    for(int i = 0; i < SZ(g[x]); i++) {
        int y = g[x][i];
        if(!dfn[y]) {
            child++;
            stk[++top] = (Edge){x, y};
            dfs(y, x);
            low[x] = min(low[x], low[y]);
            if(low[y] >= dfn[x]) {
                iscut[x] = true;
                bcc[++bcc_cnt].clear();
            }
        }
    }
}

```

```

for(;;) {
    Edge e = stk[top--];
    if(bccno[e.u]!=bcc_cnt){bcc[bcc_cnt].push_back(e.u);bccno[e.u]=bcc_cnt;}
    if(bccno[e.v]!=bcc_cnt){bcc[bcc_cnt].push_back(e.v);bccno[e.v]=bcc_cnt;}
    if(e.u == x && e.v == y) break;
}
}
else if(y != fa && dfn[y] < dfn[x]) {
    stk[++top] = (Edge){x, y};
    low[x] = min(low[x], dfn[y]);
}
}
if(fa == 0 && child == 1) iscut[x] = false;
}
void find_bcc() // 求点双连通分量, 需要时手动 1 到 n 清空, 1-based
{
    memset(dfn, 0, sizeof(dfn));
    memset(iscut, 0, sizeof(iscut));
    memset(bccno, 0, sizeof(bccno));
    dfs_clock = bcc_cnt = 0;
    for(int i = 1; i <= n; i++)
        if(!dfn[i]) dfs(i, 0);
}
void prepare() { // 建出缩点后的树
    for(int i = 1; i <= n + bcc_cnt; i++)
        G[i].clear();
    for(int i = 1; i <= bcc_cnt; i++) {
        int x = i + n;
        for(int j = 0; j < SZ(bcc[i]); j++) {
            int y = bcc[i][j];
            G[x].push_back(y);
            G[y].push_back(x);
        }
    }
}

```

Hopcroft-Karp 求最大匹配

```

int matchx[N], matchy[N], level[N];
bool dfs(int x) {
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i], w = matchy[y];
        if (w == -1 || level[x] + 1 == level[w] && dfs(w)) {
            matchx[x] = y;
            matchy[y] = x;
            return true;
        }
    }
    level[x] = -1;
    return false;
}
int solve() {
    std::fill(matchx, matchx + n, -1);
    std::fill(matchy, matchy + m, -1);
    for (int answer = 0; ; ) {
        std::vector<int> queue;
    }
}

```



```

for (int i = 0; i < n; ++i) {
    if (matchx[i] == -1) {
        level[i] = 0;
        queue.push_back(i);
    } else level[i] = -1;
}
for (int head = 0; head < (int)queue.size(); ++head) {
    int x = queue[head];
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i], w = matchy[y];
        if (w != -1 && level[w] < 0) {
            level[w] = level[x] + 1;
            queue.push_back(w);
        }
    }
}
int delta = 0;
for (int i = 0; i < n; ++i)
    if (matchx[i] == -1 && dfs(i))
        delta++;
if (delta == 0) return answer;
else answer += delta;
}
}

```

KM 带权匹配

**注意事项：**最小权完美匹配，复杂度为  $\mathcal{O}(|V|^3)$ 。

```

int DFS(int x){
    visx[x] = 1;
    for (int y = 1; y <= ny; y++){
        if (visy[y]) continue;
        int t = lx[x] + ly[y] - w[x][y];
        if (t == 0) {
            visy[y] = 1;
            if (link[y] == -1 || DFS(link[y])){
                link[y] = x;
                return 1;
            }
        }
        else slack[y] = min(slack[y], t);
    }
    return 0;
}
int KM(){
    int i, j;
    memset(link, -1, sizeof(link));
    memset(ly, 0, sizeof(ly));
    for (i = 1; i <= nx; i++)
        for (j = 1, lx[i] = -inf; j <= ny; j++)
            lx[i] = max(lx[i], w[i][j]);
    for (int x = 1; x <= nx; x++){
        for (i = 1; i <= ny; i++) slack[i] = inf;
        while (true) {
            memset(visx, 0, sizeof(visx));
            memset(visy, 0, sizeof(visy));

```

```

        if (DFS(x)) break;
        int d = inf;
        for (i = 1; i <= ny; i++)
            if (!visy[i] && d > slack[i]) d = slack[i];
        for (i = 1; i <= nx; i++)
            if (visx[i]) lx[i] -= d;
        for (i = 1; i <= ny; i++)
            if (visy[i]) ly[i] += d;
            else slack[i] -= d;
    }
}
int res = 0;
for (i = 1; i <= ny; i++)
    if (link[i] > -1) res += w[link[i]][i];
return res;
}

```

zkw 费用流

```

namespace zkw{
    struct eglist{
        int other[maxM], succ[maxM], last[maxM], cap[maxM], cost[maxM], sum;
        void clear() {
            memset(last, -1, sizeof last);
            sum = 0;
        }
        void _addEdge(int a, int b, int c, int d) {
            other[sum] = b, succ[sum] = last[a], last[a] = sum, cost[sum] = d, cap[sum++] = c;
        }
        void addEdge(int a, int b, int c, int d) {
            _addEdge(a, b, c, d);
            _addEdge(b, a, 0, -d);
        }
    };
    int n, m, S, T, tot, totFlow, totCost;
    int dis[maxN], slack[maxN], visit[maxN], cur[maxN];

    int modlable() {
        int delta = inf;
        for (int i = 1; i <= T; ++i) {
            if (!visit[i] && slack[i] < delta)
                delta = slack[i];
            slack[i] = inf;
            // cur[i] = e.last[i];
        }
        if (delta == inf)
            return 1;
        for (int i = 1; i <= T; ++i)
            if (visit[i]) dis[i] += delta;
        return 0;
    }

    int dfs(int x, int flow) {
        if (x == T) {
            totFlow += flow;

```

```

    totCost += flow * (dis[S] - dis[T]);
    return flow;
}
visit[x] = 1;
int left = flow;
for (int i = e.last[x]; ~i; i = e.succ[i])
    if (e.cap[i] > 0 && !visit[e.other[i]]) {
        int y = e.other[i];
        if (dis[y] + e.cost[i] == dis[x]) {
            int delta = dfs(y, std::min(left, e.cap[i]));
            e.cap[i] -= delta;
            e.cap[i ^ 1] += delta;
            left -= delta;
            if (!left) {visit[x] = 0; return flow;}
        } else {
            slack[y] = std::min(slack[y], dis[y] + e.cost[i] - dis[x]);
        }
    }
return flow - left;
}

std::pair<int,int> minC() {
    totFlow = totCost = 0;
    std::fill(dis + 1, dis + T + 1, 0);
    for (int i = 1; i <= T; ++i) cur[i] = e.last[i];
    do {
        do {
            std::fill(visit + 1, visit + T + 1, 0);
        } while(dfs(S, inf));
    } while(!modlable());
    return std::make_pair(totFlow, totCost);
}
}

```

## 2-SAT 问题

```

int stamp, comps, top;
int dfn[N], low[N], comp[N], stack[N];
void add(int x, int a, int y, int b) {
    edge[x << 1 | a].push_back(y << 1 | b);
}
void tarjan(int x) {
    dfn[x] = low[x] = ++stamp;
    stack[top++] = x;
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        if (!dfn[y]) {
            tarjan(y);
            low[x] = std::min(low[x], low[y]);
        } else if (!comp[y])
            low[x] = std::min(low[x], dfn[y]);
    }
    if (low[x] == dfn[x]) {
        comps++;
        do {
            int y = stack[--top];

```

```

            comp[y] = comps;
        } while (stack[top] != x);
    }
}
bool solve() {
    int counter = n + n + 1;
    stamp = top = comps = 0;
    std::fill(dfn, dfn + counter, 0);
    std::fill(comp, comp + counter, 0);
    for (int i = 0; i < counter; ++i) {
        if (!dfn[i]) tarjan(i);
    }
    for (int i = 0; i < n; ++i) {
        if (comp[i << 1] == comp[i << 1 | 1]) return false;
        answer[i] = (comp[i << 1 | 1] < comp[i << 1]);
    }
    return true;
}
}

```

## 有根树的同构

```

const unsigned long long MAGIC = 4423;
unsigned long long magic[N];
std::pair<unsigned long long, int> hash[N];
void solve(int root) {
    magic[0] = 1;
    for (int i = 1; i <= n; ++i) {
        magic[i] = magic[i - 1] * MAGIC;
    }
    std::vector<int> queue;
    queue.push_back(root);
    for (int head = 0; head < (int)queue.size(); ++head) {
        int x = queue[head];
        for (int i = 0; i < (int)son[x].size(); ++i) {
            int y = son[x][i];
            queue.push_back(y);
        }
    }
    for (int index = n - 1; index >= 0; --index) {
        int x = queue[index];
        hash[x] = std::make_pair(0, 0);
        std::vector<std::pair<unsigned long long, int> > value;
        for (int i = 0; i < (int)son[x].size(); ++i) {
            int y = son[x][i];
            value.push_back(hash[y]);
        }
        std::sort(value.begin(), value.end());

        hash[x].first = hash[x].first * magic[1] + 37;
        hash[x].second++;
        for (int i = 0; i < (int)value.size(); ++i) {
            hash[x].first = hash[x].first * magic[value[i].second] + value[i].first;
            hash[x].second += value[i].second;
        }
        hash[x].first = hash[x].first * magic[1] + 41;
        hash[x].second++;
    }
}

```

```

}
}

Dominator Tree
class Edge{
public:
    int size, begin[MAXN], dest[MAXM], next[MAXM];
    void clear(int n){
        size = 0;
        fill(begin, begin + n, -1);
    }
    Edge(int n = MAXN){ clear(n); }
    void add_edge(int u, int v){
        dest[size] = v;
        next[size] = begin[u];
        begin[u] = size++;
    }
};

class dominator{
public:
    int dfn[MAXN], sdom[MAXN], idom[MAXN], id[MAXN], f[MAXN], fa[MAXN], smin[MAXN], stamp;
    void predfs(int x, const Edge &succ){
        id[dfn[x] = stamp++] = x;
        for(int i = succ.begin[x]; ~i; i = succ.next[i]){
            int y = succ.dest[i];
            if(dfn[y] < 0)
                f[y] = x, predfs(y, succ);
        }
    }
    int getfa(int x){
        if(fa[x] == x) return x;
        int ret = getfa(fa[x]);
        if(dfn[sdom[smin[fa[x]]]] < dfn[sdom[smin[x]]])
            smin[x] = smin[fa[x]];
        return fa[x] = ret;
    }
    void solve(int s, int n, const Edge &succ){
        fill(dfn, dfn + n, -1);
        fill(idom, idom + n, -1);
        static Edge pred, tmp;
        pred.clear(n);
        for(int i = 0; i < n; ++i)
            for(int j = succ.begin[i]; ~j; j = succ.next[j])
                pred.add_edge(succ.dest[j], i);
        stamp = 0;
        tmp.clear(n);
        predfs(s, succ);
        for(int i = 0; i < stamp; ++i)
            fa[id[i]] = smin[id[i]] = id[i];
        for(int o = stamp - 1; o >= 0; --o){
            int x = id[o];
            if(o){
                sdom[x] = f[x];
                for(int i = pred.begin[x]; ~i; i = pred.next[i]){
                    int p = pred.dest[i];

```

```

                    if(dfn[p] < 0) continue;
                    if(dfn[p] > dfn[x]){
                        getfa(p);
                        p = sdom[smin[p]];
                    }
                    if(dfn[sdom[x]] > dfn[p])
                        sdom[x] = p;
                }
            tmp.add_edge(sdom[x], x);
        }
        while(~tmp.begin[x]){
            int y = tmp.dest[tmp.begin[x]];
            tmp.begin[x] = tmp.next[tmp.begin[x]];
            getfa(y);
            if(x != sdom[smin[y]]) idom[y] = smin[y];
            else idom[y] = x;
        }
        for(int i = succ.begin[x]; ~i; i = succ.next[i])
            if(f[succ.dest[i]] == x) fa[succ.dest[i]] = x;
    }
    idom[s] = s;
    for(int i = 1; i < stamp; ++i){
        int x = id[i];
        if(idom[x] != sdom[x]) idom[x] = idom[idom[x]];
    }
}
};

```

无向图最小割

```

int node[N], dist[N];
bool visit[N];
int solve(int n) {
    int answer = INT_MAX;
    for (int i = 0; i < n; ++i) node[i] = i;
    while (n > 1) {
        int max = 1;
        for (int i = 0; i < n; ++i) {
            dist[node[i]] = graph[node[0]][node[i]];
            if (dist[node[i]] > dist[node[max]]) max = i;
        }
        int prev = 0;
        memset(visit, 0, sizeof(visit));
        visit[node[0]] = true;
        for (int i = 1; i < n; ++i) {
            if (i == n - 1) {
                answer = std::min(answer, dist[node[max]]);
                for (int k = 0; k < n; ++k) {
                    graph[node[k]][node[prev]] =
                        (graph[node[prev]][node[k]] += graph[node[k]][node[max]]);
                }
                node[max] = node[--n];
            }
            visit[node[max]] = true;
            prev = max;
            max = -1;

```

```

    for (int j = 1; j < n; ++j) {
        if (!visit[node[j]]) {
            dist[node[j]] += graph[node[prev]][node[j]];
            if (max == -1 || dist[node[max]] < dist[node[j]]) max = j;
        }
    }
}
return answer;
}

```

## 带花树

```

int match[N], belong[N], next[N], mark[N], visit[N];
std::vector<int> queue;
int find(int x) {
    if (belong[x] != x) belong[x] = find(belong[x]);
    return belong[x];
}
void merge(int x, int y) {
    x = find(x); y = find(y);
    if (x != y) belong[x] = y;
}
int lca(int x, int y) {
    static int stamp = 0;
    stamp++;
    while (true) {
        if (x != -1) {
            x = find(x);
            if (visit[x] == stamp) return x;
            visit[x] = stamp;
            if (match[x] != -1) x = next[match[x]];
            else x = -1;
        }
        std::swap(x, y);
    }
}
void group(int a, int p) {
    while (a != p) {
        int b = match[a], c = next[b];
        if (find(c) != p) next[c] = b;
        if (mark[b] == 2) {
            mark[b] = 1;
            queue.push_back(b);
        }
        if (mark[c] == 2) {
            mark[c] = 1;
            queue.push_back(c);
        }
        merge(a, b); merge(b, c); a = c;
    }
}
void augment(int source) {
    queue.clear();
    for (int i = 0; i < n; ++i) {
        next[i] = visit[i] = -1;
    }
}

```

```

    belong[i] = i;
    mark[i] = 0;
}
mark[source] = 1;
queue.push_back(source);
for (int head = 0; head < (int)queue.size() && match[source] == -1; ++head) {
    int x = queue[head];
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        if (match[x] == y || find(x) == find(y) || mark[y] == 2) continue;
        if (mark[y] == 1) {
            int r = lca(x, y);
            if (find(x) != r) next[x] = y;
            if (find(y) != r) next[y] = x;
            group(x, r); group(y, r);
        } else if (match[y] == -1) {
            next[y] = x;
            for (int u = y; u != -1; ) {
                int v = next[u], mv = match[v];
                match[v] = u; match[u] = v; u = mv;
            }
            break;
        } else {
            next[y] = x; mark[y] = 2;
            mark[match[y]] = 1;
            queue.push_back(match[y]);
        }
    }
}
}
int solve() {
    std::fill(match, match + n, -1);
    for (int i = 0; i < n; ++i)
        if (match[i] == -1) augment(i);
    int answer = 0;
    for (int i = 0; i < n; ++i) answer += (match[i] != -1);
    return answer;
}

```

## 字符串

## EXKMP

//求字符串  $b[0, n)$  的每个后缀和  $a[0, m)$  的最长公共前缀。  
 //将字符串翻转后可以求回文串。

```

void ExtendedKmp(int n, int m) {
    int i, j, k;
    for (j = 0; j + 1 < m && a[j] == a[j + 1]; ++j);
    nxt[1] = j; k = 1;
    for (i = 2; i < m; ++i) {
        int pos = k + nxt[k], len = nxt[i - k];
        if (i + len < pos) nxt[i] = len;
        else {
            for (j = max(0, pos - i); i + j < m && a[j] == a[i + j]; ++j);
            nxt[i] = j; k = i;
        }
    }
}

```

```

for(j = 0; j < m && j < n && a[j] == b[j]; ++j);
f[0] = j; k = 0;
for(i = 1; i < n; ++i){
    int pos = k + f[k], len = nxt[i - k];
    if(i + len < pos) f[i] = len;
    else {
        for(j = max(0, pos - i); j < m && i + j < n && a[j] == b[i + j]; ++j);
        f[i] = j; k = i;
    }
}
}

```

AC 自动机

```

void Insert(){
    int p = 0;
    for(int i = 0, c; str[i] != '\0'; ++i){
        c = str[i] - 'a';
        if(!ch[p][c]) ch[p][c] = ++nodecnt;
        p = ch[p][c];
    }
    val[p] = 1;
}

void Build(){
    int h = 1, t = 0, p, u;
    for(int c = 0; c < 26; ++c){
        p = ch[0][c];
        if(p) fail[p] = 0, Q[++t] = p;
    }
    while(h <= t){
        u = Q[h++];
        for(int c = 0; c < 26; ++c){
            p = ch[u][c];
            if(!p) ch[u][c] = ch[fail[u]][c];
            else {
                fail[p] = ch[fail[u]][c];
                Q[++t] = p;
            }
        }
    }
}

```

SAM

```

int n, K, flag, nodecnt;
char str[maxn];
int siz[maxn], Ws[maxn], Q[maxn]; ll sum[maxn];
struct Node{
    int nx[26], len, root;
}T[maxn];
inline void Init(){
    nodecnt = 0;
    T[0].root = -1, T[0].len = 0;
}
inline int Extend(int p, int c){
    int np = ++nodecnt;
    T[np].len = T[p].len + 1, siz[np] = 1;

```

```

for(; p != -1 && !T[p].nx[c]; p = T[p].root) T[p].nx[c] = np;
if(p == -1) T[np].root = 0;
else {
    int q = T[p].nx[c];
    if(T[q].len == T[p].len + 1) T[np].root = q;
    else {
        int nq = ++nodecnt;
        T[nq] = T[q]; T[nq].len = T[p].len + 1;
        for(; p != -1 && T[p].nx[c] == q; p = T[p].root) T[p].nx[c] = nq;
        T[q].root = T[np].root = nq;
    }
}
return np;
}

void Dfs(int x){
    if(K <= siz[x]) return;
    K -= siz[x];
    for(int c = 0; c < 26; ++c){
        if(T[x].nx[c]){
            if(K <= sum[T[x].nx[c]]){
                putchar('a' + c);
                Dfs(T[x].nx[c]);
                break;
            }
            else K -= sum[T[x].nx[c]];
        }
    }
}

int main(){
    scanf("%s", str); n = strlen(str);
    scanf("%d%d", &flag, &K);
    Init();
    for(int i = 0, last = 0; i < n; ++i) last = Extend(last, str[i] - 'a');
    for(int i = 1; i <= nodecnt; ++i) Ws[T[i].len]++;
    for(int i = 1; i <= n; ++i) Ws[i] += Ws[i - 1];
    for(int i = nodecnt; i > 0; --i) Q[Ws[T[i].len] - 1] = i;
    for(int i = nodecnt, x; i > 0; --i){
        x = Q[i];
        if(!flag) siz[x] = 1;
        else siz[T[x].root] += siz[x];
    }
    siz[0] = 0;
    for(int i = nodecnt, x; i >= 0; --i){
        x = Q[i];
        sum[x] = siz[x];
        for(int c = 0; c < 26; ++c){
            if(T[x].nx[c]) sum[x] += sum[T[x].nx[c]];
        }
    }
    if(sum[0] < K) puts("-1");
    else Dfs(0);
    return 0;
}

```

## 后缀数组

```

int Len;
int sa[maxn], Rank[maxn], h[maxn], r[maxn], wa[maxn << 1], wb[maxn << 1], Ws[maxn];
bool cmp(int *y, int a, int b, int len){
    return y[a] == y[b] && y[a + len] == y[b + len];
}
void Da(int n, int m){
    int i, j, p, *x = wa, *y = wb;
    for(i = 0; i < m; ++i) Ws[i] = 0;
    for(i = 0; i < n; ++i) Ws[x[i]] = r[i]++;
    for(i = 1; i < m; ++i) Ws[i] += Ws[i - 1];
    for(i = n - 1; i >= 0; --i) sa[--Ws[x[i]]] = i;
    for(j = 1, p = 0; p < n; j <= 1, m = p){
        for(p = 0, i = n - j; i < n; ++i) y[p++] = i;
        for(i = 0; i < n; ++i){
            if(sa[i] >= j) y[p++] = sa[i] - j;
        }
        for(i = 0; i < m; ++i) Ws[i] = 0;
        for(i = 0; i < n; ++i) Ws[x[y[i]]]++;
        for(i = 1; i < m; ++i) Ws[i] += Ws[i - 1];
        for(i = n - 1; i >= 0; --i) sa[--Ws[x[y[i]]]] = y[i];
        for(swap(x, y), i = 1, p = 1, x[sa[0]] = 0; i < n; ++i){
            x[sa[i]] = cmp(y, sa[i - 1], sa[i], j) ? p - 1 : p++;
        }
    }
}
void Calheight(int n){
    int i, j, k = 0;
    for(i = 1; i <= n; ++i) Rank[sa[i]] = i;
    for(i = 0; i < n; h[Rank[i+1]] = k){
        for(k > 0 ? k-- : 0, j = sa[Rank[i] - 1]; r[i + k] == r[j + k]; ++k);
    }
}
int Log[maxn], f[maxn << 1][20];
void ST(int n){
    Log[1] = 0;
    for(int i = 2; i <= n; ++i){
        Log[i] = Log[i - 1];
        if((1 << (Log[i] + 1)) == i) Log[i]++;
    }
    memset(f, 0x3f, sizeof(f));
    for(int i = 1; i <= n; ++i) f[i][0] = h[i];
    for(int j = 1; (1 << j) <= n; ++j){
        for(int i = 1; i <= n; ++i){
            f[i][j] = min(f[i][j - 1], f[i + (1 << (j - 1))][j - 1]);
        }
    }
}
int LCP(int x, int y){
    if(x == y) return Len - x;
    x = Rank[x], y = Rank[y];
    if(x > y) swap(x, y); ++x;
    int len = y - x + 1;
    return min(f[x][Log[len]], f[y - (1 << Log[len]) + 1][Log[len]]);
}

```

## 回文自动机

```

/*
Palindrome Automaton
本质不同的回文子串的个数 = 自动机节点个数 - 2。
siz[x] 表示 x 节点代表的回文串在整个字符串中的出现次数。
Apio2014 Palindrome
*/
struct Node{
    int len, fail, siz, nx[26];
}T[maxn];
void Init(){
    nodecnt = 1;
    T[0].len = 0, T[0].fail = 1;
    T[1].len = -1;
}
int Extend(int p, int c, int len){
    for(; str[len - T[p].len - 1] != str[len]; p = T[p].fail);
    if(!T[p].nx[c]){
        int np = ++nodecnt, x;
        for(x = T[p].fail; str[len - T[x].len - 1] != str[len]; x = T[x].fail);
        T[np].fail = T[x].nx[c];
        T[p].nx[c] = np;
        T[np].len = T[p].len + 2;
    }
    T[T[p].nx[c]].siz++;
    return T[p].nx[c];
}
int main(){
    Init();
    scanf("%s", str + 1);
    for(int i = 1, last = 0; str[i] != '\0'; ++i){
        last = Extend(last, str[i] - 'a', i);
    }
    ll ans = 0;
    for(int i = nodecnt; i >= 2; --i){
        T[T[i].fail].siz += T[i].siz;
        ans = max(ans, 1LL * T[i].siz * T[i].len);
    }
}

```

## Manacher

```

void Manacher(int n){
    for(int i = n; i >= 1; --i){
        if(i & 1) str[i] = '#';
        else str[i] = str[i >> 1];
    }
    str[0] = '$'; str[n + 1] = '*';
    for(int i = 1, mx = 0, pos = 0; i <= n; ++i){
        d[i] = i < mx ? min(d[pos * 2 - i], mx - i) : 1;
        while(str[i - d[i]] == str[i + d[i]]) d[i]++;
        if(i + d[i] > mx) mx = i + d[i], pos = i;
    }
}

```

循环串的最小表示

注意事项: 0-Based 算法, 请注意下标。

```
int getmin(char *s, int n){ // 0-base
    int i = 0, j = 1, k = 0;
    while(i < n && j < n && k < n){
        int x = i + k; if(x >= n) x -= n;
        int y = j + k; if(y >= n) y -= n;
        if(s[x] == s[y]) k++;
        else{
            if(s[x] > s[y]) i += k + 1;
            else j += k + 1;
            if(i == j) j++;
            k = 0;
        }
    }
    return min(i, j);
}
```

计算几何

二维几何

```
struct Point {
    Point rotate(const double ang) { // 逆时针旋转 ang 弧度
        return Point(cos(ang) * x - sin(ang) * y, cos(ang) * y + sin(ang) * x);
    }
    Point turn90() { // 逆时针旋转 90 度
        return Point(-y, x);
    }
};

Point isLL(const Line &l1, const Line &l2) {
    double s1 = det(l2.b - l2.a, l1.a - l2.a),
           s2 = -det(l2.b - l2.a, l1.b - l2.a);
    return (l1.a * s2 + l1.b * s1) / (s1 + s2);
}

bool onSeg(const Line &l, const Point &p) { // 点在线段上
    return sign(det(p - l.a, l.b - l.a)) == 0 && sign(dot(p - l.a, p - l.b)) <= 0;
}

Point projection(const Line &l, const Point &p) { // 点到直线投影
    return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).len2());
}

double disToLine(const Line &l, const Point &p) {
    return abs(det(p - l.a, l.b - l.a) / (l.b - l.a).len());
}

double disToSeg(const Line &l, const Point &p) { // 点到线段距离
    return sign(dot(p - l.a, l.b - l.a)) * sign(dot(p - l.b, l.a - l.b)) != 1 ?
        disToLine(l, p) : min((p - l.a).len(), (p - l.b).len());
}

Point symmetryPoint(const Point a, const Point b) { // 点 b 关于点 a 的中心对称点
    return a + a - b;
}

Point reflection(const Line &l, const Point &p) { // 点关于直线的对称点
    return symmetryPoint(projection(l, p), p);
}

// 求圆与直线的交点
bool isCL(Circle a, Line l, Point &p1, Point &p2) {
```

```
double x = dot(l.a - a.o, l.b - l.a),
       y = (l.b - l.a).len2(),
       d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
if (sign(d) < 0) return false;
d = max(d, 0.0);
Point p = l.a - ((l.b - l.a) * (x / y)), delta = (l.b - l.a) * (sqrt(d) / y);
p1 = p + delta, p2 = p - delta;
return true;
}

// 求圆与圆的交面积
double areaCC(const Circle &c1, const Circle &c2) {
    double d = (c1.o - c2.o).len();
    if (sign(d - (c1.r + c2.r)) >= 0) {
        return 0;
    }
    if (sign(d - abs(c1.r - c2.r)) <= 0) {
        double r = min(c1.r, c2.r);
        return r * r * PI;
    }
    double x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d),
           t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
    return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1);
}

// 求圆与圆的交点, 注意调用前要先判定重圆
bool isCC(Circle a, Circle b, Point &p1, Point &p2) {
    double s1 = (a.o - b.o).len();
    if (sign(s1 - a.r - b.r) > 0 || sign(s1 - abs(a.r - b.r)) < 0) return false;
    double s2 = (a.r * a.r - b.r * b.r) / s1;
    double aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
    Point o = (b.o - a.o) * (aa / (aa + bb)) + a.o;
    Point delta = (b.o - a.o).unit().turn90() * newSqrt(a.r * a.r - aa * aa);
    p1 = o + delta, p2 = o - delta;
    return true;
}

// 求点到圆的切点, 按关于点的顺时针方向返回两个点
bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2) {
    double x = (p0 - c.o).len2(), d = x - c.r * c.r;
    if (d < EPS) return false; // 点在圆上认为没有切点
    Point p = (p0 - c.o) * (c.r * c.r / x);
    Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
    p1 = c.o + p + delta;
    p2 = c.o + p - delta;
    return true;
}

// 求圆到圆的外共切线, 按关于 c1.o 的顺时针方向返回两条线
vector<Line> extanCC(const Circle &c1, const Circle &c2) {
    vector<Line> ret;
    if (sign(c1.r - c2.r) == 0) {
        Point dir = c2.o - c1.o;
        dir = (dir * (c1.r / dir.len())).turn90();
        ret.push_back(Line(c1.o + dir, c2.o + dir));
        ret.push_back(Line(c1.o - dir, c2.o - dir));
    } else {
        Point p = (c1.o * -c2.r + c2.o * c1.r) / (c1.r - c2.r);
        Point p1, p2, q1, q2;
```

```

    if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) {
        if (c1.r < c2.r) swap(p1, p2), swap(q1, q2);
        ret.push_back(Line(p1, q1));
        ret.push_back(Line(p2, q2));
    }
}
return ret;
}
// 求圆到圆的内共切线, 按关于 c1.o 的顺时针方向返回两条线
vector<Line> intanCC(const Circle &c1, const Circle &c2) {
    vector<Line> ret;
    Point p = (c1.o * c2.r + c2.o * c1.r) / (c1.r + c2.r);
    Point p1, p2, q1, q2;
    if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
        ret.push_back(Line(p1, q1));
        ret.push_back(Line(p2, q2));
    }
    return ret;
}
bool contain(vector<Point> polygon, Point p) {
    // 判断点 p 是否被多边形包含, 包括落在边界上
    int ret = 0, n = polygon.size();
    for(int i = 0; i < n; ++i) {
        Point u = polygon[i], v = polygon[(i + 1) % n];
        if (onSeg(Line(u, v), p)) return true;
        if (sign(u.y - v.y) <= 0) swap(u, v);
        if (sign(p.y - u.y) > 0 || sign(p.y - v.y) <= 0) continue;
        ret += sign(det(p, v, u)) > 0;
    }
    return ret & 1;
}
vector<Point> convexCut(const vector<Point>&ps, Line l) {
    // 用半平面 (q1,q2) 的逆时针方向去切凸多边形
    vector<Point> qs;
    int n = ps.size();
    for (int i = 0; i < n; ++i) {
        Point p1 = ps[i], p2 = ps[(i + 1) % n];
        int d1 = sign(det(l.a, l.b, p1)), d2 = sign(det(l.a, l.b, p2));
        if (d1 >= 0) qs.push_back(p1);
        if (d1 * d2 < 0) qs.push_back(isLL(Line(p1, p2), l));
    }
    return qs;
}
vector<Point> convexHull(vector<Point> ps) { // 求点集 ps 组成的凸包
    int n = ps.size(); if (n <= 1) return ps;
    sort(ps.begin(), ps.end());
    vector<Point> qs;
    for (int i = 0; i < n; qs.push_back(ps[i++]))
        while (qs.size() > 1 && sign(det(qs[qs.size()-2], qs.back(), ps[i])) <= 0)
            qs.pop_back();
    for (int i = n - 2, t = qs.size(); i >= 0; qs.push_back(ps[i--]))
        while ((int)qs.size() > t && sign(det(qs[(int)qs.size()-2], qs.back(), ps[i])) <= 0)
            qs.pop_back();
    return qs;
}

```

### 阿波罗尼茨圆

硬币问题: 易知两两相切的圆半径为  $r_1, r_2, r_3$ , 求与他们都相切的圆的半径  $r_4$

分母取负号, 答案再取绝对值, 为外切圆半径

分母取正号为内切圆半径

$$// r_4^{\pm} = \frac{r_1 r_2 r_3}{r_1 r_2 + r_1 r_3 + r_2 r_3 \pm 2\sqrt{r_1 r_2 r_3 (r_1 + r_2 + r_3)}}$$

### 三角形与圆交

// 反三角函数要在  $[-1, 1]$  中, sqrt 要与 0 取 max 别忘了取正负

// 改成周长请用注释, res1 为直线长度, res2 为弧线长度

// 多边形与圆求交时, 相切精度比较差

```

D areaCT(P pa, P pb, D r) { //, D & res1, D & res2) {
    if (pa.len() < pb.len()) swap(pa, pb);
    if (sign(pb.len()) == 0) return 0;
    // if (sign(pb.len()) == 0) { res1 += min(r, pa.len()); return; }
    D a = pb.len(), b = pa.len(), c = (pb - pa).len();
    D sinB = fabs(pb * (pb - pa)), cosB = pb % (pb - pa), area = fabs(pa * pb);
    D S, B = atan2(sinB, cosB), C = atan2(area, pa % pb);
    sinB /= a * c; cosB /= a * c;
    if (a > r) {
        S = C / 2 * r * r; D h = area / c; //res2 += -1 * sgn * C * r; D h = area / c;
        if (h < r && B < pi / 2) {
            //res2 -= -1 * sgn * 2 * acos(max((D)-1., min((D)1., h / r))) * r;
            //res1 += 2 * sqrt(max((D)0., r * r - h * h));
            S -= (acos(max((D)-1., min((D)1., h / r))) * r * r - h * sqrt(max((D)0.,
            // r * r - h * h)));
        }
    } else if (b > r) {
        D theta = pi - B - asin(max((D)-1., min((D)1., sinB / r * a)));
        S = a * r * sin(theta) / 2 + (C - theta) / 2 * r * r;
        //res2 += -1 * sgn * (C - theta) * r;
        //res1 += sqrt(max((D)0., r * r + a * a - 2 * r * a * cos(theta)));
    } else S = area / 2; //res1 += (pb - pa).len();
    return S;
}

```

### 圆并

```

struct Event {
    Point p;
    double ang;
    int delta;
    Event(Point p = Point(0, 0), double ang = 0, double delta = 0) : p(p), ang(ang),
    delta(delta) {}
};
bool operator < (const Event &a, const Event &b) {
    return a.ang < b.ang;
}
void addEvent(const Circle &a, const Circle &b, vector<Event> &evt, int &cnt) {
    double d2 = (a.o - b.o).len2(),
        dRatio = ((a.r - b.r) * (a.r + b.r) / d2 + 1) / 2,
        pRatio = sqrt(-(d2 - sqr(a.r - b.r)) * (d2 - sqr(a.r + b.r)) / (d2 * d2 * 4));
    Point d = b.o - a.o, p = d.rotate(PI / 2),
        q0 = a.o + d * dRatio + p * pRatio,
        q1 = a.o + d * dRatio - p * pRatio;
    double ang0 = (q0 - a.o).ang(),

```



```

    ang1 = (q1 - a.o).ang();
    evt.push_back(Event(q1, ang1, 1));
    evt.push_back(Event(q0, ang0, -1));
    cnt += ang1 > ang0;
}
bool issame(const Circle &a, const Circle &b) { return sign((a.o - b.o).len()) == 0
    ↪ && sign(a.r - b.r) == 0; }
bool overlap(const Circle &a, const Circle &b) { return sign(a.r - b.r - (a.o -
    ↪ b.o).len()) >= 0; }
bool intersect(const Circle &a, const Circle &b) { return sign((a.o - b.o).len() -
    ↪ a.r - b.r) < 0; }
int C;
Circle c[N];
double area[N];
void solve() {
    memset(area, 0, sizeof(double) * (C + 1));
    for (int i = 0; i < C; ++i) {
        int cnt = 1;
        vector<Event> evt;
        for (int j = 0; j < i; ++j) if (issame(c[i], c[j])) ++cnt;
        for (int j = 0; j < C; ++j)
            if (j != i && !issame(c[i], c[j]) && overlap(c[j], c[i]))
                ++cnt;
        for (int j = 0; j < C; ++j) {
            if (j != i && !overlap(c[j], c[i]) && !overlap(c[i], c[j]) && intersect(c[i],
    ↪ c[j]))
                addEvent(c[i], c[j], evt, cnt);
            if (evt.size() == 0) {
                area[cnt] += PI * c[i].r * c[i].r;
            } else {
                sort(evt.begin(), evt.end());
                evt.push_back(evt.front());
                for (int j = 0; j + 1 < (int)evt.size(); ++j) {
                    cnt += evt[j].delta;
                    area[cnt] += det(evt[j].p, evt[j + 1].p) / 2;
                    double ang = evt[j + 1].ang - evt[j].ang;
                    if (ang < 0) ang += PI * 2;
                    area[cnt] += ang * c[i].r * c[i].r / 2 - sin(ang) * c[i].r * c[i].r / 2;
                }
            }
        }
    }
}

```

整数半平面交

```

typedef __int128 J; // 坐标 |1e9| 就要用 int128 来判断
struct Line {
    bool include(P a) const { return (a - s) * d >= 0; } // 严格去掉 =
    bool include(Line a, Line b) const {
        J l1(a.d * b.d);
        if (!l1) return true;
        J x(l1 * (a.s.x - s.x)), y(l1 * (a.s.y - s.y));
        J l2((b.s - a.s) * b.d);
        x += l2 * a.d.x; y += l2 * a.d.y;
        J res(x * d.y - y * d.x);
        return l1 > 0 ? res >= 0 : res <= 0; // 严格去掉 =
    }
}

```

```

    }
};
bool HPI(vector<Line> v) { // 返回 v 中每个射线的右侧的交是否非空
    sort(v.begin(), v.end()); // 按方向排极角序
    { // 同方向取最严格的一个
        vector<Line> t; int n(v.size());
        for (int i = 0, j; i < n; i = j) {
            LL mx(-9e18); int mx_i;
            for (j = i; j < n && v[i].d * v[j].d == 0; j++) {
                LL tmp(v[j].s * v[i].d);
                if (tmp > mx)
                    mx = tmp, mx_i = j;
            }
            t.push_back(v[mx_i]);
        }
        swap(v, t);
    }
    deque<Line> res;
    bool emp(false);
    for (auto i : v) {
        if (res.size() == 1) {
            if (res[0].d * i.d == 0 && !i.include(res[0].s)) {
                res.pop_back();
                emp = true;
            }
        } else if (res.size() >= 2) {
            while (res.size() >= 2u && !i.include(res.back(), res[res.size() - 2])) {
                if (i.d * res[res.size() - 2].d == 0 || !res.back().include(i, res[res.size()
    ↪ - 2])) {
                    emp = true;
                    break;
                }
                res.pop_back();
            }
            while (res.size() >= 2u && !i.include(res[0], res[1])) res.pop_front();
        }
        if (emp) break;
        res.push_back(i);
    }
    while (res.size() > 2u && !res[0].include(res.back(), res[res.size() - 2]))
    ↪ res.pop_back();
    return !emp; // emp: 是否为空, res 按顺序即为半平面交
}

```

半平面交

```

struct Point {
    int quad() const { return sign(y) == 1 || (sign(y) == 0 && sign(x) >= 0); }
};
struct Line {
    bool include(const Point &p) const { return sign(det(b - a, p - a)) > 0; }
    Line push() const { // 将半平面向外推 eps
        const double eps = 1e-6;
        Point delta = (b - a).turn90().norm() * eps;
        return Line(a - delta, b - delta);
    }
}

```

```

};
bool sameDir(const Line &l0, const Line &l1) { return parallel(l0, l1) &&
    ↪ sign(dot(l0.b - l0.a, l1.b - l1.a)) == 1; }
bool operator < (const Point &a, const Point &b) {
    if (a.quad() != b.quad()) {
        return a.quad() < b.quad();
    } else {
        return sign(det(a, b)) > 0;
    }
}
bool operator < (const Line &l0, const Line &l1) {
    if (sameDir(l0, l1)) {
        return l1.include(l0.a);
    } else {
        return (l0.b - l0.a) < (l1.b - l1.a);
    }
}
bool check(const Line &u, const Line &v, const Line &w) { return
    ↪ w.include(intersect(u, v)); }
vector<Point> intersection(vector<Line> &l) {
    sort(l.begin(), l.end());
    deque<Line> q;
    for (int i = 0; i < (int)l.size(); ++i) {
        if (i && sameDir(l[i], l[i - 1])) {
            continue;
        }
        while (q.size() > 1 && !check(q[q.size() - 2], q[q.size() - 1], l[i]))
            ↪ q.pop_back();
        while (q.size() > 1 && !check(q[1], q[0], l[i])) q.pop_front();
        q.push_back(l[i]);
    }
    while (q.size() > 2 && !check(q[q.size() - 2], q[q.size() - 1], q[0]))
        ↪ q.pop_back();
    while (q.size() > 2 && !check(q[1], q[0], q[q.size() - 1])) q.pop_front();
    vector<Point> ret;
    for (int i = 0; i < (int)q.size(); ++i) ret.push_back(intersect(q[i], q[(i + 1) %
        ↪ q.size()]));
    return ret;
}

```

### 三角形

```

Point fermat(const Point& a, const Point& b, const Point& c) {
    double ab((b - a).len()), bc((b - c).len()), ca((c - a).len());
    double cosa(dot(b - a, c - a) / ab / ca);
    double cosb(dot(a - b, c - b) / ab / bc);
    double cosc(dot(b - c, a - c) / ca / bc);
    Point mid; double sq3(sqrt(3) / 2);
    if(sgn(det(b - a, c - a)) < 0) swap(b, c);
    if(sgn(cosa + 0.5) < 0) mid = a;
    else if(sgn(cosb + 0.5) < 0) mid = b;
    else if(sgn(cosc + 0.5) < 0) mid = c;
    else mid = isLL(Line(a, c + (b - c).rot(sq3) - a), Line(c, b + (a - b).rot(sq3) -
        ↪ c));
    return mid;
    // mid 为三角形 abc 费马点, 要求 abc 非退化
}

```

```

length = (mid - a).len() + (mid - b).len() + (mid - c).len();
// 以下求法仅在三角形三个角均小于 120 度时, 可以求出 ans 为费马点到 abc 三点距离和
length = (a - c - (b - c).rot(sq3)).len();
}
Point inCenter(const Point &A, const Point &B, const Point &C) { // 内心
    double a = (B - C).len(), b = (C - A).len(), c = (A - B).len(),
        s = fabs(det(B - A, C - A)), r = s / p;
    return (A * a + B * b + C * c) / (a + b + c); // 偏心则将对应点前两个加号改为减号
}
Point circumCenter(const Point &a, const Point &b, const Point &c) { // 外心
    Point bb = b - a, cc = c - a;
    double db = bb.len2(), dc = cc.len2(), d = 2 * det(bb, cc);
    return a - Point(bb.y * dc - cc.y * db, cc.x * db - bb.x * dc) / d;
}
Point othroCenter(const Point &a, const Point &b, const Point &c) { // 垂心
    Point ba = b - a, ca = c - a, bc = b - c;
    double Y = ba.y * ca.y * bc.y,
        A = ca.x * ba.y - ba.x * ca.y,
        x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
        y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
    return Point(x0, y0);
}

```

### 经纬度求球面最短距离

```

double sphereDis(double lon1, double lat1, double lon2, double lat2, double R) {
    return R * acos(cos(lat1) * cos(lat2) * cos(lon1 - lon2) + sin(lat1) * sin(lat2));
}

```

### 长方体表面两点最短距离

```

int r;
void turn(int i, int j, int x, int y, int z, int x0, int y0, int L, int W, int H) {
    if (z==0) { int R = x*x+y*y; if (R<r) r=R;
    } else {
        if(i>=0 && i<2) turn(i+1, j, x0+L+z, y, x0+L-x, x0+L, y0, H, W, L);
        if(j>=0 && j<2) turn(i, j+1, x, y0+W+z, y0+W-y, x0, y0+W, L, H, W);
        if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0, x0-H, y0, H, W, L);
        if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0, x0, y0-H, L, H, W);
    }
}
int main(){
    int L, H, W, x1, y1, z1, x2, y2, z2;
    cin >> L >> W >> H >> x1 >> y1 >> z1 >> x2 >> y2 >> z2;
    if (z1!=0 && z1!=H) if (y1==0 || y1==W)
        swap(y1,z1), std::swap(y2,z2), std::swap(W,H);
    else swap(x1,z1), std::swap(x2,z2), std::swap(L,H);
    if (z1==H) z1=0, z2=H-z2;
    r=0x3fffffff;
    turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
    cout<<r<<endl;
}

```

### 点到凸包切线

```

P lb(P x, vector<P> &v, int le, int ri, int sg) {
    if (le > ri) le = ri;
    int s(le), t(ri);
}

```

```

while (le != ri) {
    int mid((le + ri) / 2);
    if (sign((v[mid] - x) * (v[mid + 1] - v[mid])) == sg)
        le = mid + 1; else ri = mid;
}
return x - v[le]; // le 即为下标, 按需返回
}
// v[0] 为顺时针上凸壳, v[1] 为顺时针下凸壳, 均允许起始两个点横坐标相同
// 返回值为真代表严格在凸包外, 顺时针旋转在 d1 方向先碰到凸包
bool getTan(P x, vector<P> * v, P & d1, P & d2) {
    if (x.x < v[0][0].x) {
        d1 = lb(x, v[0], 0, sz(v[0]) - 1, 1);
        d2 = lb(x, v[1], 0, sz(v[1]) - 1, -1);
        return true;
    } else if (x.x > v[0].back().x) {
        d1 = lb(x, v[1], 0, sz(v[1]) - 1, 1);
        d2 = lb(x, v[0], 0, sz(v[0]) - 1, -1);
        return true;
    } else {
        for(int d(0); d < 2; d++) {
            int id(lower_bound(v[d].begin(), v[d].end(), x,
                [&](const P & a, const P & b) {
                    return d == 0 ? a < b : b < a;
                }) - v[d].begin());
            if (id && (id == sz(v[d]) || (v[d][id - 1] - x) * (v[d][id] - x) > 0)) {
                d1 = lb(x, v[d], id, sz(v[d]) - 1, 1);
                d2 = lb(x, v[d], 0, id, -1);
                return true;
            }
        }
    }
    return false;
}

```

### 直线与凸包的交点

// a 是顺时针凸包, i1 为 x 最小的点, j1 为 x 最大的点 需保证 j1 > i1  
 // n 是凸包上的点数, a 需复制多份或写循环数组类

```

int lowerBound(int le, int ri, const P & dir) {
    while (le < ri) {
        int mid((le + ri) / 2);
        if (sign((a[mid + 1] - a[mid]) * dir) <= 0) {
            le = mid + 1;
        } else ri = mid;
    }
    return le;
}
int boundLower(int le, int ri, const P & s, const P & t) {
    while (le < ri) {
        int mid((le + ri + 1) / 2);
        if (sign((a[mid] - s) * (t - s)) <= 0) {
            le = mid;
        } else ri = mid - 1;
    }
    return le;
}

```

```

void calc(P s, P t) {
    if (t < s) swap(t, s);
    int i3(lowerBound(i1, j1, t - s)); // 和上凸包的切点
    int j3(lowerBound(j1, i1 + n, s - t)); // 和下凸包的切点
    int i4(boundLower(i3, j3, s, t));
    // 如果有交则是右侧的交点, 与 a[i4]~a[i4+1] 相交 要判断是否有交的话 就手动 check 一下
    int j4(boundLower(j3, i3 + n, t, s)); // 如果有交左侧的交点, 与 a[j4]~a[j4+1] 相交
    // 返回的下标不一定在 [0 ~ n-1] 内
}

```

### 平面最近点对

```

struct Data { double x, y; };
double sqr(double x) { return x * x; }
double dis(Data a, Data b) { return sqrt(sqr(a.x - b.x) + sqr(a.y - b.y)); }
int n; Data p[N], q[N];
double solve(int l, int r) {
    if (l == r) return 1e18;
    if (l + 1 == r) return dis(p[l], p[r]);
    int m = (l + r) / 2;
    double d = min(solve(l, m), solve(m + 1, r));
    int qt = 0;
    for(int i = l; i <= r; i++)
        if (fabs(p[m].x - p[i].x) <= d)
            q[++qt] = p[i];
    sort(q + 1, q + qt + 1, [&](const Data & a, const Data & b) { return a.y < b.y; });
    for(int i = 1; i <= qt; i++) {
        for(int j = i + 1; j <= qt; j++) {
            if (q[j].y - q[i].y >= d) break;
            d = min(d, dis(q[i], q[j]));
        }
    }
    return d;
}

```

### 三维几何

```

Point3D det(const Point3D &a, const Point3D &b) {
    return Point3D(a.y * b.z - a.z * b.y, a.z * b.x - a.x * b.z, a.x * b.y - a.y *
        b.x);
}
// 平面法向量 : 平面上两个向量叉积 点共平面 : 平面上一点与之的向量点积法向量为 0
// 点在线段 ( 直线 ) 上 : 共线且两边点积非正
// 点在三角形内 ( 不包含边界, 需再判断是与某条边共线 )
bool pointInTri(const Point3D &a, const Point3D &b, const Point3D &c, const Point3D
    &p) {
    return sign(det(a - b, a - c).len() - det(p - a, p - b).len() - det(p - b, p -
        c).len() - det(p - c, p - a).len()) == 0;
}
// 共平面的两点是否在这平面上一条直线的同侧
bool sameSide(const Point3D &a, const Point3D &b, const Point3D &p0, const Point3D
    &p1) {
    return sign(dot(det(a - b, p0 - b), det(a - b, p1 - b))) > 0;
}
// 两点在平面同侧 : 点积法向量符号相同 两直线平行 / 垂直 : 同二维
// 平面平行 / 垂直 : 判断法向量 线面垂直 : 法向量和直线平行
// 判断空间线段是否相交 : 四点共面两线段不平行相互在异侧

```

```
// 线段和三角形是否相交：线段在三角形平面不同侧 三角形任意两点在线段和第三点组成的平面的不同侧
Point3D intersection(const Point3D &a0, const Point3D &b0, const Point3D &a1, const
    Point3D &b1) { // 求空间直线交点
    double t = ((a0.x - a1.x) * (a1.y - b1.y) - (a0.y - a1.y) * (a1.x - b1.x)) / ((a0.x
    - b0.x) * (a1.y - b1.y) - (a0.y - b0.y) * (a1.x - b1.x));
    return a0 + (b0 - a0) * t;
}
Point3D intersection(const Point3D &a, const Point3D &b, const Point3D &c, const
    Point3D &l0, const Point3D &l1) { // 求平面和直线的交点
    Point3D p = pVec(a, b, c); // 平面法向量
    double t = (p.x * (a.x - l0.x) + p.y * (a.y - l0.y) + p.z * (a.z - l0.z)) / (p.x *
    (l1.x - l0.x) + p.y * (l1.y - l0.y) + p.z * (l1.z - l0.z));
    return l0 + (l1 - l0) * t;
}
// 求平面交线：取不平行的一条直线的一个交点，以及法向量叉积得到直线方向
// 点到直线距离：叉积得到三角形的面积除以底边 点到平面距离：点积法向量
// 直线间距离：平行时随便取一点求距离，否则叉积方向向量得到方向点积计算长度
// 直线夹角：点积 平面夹角：法向量点积
// 三维向量旋转操作（绕向量 s 旋转 ang 角度），对于右手系 s 指向观察者时逆时针
void rotate(const Point3D &s, double ang) {
    double l = s.len(), x = s.x / l, y = s.y / l, z = s.z / l, sinA = sin(ang), cosA =
    cos(ang);
    double p[4][4] = {CosA + (1 - CosA) * x * x, (1 - CosA) * x * y - SinA * z, (1 -
    CosA) * x * z + SinA * y, 0,
    (1 - CosA) * y * x + SinA * z, CosA + (1 - CosA) * y * y, (1 - CosA) * y * z -
    SinA * x, 0,
    (1 - CosA) * z * x - SinA * y, (1 - CosA) * z * y + SinA * x, CosA + (1 - CosA) *
    z * z, 0,
    0, 0, 0, 1};
}
// 计算版：把需要旋转的向量按照 s 分解，做二维旋转，再回到三维
```

## 其他

## 最小树形图

```
const int maxn=1100;
int n,m, g[maxn][maxn], used[maxn], pass[maxn], eg[maxn], more, queue[maxn];
void combine(int id, int &sum) {
    int tot = 0, from, i, j, k;
    for (; id!=0 && !pass[id]; id=eg[id]) {
        queue[tot++]=id; pass[id]=1;
    }
    for (from=0; from<tot && queue[from]!=id; from++);
    if (from==tot) return;
    more = 1;
    for (i=from; i<tot; i++) {
        sum+=g[eg[queue[i]]][queue[i]];
        if (i!=from) {
            used[queue[i]]=1;
            for (j = 1; j <= n; j++) if (!used[j])
                if (g[queue[i]][j]<g[id][j]) g[id][j]=g[queue[i]][j];
        }
    }
    for (i=1; i<=n; i++) if (!used[i] && i!=id) {
        for (j=from; j<tot; j++){
```

```
k=queue[j];
        if (g[i][id]>g[i][k]-g[eg[k]][k]) g[i][id]=g[i][k]-g[eg[k]][k];
    }
}
int mdst(int root) { // return the total length of MDST
    int i, j, k, sum = 0;
    memset(used, 0, sizeof(used));
    for (more=1; more; ) {
        more = 0;
        memset(eg, 0, sizeof(eg));
        for (i=1; i<=n; i++) if (!used[i] && i!=root) {
            for (j=1, k=0; j<=n; j++) if (!used[j] && i!=j)
                if (k==0 || g[j][i] < g[k][i]) k=j;
            eg[i] = k;
        }
        memset(pass, 0, sizeof(pass));
        for (i=1; i<=n; i++) if (!used[i] && !pass[i] && i!=root) combine(i, sum);
    }
    for (i=1; i<=n; i++) if (!used[i] && i!=root) sum+=g[eg[i]][i];
    return sum;
}
```

## DLX

```
int n,m,K;
struct DLX{
    int L[maxn],R[maxn],U[maxn],D[maxn];
    int sz,col[maxn],row[maxn],s[maxn],H[maxn];
    bool vis[233];
    int ans[maxn],cnt;
    void init(int m){
        for(int i=0;i<=m;i++){
            L[i]=i-1;R[i]=i+1;
            U[i]=D[i]=i;s[i]=0;
        }
        memset(H,-1,sizeof(H));
        L[0]=m;R[m]=0;sz=m+1;
    }
    void Link(int r,int c){
        U[sz]=c;D[sz]=D[c];U[D[c]]=sz;D[c]=sz;
        if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
        else{
            L[sz]=H[r];R[sz]=R[H[r]];
            L[R[H[r]]]=sz;R[H[r]]=sz;
        }
        s[c]++;col[sz]=c;row[sz]=r;sz++;
    }
    void remove(int c){
        for(int i=D[c];i!=c;i=D[i])
            L[R[i]]=L[i],R[L[i]]=R[i];
    }
    void resume(int c){
        for(int i=U[c];i!=c;i=U[i])
```

```

    L[R[i]]=R[L[i]]=i;
}
int A(){
    int res=0;
    memset(vis,0,sizeof vis);
    for(int i=R[0];i;i=R[i])if(!vis[i]){
        vis[i]=1;res++;
        for(int j=D[i];j!=i;j=D[j])
            for(int k=R[j];k!=j;k=R[k])
                vis[col[k]]=1;
    }
    return res;
}
void dfs(int d,int &ans){
    if(R[0]==0){ans=min(ans,d);return;}
    if(d+A()>=ans)return;
    int tmp=23333,c;
    for(int i=R[0];i;i=R[i])
        if(tmp>s[i])tmp=s[i],c=i;
    for(int i=D[c];i!=c;i=D[i]){
        remove(i);
        for(int j=R[i];j!=i;j=R[j])remove(j);
        dfs(d+1,ans);
        for(int j=L[i];j!=i;j=L[j])resume(j);
        resume(i);
    }
}
void del(int c){//exactly cover
    L[R[c]]=L[c];R[L[c]]=R[c];
    for(int i=D[c];i!=c;i=D[i])
        for(int j=R[i];j!=i;j=R[j])
            U[D[j]]=U[j],D[U[j]]=D[j],--s[col[j]];
}
void add(int c){ //exactly cover
    R[L[c]]=L[R[c]]=c;
    for(int i=U[c];i!=c;i=U[i])
        for(int j=L[i];j!=i;j=L[j])
            ++s[col[U[D[j]]=D[U[j]]=j]];
}
bool dfs2(int k){//exactly cover
    if(!R[0]){
        cnt=k;return 1;
    }
    int c=R[0];
    for(int i=R[0];i;i=R[i])
        if(s[c]>s[i])c=i;
    del(c);
    for(int i=D[c];i!=c;i=D[i]){
        for(int j=R[i];j!=i;j=R[j])
            del(col[j]);
        ans[k]=row[i];if(dfs2(k+1))return true;
        for(int j=L[i];j!=i;j=L[j])
            add(col[j]);
    }
    add(c);
}

```

```

    return 0;
}
}dlx;
int main(){
    dlx.init(n);
    for(int i=1;i<=m;i++)
        for(int j=1;j<=n;j++)
            if(dis(station[i],city[j])<mid-eps)
                dlx.Link(i,j);
    dlx.dfs(0,ans);
}

```

某年某月某日是星期几

```

int solve(int year, int month, int day) {
    int answer;
    if (month == 1 || month == 2) {
        month += 12;
        year--;
    }
    if ((year < 1752) || (year == 1752 && month < 9) ||
        (year == 1752 && month == 9 && day < 3)) {
        answer = (day + 2 * month + 3 * (month + 1) / 5 + year + year / 4 + 5) % 7;
    } else {
        answer = (day + 2 * month + 3 * (month + 1) / 5 + year + year / 4
            - year / 100 + year / 400) % 7;
    }
    return answer;
}

```

枚举大小为  $k$  的子集

使用条件:  $k > 0$

```

void solve(int n, int k) {
    for (int comb = (1 << k) - 1; comb < (1 << n); ) {
        int x = comb & -comb, y = comb + x;
        comb = (((comb & ~y) / x) >> 1) | y;
    }
}

```

环状最长公共子串

```

int n, a[N << 1], b[N << 1];
bool has(int i, int j) {
    return a[(i - 1) % n] == b[(j - 1) % n];
}
const int DELTA[3][2] = {{0, -1}, {-1, -1}, {-1, 0}};
int from[N][N];
int solve() {
    memset(from, 0, sizeof(from));
    int ret = 0;
    for (int i = 1; i <= 2 * n; ++i) {
        from[i][0] = 2;
        int left = 0, up = 0;
        for (int j = 1; j <= n; ++j) {
            int upleft = up + 1 + !from[i - 1][j];
            if (!has(i, j)) {
                upleft = INT_MIN;
            }
        }
    }
}

```

```

}
int max = std::max(left, std::max(upleft, up));
if (left == max) {
    from[i][j] = 0;
} else if (upleft == max) {
    from[i][j] = 1;
} else {
    from[i][j] = 2;
}
left = max;
}
if (i >= n) {
    int count = 0;
    for (int x = i, y = n; y; ) {
        int t = from[x][y];
        count += t == 1;
        x += DELTA[t][0];
        y += DELTA[t][1];
    }
    ret = std::max(ret, count);
    int x = i - n + 1;
    from[x][0] = 0;
    int y = 0;
    while (y <= n && from[x][y] == 0) {
        y++;
    }
    for (; x <= i; ++x) {
        from[x][y] = 0;
        if (x == i) {
            break;
        }
        for (; y <= n; ++y) {
            if (from[x + 1][y] == 2) {
                break;
            }
            if (y + 1 <= n && from[x + 1][y + 1] == 1) {
                y++;
                break;
            }
        }
    }
}
return ret;
}

```

LLMOD STL 内存清空开栈

```

LL multiplyMod(LL a, LL b, LL P) { // `需要保证 a 和 b 非负`
    LL t = (a * b - LL((long double)a / P * b + 1e-3) * P) % P;
    return t < 0 : t + P : t;
}

```

```

template <typename T>
__inline void clear(T& container) {
    container.clear(); // 或者删除了一堆元素
    T(container).swap(container);
}

```

```

}
register char *_sp __asm__("rsp");
int main() {
    const int size = 400 << 20; // 400MB
    static char *sys, *mine(new char[size] + size - 4096);
    sys = _sp; _sp = mine; _main(); _sp = sys;
}

```

vimrc

```

set ru nu cin ts=4 sts=4 sw=4 hls is ar acd bs=2 mouse=a ls=2 fdm=syntax fdl=100
set makeprg=g++\ %:r.cpp\ -o\ %:r\ -g\ -std=c++11\ -Wall
map <F3> :vnew %:r.in<cr>
map <F4> :!gedit %<cr>
map <F5> :!time ./%:r<cr>
map <F8> :!time ./%:r < %:r.in<cr>
map <F9> :make<cr>
map <C-F9> :!g++ %:r.cpp -o %:r -g -O2 -std=c++11<cr>
map <F10> :!gdb ./%:r<cr>

```

上下界网络流

无源汇的上下界可行流

建立超级源点  $S^*$  和超级汇点  $T^*$ , 对于原图每条边  $(u, v)$  在新网络中连如下三条边:  $S^* \rightarrow v$ , 容量为  $B(u, v)$ ;  $u \rightarrow T^*$ , 容量为  $B(u, v)$ ;  $u \rightarrow v$ , 容量为  $C(u, v) - B(u, v)$ 。最后求新网络的最大流, 判断从超级源点  $S^*$  出发的边是否都满流即可, 边  $(u, v)$  的最终解中的实际流量为  $G(u, v) + B(u, v)$ 。

有源汇的上下界可行流

从汇点  $T$  到源点  $S$  连一条上界为  $\infty$ , 下界为  $0$  的边。按照无源汇的上下界可行流一样做即可, 流量即为  $T \rightarrow S$  边上的流量。

有源汇的上下界最大流

1. 在有源汇的上下界可行流中, 从汇点  $T$  到源点  $S$  的边改为连一条上界为  $\infty$ , 下界为  $x$  的边。 $x$  满足二分性质, 找到最大的  $x$  使得新网络存在无源汇的上下界可行流即为原图的最大流。
2. 从汇点  $T$  到源点  $S$  连一条上界为  $\infty$ , 下界为  $0$  的边, 变成无源汇的网络。按照无源汇的上下界可行流的方法, 建立超级源点  $S^*$  和超级汇点  $T^*$ , 求一遍  $S^* \rightarrow T^*$  的最大流, 再将从汇点  $T$  到源点  $S$  的这条边拆掉, 求一次  $S \rightarrow T$  的最大流即可。

有源汇的上下界最小流

1. 在有源汇的上下界可行流中, 从汇点  $T$  到源点  $S$  的边改为连一条上界为  $x$ , 下界为  $0$  的边。 $x$  满足二分性质, 找到最小的  $x$  使得新网络存在无源汇的上下界可行流即为原图的最小流。
2. 按照无源汇的上下界可行流的方法, 建立超级源点  $S^*$  与超级汇点  $T^*$ , 求一遍  $S^* \rightarrow T^*$  的最大流, 但是注意这一次不加上汇点  $T$  到源点  $S$  的这条边, 即不使之改为无源汇的网络去求解。求完后, 再加上那条汇点  $T$  到源点  $S$  上界  $\infty$  的边。因为这条边下界为  $0$ , 所以

$S^*$ ,  $T^*$  无影响, 再直接求一次  $S^* \rightarrow T^*$  的最大流。若超级源点  $S^*$  出发的边全部满流, 则  $T \rightarrow S$  边上的流量即为原图的最小流, 否则无解。

上下界费用流

设汇  $t$ , 源  $s$ , 超级源  $S$ , 超级汇  $T$ , 本质是每条边的下界为 1, 上界为 MAX, 跑一遍有源汇的上下界最小费用最小流。(因为上界无穷大, 所以只要满足所有下界的最小费用最小流)

1. 对每个点  $x$ : 从  $x$  到  $t$  连一条费用为 0, 流量为 MAX 的边, 表示可以任意停止当前的剧情 (接下来的剧情从更优的路径去走, 画个样例就知道了)
2. 对于每一条边权为  $z$  的边  $x \rightarrow y$ :
  - 从  $S$  到  $y$  连一条流量为 1, 费用为  $z$  的边, 代表这条边至少要被走一次。
  - 从  $x$  到  $y$  连一条流量为 MAX, 费用为  $z$  的边, 代表这条边除了至少走的一次之外还可以随便走。
  - 从  $x$  到  $T$  连一条流量为 1, 费用为 0 的边。(注意是每一条  $x \rightarrow y$  的边都连, 或者你可以记下  $x$  的出边数  $K_x$ , 连一次流量为  $K_x$ , 费用为 0 的边)。

建完图后从  $S$  到  $T$  跑一遍费用流, 即可。(当前跑出来的就是满足上下界的最小费用最小流了)

Bernoulli 数

1. 初始化:  $B_0(n) = 1$
2. 递推公式:  $B_m(n) = n^m - \sum_{k=0}^{m-1} \binom{m}{k} \frac{B_k(n)}{m-k+1}$
3. 应用:  $\sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} n^{m+1-k}$

Java Hints

```
import java.util.*;
import java.math.*;
import java.io.*;
public class Main{
    static class Task{
        void solve(int testId, InputReader cin, PrintWriter cout) {
            // Write down the code you want
        }
    };

    public static void main(String args[]) {
        InputStream inputStream = System.in;
        OutputStream outputStream = System.out;
        InputReader in = new InputReader(inputStream);
        PrintWriter out = new PrintWriter(outputStream);
        // Scanner cin = new Scanner(System.in);
        // cin.nextLong();
        // System.out.println(AnsA+" "+AnsB);
    }

    static class InputReader {
        public BufferedReader reader;
        public StringTokenizer tokenizer;
```

```
public InputReader(InputStream stream) {
    reader = new BufferedReader(new InputStreamReader(stream), 32768);
    tokenizer = null;
}
public String next() {
    while (tokenizer == null || !tokenizer.hasMoreTokens()) {
        try {
            tokenizer = new StringTokenizer(reader.readLine());
        } catch (IOException e) {
            throw new RuntimeException(e);
        }
    }
    return tokenizer.nextToken();
}
public int nextInt() {
    return Integer.parseInt(next());
}
}
};
// Arrays
int a[];
.fill(a[, int fromIndex, int toIndex],val); | .sort(a[, int fromIndex, int toIndex])
// String
String s;
.charAt(int i); | compareTo(String) | compareToIgnoreCase () | contains(String) |
length () | substring(int l, int len)
// BigInteger
.abs() | .add() | bitLength () | subtract () | divide () | remainder () |
↳ divideAndRemainder () | modPow(b, c) |
pow(int) | multiply () | compareTo () |
gcd() | intValue () | longValue () | isProbablePrime(int c) (1 - 1/2^c) |
nextProbablePrime () | shiftLeft(int) | valueOf ()
// BigDecimal
.ROUND_CEILING | ROUND_DOWN_FLOOR | ROUND_HALF_DOWN | ROUND_HALF_EVEN | ROUND_HALF_UP
↳ | ROUND_UP
.divide(BigDecimal b, int scale , int round_mode) | doubleValue () |
↳ movePointLeft(int) | pow(int) |
setScale(int scale , int round_mode) | stripTrailingZeros ()
BigDecimal.setScale()方法用于格式化小数点
setScale(1)表示保留一位小数,默认用四舍五入方式
setScale(1,BigDecimal.ROUND_DOWN)直接删除多余的小数位,如 2.35会变成 2.3
setScale(1,BigDecimal.ROUND_UP)进位处理,2.35变成 2.4
setScale(1,BigDecimal.ROUND_HALF_UP)四舍五入,2.35变成 2.4
setScale(1,BigDecimal.ROUND_HALF_DOWN)四舍五入,2.35变成 2.3,如果是 5 则向下舍
setScale(1,BigDecimal.ROUND_CEILING)接近正无穷大的舍入
setScale(1,BigDecimal.ROUND_FLOOR)接近负无穷大的舍入,数字>0=ROUND_UP,数字<0=ROUND_DOWN
setScale(1,BigDecimal.ROUND_HALF_EVEN)向最接近的数字舍入,如果距离相等则向相邻的偶数舍入
// StringBuilder
StringBuilder sb = new StringBuilder ();
sb.append(elem) | out.println(sb)
```



数学

常用数学公式

求和公式

- 1.  $\sum_{k=1}^n (2k-1)^2 = \frac{n(4n^2-1)}{3}$
- 2.  $\sum_{k=1}^n k^3 = [\frac{n(n+1)}{2}]^2$
- 3.  $\sum_{k=1}^n (2k-1)^3 = n^2(2n^2-1)$
- 4.  $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- 5.  $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$
- 6.  $\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$
- 7.  $\sum_{k=1}^n k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
- 8.  $\sum_{k=1}^n k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$

斐波那契数列

- 1.  $fib_0 = 0, fib_1 = 1, fib_n = fib_{n-1} + fib_{n-2}$
- 2.  $fib_{n+2} \cdot fib_n - fib_{n+1}^2 = (-1)^{n+1}$
- 3.  $fib_{-n} = (-1)^{n-1} fib_n$
- 4.  $fib_{n+k} = fib_k \cdot fib_{n+1} + fib_{k-1} \cdot fib_n$
- 5.  $gcd(fib_m, fib_n) = fib_{gcd(m,n)}$
- 6.  $fib_m | fib_n^2 \Leftrightarrow n | fib_m$

错排公式

- 1.  $D_n = (n-1)(D_{n-2} - D_{n-1})$
- 2.  $D_n = n! \cdot (1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!})$

莫比乌斯函数

$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d)g(\frac{n}{d})$   $g(x) = \sum_{n=1}^{[x]} f(\frac{x}{n}) \Leftrightarrow f(x) = \sum_{n=1}^{[x]} \mu(n)g(\frac{x}{n})$

伯恩赛德引理

设  $G$  是一个有限群，作用在集合  $X$  上。对每个  $g$  属于  $G$ ，令  $X^g$  表示  $X$  中在  $g$  作用下的不动元素，轨道数（记作  $|X/G|$ ）由如下公式给出： $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$ 。

五边形数定理

设  $p(n)$  是  $n$  的拆分数，有  $p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k-1} p(n - \frac{k(3k-1)}{2})$

树的计数

- 1. 有根树计数： $n+1$  个结点的有根树的个数为  $a_{n+1} = \frac{\sum_{j=1}^n j \cdot a_j \cdot S_{n,j}}{n}$  其中， $S_{n,j} = \sum_{i=1}^{n/j} a_{n+1-ij} = S_{n-j,j} + a_{n+1-j}$
- 2. 无根树计数：当  $n$  为奇数时， $n$  个结点的无根树的个数为  $a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$  当  $n$  为偶数时， $n$  个结点的无根树的个数为  $a_n - \sum_{i=1}^{n/2} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1)$
- 3.  $n$  个结点的完全图的生成树个数为  $n^{n-2}$

- 4. 矩阵 - 树定理：图  $G$  由  $n$  个结点构成，设  $A[G]$  为图  $G$  的邻接矩阵、 $D[G]$  为图  $G$  的度数矩阵，则图  $G$  的不同生成树的个数为  $C[G] = D[G] - A[G]$  的任意一个  $n-1$  阶主子式的行列式值。

欧拉公式

平面图形的顶点个数、边数和面的个数有如下关系： $V - E + F = C + 1$  其中， $V$  是顶点的数目， $E$  是边的数目， $F$  是面的数目， $C$  是组成图形的连通部分的数目。当图是单连通图的时候，公式简化为： $V - E + F = 2$

皮克定理

给定顶点坐标均是整点（或正方形格点）的简单多边形，其面积  $A$  和内部格点数目  $i$ 、边上格点数目  $b$  的关系： $A = i + \frac{b}{2} - 1$

牛顿恒等式

设  $\prod_{i=1}^n (x - x_i) = a_n + a_{n-1}x + \dots + a_1x^{n-1} + a_0x^n$   $p_k = \sum_{i=1}^n x_i^k$  则  $a_0p_k + a_1p_{k-1} + \dots + a_{k-1}p_1 + ka_k = 0$

特别地，对于  $|A - \lambda E| = (-1)^n (a_n + a_{n-1}\lambda + \dots + a_1\lambda^{n-1} + a_0\lambda^n)$  有  $p_k = Tr(A^k)$

平面几何公式

三角形

- 1. 面积  $S = \frac{a \cdot H_a}{2} = \frac{ab \cdot \sin C}{2} = \sqrt{p(p-a)(p-b)(p-c)}$
- 2. 中线  $M_a = \frac{\sqrt{2(b^2+c^2)-a^2}}{2} = \frac{\sqrt{b^2+c^2+2bc \cdot \cos A}}{2}$
- 3. 角平分线  $T_a = \frac{\sqrt{bc \cdot [(b+c)^2 - a^2]}}{b+c} = \frac{2bc \cos \frac{A}{2}}{b+c}$
- 4. 高线  $H_a = b \sin C = c \sin B = \sqrt{b^2 - (\frac{a^2+b^2-c^2}{2a})^2}$
- 5. 内切圆半径

$$r = \frac{S}{p} = \frac{\arcsin \frac{B}{2} \cdot \sin \frac{C}{2}}{\sin \frac{B+C}{2}} = 4R \cdot \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$
$$= \sqrt{\frac{(p-a)(p-b)(p-c)}{p}} = p \cdot \tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2}$$

- 6. 外接圆半径  $R = \frac{abc}{4S} = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$

四边形

$D_1, D_2$  为对角线， $M$  对角线中点连线， $A$  为对角线夹角， $p$  为半周长

- 1.  $a^2 + b^2 + c^2 + d^2 = D_1^2 + D_2^2 + 4M^2$
- 2.  $S = \frac{1}{2} D_1 D_2 \sin A$
- 3. 对于圆内接四边形  $ac + bd = D_1 D_2$
- 4. 对于圆内接四边形  $S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$



正  $n$  边形

$R$  为外接圆半径,  $r$  为内切圆半径

1. 中心角  $A = \frac{2\pi}{n}$
2. 内角  $C = \frac{n-2}{n}\pi$
3. 边长  $a = 2\sqrt{R^2 - r^2} = 2R \cdot \sin\frac{A}{2} = 2r \cdot \tan\frac{A}{2}$
4. 面积  $S = \frac{nar}{2} = nr^2 \cdot \tan\frac{A}{2} = \frac{nR^2}{2} \cdot \sin A = \frac{na^2}{4 \cdot \tan\frac{A}{2}}$

圆

1. 弧长  $l = rA$
2. 弦长  $a = 2\sqrt{2hr - h^2} = 2r \cdot \sin\frac{A}{2}$
3. 弓形高  $h = r - \sqrt{r^2 - \frac{a^2}{4}} = r(1 - \cos\frac{A}{2}) = \frac{1}{2} \cdot \arctan\frac{A}{4}$
4. 扇形面积  $S_1 = \frac{rl}{2} = \frac{r^2 A}{2}$
5. 弓形面积  $S_2 = \frac{rl - a(r-h)}{2} = \frac{r^2}{2}(A - \sin A)$

棱柱

1. 体积  $V = Ah$   $A$  为底面积,  $h$  为高
2. 侧面积  $S = lp$   $l$  为棱长,  $p$  为直截面周长
3. 全面积  $T = S + 2A$

棱锥

1. 体积  $V = Ah$   $A$  为底面积,  $h$  为高
2. 正棱锥侧面积  $S = lp$   $l$  为棱长,  $p$  为直截面周长
3. 正棱锥全面积  $T = S + 2A$

棱台

1. 体积  $V = (A_1 + A_2 + \sqrt{A_1 A_2}) \cdot \frac{h}{3}$   $A_1, A_2$  为上下底面积,  $h$  为高正棱台侧面积  $S = \frac{p_1 + p_2}{2} l$   $p_1, p_2$  为上下底面周长,  $l$  为斜高
2. 正棱台全面积  $T = S + A_1 + A_2$

圆柱

1. 侧面积  $S = 2\pi rh$
2. 全面积  $T = 2\pi r(h + r)$
3. 体积  $V = \pi r^2 h$

圆锥

1. 母线  $l = \sqrt{h^2 + r^2}$
2. 侧面积  $S = \pi rl$  全面积  $T = \pi r(l + r)$
3. 体积  $V = \frac{\pi}{3} r^2 h$

圆台

1. 母线  $l = \sqrt{h^2 + (r_1 - r_2)^2}$
2. 侧面积  $S = \pi(r_1 + r_2)l$  全面积  $T = \pi r_1(l + r_1) + \pi r_2(l + r_2)$
3. 体积  $V = \frac{\pi}{3}(r_1^2 + r_2^2 + r_1 r_2)h$

球台

1. 侧面积  $S = 2\pi rh$  全面积  $T = \pi(2rh + r_1^2 + r_2^2)$
2. 体积  $V = \frac{\pi h[3(r_1^2 + r_2^2) + h^2]}{6}$

球扇形

1. 全面积  $T = \pi r(2h + r_0)$   $h$  为球冠高,  $r_0$  为球冠底面半径
2. 体积  $V = \frac{2}{3}\pi r^2 h$

积分表

$$\begin{aligned} \int \frac{1}{1+x^2} dx &= \tan^{-1} x \\ \int \frac{1}{a^2+x^2} dx &= \frac{1}{a} \tan^{-1} \frac{x}{a} \\ \int \frac{x}{a^2+x^2} dx &= \frac{1}{2} \ln |a^2 + x^2| \\ \int \frac{x^2}{a^2+x^2} dx &= x - a \tan^{-1} \frac{x}{a} \\ \int \sqrt{x^2 \pm a^2} dx &= \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln |x + \sqrt{x^2 \pm a^2}| \\ \int \sqrt{a^2 - x^2} dx &= \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}} \\ \int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx &= \frac{1}{2} x \sqrt{x^2 \pm a^2} \mp \frac{1}{2} a^2 \ln |x + \sqrt{x^2 \pm a^2}| \\ \int \frac{1}{\sqrt{x^2 \pm a^2}} dx &= \ln |x + \sqrt{x^2 \pm a^2}| \\ \int \frac{1}{\sqrt{a^2 - x^2}} dx &= \sin^{-1} \frac{x}{a} \\ \int \frac{x}{\sqrt{x^2 \pm a^2}} dx &= \sqrt{x^2 \pm a^2} \\ \int \frac{x}{\sqrt{a^2 - x^2}} dx &= -\sqrt{a^2 - x^2} \\ \int \sqrt{ax^2 + bx + c} dx &= \frac{b+2ax}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac-b^2}{8a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| \\ \int x^n e^{ax} dx &= \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx \\ \int \sin^2 ax dx &= \frac{x}{2} - \frac{1}{4a} \sin 2ax \\ \int \sin^3 ax dx &= -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a} \\ \int \cos^2 ax dx &= \frac{x}{2} + \frac{\sin 2ax}{4a} \\ \int \cos^3 ax dx &= \frac{3 \sin ax}{4a} + \frac{\sin 3ax}{12a} \\ \int \tan ax dx &= -\frac{1}{a} \ln \cos ax \\ \int \tan^2 ax dx &= -x + \frac{1}{a} \tan ax \\ \int x \cos ax dx &= \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \\ \int x^2 \cos ax dx &= \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \\ \int x \sin ax dx &= -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \\ \int x^2 \sin ax dx &= \frac{2-a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \end{aligned}$$

## 博弈游戏

### 巴什博弈

1. 只有一堆  $n$  个物品，两个人轮流从这堆物品中取物，规定每次至少取一个，最多取  $m$  个。最后取光者得胜。
2. 显然，如果  $n = m + 1$ ，那么由于一次最多只能取  $m$  个，所以，无论先取者拿走多少个，后取者都能够一次拿走剩余的物品，后者取胜。因此我们发现了如何取胜的法则：如果  $n = \lfloor m + 1 \rfloor r + s$  ( $r$  为任意自然数， $s \leq m$ )，那么先取者要拿走  $s$  个物品，如果后取者拿走  $k$  ( $k \leq m$ ) 个，那么先取者再拿走  $m + 1 - k$  个，结果剩下  $(m + 1)(r - 1)$  个，以后保持这样的取法，那么先取者肯定获胜。总之，要保持给对手留下  $(m + 1)$  的倍数，就能最后获胜。

### 威佐夫博弈

1. 有两堆各若干个物品，两个人轮流从某一堆或同时从两堆中取同样多的物品，规定每次至少取一个，多者不限，最后取光者得胜。
2. 判断一个局势  $(a, b)$  为奇异局势（必败态）的方法： $a_k = \lfloor k(1 + \sqrt{5})/2 \rfloor, b_k = a_k + k$

### 阶梯博弈

1. 博弈在一列阶梯上进行，每个阶梯上放着自然数个点，两个人进行阶梯博弈，每一步则是将一个阶梯上的若干个（至少一个）移到前面去，最后没有点可以移动的人输。
2. 解决方法：把所有奇数阶梯看成  $N$  堆石子，做 **NIM**。（把石子从奇数堆移动到偶数堆可以理解为拿走石子，就相当于几个奇数堆的石子在做 **Nim**）

## 图上删边游戏

### 链的删边游戏

1. 游戏规则：对于一条链，其中一个端点是根，两人轮流删边，脱离根的部分也算被删去，最后没边可删的人输。
2. 做法： $sg[i] = n - dist(i) - 1$ （其中  $n$  表示总点数， $dist(i)$  表示离根的距离）

### 树的删边游戏

1. 游戏规则：对于一棵有根树，两人轮流删边，脱离根的部分也算被删去，没边可删的人输。
2. 做法：叶子结点的  $sg = 0$ ，其他节点的  $sg$  等于儿子结点的  $sg + 1$  的异或和。

### 局部连通图的删边游戏

1. 游戏规则：在一个局部连通图上，两人轮流删边，脱离根的部分也算被删去，没边可删的人输。局部连通图的构图规则是，在一棵基础树上加边得到，所有形成的环保证不共用边，且只与基础树有一个公共点。
2. 做法：去掉所有的偶环，将所有的奇环变为长度为 1 的链，然后做树的删边游戏。